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DYNAMICS OF DENSITY FLUCTUATIONS IN A NON-MARKOVIAN BOLTZMANN-LANGEVIN MODEL

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INTRODUCTION

In the course of the past few years, the nuclear Boltzmann-Langevin (BL) model has emerged as a promising microscopic model for nuclear dynamics at intermediate energies[1,2]. The BL model goes beyond the much employed Boltzmann-Uehling-Uhlenbeck (BUU) model[3], and hence it provides a basis for describing dynamics of density fluctuations and addressing processes exhibiting spontaneous symmetry breaking and catastrophic transformations in nuclear collisions, such as induced fission and multifragmentation[4,5,6].

In these standard models , the collision term is treated in a Markovian approximation by assuming that two-body collisions are local in both space and time, in accordance with Boltzmann's original treatment. This simplification is usually justified by the fact that the duration of a two-body collision is short on the time scale characteristic of the macroscopic evolution of the system. As a result, transport properties of the collective motion has then a classical character. However, when the system possesses fast collective modes with characteristic energies that are not small in comparision with the temperature, then the quantum-statistical effects are important and the standart Markovian treatment is inadequate. In this case, it is necessary to improve the one-body transport model by including the memory effect due to the finite duration of two-body collisions[7,8].

First we briefly describe the non-Markovian extension of the BL model by including the finite memory time associated with two-body collisions [9]. Then, using this non-Markovian model in a linear response framework, we investigate the effect of the memory time on the agitation of unstable modes in nuclear matter in the spinodal zone, and calculate the collisional relaxation rates of nuclear collective vibrations.

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BOLTZMANN-LANGEVIN MODEL WITH MEMORY EFFECT

In the BL model the evolution of the phase-space density $f(\mathbf{r},\mathbf{p},t)$ is determined by a stochastic transport equation,

$$\frac{\partial}{\partial t}f(\mathbf{r},\mathbf{p},t)-\{h(f),f(\mathbf{r},\mathbf{p},t)\}=K(f)+\delta K(\mathbf{r},\mathbf{p},t). \tag{2}$$

Here, the l.h.s. describes the Vlasov propagation in terms of the self-consistent one-body Hamiltonian $h(f)$. On the r.h.s. $K(f)$ is a binary collision term and δK denotes its stochastic part, which arises from correlations and describes the fluctuating aspect of two-body collisions. In analogy with the treatment of Brownian motion, it is assumed that this equation describes a stochastic process for the evolution of the phase-space density in which δK acts like a random force characterized by a correlation function. In the standard BL model, two-body collisions are treated in a Markovian approximation by assuming the duration time of collisions is much shorter than time scale of the mean-field fluctuations and the mean-free-time between collisions, which would be appropriate if two-body collisions can be considered instantaneous. In this case, the BUU form with on-shell two-body collisions is a good approximation and the stochastic collision term can be treated as a white noise with a local correlation function[1,2].

The standard description provides a good approximation at intermediate energies when the system does not involve fast collective modes, since the weak-coupling condition is well satisfied due to relatively long mean-free-path of nucleons[3]. When the system possesses fast collective modes, for example high-frequency collective vibrations or rapidly growing unstable modes, the Markovian approximation breaks down and the memory effect due to finite duration of the collisions becomes important. The finite duration time allows for a direct coupling between two-body collisions and collective modes, which gives rise to an appropriate quantum-statistical description of the collective modes[10]. In order to improve the transport description, we propose a non-Markovian extension of the BL model by including the memory effect due to finite duration of collision , in analogy with the treatment of the quantal Brownian motion[9]. The non-Markovian binary collision term has a non-local structure of the form[8],

$$K(f)=\frac{d}{(2\pi\hbar)^3}\int d^3p_2d^3p_3d^3p_4\int_0^t d\tau W(12;34;\tau)[\tilde{f}\tilde{f}_2f_3f_4-ff_2\tilde{f}_3\tilde{f}_4]_{t-\tau} \tag{3}$$

where $\tilde{f}_j=1-f_j$ with $f_j(t-\tau)=f(\mathbf{r}-\tau\mathbf{v},\mathbf{p}+\tau\nabla U,t-\tau)$, d is the spin-isospin degeneracy factor and the binary collision kernel is given in terms of the basic two-body transition rate by $W(12;34;\tau)=w(12;34)[g_1(\tau)g_2(\tau)g_3(\tau)^*g_4(\tau)^*+c.c.]$ with the mean-field propagator

$$g_j(\tau)=\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}\langle\mathbf{p}_j+\frac{\mathbf{k}}{2}|T\cdot exp\{-i\int_{t-\tau}^t dt'h(t')\}|\mathbf{p}_j-\frac{\mathbf{k}}{2}\rangle. \tag{4}$$

