Mass Hierarchies from Anomalies: a Peek Behind the Planck Curtain

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Behind the Planck Curtain

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The masses of quarks and leptons suggest a strong hierarchical structure. We argue that their patterns can be reproduced through the introduction of a new Abelian symmetry. The data suggest that this symmetry is anomalous. We suggest that the cancellation of its anomalies occur through the Green-Schwarz mechanism. An important check of this idea is that it links the Weinberg angle to a mass ratio of the elementary fermions. The Green-Schwarz mechanism occurs naturally in many superstring compactifications, and produces a small parameter, which we use to determine the quark mass hierarchy. We show that hierarchy and mixings among the chiral fermions is a consequence of the Green-Schwarz mechanism. We present several models where this idea is realized

1 Introduction

Superstring and related theories of extended objects, to which Keiji Kikkawa has contributed so much, offer the best hope for resolving the difference between General Relativity and Quantum Mechanics. Remarkably they contain the gauge structures found in the description of low energy phenomena. Yet, in spite of this conceptual matching between the "observed" and the "fundamental", the lack of detailed predictions, such as relations among the parameters and structures of the standard model, has undermined the credibility of these theories as candidate Theories of Nature. The disparity of scales has made it difficult to equate the two: the standard model is an effective theory with a cut-off somewhere between 1 TeV and the Planck mass; superstring theories apply to physics above the Planck mass. However, as shown by 't Hooft, anomalies are impervious to scales. For example in QCD, the global chiral anomaly responsible for the decay of the π^0 "measures" the number of colors, at a scale well below that at which quarks become apparent.

As a chiral theory, the standard model could have many anomalies, but as is well known the hypercharge anomalies cancel between quarks and leptons. Curiously the mixed anomaly between the hypercharge current and the energy-momentum tensors cancels as well. Hence to use anomalies as probes of fundamental physics, we must extend the standard model.

Below we argue that the puzzling pattern of quark and lepton masses can be generated by adding an Abelian symmetry to the standard model,

following the idea of Froggatt and Nielsen[1]. Remarkably, their simple picture can reproduce the features of the CKM matrix, and the mass hierarchies, if the Abelian symmetry is anomalous[2,3].

Many superstring compactifications contain in their low energy manifestation, a chiral Abelian symmetry, with anomalies "cancelled" by the Green-Schwarz mechanism[4]. This symmetry is broken spontaneously *below* the string cut-off, automatically producing a small parameter that can be used to generate the mass hierarchies. The Weinberg angle at the string cut-off is then determined[5] in terms of the anomaly coefficients, which depend on the charges of the massless particles.

To match the two theories through this $U(1)$ [6,2], their cut-offs must be at the same scale. For the standard model, this requires its extension by low energy supersymmetry. Thus all our remarks below will be made in that context.

2   The Yukawa Sector

The regularities found in the gauge sector of the standard model, do not seem to have any counterparts in the Yukawa sector, the most studied, least understood, and therefore the most interesting sector of the standard model.

The high value of the mass of the top quark[7] enables a very attractive mechanism for electroweak breaking, triggered through the renormalization group by soft supersymmetry breaking terms. This picture predicts the existence of the superpartners of the elementary particles with masses of the order of hundred(s) of GeVs. The same theory also allows us to extrapolate the physical parameters of the standard model to its supersymmetric cut-off, as the theory is perturbative throughout that range.

The Yukawa sector contains many small parameters, such as the ratio of the bottom to top quark masses, the ratio of electron to muon masses, etc..., as well as small mixing angles. A convenient parametrization[8] of these mass parameters, introduced for the CKM matrix by Wolfenstein[9], uses the Cabibbo angle $\lambda = V_{us}$ as the small parameter. If we assume no significant matter between the TeV scale and the cut-off, as implied by the convergence of the gauge couplings, we find the following ratios, valid around 10^{16} GeV, slightly below the string cut-off:

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) \; ; \qquad \frac{m_c}{m_t} = \mathcal{O}(\lambda^4) \; ; \tag{1}$$

$$\frac{m_d}{m_b} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2) \; , \tag{2}$$

The charged lepton masses satisfy similar relations

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4) \ ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) \ . \tag{3}$$

Finally, we have the interfamily hierarchy

$$\frac{m_b}{m_t} = \mathcal{O}(\lambda^3) \ . \tag{4}$$

The Froggatt and Nielsen[1] idea relates the exponents that appear in the mass ratios are related to the dimension of the operators that generated them. In the following we present a general framework to implement this idea, and then present several models, using invariance under the anomalous $U(1)_X$ and under other possible Abelian symmetries present in superstring-generated theories.

