Yukawa textures with an anomalous horizontal abelian symmetry

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Abstract

The observed hierarchy of quark and lepton masses and mixings may be obtained by adding an abelian family symmetry to the Minimal Supersymmetric Model and coupling quarks and leptons to an electroweak singlet scalar field. In a large class of such models, this symmetry suffers from anomalies which must be compensated by the Green-Schwarz mechanism; this in turn fixes the electroweak mixing angle to be $\sin^2 \theta_W = 3/8$ at the string scale, without any assumed GUT structure. The analysis is extended to two distinct generalisations of the Standard Model: neutrino masses and mixings and R-parity violating interactions.
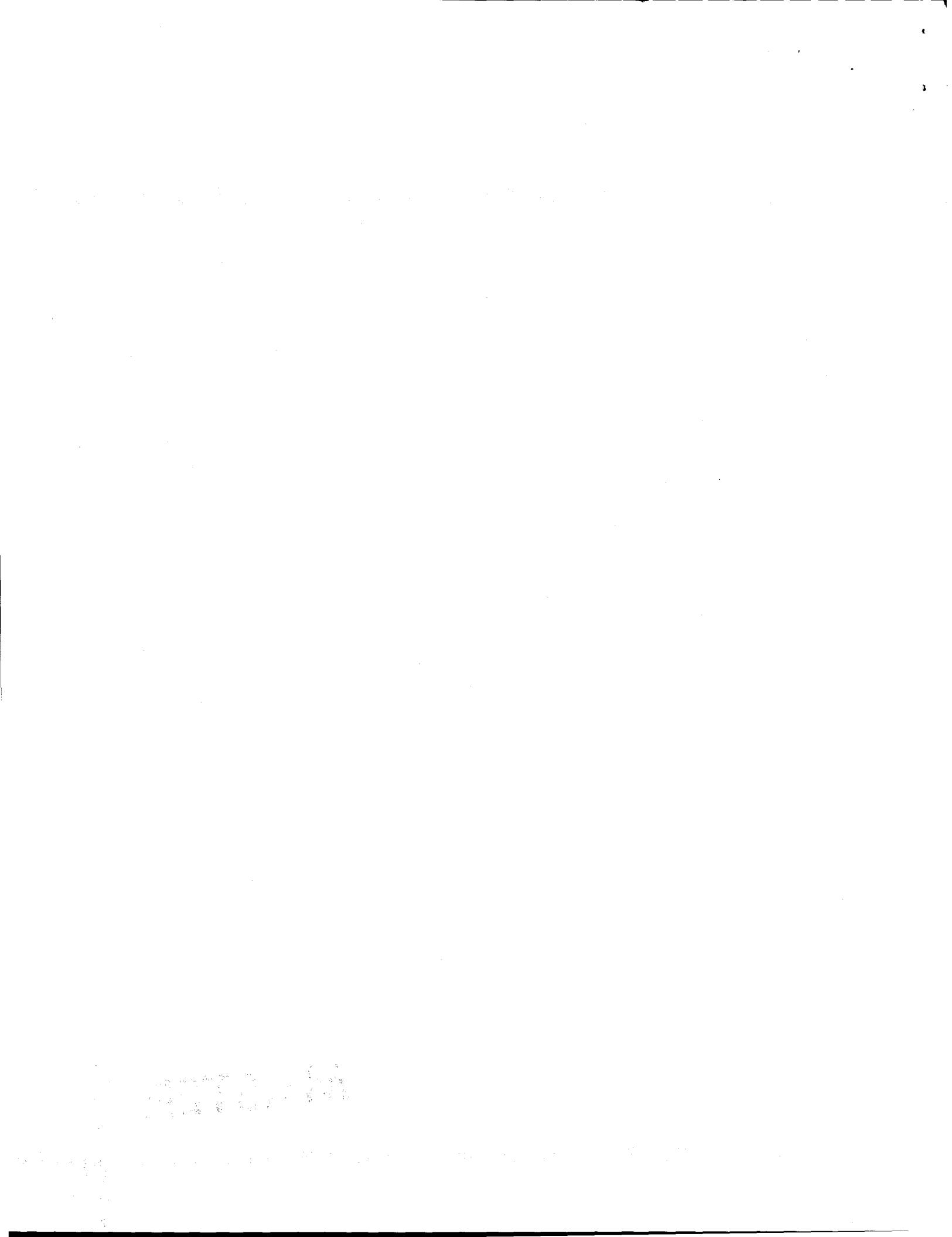
1 Introduction.

The problem of quark and lepton mass hierarchies and mixing is not addressed by the Standard Model and has been a thorn in particle theorists side. Recent developments, both experimental and theoretical, might shed new light on this long standing issue. On the experimental side, it is the discovery of the top quark [1] in the mass range of the electroweak scale: in more technical terms, the top Yukawa coupling is found to be of the order of the gauge couplings. On the theoretical side, the emergence of string theories as a universal theory encompassing all known fundamental interactions including gravity provides a unique framework which allows to relate features of the effective low energy theory which seemed heretofore uncorrelated. Of special interest for the problem that we are discussing are: the presence of non-renormalisable interactions (which can in principle be computed within a given string model); an often large number of horizontal gauge symmetries, especially abelian, which are spontaneously broken at scales which may vary between the electroweak scale and the Planck scale; a large number of Standard Model singlet scalar fields whose couplings to ordinary matter are fixed by the latter symmetries.

All these properties have induced several groups to reconsider the original idea of Froggatt and Niesen [2] which uses nonrenormalisable couplings of quarks and leptons to electroweak gauge single fields and an horizontal symmetry to constrain these couplings in order to generate mass hierarchies.The first results are promising and lead to new theoretical developments and new ways to test experimentally these ideas.

We address some aspects of this program in this paper. In section 2, we recall the basic concepts and stress the relevance of some parameters, such as the supersymmetric μ-term. We then proceed to discuss the connection between the phenomenological constraint coming from the quark and lepton mass spectrum and the more fundamental issue of the anomaly structure of the horizontal family. We show that, for a large class of models, phenomenology requires our abelian symmetry to be anomalous, this anomaly being cancelled by a Green-Schwarz mechanism [3]. This property obviously points towards string theories. Section 3 is devoted to the the study of neutrino masses and mixings when one adds to the particle content of the Standard Model right-handed neutrinos. It is shown that the abelian horizontal symmetry provides constraints on the neutrino mass spectrum as well as on the mixing angle. In section 4, we consider another extension of the Standard Model: the spectrum remains minimal but we allow for couplings which violate R-parity. Again, the horizontal symmetry constrains these new couplings. Finally, section 5 gives our conclusions.

DISCLAIMER

2 Strategies with chiral scalars.

The basic idea, which dates back to the early papers of Froggatt and Nielsen [2] is to use an abelian horizontal symmetry $U(1)_X$ in order to forbid most Yukawa couplings: in practice all but the top quark coupling or all but the third family couplings. The hierarchies of fermion masses and mixings are then generated through higher dimensional operators involving one or several electroweak singlet scalar fields. These fields acquire a vacuum expectation value which breaks the horizontal symmetry at some large scale and gives rise to the ordinary Yukawa couplings. More specifically, if θ is one such field of X-charge -1, X-charge conservation allows for example the non-renormalisable term in the superpotential:

$$\lambda_{ij}^U Q_i \bar{u}_j H_u \left(\frac{\theta}{M} \right)^{n_{ij}} \tag{1}$$

where Q_i is the quark isodoublet of the ith generation, $\bar{u}_j$ is the u quark-type isosinglet of the j-th generation, H_u is one of the two Higgs doublets of the supersymmetric standard model. The coupling λ_{ij}^U is expected to be of order one and the mass M is a large mass scale, the order of which we will discuss later. The positive rational number n_{ij} is nothing but the sum of the X-charges of the standard model fields involved, namely Q_i, $\bar{u}_j$ and H_u:

$$n_{ij} = q_i + u_j + h_u. \tag{2}$$

Once θ gets a vacuum expectation value, one obtains an effective Yukawa coupling:

$$Y_{ij}^U = \lambda_{ij}^U \left(\frac{<\theta>}{M} \right)^{n_{ij}}. \tag{3}$$

If $<\theta>/M$ is a small number, and if the array of X-charges is sufficiently diversified, one may implement in the theory various hierarchies of masses and mixings. Our goal is to select a class of models where such a strategy proves to be efficient as well as it leads to specific predictions. In this Section, we will review the possibilities that are open to us in order to decide which lead to the most interesting and fruitful properties.

The electroweak singlet fields θ may appear in vectorlike pairs or as chiral individuals. In the latter case, the low energy Yukawa matrix will contain zeroes whenever the excess charge n_{ij} turns out to be negative, since, in this case, the holomorphy of the superpotential prevents [4] a coupling of the type (1): we will thus refer to them as *supersymmetric zeroes*. Such a property may or may not be a welcome feature, since it may yield too many zeroes in the mass matrix. One may thus prefer to introduce a vectorlike pair $(\theta, \bar{\theta})$ of electroweak singlets of respective X-charge (-1) and $(+1)$. If they correspond to D-flat directions, then naturally $<\theta> = <\bar{\theta}>$ and the low energy Yukawa couplings will be of order $(<\theta>/M)^{|n_{ij}|}$, irrespective of the sign of the excess charge n_{ij} [5].