The stochastic collision term is characterized by a nonlocal correlation function

$$\langle\delta K(\mathbf{r},\mathbf{p},t)\delta K(\mathbf{r}',\mathbf{p}',t')\rangle=\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}C(\mathbf{p},\mathbf{p}';\omega)\delta(\mathbf{r}-\mathbf{r}'). \tag{5}$$

Here $C(\mathbf{p},\mathbf{p}';\omega)$ denotes the spectral density of the correlation function, which can be expressed in terms of one-body properties as in the Markovian case,

$$C(\mathbf{p},\mathbf{p}';\omega) = \frac{1}{2}\int d\tau e^{i\omega\tau}\int d^3p_3 d^3p_4 \quad W(11';34;\tau)[\tilde{f}_1\tilde{f}_{1'}f_3f_4 + f_1f_{1'}\tilde{f}_3\tilde{f}_4]_{t-\tau} \quad (6)$$
$$-\int d\tau e^{i\omega\tau}\int d^3p_2 d^3p_4 \quad W(12;1'4;\tau)[\tilde{f}_1\tilde{f}_2 f_{1'}f_4 + f_1f_2\tilde{f}_{1'}\tilde{f}_4]_{t-\tau}$$
$$+\delta(\mathbf{p}-\mathbf{p}')\frac{1}{2}\int d\tau e^{i\omega\tau}\int d^3p_2 d^3p_3 d^3p_4 \quad W(12;34;\tau)[\tilde{f}_1\tilde{f}_2 f_3f_4 + f_1f_2\tilde{f}_3\tilde{f}_4]_{t-\tau}.$$

The basic transition rate $w(12;34)$ may be expressed in terms of the scattering cross-section as

$$w(12;34) = \frac{2}{\pi}\frac{1}{m^2}\frac{d\sigma}{d\Omega}\delta(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_3}-\mathbf{p_4})\exp[-\frac{1}{2}(t_c\Delta\epsilon)^2] \quad (7)$$

where $\Delta\epsilon = \epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4$ is the energy exchange experienced by the colliding pair and t_c is the duration tine of a two-body collision. The Gaussian factor specified by the duration time acts as a cut-off for the off-shell scattering and modulates the frequency spectrum of the correlation function. In the mean-field dominated regime when the characteristic time associated with the mean-field fluctuations is short in comparison with the nucleon mean-free-path, the τ-dependence of the phase-space density in the collision term (3) and the correlation function (6) can be neglected, $f_j(t-\tau) \approx f_j(t)$. Then , the collision term takes essentially a Markovian form with an effective transition rate $\int_0^t d\tau W(12;34;\tau)$, and the spectral density $C(\mathbf{p},\mathbf{p}';\omega)$ of the correlation function reduces to a form similar to the standard expression in the Markovian limit[1], but expressed with a frequency dependent transition density $W(12;34;\omega) = \int d\tau e^{i\omega\tau}W(12;34;\tau)$.

LINEAR RESPONSE TREATMENT

The model developed in the previous section can be applied to study the small amplitude density fluctuations around a stable or unstable equilibrium in the linear response approximation. The small deviations of the phase-space density $\delta f(\mathbf{r},\mathbf{p},t) = f(\mathbf{r},\mathbf{p},t) - f_0(\mathbf{r},\mathbf{p}) = \{Q(\mathbf{r},\mathbf{p},t), f_0\}$ is determined by the linearized BL equation,

$$\frac{\partial}{\partial t}\delta f - \{\delta h, f_0\} - \{h_0, \delta f\} = I_0\cdot\delta f + \delta K_0 \quad (8)$$

where f_0 is the Fermi-Dirac density representing the equilibrium state, $I_0\cdot\delta f$ is the linearize approximation to the non-Markovian collision term and δK_0 describes the rate of fluctuations generated in the equilibrium state.