3   Profile of Superstring-Generated Effective Theories

At a time when dualities suggest that all superstring theories are manisfestations of one unknown theory, we still do not know the reason nor the dynamical mechanism by which they compactify to four space-time dimensions. Such ignorance forces us to consider compactifications on all types of manifolds, generating a large number of candidate four dimensional theories.

While we cannot put much credence on the detailed predictions of any one of these compactifications, many contain similar characteristic features. In that spirit, we study a class of compactifications[10] which contains a gauged phase symmetry, which has anomalies cancelled in a way specific to superstring theories, first discovered by Green and Schwarz[4].

Compactified superstring theories engender at and below a string cutoff, M_{string} an effective "low energy" theory. This theory is a gauge theory, generically invariant under $N = 1$ supersymmetry, with one universal gauge coupling α_{string}. These two parameters are related by the Planck mass. The gauge structure of these theories is separated in the "visible" and "hidden" sectors, connected to one another by an "anomalous" phase symmetry, $U(1)_X$, and possible other straddling but non-anomalous phase symmetries:

$$\mathcal{G}^{visible} \times U(1)_X \times U(1) \times \cdots \times U(1) \times \mathcal{G}^{hidden} \ . \tag{5}$$

The first gauge group describes the "visible sector", with gauge group which may be a GUT group, but must at least contain the standard model structure

$$\mathcal{G}^{visible} \subset SU(3)^C \times SU(2)^W \times U(1)^Y \times \cdots \ . \tag{6}$$

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In superstring compactifications, the dots stand for Abelian symmetries. The gauge group of the hidden sector is $\mathcal{G}^{hidden}$. Its precise form is model dependent. The matter chiral superfields which transform under $\mathcal{G}^{hidden}$ are singlets under $\mathcal{G}^{visible}$, and vice-versa. The two sectors interact through the universal gravitational interactions, and a set of Abelian forces. The interactions of the hidden sectors are assumed to be strong, generating perhaps dynamical breaking of supersymmetry by means of gaugino condensation.

When viewed below the cut-off of an effective field theory, the straddling symmetry, $U(1)_X$, is anomalous: the massless fermions produce anomalous divergences of its current, to be compensated by cut-off dependent terms. For superstrings, this term is an axion-like coupling term of dimension five. This mechanism is the four-dimensional analogue of the Green-Schwarz anomaly cancellation of the mother superstring theory in ten dimensions. The theory may contain other straddling $U(1)$s, they are not anomalous, although model dependent.

In addition, the anomalous $U(1)_X$ is spontaneously broken by a stringy mechanism a few orders of magnitude *below* the string cut-off. This mechanism, first noticed by Dine, Seiberg, and Witten[11], generates a one-loop finite contribution to its D-term

$$D_X = D_X^{tree} + \frac{g_{string}^3}{192\pi^2}\, M_{string}^2 C_{\text{grav}} \,, \tag{7}$$

where D_X^{tree} is the tree level D term, and C_{grav} is the sum over helicities of the X-charges of all the fermions, the mixed gravitational anomaly of the $U(1)_X$ charge

$$C_{\text{grav}} = \text{Tr}\,(XTT) \,, \tag{8}$$

where T stands for the energy momentum tensor. As the breaking occurs below the cut-off, the effective low energy theory is invariant under $U(1)_X$, imposing constraints on the form of the low energy theory, and generating a small parameter, the ratio of the breaking scale to the string cut-off. Below, we argue that this small parameter is at the origin of the fermion mass hierarchies.

Let us specialize these equations to theories that contain the standard model. We take the non-Abelian visible gauge group to be $\mathcal{G}^{visible} = SU(3)^C \times SU(2)^W \times U(1)_1 \times U(1)_2 \times \cdots \times U(1)_K$. The hypercharge Y is a linear combination of these Abelian charges, generated by currents Y_k, $k = 1, 2 \ldots, K$.

We denote the mixed anomaly between the X current and the non-Abelian gauge currents by C_{color} and C_{weak}

$$\text{Tr}\,(XG^A G^B) = \delta^{AB} C_{\text{color}} \,; \qquad \text{Tr}\,(XW^a W^b) = \delta^{ab} C_{\text{weak}} \,, \tag{9}$$

