Quantification of ^{238}U Holdup using Iterative, Point-Cloud-Based Compton Imaging

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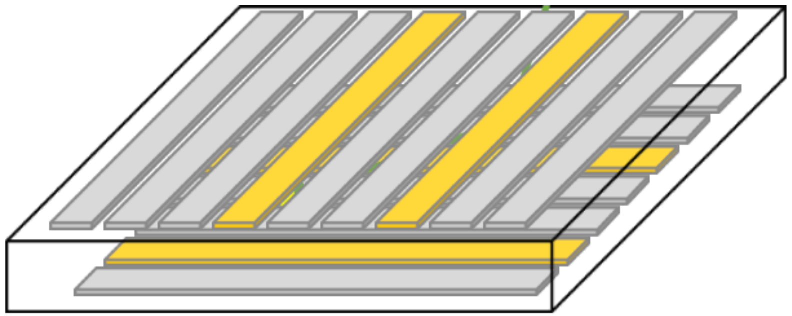
Motivation

- U holdup has impacts on several aspects of operation:
  - Worker dose
  - Criticality safety
  - Safeguards
  - Outage planning
- Generalized Geometry Holdup (GGH) currently estimates U mass within a high uncertainty band (± 50%)
- *This method seeks to improve this uncertainty via quantitative Compton imaging*

Holdup measurement with collimated NaI detector

Image from the PANDA manual [1]

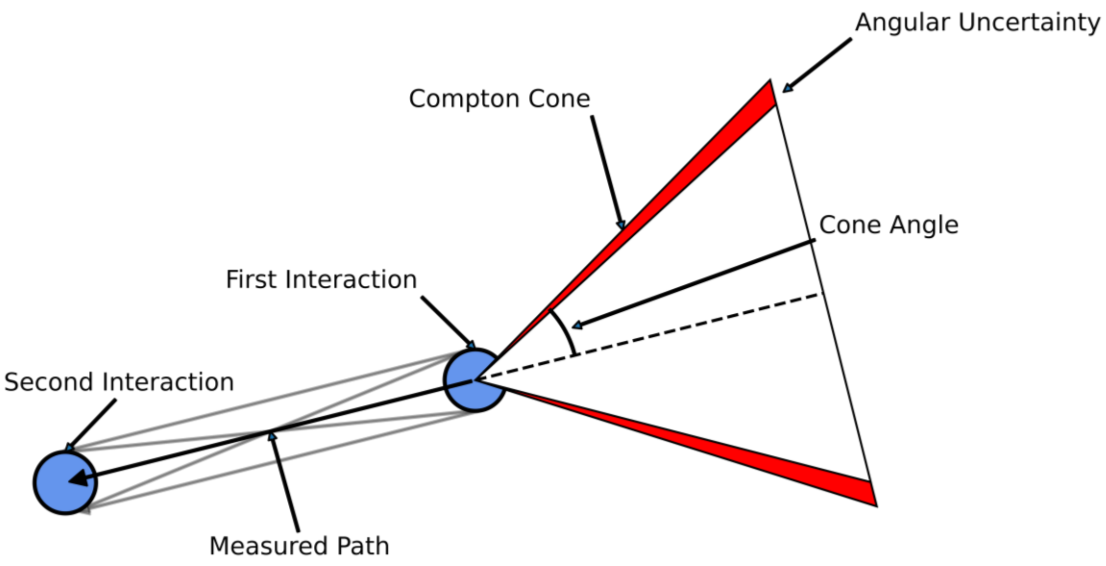
GeGI^{TM} Detector



- Double-sided Strip Detector (DSSD)
  - High Purity Germanium (HPGe)
  - 16 strips on either side
  - 4.5 cm radius, 1.1 cm thickness for active detector volume
- Strip hits contain information on:
  - Interaction energies
  - Interaction positions
- Sub-strip localization enabled via transient signals

GeGI^{TM} Image taken from [2]