The linearized collision term involves two different contributions: one part comes from the deviation $\delta f(\mathbf{r},\mathbf{p},t)$ of the phase-space density, and the other part arises from the fluctuating part of the mean-field propagator $g_j(\tau)$, which is usually neglected in the Markovian limit. In the mean-field dominated regime when the collisional damping is weak, the deviation $\delta f(\mathbf{r},\mathbf{p},t)$ can be determined in terms of the fluctuating part of the mean-field by setting the r.h.s. of eq.(8) to equal to zero. As a result, these two contributions can be combined to give [11,12],

$$I_0\cdot\delta f = \frac{d}{(2\pi\hbar)^3}\int d^3p_2 d^3p_3 d^3p_4 \delta W[\tilde{f}^0\tilde{f}^0_2 f^0_3 f^0_4 - f^0 f^0_2\tilde{f}^0_3\tilde{f}^0_4] \quad (9)$$

where the transition rate is given by

$$\delta W = \int\frac{d\omega}{2\pi}e^{-i\omega t}w(12;34)\Delta Q(\omega)[\delta(\Delta\epsilon-\omega) - \delta(\Delta\epsilon+\omega)] \quad (10)$$

with $\Delta Q = Q_3(\omega) + Q_4(\omega) - Q_1(\omega) - Q_2(\omega)$ and $Q_j(\omega) = Q(\mathbf{r}, \mathbf{p_j}, \omega)$ is the Fourier transform of the distortion function $Q(\mathbf{r}, \mathbf{p}, \mathbf{t})$. The spectral density of the fluctuating collision term δK_0 is given by the equilibrium limit of the expression (6) by replacing f_j with f_j^0, and the full propagator with the free propagator in the collision kernel $W(12;34;\tau)$ yielding a frequency dependent transition density [9] $W(12;34;\omega) = w(12;34)[\delta(\Delta\epsilon - \omega) + \delta(\Delta\epsilon + \omega)]$.

The frequency dependence in the collision term and the correlation function of the stochastic part represents emission and absorption of collective phonons with energy $\hbar\omega$ in direct coupling with two-body collision. The standard results of the Markovian description are obtained as zero frequency limit of these expressions. However, when the characteristic energies are large as compared to the temperature of the system the non-Markovian extension provides an appropriate description of the transport properties of collective modes in accordance with the quantal fluctuation-dissipation relation.

UNSTABLE NUCLEAR MATTER

The early evolution of the unstable nuclear matter in the spinodal zone has been recently addressed in the framework of the BL model in [13,14]. The system is mechanically unstable and the density fluctuations generated by the stochastic collision term may be amplified by the self-consistent mean-field, leading towards a transformation of the system into an assembly of nuclear clusters. Here, we consider this problem on the basis of the non-Markovian BL model and investigate the effect of finite memory time on the development of the unstable collective modes in the RPA framework[15]. Because of translational symmetry, collective modes in matter are characterized with a wave number k. For each wave number there are two unstable collective modes with imaginary frequencies $\omega = \pm i\gamma_k$ determined by a semi-classical dispersion relation,

$$\frac{\partial U_k}{\partial \rho} \int \frac{d^3p}{(2\pi\hbar)^3} \frac{\mathbf{k}\cdot\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \frac{\partial f^0}{\partial \epsilon} = 1 \tag{11}$$

where $U_k(\rho)$ is the Fourier transform of the self-consistent mean-field. The associated RPA amplitudes of these modes are given by $Q_k^{\pm} = N_k/\mathbf{k}\cdot\mathbf{v} \pm i\gamma_k$, with N_k as a normalization factor.

The fluctuating phase-space density can be expanded in terms of RPA modes as

$$\delta f(\mathbf{r}, \mathbf{p}, t) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} [A_k^+ Q_k^+ + A_k^- Q_k^-] \mathbf{k}\cdot\mathbf{p} \frac{\partial}{\partial \epsilon} f^0 \tag{12}$$

where A_k^+ and A_k^- represent the amplitudes of the growing and decaying modes, respectively. Integrating this equation over the momentum $\mathbf{p}$ and using the dispersion relation (11), the fluctuating density can be expressed in terms of the collective amplitudes as

$$\delta\rho(\mathbf{r}, t) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} (A_k^+ + A_k^-). \tag{13}$$