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where G^A are the QCD currents, and W^a the weak isospin currents. The Green-Schwarz mechanism requires

$$\text{Tr } (XY_iY_j) = \delta_{ij}C_i \ . \tag{10}$$

All the other anomaly coefficients must vanish by themselves

$$\text{Tr } (Y_iY_jY_k) = \text{Tr } (Y_iXX) = 0 \ . \tag{11}$$

The Green-Schwarz anomaly cancellation mechanism is understood in the language of an effective theory by writing its Lagrangian as

$$\mathcal{L} = \frac{1}{g_{string}^2}\sum_j k_j F_{\mu\nu}^{[j]}F_{\mu\nu}^{[j]} + i\frac{\eta}{M_{string}}\sum_j k_j F_{\mu\nu}^{[j]}\tilde{F}_{\mu\nu}^{[j]} + \mathcal{L}_{matter} + \cdots \ , \tag{12}$$

where the k_j are the Kac-Moody levels, the index j =color, weak, k,...,hidden, denotes all possible gauge groups of the model, with field strengths $F_{\mu\nu}^{[j]}$, and η is the axion, part of the dilaton supermultiplet, the transverse remnant of the Kalb-Ramond antisymmetric tensor field. The axion also couples to the gravitational field through a term symbolically denoted by $k_g\eta R\tilde{R}$. The divergence of the $U(1)_X$ current is altered by the anomalies

$$\partial_\mu J_\mu^X \sim \sum_j C_j F_{\mu\nu}^{[j]}\tilde{F}_{\mu\nu}^{[j]} + C_{\text{grav}}R\tilde{R} \ , \tag{13}$$

including for completeness the gravitational contribution to the anomaly.

Under a gauge transformation of $U(1)_X$, the Lagrangian changes by an amount proportional to the divergence of this current, thereby creating an apparent lack of invariance. However, the axion field shifts at the same time, which generates a term of exactly the same form as that from the divergence of the current, provided that

$$\frac{C_{\text{grav}}}{12} = \frac{C_{\text{color}}}{k_{\text{color}}} = \frac{C_{\text{weak}}}{k_{\text{weak}}} = \cdots = \frac{C_i}{k_i} = \cdots \neq 0 \ . \tag{14}$$

The normalization between the mixed gravitational and mixed gauge anomalies is fixed in superstring theories. In any superstring derived theory, these equations are always satisfied since the underlying theory is anomaly-free. It is just that an apparent anomaly has been introduced by the cut-off procedure.

One can show [12] that the Green-Schwarz mechanism is unaffected by a canonical change of basis among the Abelian factors, $Y_i \to \tilde{Y}_i$, that is

$$\text{Tr } (X\tilde{Y}_i\tilde{Y}_j) = \delta_{ij}\bar{C}_i \qquad \frac{\bar{C}_i}{\bar{k}_i} = \frac{C_i}{k_i} \ . \tag{15}$$

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It follows that we can always choose the hypercharge as one of the Y_i's, say Y_1, and drop the tildes.

The implications of these equalities are far reaching. As noted by Ibáñez[5], they can be used to relate the Weinberg angle at the cut-off to the anomaly coefficients. Indeed, we have

$$\tan^2 \theta_W = \frac{g_Y^2}{g_{\text{weak}}^2} = \frac{k_Y}{k_{\text{weak}}} = \frac{C_Y}{C_{\text{weak}}} \;. \tag{16}$$

All the Kac-Moody levels are the same for the non-Abelian groups.

The second consequence of these equations is that the mixed anomaly with the hidden sector gauge groups do not vanish, implying the existence of massless particles with hidden non-Abelian charges. If none are to survive as massless particles, they must all be confined, implying the existence of strong hidden forces, which might break supersymmetry dynamically. It also means that the mixed gravitational anomaly contains contributions from hidden sector particles.

4 Anomaly Constraints on the Visible Sector

Consider the minimal supersymmetric standard model with three chiral families. Out of its chiral supermultiplets, it is possible to form analytic combinations that are both electroweak and R-invariants:

$$
\begin{aligned}
\mathcal{O}_{ij}^{[u]} &= \mathbf{Q}_i \overline{\mathbf{u}}_j H_u \;; & \mathcal{O}_{ij}^{[d]} &= \mathbf{Q}_i \overline{\mathbf{d}}_j H_d \;; \\
\mathcal{O}_{ij}^{[e]} &= L_i \bar{e}_j H_d \;; & \mathcal{O}^{[\mu]} &= H_u H_d \;,
\end{aligned}
\tag{17}
$$

where i, j are the family indices. With one right-handed neutrino for each chiral family, the list includes more terms

$$\mathcal{O}_{ij}^{[\nu]} = L_i \overline{N}_j H_u \;; \qquad \mathcal{O}_{ij}^{[0]} = \overline{N}_i \overline{N}_j \;. \tag{18}$$

In the visible sector, there could be other chiral multiplets. So as not to contradict data, they should appear as vector-like pairs under the standard model gauge groups, although they could be chiral with respect to symmetries beyond. Their presence would not affect C_{color}, C_{weak}, nor C_Y, but would affect C_k.

