

ELECTROMAGNETIC MODELING OF SUBSURFACE 3D STRUCTURES

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ABSTRACT

A 3D frequency domain electromagnetic numerical solution has been implemented for sensing buried structures in a lossy earth. Because some structures contain metal, it is necessary to treat them as very good conductors residing in a complicated lossy earth background. To model these scenarios and to avoid excessive gridding in the numerical solution, we assume the structures to be perfectly conducting, which forces the total electric field to zero within the conductor. This is accomplished by enforcing internal boundary conditions on the numerical grid.

The numerical solution is based on a vector Helmholtz equation for the scattered electric fields, which is approximated using finite differences on a staggered grid. After finite differencing, a complex-symmetric matrix system of equations is assembled and preconditioned using Jacobi scaling before it is iteratively solved using the quasi-minimum residual (qmr) or bi-conjugate gradient (bicg) methods. For frequencies approaching the static limit (< 10 kHz), the scheme incorporates a static-divergence correction to accelerate solution convergence. This is accomplished by enforcing the divergence of the scattering current within the earth as well as the divergence of the scattered electric field in the air.

INTRODUCTION

An important application of electromagnetic (EM) sensing involves detecting buried manmade structures. Because these structures often include metal in their fabrication as well as electrical wiring, they can sometimes be modeled as highly conducting bodies and wires in a lossy earth. Sheet models have been useful to model such cases, [1], but are limited in their complexity and the complexity of the earth background that can be incorporated in the model. To overcome these limitations, finite element and difference solutions to the EM field can be implemented. However because of the highly conductive nature of the above mentioned structures it appears that excessive gridding is needed. Still, this problem can be avoided if we treat the structures as perfectly conducting, and in effect replace them with an internal boundary condition that enforces zero electric field.

In this paper we demonstrate how this idea can be implemented using an iterative solution to the vector Helmholtz equation based on finite difference approximations. In addition we will show how this solution can be accelerated at low frequencies by incorporating a

static-divergence correction which enforces auxiliary divergence conditions on the fields. Fast low frequency simulations are important because some targets can be buried at depths where lower frequencies are required for their detection.

THEORETICAL FORMULATION

The Vector Helmholtz Equation

Assuming a time harmonic dependence of $e^{i\omega t}$ where $i = \sqrt{-1}$ the vector Helmholtz equation for the scattered electric field given by [2] is written here as

$$\nabla \times \frac{\mu_p}{\mu} \nabla \times \mathbf{E}_s + i\omega\mu_p(\sigma + i\omega\epsilon)\mathbf{E}_s = -i\omega\mu_p \left[(\sigma - \sigma_p) + i\omega(\epsilon - \epsilon_p) \right] \mathbf{E}_p - i\omega\mu_p \nabla \times \left[\left(\frac{\mu - \mu_p}{\mu} \right) \mathbf{H}_p \right] \quad (1)$$

In this expression the electrical conductivity, magnetic permeability and dielectric permittivity are denoted by σ , μ and ϵ , respectively, where the subscripts "p" and "s" designates background, or primary values, and scattered values respectively. Given this definition of primary and scattered fields, the total electric and magnetic fields are determined from $\mathbf{E}_t = \mathbf{E}_p + \mathbf{E}_s$ and $\mathbf{H}_t = \mathbf{H}_p + \mathbf{H}_s$. In addition note that $[(\sigma - \sigma_p) + i\omega(\epsilon - \epsilon_p)]\mathbf{E}_p$ and $[(\mu - \mu_p)]\mathbf{H}_p$ can be thought of as equivalent source vectors of the background medium. The primary fields and background values found in these source vectors can be that of a whole space, a layered half-space, or some previously run model for which results were saved to be incorporated as a primary field. For all examples presented here the primary field is assumed to arise from impressed dipole sources in a whole space background. When equation (1) is approximated with finite differences on a staggered grid a sparse linear system results in which the matrix is complex symmetric. This system can be efficiently solved iteratively using Krylov sub-space methods, including the quasi minimum residual (qmr) and bi-conjugate gradient (bicg) solvers. The reader is referred to [2] and [3] for details on how these solvers are implemented.