Compton Imaging



- Compton imaging requires:
  - Compton cone origin, axis, and angle
- Angular uncertainty is inherent
- Cone origin and axis defined off first two interaction positions
- Angle is given by Compton scatter equation (below)
- Angular uncertainty derived from energy and position uncertainty contributions (in appendix)

$$\cos(\theta) = 1 + \frac{m_e c^2}{E_p} - \frac{m_e c^2}{E_p - E_1}$$

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Compton Imaging for U Holdup Characterization vs Coded Aperture Imaging

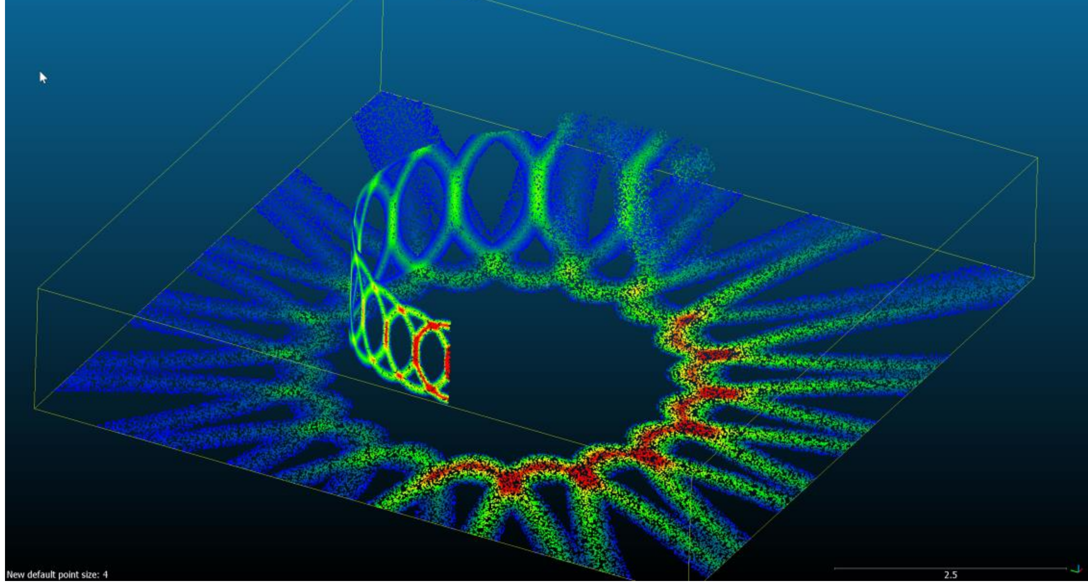
Advantages

- Can image in 4π
- Can image high-energy photons from ^{238}U
- Less impacted by attenuation
  - Self-attenuation
  - Environmental attenuation

Disadvantages

- Poor sensitivity
  - Requires full energy deposition and at least two interactions
- Poor emission rate
  - ^{238}U has associated 0.842% emission probability at 1001 keV
  - Requires long count time
  - Lower signal-background ratio
- Low angular resolution

Developing an Image


Sample cones drawn in example environment

- Point-cloud-based source environment
  - Points in space represent possible sources
  - Points have associated energy spectra
- Once a Compton cone is generated, counts can then be attributed to points in the point cloud
  - Counts depends on angular separation between point and Compton cone
- Counts from several Compton cones comprise an image

Iterative Analysis

- For first iteration, perform simple back-projection (SBP) of Compton events
  - Generate initial count image
- For next iteration, distribute counts per cone according to previous count image
  - Previous image serves to “weight” points
  - Points with more counts get attributed more in following iterations
- Cease iteration when counts/mass in ROI stabilizes

Mass Estimations

- U point mass derived using point intensities (counts) at photopeak (1001 keV):

$$M_p = \frac{I_p}{t * BR * SA * R_p * (1 - f_{SA})\Omega_{det}} C_{emp}$$

- M_p = Point mass, I_p = Point intensity, t = measurement time, BR = Branching Ratio, SA = Specific Activity, R_p = Detector response, f_{SA} = Self-attenuation fraction, Ω_{det} = Detector solid angle, C_{emp} = empirical correction factor
- Response is a function of imaging efficiency and localization efficiency
- This analysis currently requires a constant, empirical factor (see above) to accurately estimate mass