These combinations are the lowest dimension invariants. Without any symmetry beyond the minimal supersymmetric standard model, all can appear in the superpotential, providing no explanation for any hierarchy among the Yukawa couplings.

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As we have just discussed, low energy theories derived from superstrings can be expected to contain extra Abelian symmetries. The above invariants that do not respect these symmetries may still appear as higher dimension operators in the effective superpotential, dressed with new operators required by the Abelian invariances, but suppressed by inverse powers of the cut-off. This will produce, upon the breaking of the phase symmetries, in a hierarchical structure of Yukawa couplings. Remarkably, it turns out that a hierarchy is generated if one of these extra charges is the Green-Schwarz symmetry.

The charges of the chiral superfields, with family index $i = 1, 2, 3$ are summarized in the following table, where we have separated the hypercharge from the other non-anomalous $U(1)$'s.

Charge	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	$\overline{N}_i$	H_u	H_d	θ_a
Y	1/3	$-4/3$	2/3	-1	2	0	1	-1	0
X	a_i	b_i	c_i	d_i	e_i	f_i	s_1	s_2	x_a
Y_k	$a_{[k]i}$	$b_{[k]i}$	$c_{[k]i}$	$d_{[k]i}$	$e_{[k]i}$	$f_{[k]i}$	$t_{[k]1}$	$t_{[k]2}$	$y_{[k]a}$

We have supplemented the list of chiral superfields by an arbitrary number of chiral superfields θ_a; they are electroweak and color singlets, but are assumed to have arbitrary charges under the extra Abelian symmetries. Some of these fields may have hidden quantum numbers as well.

The mixed anomaly coefficients that involve one X and two of the same non-anomalous symmetries do not vanish, and are given by

$$
\begin{aligned}
C_Y &= \tfrac{1}{3}\sum_i(a_i + 8b_i + 2c_i + 3d_i + 6e_i) + s_1 + s_2 \,, \\
C_{[k]} &= \tfrac{1}{3}\sum_i(a_{[k]i} + 8b_{[k]i} + 2c_{[k]i} + 3d_{[k]i} + 6e_{[k]i}) + t_{[k]1} + t_{[k]2} \,, \\
C_{\text{weak}} &= \sum_i(3a_i + d_i) + s_1 + s_2 \,, \\
C_{\text{color}} &= \sum_i(2a_i + b_i + c_i) \,.
\end{aligned}
\tag{19}
$$

The mixed gravitational anomaly is the sum of the X charge times the multiplicity

$$
C_{\text{grav}} = \sum_i [3(2a_i + b_i + c_i) + 2d_i + e_i + f_i] + 2s_1 + 2s_2 + \sum_a x_a \,.
\tag{20}
$$

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None of these coefficients vanish, and are related to one another by the Green-Schwarz constraints.

These mixed anomalies can now be used to draw inferences on the structure of the standard model because they are all related to the (anomalous) charges of the electroweak invariant combinations made out of the standard model superfields

$$
\begin{aligned}
C_{\text{color}} &= X_{\mathcal{O}_{ii}^{[u]}} + X_{\mathcal{O}_{ii}^{[d]}} - 3X_{\mathcal{O}^{[\mu]}} \ , \\
C_Y + C_{\text{weak}} - \tfrac{8}{3}C_{\text{color}} &= 2X_{\mathcal{O}_{ii}^{[e]}} - 2X_{\mathcal{O}_{ii}^{[d]}} + 2X_{\mathcal{O}^{[\mu]}} \ , \\
C_{\text{grav}} &= 3C_{\text{color}} + X_{\mathcal{O}_{ii}^{[e]}} + X_{\mathcal{O}_{ii}^{[\nu]}} + 2X_{\mathcal{O}^{[\mu]}} + \sum_a x_a \ ,
\end{aligned}
$$

(21)

with the sum taken over family space. Since the Kac-Moody levels of the non-Abelian factors are the same in string theories, the Green-Schwarz mechanism requires $C_{\text{weak}} = C_{\text{color}}$, which generates the further relation

$$
C_Y = \frac{5}{3}X_{\mathcal{O}_{ii}^{[u]}} - \frac{1}{3}X_{\mathcal{O}_{ii}^{[d]}} + 2X_{\mathcal{O}_{ii}^{[e]}} - 3X_{\mathcal{O}^{[\mu]}} \ .
\tag{22}
$$

