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Final Report on Subcontract B656491: Block smoothers and generalized ideal interpolation in AMG

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Progress has been made in several of the proposed areas of research, as summarized below.

The Pennsylvania State University ("Subcontractor") worked on developing new parallel algebraic multilevel methods suitable for solving PDEs. Specifically, work on the design of multigrid solvers for coupled systems of partial differential equations arising in numerical modeling of various applications was completed. A main emphasis was on the design of new ideal algebraic multigrid interpolation for problems such as Maxwell's equations where block smoothers are needed and the standard form of ideal interpolation is not an effective choice.

Block smoothers in AMG

The team studied and developed an approach for automatically constructing block smoothers in an AMG algorithm for Maxwell's equations, Grad-Div systems and lower-order refined discretizations of higher-order approximations of the Poisson equation. The overall algorithm finds the local near kernel components of the system matrix by solving local eigen-problems restricted to neighborhoods defined in terms of the matrices nonzero structure. Here, local neighborhoods are defined wrt the distance (or power of the system matrix) used in selecting the coarse variable sets.

Generalized ideal interpolation in AMG

The team also developed a new generalized form of ideal interpolation for the Maxwell and Grad-Div problems. The approach together with the new block smoother and CR process described below yields a completely algebraic setup, using only the local near kernel components that are computed to define the block smoother; it approximates optimal AMG interpolation, the latter directly minimizes the two-grid convergence rate for the Maxwell problem; and it results in an optimal two-grid method for the various model problems. Various theoretical results on this new ideal interpolation were derived. Also, the team observed that for a proper choice of the coarse variables, a sparse approximation of this new interpolation is equivalent to the standard geometric interpolation for the Maxwell problem.

Compatible relaxation

Lastly, compatible relaxation type estimates for measuring the quality of the coarse grid and the block smoother were studied . A new sharp measure using the new optimal form of interpolation was studied together with the less predictive but practical habituated form of CR. Interestingly, constructing an optimal form of CR is practical

using the local near kernel components that define the block smoother. A new adaptive coarsening algorithm was developed and implemented for these block smoothers. The algorithm
was tested for scalar and PDE systems, showing promising results.

Together with R. Falgout from CASC, the team is currently in the process of writing a journal article on this research.