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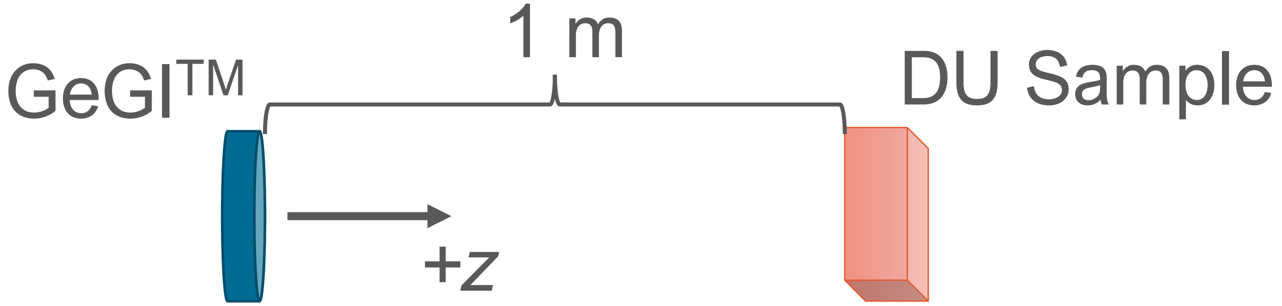
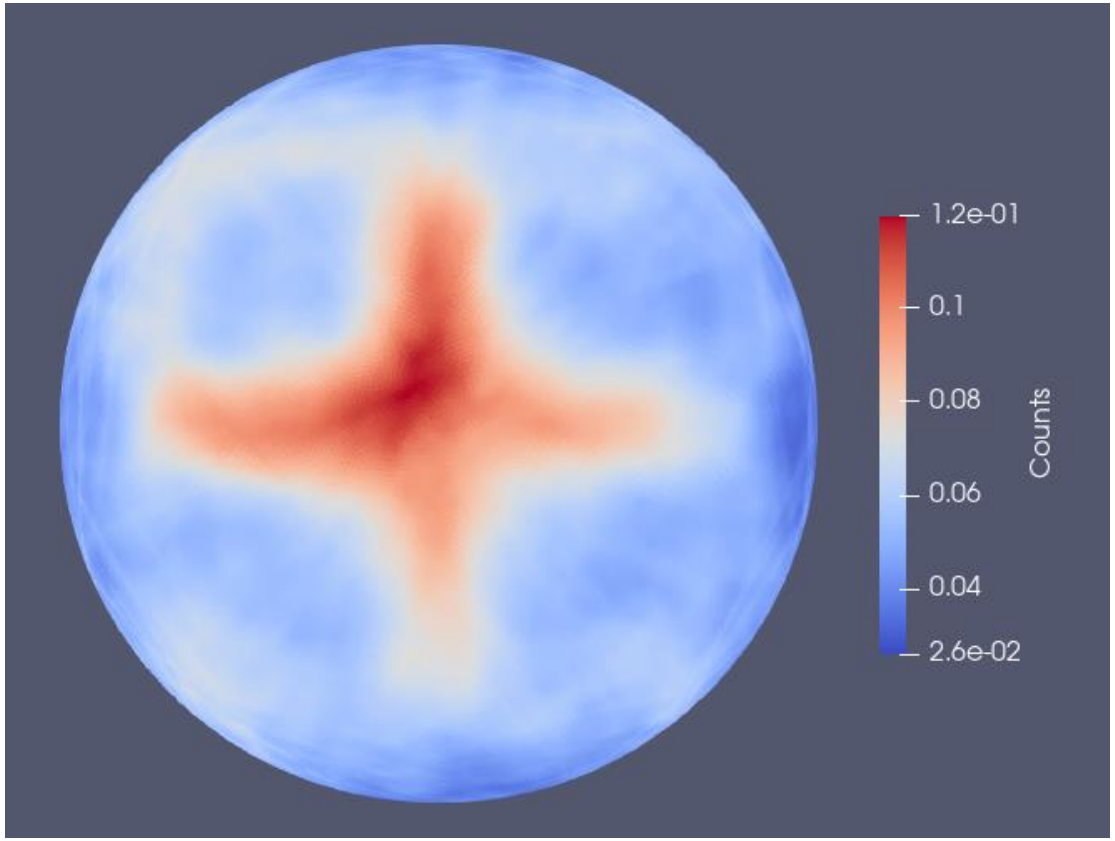
Measurement/Analysis Setup