Inserting the expansion (12) into eq.(8) and projecting the resultant equation by $Q_k^+ \exp(i\mathbf{k}\cdot\mathbf{r})$ and $Q_k^- \exp(i\mathbf{k}\cdot\mathbf{r})$, respectively, we obtain stochastic equations for the amplitudes A_k^+ and A_k^-. Here for simplicity, we consider only the amplitudes $A_k^+ = A_k$ of the growing modes,

$$\frac{d}{dt} A_k - \gamma_k A_k = -\frac{1}{2}\Gamma_k A_k + F_k \tag{14}$$

where Γ_k and F_k are the collisional damping width and the stochastic force associated with the mode. If the initial system consists of uniform nuclear matter, the initial amplitudes all vanish, $A_k(0) = 0$. It then follows that the amplitudes remain zero on the average $\langle A_k(t)\rangle = 0$. However, each individual history displays a random evolution and the development of the average magnitude of the amplitudes is described by the associated variances $\sigma_k(t) = \langle A_k(t)^* A_k(t)\rangle$. Then, it is convenient to convert eq.(14) for the amplitudes into an equation for the variances,

$$\frac{d}{dt}\sigma_k - 2\gamma_k\sigma_k = -\Gamma_k\sigma_k + 2D_k. \tag{15}$$

Here, both the damping width Γ_k and the diffusion coefficient D_k can be expressed in terms of the correlation function $C_k(\omega)$ of the stochastic force F_k,

$$D_k = \int_0^t d\tau e^{\gamma_k \tau} \int \frac{d\omega}{2\pi} e^{-i\omega\tau} C_k(\omega) \tag{16}$$

and a similar expression for Γ_k, and the correlation function of the stochastic force is

$$C_k(\omega) = \int d^3p_1 d^3p_2 d^3p_3 d^3p_4 W(12;34;\omega) |\frac{\Delta Q_k}{2}|^2 f_1^0 f_2^0 \tilde{f}_3^0 \tilde{f}_4^0 \tag{17}$$

where $\Delta Q_k = Q_k(\mathbf{p}_1) + Q_k(\mathbf{p}_2) - Q_k(\mathbf{p}_3) - Q_k(\mathbf{p}_4)$ represents the change in the quantity $Q_k(\mathbf{p})$ during a two-body collision. Here , we neglect the damping and study the diffusion coefficient which acts as a source term for exciting the collective modes.

The four momentum integrals in the expression (17) are constraint by the energy conserving δ-functions in the transition rate $W(12;34;\omega)$. For temperatures small in comparison with the Fermi energy $T \ll \epsilon_F$, the integrand is effectively confined to the region near the Fermi surface. When, furthermore, the energy exchange is small as well, $\hbar\omega \ll \epsilon_F$, then the energy and angular parts of the integration approximately decouple. It is then possible to factor out the ω dependence of the correlation function[16], $C_k(\omega) = C_k(0)\ \chi(\omega)$, where $C_k(0)$ is the correlation function obtained in the standard treatment without the memory effect, and the influence of the finite memory is contained in the frequency modulation function

$$\chi(\omega) = \frac{\hbar\omega}{2T} coth(\frac{\hbar\omega}{2T})[1 + (\frac{\hbar\omega}{2\pi T})^2] exp[-\frac{1}{2}(t_c\omega)^2]. \tag{18}$$

This function is displayed in figure 1a for a range of temperatures T and the duration time $t_c = 6$ fm/c. The modulation function $\chi(\omega)$ strongly depends on temperature and approaches to the classical result at high temperatures, as expected. It then follows the diffusion coefficient associated with a collective mode becomes $D_k = D_k^0\ \chi_k(t)$, where D_k^0 is the standard expression in the Markovian limit and $\chi_k(t)$ is a time dependent factor related to the frequency modulation function $\chi(\omega)$ according to eq.(16). The influence of the memory time on the growth of density fluctuations can be illustrated from the solution of eq.(15), which can be expressed as $\sigma_k(t) = \sigma_k^0(t)\bar{\chi}_k(t)$, where $\sigma_k^0(t)$ is the standard result and $\bar{\chi}_k(t)$ is a correction factor due to the finite memory time,

$$\bar{\chi}_k(t) = \frac{\int_0^t 2dt' exp(-2\gamma_k t')\chi_k(t')}{\int_0^t dt' exp(-2\gamma_k t')} \tag{19}$$

This correction factor is plotted in figure 1b for the most unstable mode corresponding to $\rho = 0.3\rho_0$ and $T = 4$ MeV. As seen from the figure, the correction factor can deviate significantly from unity particularly in the domain where the fastest growth occurs, hence the density undulation grows larger in the course of a given time interval. It, therefore appears important to incorporate such memory effect in the BL simulations.