We can get another relation by requiring that the Weinberg angle assume its canonical ($SU(5)$) value, $\sin^2\theta_w = 3/8$. This yields $3C_Y = 5C_{\text{weak}}$, which enables us to express the excess charge of the μ term in terms of those of the Yukawas

$$
X_{\mathcal{O}^{[\mu]}} = X_{\mathcal{O}_{ii}^{[d]}} - X_{\mathcal{O}_{ii}^{[e]}} \ .
\tag{23}
$$

We can now rewrite the mixed color anomaly in terms of the excess charges of the the Yukawa operators only

$$
C_{\text{color}} = \sum_i \left[X_{\mathcal{O}_{ii}^{[u]}} - 2X_{\mathcal{O}_{ii}^{[d]}} + 3X_{\mathcal{O}_{ii}^{[e]}} \right] \ .
\tag{24}
$$

We can also relate the mixed anomalies to the excess charges of the off-diagonal Yukawa operators, obtaining a similar conclusion concerning the interfamily couplings,

$$
C_{\text{color}} = \frac{1}{2} \sum_{i \neq j} \left[X_{\mathcal{O}_{ij}^{[u]}} - 2X_{\mathcal{O}_{ij}^{[d]}} + 3X_{\mathcal{O}_{ij}^{[e]}} \right] \ ,
\tag{25}
$$

implying mixing among the families. These relations enable us to draw important conclusions that are independent of the details of the model.

8

Since C_{color} is not zero, the X charges of some of the electroweak invariants do not vanish, and they cannot appear in the tree level superpotential. They can, however, appear as higher dimension operators, but then they will be suppressed by inverse powers of the cut-off, which produces hierarchy and mixing among the fermions masses of the standard model to satisfy Eqs. (24) and (25)! At this stage, the argument is not specific enough to determine in which sector the mixing takes place. To answer that question, we need more detailed models with several additional non-anomalous phase symmetries. Without right-handed neutrinos, the off-diagonal charged lepton entries can be set to zero, and in this case, the above equation does imply mixing among the quarks.

The remaining symmetries are restricted since all of their anomaly coefficients must vanish. The mixed gravitational anomalies of Y already vanishes, and that of any Y_k must as well. The vanishing of the other anomalies yield relations among the Y_k charge of the Yukawa operators

$$Y^{[k]}_{\mathcal{O}^{[\mu]}} = \frac{1}{3}\left[Y^{[k]}_{\mathcal{O}^{[u]}_{ii}} + Y^{[k]}_{\mathcal{O}^{[d]}_{ii}} \right] = Y^{[k]}_{\mathcal{O}^{[d]}_{ii}} - Y^{[k]}_{\mathcal{O}^{[e]}_{ii}} \; . \tag{26}$$

The same equations for the hypercharges are of course trivially satisfied. We have the same equations for the sum of the off-diagonal elements

$$Y^{[k]}_{\mathcal{O}^{[\mu]}} = \frac{1}{3}\sum_{i \neq j}\left[Y^{[k]}_{\mathcal{O}^{[u]}_{ij}} + Y^{[k]}_{\mathcal{O}^{[d]}_{ij}} \right] = \sum_{i \neq j}\left[Y^{[k]}_{\mathcal{O}^{[d]}_{ij}} - Y^{[k]}_{\mathcal{O}^{[e]}_{ij}} \right] \; . \tag{27}$$

Finally, we note that we can always consider adding to X any linear combination of Y and Y_k. This does not affect most of the anomaly conditions except Tr $(YXX) = 0$ and Tr $(Y_kXX) = 0$. Thus we can choose to take $s_1 = s_2$ and $t_{[k]1} = t_{[k]2}$, respectively, but then can no longer use these two anomaly conditions.

5 Models

5.1 Le Petit Modèle

This is the first in a series of simple models with Abelian symmetries beyond the standard model. It contains one family of quarks and leptons, with a right-handed neutrino and see-saw mechanism, as well as one electroweak singlet field θ. We assume one Abelian symmetry beyond the standard model, with its anomalies cancelled by the Green-Schwarz mechanism. While very elementary, this model serves as an example of our ideas and procedures for

generating fermion mass hierarchies. The superpotential, which assumes tree-level Yukawas only for the top quark[13] and right-handed neutrino, is of the form

$$W = \mathbf{Q}\overline{\mathbf{u}}H_u + L\overline{N}H_u + \mathbf{Q}\overline{\mathbf{d}}H_d \left(\frac{\theta}{M}\right)^{n_d} + L\overline{e}H_d \left(\frac{\theta}{M}\right)^{n_e} + M\overline{N}\,\overline{N} \left(\frac{\theta}{M}\right)^{n_o} .$$

(28)