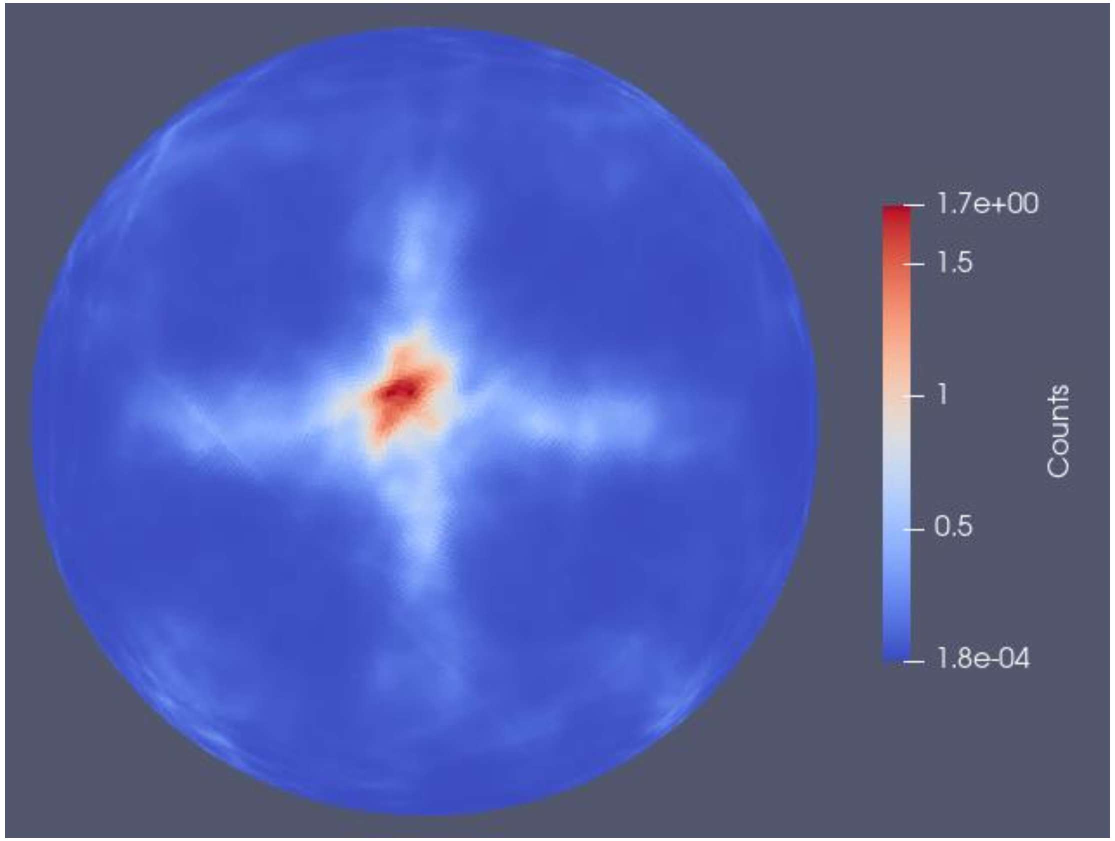
- Depleted uranium (DU) sources placed 1 m in front of detector
  - Density, thickness, and mass of DU known beforehand[3]
- Point cloud initialized as 100,000 points placed uniformly on unit sphere (1 m radius)
- Source region-of-interest ROI chosen as 45° cone in front of detector
- Empirical correction evaluated such that mass calculated for 300 g ^{238}U source is accurate