The Static-Divergence Correction

As has been demonstrated by [4], a staggered grid is a natural grid to use in the numerical solution since it implicitly enforces the auxiliary divergence conditions on the

fields. With a scattered electric field formulation the critical condition is

$$\nabla \cdot \{(\sigma + i\omega\epsilon)\mathbf{E}_s\} = -\nabla \cdot \left\{ \left[(\sigma - \sigma_p) + i\omega(\epsilon - \epsilon_p) \right] \mathbf{E}_p \right\}. \quad (2)$$

Even with the benefits of a staggered grid we have observed that the time needed to solve for the fields greatly increase with falling frequency as one approaches the static limit. The reason can be traced to the fact that the numerical solution poorly approximates equation (2). To overcome this difficulty a correction can be implemented following a procedure developed by [4] for a total field formulation. Here we will modify the procedure for the scattered fields such that equation (2) is enforced periodically during the iterative solution. To accomplish this let us define an *error* where,

$$\begin{aligned} \text{error} = & -\nabla \cdot \{(\sigma + i\omega\epsilon)\mathbf{E}_s\} \\ & -\nabla \cdot \left\{ \left[(\sigma - \sigma_p) + i\omega(\epsilon - \epsilon_p) \right] \mathbf{E}_p \right\}. \end{aligned} \quad (3)$$

Ideally the *error* should be zero, but because of precision problems it is not, particularly as frequency approaches the static limit. Next we define a scalar potential, ϕ , such that

$$\text{error} = \nabla \cdot \{(\sigma + i\omega\epsilon)\nabla\phi\}. \quad (4)$$

Difficulties still arise with this equation when air is present in the model because it cannot support electric currents in the static limit. To overcome this difficulty, we redefine equation (4) in the air to read

$$\text{error}' = \nabla \cdot \nabla\phi. \quad (5)$$

When the sources are buried in the earth we need to enforce the condition that total electric field is divergence free in the air, thus

$$\text{error}' = -\nabla \cdot \mathbf{E}_s - \nabla \cdot \mathbf{E}_p. \quad (6)$$

On the other hand if the sources are in the air we force the scattered electric field to be divergence free, hence

$$\text{error}' = -\nabla \cdot \mathbf{E}_s. \quad (7)$$

To enforce equation (2) and the divergence conditions on the electric field in the air we solve equations (4) and (5) for the potential on the staggered grid using a finite difference approximation. The corrected version of the scattered electric field is then given by

$$\mathbf{E}_s' = \mathbf{E}_s + \nabla\phi. \quad (8)$$

Solution to equations (4) and (5) take much less time than solving equation (1) does. To obtain the potential solution we assemble the scalar equations into a single system, precondition it with simple Jacobi scaling and solve it using a bi-conjugate gradient algorithm for non-symmetric matrices. This correction procedure is alternated with a series of qmr or bicg iterations on the vector Helmholtz equation. At each correction step the scalar equations only need to be solved very approximately to dramatically improve the convergence of the Helmholtz equation at low frequency as in Figure 1.

The number of sub iterations operating on the Helmholtz system is critical for effective acceleration of the solution when using the static-divergence correction. If too few sub iterations are specified there is a possibility that the solution will never converge to an acceptable error level. To overcome this difficulty an adaptive scheme has been implemented, which will continue to double the number of sub iterations if the error has not been reduced by a factor of two from the previous set of sub iterations. This procedure is continued until convergence to a given error level is achieved or a maximum number of iterations (including all sub iterations) has been attained.

Simulations Involving Very Good Conductors

To model very good conductors and to avoid excessive gridding in the numerical solution, we assume the structures to be perfectly conducting. Large-scale good conductors can be treated with this approach as well as wire conductors since wires can easily be accommodated in a staggered grid.

With a perfectly conducting boundary condition, the total electric field is forced to zero within the conductor as well as on its surface. Instead of explicitly enforcing this condition, we have designed it implicitly on internal portions of the grid due to ease of implementation. At or inside a very good conducting region, equation (1) can be written as

$$\nabla \times \frac{\mu_p}{\mu} \nabla \times \mathbf{E}_s + i\omega\mu_p\sigma_{large}\mathbf{E}_s = -i\omega\mu_p\sigma_{large}\mathbf{E}_p \quad (9)$$

where conductivity property dominates over the dielectric and magnetic permeability properties of the medium. Because the medium is highly conducting the following inequality is also true;

$$\left| \nabla \times \frac{\mu_p}{\mu} \nabla \times \mathbf{E}_s \right| \ll \left| \omega\mu_p\sigma_{large}\mathbf{E}_s \right|. \quad (10)$$

Considering equation (10) with equation (9) numerically yields

$$(\sigma_{large})E_s = -(\sigma_{large})E_p \quad (11)$$

or effectively $E_s = -E_p$. Thus $E_t = 0$ which is the condition required in and on a perfect conductor.