In order to comply with the analyticity of the superpotential, n_e, n_d, and n_0 must be positive integers. We further assume that the μ term is generated through the Kähler potential, in the way suggested by Giudice and Masiero[14],

$$K_{GM} = H_u H_d \left(\frac{\overline{\theta}}{M}\right)^N .$$

(29)

Beyond the usual hypercharge, these couplings are required to be invariant under an anomalous charge X. This fixes N to be a positive integer, and forbids the appearance of a μ term in the superpotential. The X and Y charges are given by

Charge	$\mathbf{Q}$	$\overline{\mathbf{u}}$	$\overline{\mathbf{d}}$	L	$\overline{e}$	$\overline{N}$	H_u	H_d	θ
Y	1/3	−4/3	2/3	−1	2	0	1	−1	0
X	a	b	c	d	$.e$	f	s_1	s_2	x

This extra symmetry can be anomalous with its mixed anomalies given by

$$\begin{aligned}
C_Y &= \tfrac{1}{3}(a + 8b + 2c + 3d + 6e) + s_1 + s_2 , \\
C_{\text{weak}} &= 3a + d + s_1 + s_2 , \\
C_{\text{color}} &= 2a + b + c , \\
C_{\text{grav}} &= 3(2a + b + c) + 2d + e + f + 2s_1 + 2s_2 + x
\end{aligned}$$

(30)

From the superpotential, the equations

$$a + b + s = 0 , \qquad d + f + s = 0 ,$$

(31)

determine two charges, and

$$n_d = -\frac{a + c + s}{x} , \qquad n_e = -\frac{d + e + s}{x} , \qquad n_0 = -\frac{2f}{x} , \qquad N = \frac{2s}{x} ,$$

(32)

are the positive integers which fix the dimensions of the non-renormalizable interactions.

The electroweak singlet field θ takes on a vacuum value determined by the DSW mechanism, through the vanishing of its D term,

$$D_X = xg|\theta|^2 + \frac{g^3}{192\pi^2} M_{str}^2 C_{grav} , \qquad (33)$$

where g is the string gauge coupling, and M_{str} is the string unification scale. Clearly C_{grav}/x has to be negative, otherwise supersymmetry is broken. It follows that

$$\lambda \equiv \frac{<\theta>}{M_{str}} = \sqrt{\frac{g^2}{192\pi^2} \frac{-C_{grav}}{x}} , \qquad (34)$$

is less than one for reasonable values of the mixed gravitational anomaly, and can be used as an expansion parameter in the mass ratios

$$\frac{m_b}{m_t} = \cot\beta\lambda^{n_d} ; \qquad \frac{m_b}{m_\tau} = \lambda^{n_d-n_e} , \qquad (35)$$

where the angle β parametrizes the ratio of the vacuum values of the two Higgs doublets

$$\tan\beta = \frac{<H_u>}{<H_d>} . \qquad (36)$$

We also have the additional relation that expresses the μ parameter in terms of the gravitino mass

$$\mu = m_{3/2}\lambda^N . \qquad (37)$$

Using the invariances of the superpotential, we can express two of the anomalies in terms of the integer powers

$$C_{color} = -(N + n_d)x ; \qquad C_{grav} = (1 - 3n_d - n_e - 2N)x , \qquad (38)$$

so that the Kac-Moody level is given by

$$k_{color} = 12\frac{N + n_d}{3n_d + n_e + 2N - 1} . \qquad (39)$$

For consistency it must be a positive integer. Using Eq. (38), the non-vanishing of the color anomaly implies $N \geq 0$, or $n_d \geq 0$, or both. Also, it tells us that, when N and n_d are positive integers, C_{grav}/x is negative and the DSW mechanism preserves supersymmetry. X-symmetry is broken, producing the expansion parameter

$$\lambda \equiv \frac{<\theta>}{M_{str}} = \sqrt{\frac{g^2}{192\pi^2} M_{str} \frac{2N + 3n_d + n_e - 1}{N + n_d}} . \qquad (40)$$

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In terms of anomalies, the mass ratio becomes[2]

$$\frac{m_b}{m_\tau} = \lambda^{(C_Y + C_{\text{weak}} - 8/3 C_{\text{color}})/2x - N} \ . \tag{41}$$

We can express this equation in more physical terms by appealing to the Green-Schwarz mechanism, according to which the anomalies must satisfy Eq. (14)

$$\frac{C_{\text{grav}}}{12} = \frac{C_Y}{k_Y} = \frac{C_{\text{weak}}}{k_{\text{weak}}} = \frac{C_{\text{color}}}{k_{\text{color}}} = \frac{C_X}{k_X} \ . \tag{42}$$