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The problem with this approach is that a supersymmetric mass term $\tilde{M}\theta\bar{\theta}$ is perfectly allowed by the symmetries (unless one assumes an unwelcome fine tuning, $\tilde{M}$ is a large mass scale) and spoils the D-flat direction, leading to a large hierarchy between the vacuum expectation values.

On the other hand, we have shown [6] that, in a large class of models with a chiral θ field, there exists an interesting connection between the fermion mass spectrum and the value of the Weinberg angle. More precisely, the fermion mass spectrum puts such constraints on the X-charges that the mixed anomalies of the $U(1)_X$ symmetry are necessarily nonzero and must be cancelled using the Green-Schwarz mechanism [3]. As generically stressed by Ibáñez [7], this in turn fixes also the weak mixing angle which we find equal to its standard value of $\sin^2\theta_W = 3/8$ at the superheavy scale. We will return to this question below but this attractive feature leads us to concentrate in the rest of this paper on the class of models with only chiral electroweak singlet scalars (*i.e.* no vector-like pair).

2.1 Filling the supersymmetric zeroes through wave function renormalisation

It has been stressed before [4, 8] that, in this class of models, non-renormalizable contributions to the fermion kinetic terms may lead to filling the zeroes imposed by supersymmetry (corresponding to $n_{ij} < 0$). Let us take this opportunity to discuss our general strategy. We are considering the effective theory obtained from a more fundamental theory of typical scale M, well below this scale M. The fields of the effective theory are, by assumption, those of the Minimal Supersymmetric Model plus electroweak singlet chiral scalars, generically denoted as θ. We are writing the most general couplings including non-renormalizable terms proportional to negative powers of M, compatible with the symmetries of the effective theory, namely $SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$. This yields terms of the type (1), and similar couplings for the charge $(-1/3)$ quarks and charge (-1) leptons. It may also give rise to R-parity breaking interactions. We will study this possibility in Section 4. Our concern here is that it also gives kinetic terms for the fermions with a θ dependent normalisation. The low energy fermion fields are therefore obtained through a θ dependent redefinition, which may modify the θ dependence of the Yukawa couplings.

For concreteness, let us consider the Yukawa couplings arising from (1). The normalized kinetic terms originate from a diagonal quadratic Kähler potential of the form

$$K_0(Q_i, \bar{u}_j, \cdots) = Q_i^+ Q_i + \bar{u}_i^+ \bar{u}_i + \cdots \tag{4}$$

In our case, the Kähler potential as well receives non-renormalisable contribu-

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tions; it reads, to lowest order in powers of $1/M$:

$$K(Q_i, \bar{u}_j, \cdots, \theta) = Q_i^+ Q_j \left[H(q_i - q_j) \left(\frac{\theta^+}{M} \right)^{q_i - q_j} + H(q_j - q_i) \left(\frac{\theta}{M} \right)^{q_j - q_i} \right]$$

$$+ \bar{u}_i^+ \bar{u}_j \left[H(u_i - u_j) \left(\frac{\theta^+}{M} \right)^{u_i - u_j} + H(u_j - u_i) \left(\frac{\theta}{M} \right)^{u_j - u_i} \right]$$

$$+ \cdots \tag{5}$$

where $H(x)$ is the Heaviside function ($H(x) = x$ if $x \geq 0$, $H(x) = 0$ otherwise). To bring the kinetic terms to their canonical form, we have to redefine the matter fields Φ_i ($\Phi = Q, \bar{u}, \bar{d}, L, \bar{e}$):

$$\Phi_i \rightarrow V_{ij}^\Phi \Phi_j \tag{6}$$

where the order of magnitude of the matrix elements of V^Φ depends on the relative charges ϕ_i of the Φ_i fields:

$$V_{ij}^\Phi \sim \left(\frac{<\theta>}{M} \right)^{|\phi_i - \phi_j|} . \tag{7}$$

It is useful to note that the structure ot the matrix V_Φ is simply that of the identity matrix corrected by positive powers of $<\theta>/M$.

The Yukawa couplings in the canonical basis

$$\hat{Y}^U = V^{QT} Y^U V^{\bar{u}} \tag{8}$$

are now a sum of terms

$$\hat{Y}_{ij}^U = \sum_{kl} Y_{ij,kl} \tag{9}$$

where

$$Y_{ij,kl} \sim H(q_k + u_l + h_u) \left(\frac{<\theta>}{M} \right)^{|q_i - q_k| + |u_l - u_j| + q_k + u_l + h_u} . \tag{10}$$

One immediately infers that $\hat{Y}_{ij}^U$ is at most of the order of magnitude that would be obtained with a vectorlike pair of θ fields: $(<\theta>/M)^{|n_{ij}|}$. This means that, as far as hierarchies are concerned, one does not gain much by going to a vectorlike pair scenario, the weaknesses of which we stressed earlier.

In the case where $n_{ij} \geq 0$, one deduces from (10) that

$$\hat{Y}_{ij}^U \sim \left(\frac{<\theta>}{M} \right)^{n_{ij}} . \tag{11}$$

In other words, non-zero entries to the Yukawa matrix are left untouched by the process of normalizing the kinetic terms.

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On the other hand, in the case where $n_{ij} < 0$, one can easily show from (10) that

$$Y_{ij,kl} = H(n_{kl}) \left(\frac{<\theta>}{M} \right)^{|n_{ij}| + 2max(n_{kl}, n_{kj}, n_{il})}, \qquad (12)$$

which shows that $Y_{ij}^{U'}$ is of order $(<\theta>/M)^{|n_{ij}|}$ or smaller.

As an example, we can apply the above results to the case where the (3,3) entry to the Yukawa matrix is allowed by the $U(1)_X$ symmetry, *i.e.* $n_{33} = 0$. Then applying (12) with all indices equal to 3 except for either i or j, one finds

$$\hat{Y}_{i3}^U \sim \left(\frac{<\theta>}{M} \right)^{|n_{i3}|}, \quad \hat{Y}_{3j}^U \sim \left(\frac{<\theta>}{M} \right)^{|n_{3j}|}. \qquad (13)$$

Similarly, for i and j different from 3, if both n_{i3} and n_{3j} are negative

$$\hat{Y}_{ij}^U \sim \left(\frac{<\theta>}{M} \right)^{|n_{ij}|}. \qquad (14)$$

Since $n_{ij} = n_{i3} + n_{3j}$, the corresponding zero in the original Yukawa matrix results in this case from the simultaneous presence of zeroes in the third line ($n_{3j} < 0$) and third column ($n_{i3} < 0$). If, on the other hand, only one is negative, say $n_{i3} < 0$, $n_{3j} \geq 0$, then one shows that

$$\hat{Y}_{ij}^U \sim \left(\frac{<\theta>}{M} \right)^{|n_{ij}| + 2min(n_{i'j}, n_{ij'}, n_{3j})}. \qquad (15)$$

where $i' \neq i \neq 3$ and $j' \neq j \neq 3$ and one used the fact that $det\, Y_U \neq 0$.

2.2 Horizontal abelian charges and the quark and lepton masses

As introduced in Ref. [6], the most general assignment for an Abelian horizontal charge to the particles of the Supersymmetric Standard Model reads

$$X = X_0 + X_3 + \sqrt{3}X_8, \qquad (16)$$

where X_0 is the family-independent part, X_3 is along λ_3, and X_8 is along λ_8, the two diagonal Gell-Mann matrices of the $SU(3)$ family space in each charge sector. In a basis where the entries correspond to the components in the family space of the fields Q, $\overline{u}$, $\overline{d}$, L, and $\overline{e}$, we can write the different components in the form

$$X_i = (a_i, b_i, c_i, d_i, e_i), \qquad (17)$$

for $i = 0, 3, 8$. The Higgs doublets $H_{u,d}$ have X-charges h_u and h_d respectively. These could be assumed to be equal since, using $U(1)_Y$, we have the freedom to redefine the horizontal symmetry in order to make these two X-charges equal.