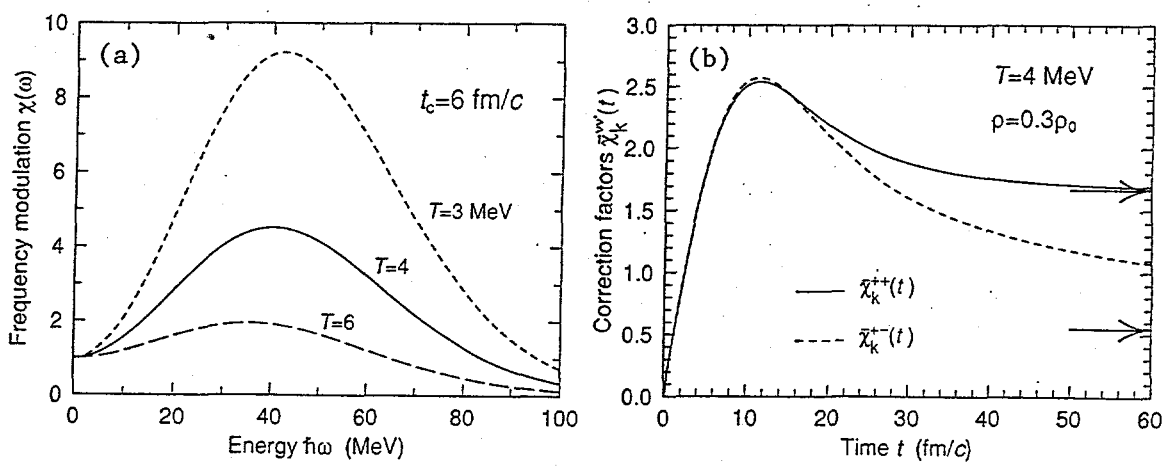

Figure 1. (a) Frequency modulation function at $T = 3, 4, 6$ MeV and $t_c = 6$ fm/c. (b) the correction factors for the most unstable modes at $\rho = 0.3\rho_0$ and $T = 4$ MeV. Taken from[15].

DAMPING OF COLLECTIVE VIBRATIONS

Semi-classical transport models of BUU-type are often employed for studying nuclear collective vibrations. Although these models give a good description of the average resonance energies, for a proper description of the collisional relaxation rate of collective vibrations it is necessary to use these models with a non-Markovian collision term as described in section 2. As a result, the collisional damping width of an iso-scalar collective vibration with a mean frequency Ω can be expressed as[8,11,12]

$$\Gamma = \frac{\int d^3r d^3p_1 d^3p_2 d^3p_3 d^3p_4 W Z(\Omega)(\Delta\chi)^2 f_1^0 f_2^0 \tilde{f}_3^0 \tilde{f}_4^0}{2\int d^3r d^3p \chi^2 \frac{\partial}{\partial\epsilon} f_0} \tag{20}$$

where $Z(\Omega) = [\delta(\Delta\epsilon - \Omega) - \delta(\Delta\epsilon + \Omega)]/2\Omega$ and $\chi \equiv \chi(\mathbf{r}, \mathbf{p}, \Omega)$ denotes the distortion function of the local Fermi surface associated with the collective mode. It is possible to derive a similar expression for the collisional widths of isovector vibrations by considering the proton and neutron degrees of freedom explicitly. In order to calculates the relaxation rates , we need to know the corresponding distortion functions $\chi \equiv \chi(\mathbf{r}, \mathbf{p}, \Omega)$, which, in principle, should be determined by solving the linearized BL eq. (8). However, an estimate of the collisional widths can be obtained by determining the distortion functions according to the scaling model description of collective vibrations. The collective vibrations induces coherent distortions into the momentum space , and the distortion function can be expressed in terms of the velocity field $\Phi(\mathbf{r})$ as $\chi = (\mathbf{p} \cdot \nabla)(\mathbf{p} \cdot \nabla)\Phi(\mathbf{r})$. Then, it is possible to derive an analytical approximation for the collisional widths assuming a constant cross-section and neglecting the surface effects [8,11].