In string compactifications, we have $k_{\text{weak}} = k_{\text{color}}$, so that

$$C_{\text{weak}} = C_{\text{color}} \ . \tag{43}$$

The Ibáñez relation

$$k_Y = \tan^2 \theta_w k_{\text{weak}} \ , \qquad C_Y = \tan^2 \theta_w C_{\text{weak}} \ , \tag{44}$$

is then used to fix the Weinberg angle. We then find that[2,16]

$$\frac{m_b}{m_\tau} = \frac{\mu}{m_{3/2}} \lambda^{(C_{\text{weak}}/2x)(\tan^2 \theta_w - 5/3)} \tag{45}$$

This equation is consistent with the data. The left-hand side is of order one. Numerical simulations of the MSSM with soft supersymmetry breaking suggest that the μ parameter is also of the order of the gravitino mass. In this case, this equation implies that $\tan^2 \theta_w = 5/3$, which agrees very well with the extrapolated value at unification! This also happens to be the value of compactifications that go through $SO(10)$. The agreement of this formula with experiment lends credence to our scenario of mass hierarchies.

We can proceed, using Eq. (43), to express all the charges and anomalies in terms of the positive integers N, n_e, n_d, n_0,

$$a = -\frac{x}{6}(3N + (n_o + 2n_d)) \ , \qquad b = \frac{x}{6}(n_o + 2n_d) \ , \qquad c = \frac{x}{6}(n_o - 4n_d) \ , \tag{46}$$

$$d = \frac{x}{2}(n_o - N) \ , \qquad e = -\frac{x}{2}(n_o + 2n_e) \ , \qquad f = -\frac{x}{2}n_o \ , \qquad s = \frac{x}{2}N \ . \tag{47}$$

In particular it follows that

$$C_Y = x\left(\frac{N + n_d}{3} - 2n_e\right) \ . \tag{48}$$

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If we fix the Weinberg angle at its $SU(5)$ value, we can generate one more relation between the integers

$$N + n_d = n_e \ . \tag{49}$$

Hence $n_e \geq n_d$, and from 38 $n_e \neq 0$, resulting in a hierarchy between lepton and quark masses. The Kac-Moody level number is now given by

$$k_{\text{color}} = 12 \frac{n_e}{3n_e + n_d - 1} \ . \tag{50}$$

There is no solution with $k_{\text{color}} = 1, 2, 3$. The lowest interesting value, $k_{\text{color}} = 4$, requires $n_d = 1$.

In general, however we expect to have a hidden sector with non-Abelian gauge groups. The Green-Schwarz mechanism requires that the mixed anomalies with the hidden sector gauge group not vanish, but be proportional to C_{color}. This implies that there are massless fermions in the hidden sector with X charges. These fermions will in turn contribute to the mixed gravitational anomaly. In the above we have not taken into account the contribution of these fermions. We should write instead

$$C_{\text{grav}} = (C^{hid} + 1 - 3n_d - n_e - 2N)x \ , \tag{51}$$

so that

$$k_{\text{color}} = 12 \frac{N + n_d}{3n_d + n_e + 2N - 1 - C^{hid}} \ , \tag{52}$$

where $C^{hid}x$ is the contribution from the hidden sector massless fermions. This shows that this contribution must be present in order to have a low value for the Kac-Moody integer k_{color}. Alternatively, if we require $k_{\text{color}} = 1$, as in models that do not have grand unified groups, we find that the contribution to the mixed gravitational anomaly from the massless particles in the hidden sector is fixed to be

$$C^{hid} = n_d - 9n_e - 1 \ . \tag{53}$$

If the hidden sector contains a non-Abelian symmetry, its Kac-Moody level will also be equal to one. Thus we know from the Green-Schwarz cancellation that its mixed anomaly coefficient is just $-n_e x$. Thus these equations give us a glimpse of the physics of the hidden sector as well.

This instructive example serves as an illustration of the power of the Green-Schwarz restrictions. It yields the correct sign of the mixed gravitational anomaly. If we fix the Weinberg angle, we find this model to be very restrictive. First the expansion parameter is found to be

$$\lambda = \sqrt{\frac{g^2 n_e}{192\pi^2}} \ . \tag{54}$$

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Since

$$\frac{m_b}{m_\tau} = \lambda^N = \frac{\mu}{m_{3/2}} \ ,$$

(55)

to get $m_b \sim m_\tau$ [15], we deduce $N = 0$, the original proposal of Giudice and Masiero. Then

$$\frac{m_b}{m_t} = \cot\beta\lambda^{n_e} \ .$$

(56)

Since n_e is a positive integer, we see that the bottom mass is suppressed, suggesting that $\tan\beta$ need not be that large. Finally we note that all the predictions of this model are, once the Weinberg angle is fixed, in terms of one positive integer n_e.