We will return to this later. In any case, most of the following discussions depend only on the sum of these charges and we thus define

$$2h \equiv h_u + h_d. \tag{18}$$

Then the excess X-charges n_{ij} defined in (2) read for the charge 2/3 quarks:

$$\left[\frac{U_0}{3} - 2(a_8 + b_8)\right] \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 3(a_8 + b_8) + a_3 + b_3 & 3(a_8 + b_8) + a_3 - b_3 & 3a_8 + a_3 \\ 3(a_8 + b_8) - a_3 + b_3 & 3(a_8 + b_8) - a_3 - b_3 & 3a_8 - a_3 \\ 3b_8 + b_3 & 3b_8 - b_3 & 0 \end{pmatrix} \tag{19}$$

and similarly for the charge $-1/3$ quarks with the replacement $(a, b) \to (a, c)$ and for the charge -1 leptons with $(a, b) \to (d, e)$. In (19) and the corresponding matrices for the charge $-1/3$ and -1 sectors, we define the family-independent overall charges:

$$\begin{aligned} U_0 &= 3(a_0 + b_0 + h_u) \\ D_0 &= 3(a_0 + c_0 + h_d) \\ E_0 &= 3(d_0 + e_0 + h_d). \end{aligned} \tag{20}$$

Some of the excess charges in (19) might be negative leading to supersymmetric zeroes in the Yukawa matrix, to be filled in the way described in the previous subsection. But a very generic result, independent to a large extent of this filling procedure, applies to the determinant of the Yukawa coupling matrices:

$$\begin{aligned} det\,\hat{Y}^U &\sim (<\theta>/M_U)^{U_0} \\ det\,\hat{Y}^D &\sim (<\theta>/M_D)^{D_0} \\ det\,\hat{Y}^E &\sim (<\theta>/M_E)^{E_0}. \end{aligned} \tag{21}$$

The only assumption is that there are not enough supersymmetric zeroes to make these determinants vanish (hence the u quark mass is nonzero [9]). In these equations, we allowed for different scales M in the three different sectors. We will come back to this in a later subsection.

The experimental values of the quark and lepton masses, extrapolated near the Planck scale, satisfy the order of magnitude estimates [10]

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) , \quad \frac{m_c}{m_t} = \mathcal{O}(\lambda^4) , \tag{22}$$

$$\frac{m_d}{m_b} = \mathcal{O}(\lambda^4) , \quad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2) , \tag{23}$$

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4) , \quad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) , \tag{24}$$

where, following Wolfenstein's parametrization [11], we use the Cabibbo angle λ, as expansion parameter. Thus, the mass hierarchy appears to be geometrical in each sector. The equality

$$m_b = m_\tau \; , \tag{25}$$

known to be valid in the ultraviolet [12], then yields the estimate

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} = \mathcal{O}(1) \; . \tag{26}$$

Of course, all these estimates should be taken with some precaution since λ is not such a small parameter (thus $2\lambda^n \sim \lambda^{n-1}/2$) and the exponents in (22-24) should be considered as valid up to a unit. In particular, the ratio m_e/m_τ is somewhat closer to λ^5 [13], which, all other mass ratios being kept unchanged, gives a ratio (26) of order λ . We nevertheless find the geometrical hierarchy an attractive mass pattern. Comparison of (26) with (21) yields in this case the simple phenomenological constraint:

$$D_0 = E_0 \tag{27}$$

which, from now on, we will refer to as the *geometrical hierarchy constraint*.

Another low energy mass scale which will play an important role in the discussion that follows is the so-called μ-term. The origin of such a low energy scale in any theory whose fundamental scale is of the order of the Planck scale poses problem. The following solutions have been proposed:

(i) introduce a field N singlet under the Standard Model gauge symmetries which has a trilinear couplings to the Higgs doublets [15]: $\delta W = \lambda N H_u H_d$.

(ii) introduce additional terms in the Kähler potential which are quadratic in the Higgs fields [16, 17]:

$$\delta K = G(M, M^+) H_u H_d + \text{ h.c.} \tag{28}$$

where G is some function of gauge singlet scalars M and their complex conjugates M^+. If the function G turns out to be some function analytic in the scalars M, then, through a Kähler transformation, this can be rephrased as follows:

(iii) add a nonrenormalisable contribution to the superpotential quadratic in the Higgs fields [18, 19]:

$$\delta W = F(M) H_u H_d. \tag{29}$$

In the context of string models, it is quite plausible that the singlet fields involved are moduli fields which are neutral under the horizontal symmetry that we consider. In this case, for any of these scenario to work, we need to impose that $h = 0$. We will thus refer to it in the sequence as the $h = 0$ *option*. This was the solution that we adopted in Ref.[6].

On the other hand, as emphasized by Nir [13] (see also Ref. [14]), the singlet field θ that we use might provide itself the solution to the μ-problem [4], following the same scenarios. In cases (i) and (iii), the following interaction would be allowed by the horizontal symmetry:

$$\delta W = M H_u H_d \left(\frac{\theta}{M}\right)^{2h} \tag{30}$$

where the holomorphy of the superpotential imposes that $h > 0$. The μ term thus obtained is of order $M(<\theta>/M)^{2h}$ and since, as we will see in subsection 2.5, M is a scale close to the Planck scale, one needs a large positive value for h.

In case (ii), the Kähler potential includes a term[4]

$$\delta K = H_u H_d \left(\frac{\theta^+}{M}\right)^{-2h} , \tag{31}$$

which obviously requires $h < 0$. The μ term is then of order $m_{3/2}(<\theta>/M)^{-2h}$ and thus such an option works for values of h moderately negative.

2.3 Anomalies

In Ref.[6], we stressed the important connection between the anomaly issue and the phenomenological constraints coming from the fermion masses. We will repeat the analysis here in the more general framework that we have adopted [8, 13].

The three chiral families contribute to the mixed gauge anomalies as follows

$$\begin{align}
C_3 &= 3(2a_0 + b_0 + c_0) , \tag{32}\\
C_2 &= 3(3a_0 + d_0) + 2h , \tag{33}\\
C_1 &= a_0 + 8b_0 + 2c_0 + 3d_0 + 6e_0 + 2h . \tag{34}
\end{align}$$

The subscript denotes the gauge group of the Standard Model, *i.e.* $1 \sim U(1)$, $2 \sim SU(2)$, and $3 \sim SU(3)$. The important feature of these three anomaly coefficients is that they depend only on the family independent charges X_0 and thus can be directly related to the determinant of the Yukawa matrices through (20,21). The relation depends on the charge h whose connection with the μ parameter we have stressed in the previous subsection.

The X-charge also has a mixed gravitational anomaly, which is simply, up to a normalisation, the trace of the X-charge,

$$C_g = 3(6a_0 + 3b_0 + 3c_0 + 2d_0 + e_0) + 4h + C'_g , \tag{35}$$

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[4] A similar term involving the field θ itself can be cast into the preceding form (30), through a Kähler transformation; and terms involving both θ and θ^+ are of higher order in $1/M$.

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where C_g' is the contribution from the massless particles that do not appear in the minimal $N = 1$ model. One must also account for the mixed YXX anomaly, given by

$$C_{YXX} = 6(a_0^2 - 2b_0^2 + c_0^2 - d_0^2 + e_0^2) + 2(h_u^2 - h_d^2) + 4A_T \, , \qquad (36)$$

with the texture-dependent part given by

$$A_T = (3a_8^2 + a_3^2) - 2(3b_8^2 + b_3^2) + (3c_8^2 + c_3^2) - (3d_8^2 + d_3^2) + (3e_8^2 + e_3^2) \, . \qquad (37)$$

The last anomaly coefficient is that of the X-charge itself, C_X, the sum of the cubes of the X-charge.

As just emphasized, it is of interest for our purposes that C_1, C_2, C_3 and $C_g - C_g'$ only depend on the family-independent charges and can thus be related to the determinants of the mass matrices through (21) [6]. Indeed, one can easily show that the only two independent combinations of these anomaly coefficients which can be expressed in terms of U_0, D_0, E_0 and h are

$$\begin{aligned}
C_3 &= (U_0 + D_0) - 6h, \\
C_1 + C_2 &= \frac{8}{3}(U_0 + D_0) + 2(E_0 - D_0) - 12h, \qquad (38)
\end{aligned}$$

which involve only $(U_0 + D_0)$ and $(E_0 - D_0)$.

Interesting combinations are $C_1 + C_2 - 8C_3/3$ which depends only on h and $E_0 - D_0$ and plays a role in the models with a geometrical hierarchy [6]; and $C_1 + C_2 - 2C_3$ which does not depend on h [8, 13].

It is interesting to express in turn the family independent charges in terms of the anomaly coefficients and the Higgs charges:

$$\begin{aligned}
a_0 &= +\frac{1}{3}(\frac{D_0}{3} - h_d) \quad +\frac{1}{3}\mathcal{C}_D \\
b_0 &= -\frac{4}{3}(\frac{D_0}{3} - h_d) \quad -\frac{1}{3}\mathcal{C}_D \qquad\qquad +\frac{1}{3}C_3 \\
c_0 &= +\frac{2}{3}(\frac{D_0}{3} - h_d) \quad -\frac{1}{3}\mathcal{C}_D \\
d_0 &= -1(\frac{D_0}{3} - h_d) \quad -1\mathcal{C}_D \quad +\frac{1}{3}C_2 - \frac{2}{3}h \\
e_0 &= +2(\frac{D_0}{3} - h_d) \quad +1\mathcal{C}_D \quad -\frac{1}{3}C_2 + \frac{1}{6}(C_1 + C_2 - \frac{8}{3}C_3), \qquad (39)
\end{aligned}$$

where $\mathcal{C}_D = -(C_g - C_g')/3 + C_1/6 + C_2/2 + 5C_3/9$ and we have arranged the right-hand side of these equations such that contributions proportional respectively to the Y, $B - L$ and L charges of the corresponding fields appear in columns.