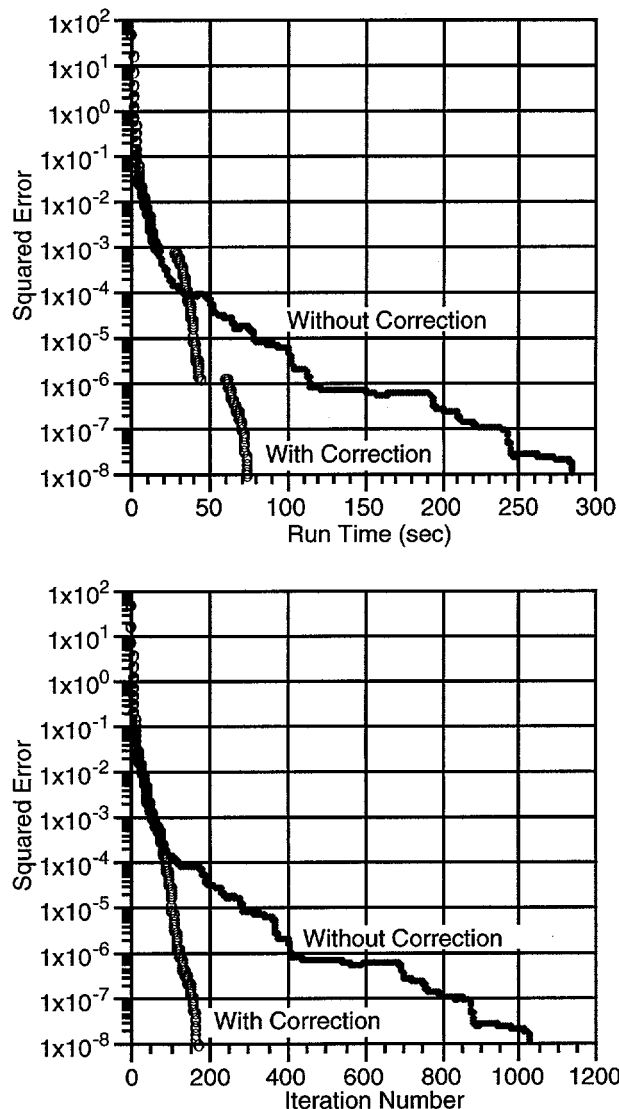


Fig. 1. Comparison between solution run times and number of iterations needed to achieve convergence with and without the static-divergence accelerator. The model consists of 96 338 cells and simulates a 100 Ω m half space with a vertical dipole magnetic source positioned 20 m above the surface and operating at a frequency of 90 Hz. The numerical solution is assumed to have converged when a squared error level of 10^{-8} is attained. Time gaps in the corrected version of the solution correspond to the times needed to make the static-divergence correction. Sixty sub iterations were used with the corrected version.

Through numerical experiments we have determined that σ_{large} assigned a numerical value of about 10^{+8} appears to work best. Larger values introduce numerical instability. Figure 2 shows an excellent check on the technique for a perfectly conducting layer. In Figure 3 we show a check against a highly conducting sheet model, and although the comparison is not as good as that of the layered model, the shapes of the curves agree quite well. We believe that discrepancy between the two solutions occurs due to discretization problems with the sheet model. The reasons for this belief are 1) with increasing discretization of the sheet the comparison continues to improve, and 2) the excellent check against the layered model. Note, we could not discretize the sheet model beyond 1600 cells due to computer memory limitations.

While this technique works well for detecting buried structures from the surface or from the air it is not ideal for computing fields in the shadow zones of very good conductors. This limitation is caused by subtractive cancellation between the scattered and primary fields. For problems of this sort a total field solution would appear to work best. We are now in the progress of implementing such a solution which could be of great use in mapping the lateral extent of highly conducting ore bodies with crosswell configurations.

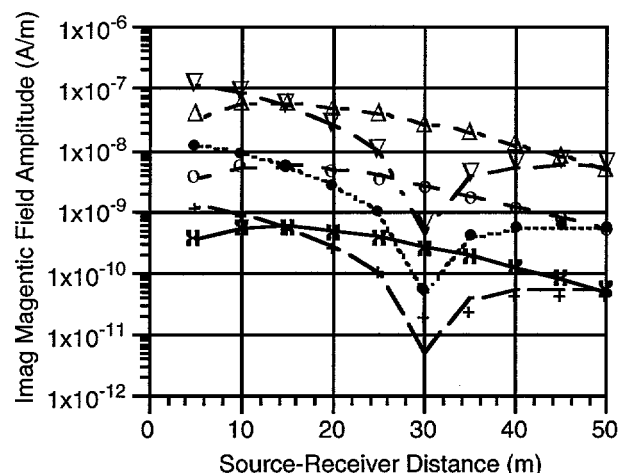
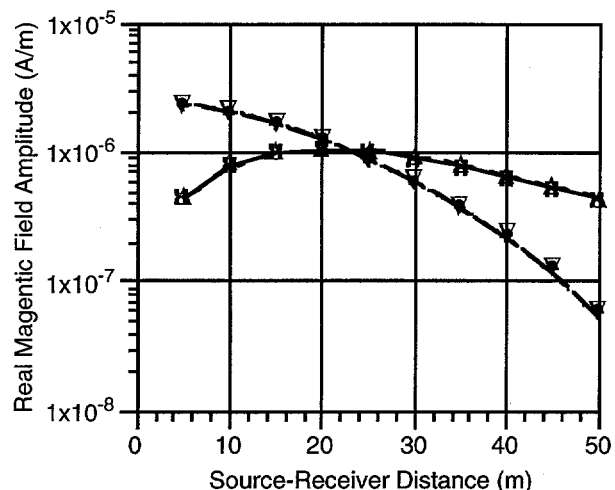
CONCLUDING REMARKS