5.2 *Le Moins Petit Modèle*

We can improve the petit Modèle in several ways. One is to increase the number of chiral families to the observed three. Less obvious is the addition of pairs of fermions which are vector-like with respect to the standard model, but chiral with respect to symmetries beyond. These fermions appear as chiral fermions in the effective theory and contribute to anomalies that do not involve electroweak and color quantum numbers. Below the scale at which the non-standard symmetries break, these fermions acquire masses. A second way to improve on the model is to increase the gauge symmetry by adding to the X symmetry non-Abelian or Abelian non-anomalous symmetries.

Thus consider a standard model with one chiral family and two Abelian symmetries, one of which is anomalous. Assume two electroweak singlet fields, θ_a, $a = 1, 2$. Its superpotential is

$$
\begin{aligned}
W = \mathbf{Q}\overline{\mathbf{u}}H_u \ &+ \ L\overline{N}H_u + \mathbf{Q}\overline{\mathbf{d}}H_d \left(\tfrac{\theta_1}{M}\right)^{n_d^{(1)}} \left(\tfrac{\theta_2}{M}\right)^{n_d^{(2)}} \\
&+ \ L\overline{e}H_d \left(\tfrac{\theta_1}{M}\right)^{n_e^{(1)}} \left(\tfrac{\theta_2}{M}\right)^{n_e^{(2)}} + M\overline{N}N \left(\tfrac{\theta_1}{M}\right)^{n_o^{(1)}} \left(\tfrac{\theta_2}{M}\right)^{n_o^{(2)}} \ ,
\end{aligned}
$$

(57)

where $n_e^{(a)}, n_d^{(a)}$, and $n_0^{(a)}$ are positive integers. The μ term is generated through the Kähler potential

$$K_{GM} = H_u H_d \left(\frac{\overline{\theta}_1}{M}\right)^{N_1} \left(\frac{\overline{\theta}_2}{M}\right)^{N_2} \ .$$

(58)

The scales at which the extra symmetries are broken are determined by the DSW mechanism

$$x_1|\theta_1|^2 + x_2|\theta_2|^2 + \frac{g^2}{192\pi^2}M_{\mathrm{Planck}}^2 C_{\mathrm{grav}} = 0 \ .$$

(59)

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The D term for the non-anomalous symmetry must vanish so as not to break supersymmetry

$$y_1|\theta_1|^2 + y_2|\theta_2|^2 + \cdots = 0 . \qquad (60)$$

Thus unless the charges are very large, we expect both X and Y_1 to break at similar scales.

We note that it is possible to generate a much lower breaking scale for Y_1 by taking into account soft supersymmetry breaking. For instance, the tree-level hypercharge D term does not vanish (unless $\beta = \pi/4$), because the soft supersymmetry breaking terms induces a negative mass squared.

With only one chiral family, the most general non-anomalous Y_1, consistent with the tree-level superpotential, is of the form

$$Y_1 = w(B - L) + zI_{3R} + y(n_{\theta_1} - n_{\theta_2}) , \qquad (61)$$

where the first two symmetries are contained within $SO(10)$. Since this symmetry is vector-like with respect to the θ fields, the vanishing of its D-term requires

$$< |\theta_1| > = < |\theta_2| > \equiv < |\theta| > . \qquad (62)$$

Hence both X and Y_1 are broken at the same scale. Invariance under Y_1 yields

$$n_{d,e}^{(1)} = n_{d,e}^{(2)} \equiv n_{d,e} , \qquad (63)$$

and there is only one expansion parameter. This case is not very different from the petit Modèle. We find that

$$C_{\text{grav}} = (1 - 3n_d - n_e - 2N)(x_1 + x_2) , \qquad (64)$$

which has the proper sign to preserve supersymmetry. Here $N = N_1 = N_2$. however we are left with much arbitrariness.

In special cases, this model has some distinctive features. For instance if Y_1 is chosen so that $y = 0$, it can be broken at a very different scale, say when the right-handed sneutrino gets a vacuum value, perhaps around 10^{11} GeV. Another choice is when $z = 0$, in which case, the X symmetry loses its anomaly, contradicting our assumptions. Finally, when $w = 0$, we find that $n_d = 0$ indicating no suppression of the bottom to top ratio. Thus unless the non-anomalous symmetry is determined in a special way, we do not have enough data to narrow down the anomaly structure. With several families, this situation may be different. We leave to a future publication [12] the study of such models.

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