This shows that one can set $a_0 = -c_0$ by using the $U(1)_Y$ symmetry to redefine the X charges. In this case, the first column is suppressed and all charges are expressed in terms of the anomaly coefficients and of the two Higgs

charges (this does not mean of course that D_0 can be made to vanish; instead we have $D_0 = 3h_d$).

If the theory also has a $U(1)_{B-L}$ symmetry, one can further set $a_0 = 0$. Moreover, since the gravitational anomaly $C_g - C_g{}'$ is exactly along the $B - L$ charge, one can altogether cancel it if one includes a right-handed neutrino to make the $U(1)_{B-L}$ symmetry non-anomalous (*i.e.* traceless).

The parametrisation (39) allows to treat easily the case with no mixed gauge anomalies: $C_1 = C_2 = C_3 = 0$. Indeed, one immediately reads off the charges (with the Y component in the first column subtracted) and deduces that $U_0 = 3h_u$, $D_0 = 3h_d$ and $E_0 = 2h_d - h_u$. Assuming a geometric hierarchy (27) yields $-U_0 = D_0 = E_0$ ($h = 0$) which is easily seen not to hold.

We thus turn to the models where the anomaly coefficients are non-zero. In this case, the anomalies must be cancelled by the Green-Schwarz mechanism [3]. String theories contain an antisymmetric tensor field which, in 4 dimensions, couples in a universal way to the divergence of the anomalous currents. One can therefore use the Green-Schwarz mechanism to cancel the anomalies. Due to the universality of the couplings of this axion-like field, this is only possible if the mixed anomaly coefficients appear in commensurate ratios:

$$\frac{C_i}{k_i} = \frac{C_X}{k_X} = \frac{C_g}{12} \tag{40}$$

where the k's are the Kac-Moody levels at which the corresponding group structures appear. They are integers in the case of non-abelian groups and all string models constructed so far have $k_2 = k_3$, which implies

$$C_2 = C_3. \tag{41}$$

These Kac-Moody levels appear themselves in the gauge coupling unification condition which is valid at the string scale, without any assumed GUT structure. This condition reads:

$$k_i g_i^2 = k_X g_X^2 = g_{string}^2. \tag{42}$$

As mentioned earlier, one can relate the ratio of d-type quark masses to charged lepton masses with a combination of anomaly coefficients which can be turned, using (40), into a combination of Kac-Moody levels, and, using (42), into a combination of gauge couplings.

More precisely, using (20,21), one obtains, assuming $M_D = M_E$,

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} = \frac{det\hat{Y}^D}{det\hat{Y}^E} = \left(\frac{<\theta>}{M_D}\right)^{3(a_0 + c_0 - d_0 - e_0)}. \tag{43}$$

Hence, through (39),

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} = \left(\frac{<\theta>}{M_D}\right)^{2h - (C_1 + C_2 - 8/3C_3)/2}. \tag{44}$$

In the $h = 0$ option, the geometrical hierarchy discussed above which gives a mass ratio of order one yields the following relation among anomaly coefficients [6]:

$$C_1 + C_2 = \frac{8}{3}C_3, \tag{45}$$

or, using (40-42),

$$\frac{C_1}{C_2} = \frac{g_2^2}{g_1^2} = 5/3. \tag{46}$$

This fixes the value of the electroweak angle to its standard GUT value, without any underlying GUT structure:

$$\sin^2 \theta_W = \frac{3}{8}. \tag{47}$$

Alternatively, one can start from (44) and impose the standard value for the electroweak angle. This is only possible for a vanishing h in which case one recovers the geometrical hierarchy, or a moderately negative h (in fact $h = -1/2$) when one departs slighly from a geometrical hierarchy ($m_e/m_\tau \sim \lambda^5$) [13]. As discussed above, in the latter case, one may use the θ field to account for the μ term; using (31) and (44), one obtains

$$\frac{m_e m_\mu m_\tau}{m_d m_s m_b} = \frac{\mu}{m_{3/2}}. \tag{48}$$

The former case necessarily involves another gauge singlet field in order to generate a μ term.

2.4 Eigenvalues and mixing angles

In Ref. [6], we presented a result on the hierarchy of mass matrix eigenvalues in models with a vectorlike pair $(\theta, \bar{\theta})$ of singlet scalars. This result can be generalized to the class of models that we are considering in this paper, namely models with a chiral singlet scalar θ. After filling the supersymmetric zeroes, the orders of magnitude of the Yukawa couplings are:

$$\hat{Y}_{ij} \sim \left(\frac{<\theta>}{M}\right)^{\rho_{ij}} \tag{49}$$

where ρ_{ij} is the power of the dominant term in the sum (9). This hierarchical structure results in a strong hierarchy between the eigenvalues of Y. Provided that $\rho_{33} \leq \rho_{ij}$, this hierarchy can be expressed in terms of the two following quantities:

$$p = \min(\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}) \tag{50}$$
$$q = \min(\rho_{11} + \rho_{22}, \rho_{12} + \rho_{21}) \tag{51}$$

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Normalized to the largest eigenvalue, whose order of magnitude is given by $\hat{Y}_{33}$, the mass eigenvalues are:

$$
\begin{array}{llll}
1 & \mathcal{O}(<\theta>/M)^{\frac{q}{2}} & \mathcal{O}(<\theta>/M)^{\frac{q}{2}} & \text{if} \quad p \geq \frac{q}{2} \\[2mm]
1 & \mathcal{O}(<\theta>/M)^p & \mathcal{O}(<\theta>/M)^{q-p} & \text{if} \quad p < \frac{q}{2}
\end{array}
\tag{52}
$$

the only case of phenomenological interest being $p < q/2$.

In the simple case studied by Froggatt and Nielsen [2] where (a) all excess charges are positive (b) $n_{33} = 0$ (c) $n_{ij} \geq n_{i'j'}$ for $i \geq i', j \geq j'$, we obtain from (19):

$$
\begin{aligned}
p &= 3(a_8 + b_8) - a_3 - b_3, \\
q &= 6(a_8 + b_8).
\end{aligned}
\tag{53}
$$

Hence the eigenvalues are simply of order

$$
O\left(\left(\frac{\theta}{M_U}\right)^{3(a_8+b_8)+a_3+b_3}\right), O\left(\left(\frac{\theta}{M_U}\right)^{3(a_8+b_8)-a_3-b_3}\right), O(1).
\tag{54}
$$

We will refer to this case as the *Froggatt-Nielsen hierarchical structure*.

Like the fermion mass ratios, the measured quark mixing angles show a clear hierarchy, which is obvious in Wolfenstein's parametrization of the CKM matrix [11]:

$$
V_{CKM} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho + i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho + i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\tag{55}
$$

where λ is the Cabibbo angle and $A \simeq 0.9 \pm 0.1$. When extrapolated near the Planck scale, V_{CKM} keeps the same structure: the only parameter affected by the renormalization is A, which is reduced by $\simeq 30\%$ [10]. For our purpose, only the order of magnitude of the mixing angles is of interest.

In order to determine the CKM matrix, we have to diagonalize both Y^U and Y^D:

$$
\begin{aligned}
Diag(m_u, m_c, m_t) &= R_L^U Y^U R_R^{U\dagger} \\
Diag(m_d, m_s, m_b) &= R_L^D Y^D R_R^{D\dagger} \\
V_{CKM} &= R_L^U R_L^{D\dagger}
\end{aligned}
\tag{56}
$$

This task becomes simpler if we assume that, in both charge sectors, the rotation matrices R_L and R_R can be decomposed into three small rotations:

$$
R_L = \begin{pmatrix}
1 & -s_{12} & 0 \\
s_{12} & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & -s_{13} \\
0 & 1 & 0 \\
s_{13} & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & -s_{23} \\
0 & s_{23} & 1
\end{pmatrix}
\tag{57}
$$

and similarly for R_R, with rotation angles s'_{12}, s'_{13}, and s'_{23}. In this parametrization, the CKM matrix reads, at leading order [20]:

$$V_{CKM} \simeq \begin{pmatrix} 1 & -s_{12} - s^U_{13}s_{23} & -s_{13} + s^U_{12}s_{23} \\ s_{12} + s^D_{13}s_{23} & 1 & -s_{23} - s^U_{12}s_{13} \\ s_{13} - s^D_{12}s_{23} & s_{23} + s^D_{12}s_{13} & 1 \end{pmatrix} \quad (58)$$

where $s_{ij} = s^U_{ij} - s^D_{ij}$. With the additional assumption that, in each Yukawa matrix, the coefficient in the (3,3) entry dominates over all other coefficients, one can express the rotation angles in terms of the Yukawa matrix coefficients. Unfortunately, these expressions are rather complicated [20, 4, 8], unless the Yukawa matrices possess the Froggatt-Nielsen hierarchical structure. In this case,

$$R^L_U \simeq \begin{pmatrix} O(1) & O(\epsilon^{2a_3}_U) & O(\epsilon^{3a_8+a_3}_U) \\ O(\epsilon^{2a_3}_U) & O(1) & O(\epsilon^{3a_8-a_3}_U) \\ O(\epsilon^{3a_8+a_3}_U) & O(\epsilon^{3a_8-a_3}_U) & O(1) \end{pmatrix} \quad (59)$$

with $\epsilon_U = <\theta>/M_U$; and similarly for R^D_L with ϵ_U replaced by ϵ_D, and V_{CKM} with ϵ_U replaced by $\max(\epsilon_U, \epsilon_D)$.