Results



Iteration 1 (SBP)



Iteration 10

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Results

- Corrected results agree well with reference DU source masses
  - Uncertainties within roughly ±10%
- Percent difference increases with decreasing DU mass
  - Indicative of background influence

Estimated ^{238}U Mass (g)	True ^{238}U Mass (g)	Percent Difference (%)
301.74	301.74	N/A (reference)
213.09	213.88	-0.37
171.90	167.16	2.84
118.65	116.03	2.26
66.19	60.35	9.68

Method Weaknesses and Assumptions

- Empirical correction factor is significant
  - Roughly an order of magnitude (~8.9x)
  - Background assumed small in analysis (contributor correction factor)
  - Underlying physics obscured
- Accurate response characterization required to reduce correction factor
  - Provides physical justification for mass estimation
- Assumes self-attenuation known beforehand
  - Influence accounted for here based on density and thickness of source
- Cross pattern present in images

In-Progress Response Characterization

- Use known lab sources to characterize response
- For each lab source:
  - Calculate counts in source region
  - Subtract background
  - Calculate efficiency
    - Known source-detector geometry and source activity
- Develop efficiency energy curve via several lab sources
- Use efficiency at 1001 keV to calibrate detector response
  - Analysis dependent on chosen source ROI

Sources of Error

- Counting statistics on DU data (Significant)
  - Background is significant
- Error in response estimation (Moderate)
  - Estimation of lab source activity
  - Counting statistics for lab sources
  - Energy-based response
- Localization efficiency (Moderate)
  - Lowers effective counting statistics in source region
- Uncertainties in detector-source geometry (Small)

Conclusions

- Point-cloud-based imaging method developed to localize and quantify ^{238}U with Compton imaging
- With corrections, mass of reference DU samples evaluated accurately
- Improved response characterization required to reduce or eliminate correction factor
- New analysis improves characterization of:
  - Background
  - Response
  - Localization efficiency

Appendix (Angular Uncertainty Equations)

$$\sigma_{\theta,E} = \sqrt{\left(\frac{m_e c^2}{\sqrt{1 - cos^2(\theta)}}\left(\frac{1}{\left(E_p - E_1\right)^2} - \frac{1}{E_p^2}\right)\delta E_p\right)^2 + \left(\frac{m_e c^2}{\sqrt{1 - cos^2(\theta)}}\left(\frac{1}{\left(E_p - E_1\right)^2}\right)\delta E_1\right)^2}$$

$$\begin{cases} \sigma_{\theta,p} = 1.367e^{-3.403d} + 0.199\,;\ d < 0.3293\ cm \\ \sigma_{\theta,p} = -1.45619d^3 + 3.7374d^2 - 3.50492d + 1.44558\ ;\ 0.3293\ cm < d < 0.8871\ cm \\ \sigma_{\theta,p} = 0.2274d^{-1.07} + 0.002479\,;\ d > 0.8871\ cm \end{cases}$$

$$\sigma_\theta = \sqrt{\left(\sigma_{\theta,E}^2 + \sigma_{\theta,p}^2\right)}$$

References

- [1]: Russo, Phyllis. "*Passive Nondestructive Assay Of Nuclear Materials 2007 Addendum*."
- [2]: PHDS Co. “The GeGI | The World’s Mode Sophisticated Gamma Ray Imager” https://phdsco.com/products/gegi
- [3]: Preston, Jeff, Bennett, Brittany, Connatser, R. Maggie, Venkataraman, Ramkumar, Ziock, Klaus Peter, Lefebvre, Jordan, Bledsoe, Keith, Garishvili, Irakli, and Lousteau, Angela. *Uranium Holdup Monitoring With Compton Imaging As Function Of Depth And Mass*. United States: N. p., 2021. Web.