We have demonstrated several extensions to the solution published by [2] on 3D wideband EM modeling. The inclusion of a static divergence condition has been demonstrated to cause significant speed up in solution run times at low frequencies, and a novel technique, which is implicit in nature yet simple to implement, for simulating responses of perfectly conducting bodies has been demonstrated.

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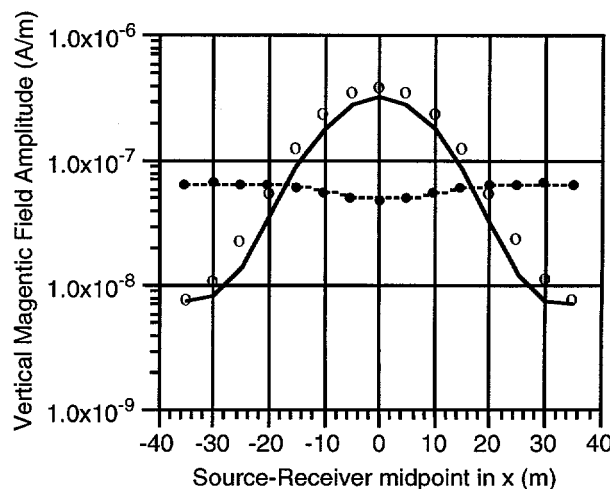
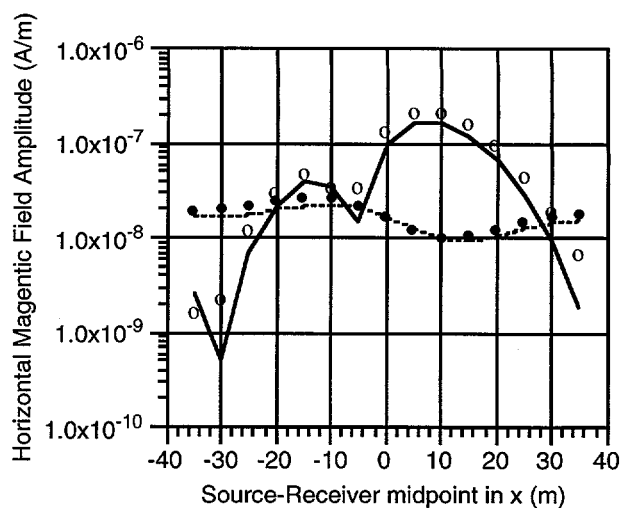
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Fig. 2. Check on the perfectly conducting boundary condition implemented implicitly on the internal portions of the finite difference mesh. The model consists of a 10 m thick perfectly conducting layer buried 10 m deep in a 100 Ωm half-space. Checks we carried out for three different frequencies 100, 1 000 and 10 000 Hz, where a vertical magnetic dipole transmitter is deployed 10 m above the surface and magnetic fields computed horizontally away from the source point. Symbols depict the 3D solution with the free space response removed, which uses 381 217 cells and lines the results from a semi-analytic 1D Hankel transform solution of Ki Ha Lee.



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Fig. 3. Check of the perfectly conducting boundary condition against the sheet model of Weidelt [1]. The model consists of a 20 m by 40m sheet buried at 15 m depth in a 100 Ωm half-space. The conductance of the sheet is 1x10¹⁰ S and a frequency of 1 000 Hz is employed. The calculations simulate measurements of the vertical and horizontal magnetic fields at a distance of 10 from a VMD source. Fifteen source positions were employed in the x direction from -40m to +30m. Symbols depict the 3D solution with the free space response removed, which uses 381 217 cells and lines the sheet results. At x=0 the discrepancy between the real parts of the two solutions is about 24%.

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