In the general case, it is more convenient for practical use to solve the equations derived from the requirement that the matrix $R_L Y R^\dagger_R$ be diagonal. The rotation angles in the (1,3) and (2,3) sectors satisfy the following set of approximate equations:

$$\begin{cases} Y_{11}s'_{13} + Y_{12}s'_{23} - Y_{33}s_{13} & \simeq & -Y_{13} \\ Y_{21}s'_{13} + Y_{22}s'_{23} - Y_{33}s_{23} & \simeq & -Y_{23} \\ Y_{11}s_{13} + Y_{21}s_{23} - Y_{33}s'_{13} & \simeq & -Y_{31} \\ Y_{12}s_{13} + Y_{22}s_{23} - Y_{33}s'_{23} & \simeq & -Y_{32} \end{cases} \quad (60)$$

Due to the hierarchical structure of the Yukawa matrices, it is easy to solve these equations for a given Y at leading order. The rotation angles in the (1,2) sector have more complicated expressions, involving the rotation angles of the two other sectors. However, when $s_{13} \leq \mathcal{O}(Y_{13})$ and $s_{23} \leq \mathcal{O}(Y_{23})$ (this is the case for most phenomenologically interesting Yukawa matrices), the expressions of s_{12} and s'_{12} reduce to the simple form:

$$s_{12} \sim \frac{Y_{11}Y_{21} + Y_{12}Y_{22}}{Y^2_{22} - Y^2_{11} + Y^2_{21} - Y^2_{12}} \quad (61)$$

$$s'_{12} \sim \frac{Y_{11}Y_{12} + Y_{21}Y_{22}}{Y^2_{22} - Y^2_{11} + Y^2_{12} - Y^2_{21}} \quad (62)$$

Since our motivation for introducing an additional $U(1)$ symmetry with a chiral singlet scalar is to explain the observed hierarchies of fermion masses and mixings, we must check that this class of models actually generates phenomenologically viable Yukawa matrices. We will restrict ourselves here to the quark

sector, which is much more constrained than the lepton sector. We assume that the scale M is the same in both charge sectors ($M_U = M_D = M$). In order to reproduce the experimental value for the Cabibbo angle, we also assume $< \theta > /M \simeq \lambda$. Using the result on the hierarchy of mass eigenvalues (52) and the equations (60) and (61) for the mixing angles, one can search systematically for all quark Yukawa matrices ($\hat{Y}^U, \hat{Y}^D$) reproducing the measured quark masses and mixing angles. They turn out to be very few. In fact, the number of phenomenologically viable Yukawa matrices is considerably reduced by the requirement that they originate from a broken abelian symmetry with a chiral singlet. Indeed, the excess charges n_{ij} then satisfy the relations

$$n_{ij} + n_{kl} = n_{il} + n_{kj} \tag{63}$$

which are valid for both the charge -1/3 and +2/3 sectors, and

$$n_{13}^U - n_{33}^U = n_{13}^D - n_{33}^D \qquad n_{23}^U - n_{33}^U = n_{23}^D - n_{33}^D \tag{64}$$

which relate the excess charges of the two charge sectors. In addition, the number of negative n_{ij} is restricted by the condition $\det \hat{Y} \neq 0$.

In practice, we only found two sets of quark Yukawa matrices ($\hat{Y}^U, \hat{Y}^D$) reproducing the measured quark masses and mixing angles. In the first one, $\hat{Y}^U$ and $\hat{Y}^D$ have no supersymmetric zeroes (all excess charges are positive) and are of the form proposed by Froggatt and Nielsen($a_3 = c_3 = 1/2, b_3 = 3/2; a_8 = 5/6, b_8 = 7/6, c_8 = 1/6$):

$$n^U = \begin{pmatrix} 8 & 5 & 3 \\ 7 & 4 & 2 \\ 5 & 2 & 0 \end{pmatrix} \qquad n^D = \begin{pmatrix} 4 & 3 & 3 \\ 3 & 2 & 2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\hat{Y}^U \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \qquad \hat{Y}^D \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \tag{65}$$

In the second one, both $\hat{Y}^U$ and $\hat{Y}^D$ have two supersymmetric zeroes, which are filled in the way described in Subsection 2.1:

$$n^U = \begin{pmatrix} 8 & -1 & -3 \\ 13 & 4 & 2 \\ 11 & 2 & 0 \end{pmatrix} \qquad n^D = \begin{pmatrix} 4 & -3 & -3 \\ 9 & 2 & 2 \\ 7 & 0 & 0 \end{pmatrix}$$

$$\hat{Y}^U \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^{13} & \lambda^4 & \lambda^2 \\ \lambda^{11} & \lambda^2 & 1 \end{pmatrix} \qquad \hat{Y}^D \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^9 & \lambda^2 & \lambda^2 \\ \lambda^7 & 1 & 1 \end{pmatrix} \tag{66}$$

Both sets of quark Yukawa matrices (65) and (66), together with any phenomenologically acceptable lepton Yukawa matrix, can be generated from an anomalous $U(1)_X$ with its anomalies compensated for à la Green-Schwarz.

As written above, both (65) and (66) verify $n_{33}^U = n_{33}^D = 0$, which implies that the Yukawa couplings of the top and the bottom quarks are of the same order at high energy: $\hat{Y}_{33}^U \sim \hat{Y}_{33}^D \sim 1$. Now if we translate the down quark excess charges by a positive integer x:

$$n_{ij}'^D = n_{ij}^D + x \tag{67}$$

the down quark Yukawa matrix is simply modified by a factor λ^x, keeping the same hierarchical structure:

$$\hat{Y}'^D = \lambda^x \hat{Y}^D \tag{68}$$

However, the presence of supersymmetric zeroes in (66) spoils this relation for $x > 2$, so we can safely translate the n_{ij}^D only by $x = 1$ or 2. Since $\hat{Y}'^D$ and $\hat{Y}^D$ have the same eigenvalues and rotation angles, $(\hat{Y}^U, \hat{Y}'^D)$ is still a phenomenologically viable set of quark Yukawa matrices, with $\hat{Y}_{33}^U \sim 1$ and $\hat{Y}_{33}'^D \sim \lambda^x$ at high energy. As suggested by Jain and Shrock [22], this can explain the low-energy hierarchy between the top and bottom quark masses in a natural way, without requiring a large $\tan\beta$. On the contrary, the high-energy relation $\hat{Y}_{33}^U \sim \hat{Y}_{33}^D \sim 1$ is compatible with the low-energy top-bottom hierarchy only for large values of $\tan\beta$ ($\tan\beta \sim m_t/m_b$) [21].

2.5 Mass scales

The fact that the horizontal symmetry that we consider is anomalous has important consequences on the scale at which we might expect its breaking.

Indeed, as a result of suming over the masless states, there is a tadpole "anomalous" contribution to the D-term of the $U(1)_X$ anomalous symmetry. The complete D-term reads [23]

$$D_X = \frac{g^3 M_{Pl}^2}{192\pi^2} C_g + g \sum_i \phi_i \Phi_i^\dagger \Phi_i \tag{69}$$

where g is the string coupling constant and ϕ_i is the X-charge of the scalar field Φ_i (the tadpole term could alternatively be written in terms of $M_{string} = g M_{Pl}$).

This provides a natural scale for the breaking of the anomalous $U(1)_X$ through a non-zero vacuum expectation value of our θ field of X-charge -1 given directly in terms of the anomaly coefficient:

$$\frac{<\theta^\dagger\theta>}{M_{Pl}^2} = \frac{g^2}{192\pi^2} C_g. \tag{70}$$

Thus, if C_g is not too large, the anomalous $U(1)$ symmetry is broken one or two orders of magnitude below the string scale. This provides us with an expansion parameter

$$\epsilon = \frac{|<\theta>|}{M_{Pl}} \tag{71}$$

which is naturally small and not too small – both properties are welcome if one wants to relate this parameter with the Cabibbo angle.