Here, we present more accurate calculations of the collisional widths, which are carried out with energy-angle dependent cross-sections, and also by incorporating surface effects in a local density approximation[12]. We take the velocity field associated with the giant quadrupole vibrations as $\Phi_Q = r^2 P_2(\theta)$, and the velocity field associated with the giant monopole vibrations as $\Phi_M = r^2$ and $j_0(kr)$ where the first and second choices correspond to the scaling description and the hydrodynamical descriptions with a wave number $k = \pi/R$, respectively. Then the collisional widths are calculated by evaluating the momentum integrals in the expression (20) with the Monte-Carlo method.

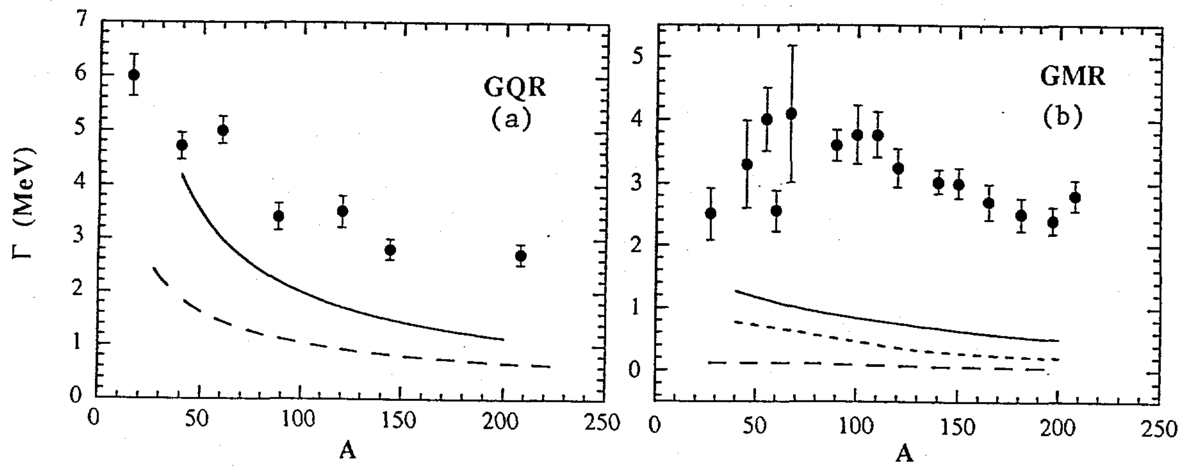

Figure 2. The collisional widths of giant quadrupole (a) and giant monopole (b) vibrations as a function of mass number at $T = 0$ MeV. Taken from[12].

The result of the Monte-Carlo calculations for the quadrupole vibrations is shown as a function of mass number by solid line in figure 2a. In the same figure, the analytical estimates with a constant cross-section $\sigma = 40$ mb are indicated by long dashed-line. In figure 2b, the Monte-Carlo results for the width of monopole vibrations are shown in the hydrodynamic model by solid-line and in the scaling model by short dashed-line. In the same figure, long dashed-line indicate the corresponding analytical estimates. The Monte-Carlo calculations yield larger collisional widths than those obtained by the analytical estimate. This increase comes out as a result of the combined effect of the diffuse nuclear surface and energy-angle dependence of the cross-section. In the vicinity of the nuclear surface the Fermi motion is reduced, and hence the cross-section becomes larger , which leads to a more effective collisional damping.

CONCLUSIONS

The standard models such as BUU and its stochastic extension BL, provides a good approximation for describing the transport properties of collective modes at low frequency-high temperature limit. However, when the system possesses fast collective modes the standard description breaks down, and it is necessary to incorporate the memory effect due to finite duration of two-body collision. This yields a non-Markovian extension of the BL model with a modified transition rate involving a direct coupling between collective modes and two-body collisions. Consequently, the extended model leads to a description of the transport properties of collective modes that is in accordance with the quantal fluctuation-dissipation relation. This is illustrated in the case of agitation of unstable collective modes in nuclear matter, and it is shown that the magnitude of the source term for exciting the most unstable collective modes is significantly modified as compared to the standard treatment. The extended model is applied to calculate the collisional damping width of the giant resonance excitations. The standard treatment with a Markovian collision term leads to vanishing collisional widths at zero temperature. Where as in the non-Markovian model the collisional widths are finite and consistent with the Landau's expression of damping of high frequency vibrations, however account for a part of the observed damping widths.

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