3   The neutrino sector.

In this section, we consider generalisations of the Minimal Supersymmetric Standard Model spectrum which include right-handed neutrinos, thus allowing for non-zero neutrino masses and mixings. We study how the horizontal abelian symmetry discussed above constrains the neutrino spectrum [24, 25, 26]. For simplicity, we will assume only one right-handed neutrino per family.

Suppose that we have three such fields, $\overline{N}_i$, each carrying X-charge. The superpotential now contains the new interaction terms

$$L_i \overline{N}_j H_u \left(\frac{\theta}{M_\nu} \right)^{p_{ij}} + M_0 \overline{N}_i \overline{N}_j \left(\frac{\theta}{M_0} \right)^{q_{ij}} , \tag{72}$$

multiplied by couplings of order one. The first term is a Dirac mass term whereas the second one is a Majorana mass term and involves the scale M_0 which is some mass of the order of the GUT scale or the string scale. In a standard E_6 description, the fields $\overline{N}_i$ may be found among the $SO(10)$ singlets or among the $SU(5)$ singlets in the **16** of $SO(10)$, in which case they are part of a doublet under a right-handed $SU(2)_R$.

We will assume here that the excess charges p_{ij} and q_{ij} are all positive and that q_{33} (resp. p_{33}) is the smallest of the q_{ij} (resp. p_{ij}) charges: $p_{ij} \geq p_{33} \geq 0$, $q_{ij} \geq q_{33} \geq 0$. In other words, the 3-3 entry of the heavy and light neutrino mass matrices are dominant. We denote the X-charges of the right-handed neutrinos by f_0, f_3, f_8.

For three families, the 6×6 Majorana mass matrix is of the form

$$\left(\begin{array}{cc} 0 & \mathcal{M} \\ \mathcal{M}^T & \mathcal{M}_0 \end{array} \right) \tag{73}$$

In the above $\mathcal{M}$ is the $\Delta I_w = 1/2$ mass matrix with entries not larger than the electroweak breaking scale, and $\mathcal{M}_0$ is the unrestricted $\Delta I_w = 0$ mass matrix. Assuming that the order of magnitude of the $\Delta I_w = 0$ masses is much larger than the electroweak scale, we obtain the generalized "see-saw" mechanism.

The calculation of the light neutrino masses and mixing angles proceeds in two steps. Let U_0 be the unitary matrix which diagonalizes the heavy neutrino mass matrix $\mathcal{M}_0$, that is

$$\mathcal{M}_0 = U_0 D_0 U_0^T , \tag{74}$$

where D_0 is diagonal. The orders of magnitude of this matrix are, using the

invariance of the Yukawa couplings (72) under $U(1)_X$,

$$\mathcal{M}_0 = M_0 \, \mathcal{O} \begin{pmatrix} \epsilon_0^{2(f_0+f_3+f_8)} & \epsilon_0^{2(f_0+f_8)} & \epsilon_0^{2f_0+f_3-f_8} \\ \epsilon_0^{2(f_0+f_8)} & \epsilon_0^{2(f_0-f_3+f_8)} & \epsilon_0^{2f_0-f_3-f_8} \\ \epsilon_0^{2f_0+f_3-f_8} & \epsilon_0^{2f_0-f_3-f_8} & \epsilon_0^{2(f_0-2f_8)} \end{pmatrix} . \quad (75)$$

where $\epsilon_0 = <\theta>/M_0$. Its diagonalization yields the three eigenvalues

$$M_1 = M_0 \mathcal{O}(\epsilon_0^{2(f_0+f_3+f_8)}), \quad M_2 = M_0 \mathcal{O}(\epsilon_0^{2(f_0-f_3+f_8)}), \quad M_3 = M_0 \mathcal{O}(\epsilon_0^{2(f_0-2f_8)}) . \quad (76)$$

Under our assumptions the charges satisfy the inequalities

$$f_0 \geq 2f_8 , \qquad 3f_8 \geq |f_3| , \quad (77)$$

which allows to use immediately the results of section 2.4. The diagonalizing matrix is

$$U_0 = \mathcal{O} \begin{pmatrix} 1 & \epsilon_0^{2|f_3|} & \epsilon_0^{3f_8+f_3} \\ \epsilon_0^{2|f_3|} & 1 & \epsilon_0^{3f_8-f_3} \\ \epsilon_0^{3f_8+f_3} & \epsilon_0^{3f_8-f_3} & 1 \end{pmatrix} . \quad (78)$$

and the inverse mass matrix reads

$$\mathcal{M}_0^{-1} = U_0^* D_0^{-1} (U_0^*)^T = \frac{1}{M_1} \mathcal{O} \begin{pmatrix} 1 & \epsilon_0^{2f_3} & \epsilon_0^{3f_8+f_3} \\ \epsilon_0^{2f_3} & \epsilon_0^{4f_3} & \epsilon_0^{3f_8+3f_3} \\ \epsilon_0^{3f_8+f_3} & \epsilon_0^{3f_8+3f_3} & \epsilon_0^{2(3f_8+f_3)} \end{pmatrix} . \quad (79)$$

which is thus obtained from $\mathcal{M}_0$ simply by replacing m_0 and ϵ_0 by their respective inverses. Then in the "see-saw" limit, the 3×3 mass matrix for the light neutrinos reads

$$\hat{Y}_\nu = -\mathcal{M} \mathcal{M}_0^{-1} \mathcal{M}^T = -(\mathcal{M} U_0^*) D_0^{-1} (\mathcal{M} U_0^*)^T. \quad (80)$$

The electroweak breaking mass term yields the matrix

$$\mathcal{M} = m \, \epsilon_\nu^{p_{33}} \, \mathcal{O} \begin{pmatrix} \epsilon_\nu^{3(d_8+f_8)+d_3+f_3} & \epsilon_\nu^{3(d_8+f_8)+d_3-f_3} & \epsilon_\nu^{3d_8+d_3} \\ \epsilon_\nu^{3(d_8+f_8)-d_3+f_3} & \epsilon_\nu^{3(d_8+f_8)-d_3-f_3} & \epsilon_\nu^{3d_8-d_3} \\ \epsilon_\nu^{3f_8+f_3} & \epsilon_\nu^{3f_8-f_3} & 1 \end{pmatrix} , \quad (81)$$

where $\epsilon_\nu = <\theta>/M_\nu$, and m is a mass of electroweak breaking size. We write $\epsilon_0 = \epsilon_\nu^z$, with $z > 0$. We find that

$$\hat{Y}_\nu = \frac{\hat{m}^2}{M_3} \, \mathcal{O} \begin{pmatrix} \epsilon_\nu^{6d_8+2d_3} & \epsilon_\nu^{6d_8} & \epsilon_\nu^{3d_8+d_3} \\ \epsilon_\nu^{6d_8} & \epsilon_\nu^{6d_8-2d_3} & \epsilon_\nu^{3d_8-d_3} \\ \epsilon_\nu^{3d_8+d_3} & \epsilon_\nu^{3d_8-d_3} & 1 \end{pmatrix} , \quad (82)$$

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where

$$\hat{m} = m\, \epsilon_\nu^{p33} \quad \text{if } z \leq 1,$$
$$\hat{m} = m\, \epsilon_\nu^{p33}\, \mathcal{O}(\epsilon_\nu^{(1-z)(3f_8+|f_3|)}) \quad \text{if } z \geq 1, \tag{83}$$

is the matrix whose eigenvalues yield the light neutrino masses and their mixing angles. It is diagonalized by the unitary matrix U_ν:

$$\hat{Y}_\nu = U_\nu D_\nu U_\nu^T, \tag{84}$$

in much the same way as the heavy neutrino mass matrix $\mathcal{M}_0$. Assuming again $3d_8 > |d_3|$, one finds

$$U_\nu = O \begin{pmatrix} 1 & \epsilon_\nu^{2|d_3|} & \epsilon_\nu^{3d_8+d_3} \\ \epsilon_\nu^{2|d_3|} & 1 & \epsilon_\nu^{3d_8-d_3} \\ \epsilon_\nu^{3d_8+d_3} & \epsilon_\nu^{3d_8-d_3} & 1 \end{pmatrix} . \tag{85}$$

The light neutrino masses are then

$$m_{\nu_1} = \frac{\hat{m}^2}{M_3} \mathcal{O}(\epsilon_\nu^{2(3d_8+d_3)}) ,$$
$$m_{\nu_2} = \frac{\hat{m}^2}{M_3} \mathcal{O}(\epsilon_\nu^{2(3d_8-d_3)}) , \tag{86}$$
$$m_{\nu_3} = \frac{\hat{m}^2}{M_3} .$$

In order to obtain the mixing matrix which appears in the charged lepton current, we must fold this matrix with that which diagonalizes the charged lepton masses. If we let $\epsilon_\nu = \epsilon_e^w$, with $w > 1$, the result is

$$V = O \begin{pmatrix} 1 & \epsilon_e^{2|d_3|} & \epsilon_e^{3d_8+d_3} \\ \epsilon_e^{2|d_3|} & 1 & \epsilon_e^{3d_8-d_3} \\ \epsilon_e^{3d_8+d_3} & \epsilon_e^{3d_8-d_3} & 1 \end{pmatrix} . \tag{87}$$

When $0 < w < 1$, the matrix has the same form with ϵ_e replaced by ϵ_ν. It is similar to the CKM matrix. We note that its elements satisfy

$$V_{e\nu_\mu} V_{\mu\nu_\tau} \sim V_{e\nu_\tau} . \tag{88}$$

Unlike quark masses and mixing, we have little solid experimental information on the values of these parameters. The most compelling evidence for neutrino masses and mixings comes from the MSW interpretation of the deficit observed in various solar neutrino fluxes. In this picture, the electron neutrino mixes with another neutrino (assumed here to be the muon neutrino) with a mixing angle θ_{12} such that

$$|m_{\nu_1}^2 - m_{\nu_2}^2| \sim 7 \times 10^{-6} \text{ eV}^2 ; \quad \sin^2 2\theta_{12} \sim 5 \times 10^{-3} . \tag{89}$$

The other piece of evidence comes from the deficit of muon neutrinos in the collision of cosmic rays with the atmosphere. If taken at face value, these suggest that the muon neutrinos oscillate into another species of neutrinos, say τ neutrinos, with a mixing angle θ_{23}, and masses such that

$$|m_{\nu_2}^2 - m_{\nu_3}^2| \sim 2 \times 10^{-2} \text{ eV}^2 \; ; \quad \sin^2 2\theta_{23} \geq .5 \; . \tag{90}$$

Fitting the parameters coming from the solar neutrino data is rather easy, suggesting that

$$V_{e\nu_\mu} \sim \epsilon_e^{2d_3} \sim \lambda^2 \; , \tag{91}$$

together with $m_{\nu_2} \approx 10^{-3}$ eV. The atmospheric neutrino data would imply

$$V_{\mu\nu_\tau} \sim \epsilon_e^{3d_8 - d_3} = \mathcal{O}(1) \; . \tag{92}$$

The relation

$$\frac{m_{\nu_2}}{m_{\nu_3}} \approx (V_{\mu\nu_\tau})^{2w} \; , \tag{93}$$

would then imply that $w > 1$. For example the value $\theta_{23} \sim \frac{\pi}{8}$ yields $m_{\nu_2}/m_{\nu_3} \sim .02$, for $w = 2$. Thus we could marginally reproduce the "data". The heaviest neutrino weighs one tenth of an eV, not enough to be of use for structure formation.

Generically, though, it is difficult to understand mixing angles of order one, as suggested by the atmospheric neutrino data. The existence of only small mixing angles in the quark sectors suggests either that the interpretation of the atmospheric neutrino data is premature, or that there is fine tuning in the neutrino matrices.

4 R-parity breaking interactions.

The gauge and Yukawa couplings are not the only interactions allowed by the gauge symmetries and supersymmetry. The following terms, which violate either B or L, can also be present in the superpotential:

$$\Lambda_{ijk} L_i L_j \bar{e}_k + \Lambda'_{ijk} L_i Q_j \bar{d}_k + \Lambda''_{ijk} \bar{d}_i \bar{d}_j \bar{u}_k \tag{94}$$

The two last ones are the most dangerous because they give rise to proton decay, if simultaneously present. In the MSSM, R-parity is assumed in order to forbid them. We consider here the most general case where R-parity may be broken, and we therefore take into account all these terms (the $L_i H_u$ term, which one usually eliminate by a redefinition of H_d, will be discussed later on). The couplings Λ_{ijk}, Λ'_{ijk} and Λ''_{ijk} must then be very small, otherwise they would induce proton decay and lepton number violation at an unacceptable

level. The upper bounds, due to the experimental limits on respectively proton decay, lepton number violation and neutron-antineutron oscillations, are [27]:

$$\sqrt{\Lambda'\Lambda''} \leq \mathcal{O}\left[\left(\frac{M_{Susy}}{1TeV}\right)10^{-13}\right] \tag{95}$$

$$\Lambda \leq \mathcal{O}\left[\left(\frac{M_{Susy}}{1TeV}\right)10^{-3}\right] \tag{96}$$

$$\Lambda'' \leq \mathcal{O}\left[\left(\frac{M_{Susy}}{1TeV}\right)^{5/2}10^{-5}\right] \tag{97}$$

Note that the most stringent constraint comes from proton decay (95). It is satisfied if one of the two terms $LQ\bar{d}$ or $\bar{d}\bar{d}\bar{u}$ is highly suppressed, or if both are. Another possibility is that either $LQ\bar{d}$ or $\bar{d}\bar{d}\bar{u}$ do not appear in the superpotential. In the following, we shall look at both possibilities.

The horizontal symmetry $U(1)_X$ discussed in the previous sections naturally generates small couplings [28]. Let us consider, for example, the $L_iQ_j\bar{d}_k$ term. It carries the excess charge $x_{ijk} = l_i + q_j + d_k$. If $x_{ijk} > 0$, $L_iQ_j\bar{d}_k$ will be generated from the non-renormalizable interaction:

$$a_{ijk}L_iQ_j\bar{d}_k\left(\frac{\theta}{M}\right)^{x_{ijk}} \tag{98}$$

where a_{ijk} is a factor of order one. The effective Λ_{ijk} coupling will then be of order $(<\theta>/M)^{x_{ijk}}$. If $x_{ijk} < 0$, the $L_iQ_j\bar{d}_k$ term will not appear in the superpotential. But, in the same way as the Yukawa couplings whose excess charges are negative (see subsection 2.1), it can be induced by non-renormalizable contributions to the kinetic terms. The effective Λ_{ijk} coupling is then:

$$\Lambda_{ijk} = \sum_{l,m,n} H(x_{lmn})\Lambda_{ijk;lmn} \tag{99}$$

where $\Lambda_{ijk;lmn}$ is the contribution of the nonzero $L_lQ_m\bar{d}_n$ term to the $L_iQ_j\bar{d}_k$ term. It is given by:

$$\Lambda_{ijk;lmn} \sim \left(\frac{<\theta>}{M}\right)^{|l_l-l_i|+|q_m-q_j|+|d_n-d_k|+x_{lmn}} \tag{100}$$

One deduces from (100) that Λ_{ijk} is at most of the order of magnitude that would be obtained with a vectorlike pair of θ fields:

$$\Lambda_{ijk} \leq \mathcal{O}\left(\left(\frac{<\theta>}{M}\right)^{|x_{ijk}|}\right) \tag{101}$$

If $x_{ijk} \geq 0$, this bound is saturated because $\Lambda_{ijk;ijk} = 1$. Therefore, the diagonalization of the kinetic terms does not affect the order of the couplings which are initially nonzero.

The only difference with the Yukawa couplings is that the number of negative excess charges is not limited by the condition of requiring a nonzero determinant: a single positive x_{ijk} is then sufficient to generate all other R-parity violating terms of the same type. However, this mechanism tends to produce small couplings. For example, in the particular case where there is a single positive excess charge x_{lmn}, one can easily show that the Λ_{ijk} induced by the diagonalization of the kinetic terms are of the order of:

$$\Lambda_{ijk} \sim \left(\frac{<\theta>}{M} \right)^{|x_{ijk}|+2x_{lmn}} \tag{102}$$

while, of course, $\Lambda_{lmn} \sim (<\theta>/M)^{x_{lmn}}$. If x_{lmn} is large enough, this leads to very small couplings. This property holds when there are several positive x_{ijk}, provided that all of them are large compared to unity.

We conclude that, in order to obtain small R-parity violating couplings, we must choose the X-charges of the MSSM fields so that all positive x_{ijk} are large. The number of negative x_{ijk} does not matter; the important point is that the smallest positive excess charge be large. Thus all effective Λ_{ijk} will be small. We require that all of them be very small, because the physical couplings, which enter the proton decay rate, involve mass eigenstates and therefore mix the Λ_{ijk}. This mixing tends to attenuate the hierarchy between R-parity violating couplings of the same type (say $LQ\bar{d}$), in disagreement with what is usually assumed in phenomenological analysis.

In practice, it is not so easy to obtain large positive excess charges for the Λ_{ijk}. Indeed, the family-dependent part of the X-charge is very constrained by the quark phenomenology, and its family-independent part is fixed by the Green-Schwarz compensation of its anomalies. These constraints disfavor large values of the x_{ijk}. The only freedom we have, provided that the neutrinos are massless, is to choose the lepton charges. Unfortunately, they must have very large values, which seems to be rather unnatural.

This is shown by the following example, where Y_U and Y_D have the form proposed by Froggatt and Nielsen [2]. The charge assignment is the following:

Table 1: X-charges of the MSSM fields according to the family index $i = 1, 2, 3$ (first example).

i	q_i	u_i	d_i	l_i	e_i	$h_u = h_d$
1	2/3	22/3	16/3	-12	18	
2	-1/3	13/3	13/3	-13	17	0
3	-7/3	7/3	13/3	55	-53	

The corresponding Yukawa matrices are:

$$Y_U \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \quad Y_D \sim \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$Y_E \sim \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^{67} \\ \lambda^3 & \lambda^2 & \lambda^{68} \\ \lambda^{71} & \lambda^{70} & 1 \end{pmatrix} \tag{103}$$

where $\lambda = (< \theta > /M)$ is assumed to be the Cabibbo angle. The constraints (95) to (97) are widely satisfied by a strong suppression of L violation:

$$\begin{aligned} \Lambda &\leq \mathcal{O}\left(10^{-38}\right) \\ \Lambda' &\leq \mathcal{O}\left(10^{-38}\right) \\ \Lambda'' &\leq \mathcal{O}\left(10^{-7}\right) \end{aligned} \tag{104}$$

but, as stressed above, the lepton charges are large, which gives rise to very small coefficients in the lepton Yukawa matrix. Ben-Hamo and Nir [28] did not encounter this problem because they did not consider the anomalies of $U(1)_X$.

As mentioned above, another possibility for avoiding proton decay is that one of the two dangerous terms $LQ\bar{d}$ and $\bar{d}\bar{d}\bar{u}$ be absent from the superpotential. This happens when all excess charges for this term are negative, because all corresponding couplings are then zero. For example, one can find a large class of $U(1)_X$ models, in which there is no $\bar{d}\bar{d}\bar{u}$ term. These models are interesting, because the experimental constraints then reduce to (96), which is very easy to satisfy. Unfortunately, they also have very large values for the lepton charges.

Our second example belongs to this class of models. The charge assignment is the following:

Table 2: X-charges of the MSSM fields according to the family index $i = 1, 2, 3$ (second example).

i	q_i	u_i	d_i	l_i	e_i	$h_u = h_d$
1	23/3	1/3	-5/3	23	-17	
2	20/3	-8/3	-8/3	22	-18	0
3	14/3	-14/3	-8/3	-78	80	

The corresponding Yukawa matrices are:

$$Y_U \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \quad Y_D \sim \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$Y_E \sim \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^{101} \\ \lambda^3 & \lambda^2 & \lambda^{100} \\ \lambda^{97} & \lambda^{98} & 1 \end{pmatrix} \tag{105}$$

As stressed above, there is no B violation from renormalizable operators, and the remaining constraint (96) is widely satisfied:

$$\begin{aligned} \Lambda &\leq \mathcal{O}\left(10^{-16}\right) \\ \Lambda' &\leq \mathcal{O}\left(10^{-16}\right) \\ \Lambda'' &= 0 \end{aligned} \tag{106}$$

We must also consider the possibility that the x_{ijk} be fractionnary, which is generally the case. The effective couplings are then zero, unless they are due to non-perturbative effects. Indeed, if one of the x_{ijk} is fractionnary, all of them are fractionnary. All Λ_{ijk} are then initially zero, and remain zero after diagonalization of the kinetic terms. This follows from the fact that the excess charges of the Yukawa couplings are integers. Consider now the three terms in (94). One can easily show that the excess charges of the first two terms are simultaneously fractionnary or integers, while the excess charges of the third

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term can be fractionnary or integers independently from the first two terms. We can therefore choose the lepton charges so that only the L violating terms (resp. only the B violating term) are present in the superpotential, which makes proton decay impossible in the absence of higher dimension operators.

So far, we did not consider the higher dimension R-parity violating operators. Two of them give a significant contribution to proton decay [27, 28]:

$$\frac{\kappa_{ijkl}}{M}Q_iQ_jQ_kL_l + \frac{\kappa'_{ijkl}}{M}\bar{u}_i\bar{u}_j\bar{d}_k\bar{e}_l \tag{107}$$

The upper bounds on the κ couplings are:

$$\kappa \leq \mathcal{O}\left[\left(\frac{M_{Susy}}{1TeV}\right)\left(\frac{M}{M_P}\right)10^{-7}\right] \tag{108}$$

$$\kappa'(K^U_{RR})_{1i} \leq \mathcal{O}\left[\left(\frac{M_{Susy}}{1TeV}\right)\left(\frac{M}{M_P}\right)10^{-8}\right] \tag{109}$$

where K^U_{RR} is the quark-squark mixing matrix for the right-handed up quarks. When there is no mixing (no FCNC), $K^U_{RR} \equiv 0$ and there is no constraint over κ'. These constraints are easily satisfied as soon as the lepton charges are large. This is the case as well in the first example (103):

$$\kappa \leq \mathcal{O}\left(10^{-32}\right)$$
$$\kappa' \leq \mathcal{O}\left(10^{-17}\right) \tag{110}$$

as in the second one (105):

$$\kappa \leq \mathcal{O}\left(10^{-24}\right)$$
$$\kappa' \leq \mathcal{O}\left(10^{-45}\right) \tag{111}$$

In the previous discussion, we did not mention the LH_u term, which should be present in the superpotential, in addition to the three terms of (94). One usually eliminate it by a redefinition of H_d. Starting with the following quadratic part of the superpotential:

$$\mu H_u H_d + \alpha_i L_i H_u \tag{112}$$

and redefining $H'_d = H_d + \sum_i(\alpha_i/\mu)L_i$, one ends up with a single quadratic term, $\mu H_u H'_d$. It is important to note that, in our model, this can be done only after the breaking of $U(1)_X$, because the H_d and L_i superfields carry different X-charges. Now the redefinition of H_d also modifies the Yukawa terms of the down quarks:

$$\lambda^D_{jk}H_dQ_j\bar{d}_k \quad \rightarrow \quad \lambda^D_{jk}H'_dQ_j\bar{d}_k - \left(\frac{\alpha_i}{\mu}\right)\lambda^D_{jk}L_iQ_j\bar{d}_k \tag{113}$$

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which gives a new contribution to the $L_i Q_j \bar{d}_k$ term (in a similar way, $L_i L_j \bar{e}_k$ receives a contribution from $H_d L_j \bar{e}_k$). The effective Λ_{ijk} is then modified as follows:

$$\Lambda_{ijk} \quad \rightarrow \quad \Lambda_{ijk} + \left(\frac{\alpha_i}{\mu}\right) \lambda_{jk}^D \tag{114}$$

Thus the $L_i H_u$ term, if present, contributes to the L violating couplings, and we must take it into account in our analysis. Note that, since the α_i are generated in the same way as the Λ_{ijk}, they are zero as soon as the excess charges $(l_i + h_u)$ are fractionnary or all negative. In this case, the LH_u term does not appear in the superpotential. Otherwise, it may give their dominant contribution to the Λ_{ij} . In particular, when the excess charges of the $LL\bar{e}$ and $LQ\bar{d}$ terms are fractionnary, only LH_u contributes to the L violating couplings.

We can distinguish between two cases:

1. if $LL\bar{e}$ and $LQ\bar{d}$ have fractionnary excess charges, the L violating couplings Λ and Λ' are generated from the LH_u term. However, when $h_u \in Z$, LH_u is absent, and there is no L violation from renormalizable operators.

2. if $LL\bar{e}$ and $LQ\bar{d}$ have integer excess charges, the LH_u term is present only if $h_u \in Z$. When $h_u = 0$ however, its contribution does not modify the order of magnitude of the L violating couplings (this is the case in both examples given).

5 Conclusions

Trying to explain fermion mass hierarchies and mixings by an *ad hoc* local abelian gauge symmetry might seem, at first glance, an honest but somewhat groundless attempt. Surprisingly, this leads to a very special type of abelian symmetry, namely the anomalous $U(1)$ whose anomalies may be cancelled by the Green-Schwarz mechanism. This leaves some hope that, in the context of string models, one may be able to make definite statements about mass hierarchies. Indeed, because of the uniqueness of the dilaton field, such a $U(1)$ symmetry is unique and plays a central rôle. One may therefore relate the charges of the matter fields under this $U(1)$ to central properties of the model. Such a $U(1)$ has already been advocated [29] to explain why a nonvanishing top Yukawa coupling may appear at string tree level. Its properties may also allow to relate the horizontal symmetry approach to the modular symmetries of the underlying string theory [30].

Surprisingly little information from the anomaly structure of this symmetry is used to derive the Weinberg angle – or, in a correlated way, the order of magnitude of the μ term in a certain class of models –. One may expect that the rest of the information, in particular the mixed gravitational anomaly which

plays a rôle in fixing the scale at which this symmetry breaks, can be used to constrain further the models [31].

We have also studied two types of extended supersymmetric standard models – massive neutrinos and R-parity breaking interactions –, where this approach proves to be (mildly) constraining. It is for instance interesting to see that, when trying to implement in this framework a generalized seesaw mechanism for neutrinos, one ends up with a light neutrino mass spectrum which cannot satisfy at the same time the cosmological and atmospheric neutrino constraints.

Acknowledgements

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