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This paper presents an unsteady flow theory for flow-induced vibration and instability of tube arrays in crossflow. It includes measurements of motion-dependent fluid forces, mathematical model, and experiments on nonlinear response of tube arrays. The unsteady flow theory can be used to provide answers to complex vibration problems in steam generators.

INTRODUCTION

The components of many heat exchangers and steam generators comprise a group of tubes submerged in crossflow. Flow-induced vibration and instability of these components continue to be a problem (Chen 1987, Paidoussis 1987, Axisa 1993). Various mathematical models have been developed to predict flow-induced vibration and instability. At this time a number of issues have not been addressed, and several aspects of the problems cannot be resolved. This paper presents a unified theory based on the unsteady flow theory for linear and nonlinear response of tubes in crossflow.

One of the key elements is motion-dependent fluid forces (Lever et al 1982, 1984). A water channel is used to measure fluid forces on all tubes due to the motion of a tube. From the measured fluid forces, fluid damping and stiffness for various tube arrays are obtained. Tests have been performed for a single tube, two tubes, tube rows, and tube arrays (Chen et al 1993, Chen et al 1995, Zhu et al 1995a, 1995b). Once fluid-damping and fluid-stiffness coefficient matrices are know, a mathematical model simulating practical tube arrays can be established and analyzed. The unsteady flow theory based on the measured fluid forces can be used to study the detailed tube motions, including subcritical vibration, instability threshold, and post-instability oscillations. The unsteady flow theory has also been applied to study the nonlinear vibration of tube arrays. Tube displacements were analyzed to characterize the tube behavior. The analytical results and experimental agree reasonably well. This demonstrates the usefulness of the unsteady flow theory (Chen et al 1993, Eisinger et al 1991).

MOTION-DEPENDENT FLUID FORCES

Unsteady Flow Theory

Consider a group of n tubes vibrating in a flow as shown in Fig. 1. The axes of the tubes are parallel to one another and perpendicular to the x-y plane. The radius R of each tube is the same, and the fluid is flowing with a gap flow velocity U. The displacement components of tube j in the x and y directions are u_j and v_j, respectively. The motion-dependent fluid-force components acting on tube j in the x and y directions are, respectively, f_j and g_j, and are given by Chen (1987) as

$$f_j = -\rho\pi R^2 \sum_{k=1}^{n}\left(\alpha_{jk}\frac{\partial^2 u_k}{\partial t^2} + \sigma_{jk}\frac{\partial^2 v_k}{\partial t^2}\right) + \frac{\rho U^2}{\omega}\sum_{j=1}^{n}\left(\alpha'_{jk}\frac{\partial u_j}{\partial t} + \sigma'_{jk}\frac{\partial v_k}{\partial t}\right) + \rho U^2 \sum_{j=1}^{n}\left(\alpha''_{jk}u_k + \sigma''_{jk}v_k\right) \qquad (1)$$

and

$$g_j = -\rho\pi R^2 \sum_{k=1}^{n}\left(\tau_{jk}\frac{\partial^2 u_k}{\partial t^2} + \beta_{jk}\frac{\partial^2 v_k}{\partial t^2}\right) + \frac{\rho U^2}{\omega}\sum_{k=1}^{n}\left(\tau'_{jk}\frac{\partial u_j}{\partial t} + \beta'_{jk}\frac{\partial v_k}{\partial t}\right) + \rho U^2 \sum_{j=1}^{n}\left(\tau''_{jk}u_k + \beta''_{jk}v_k\right), \qquad (2)$$

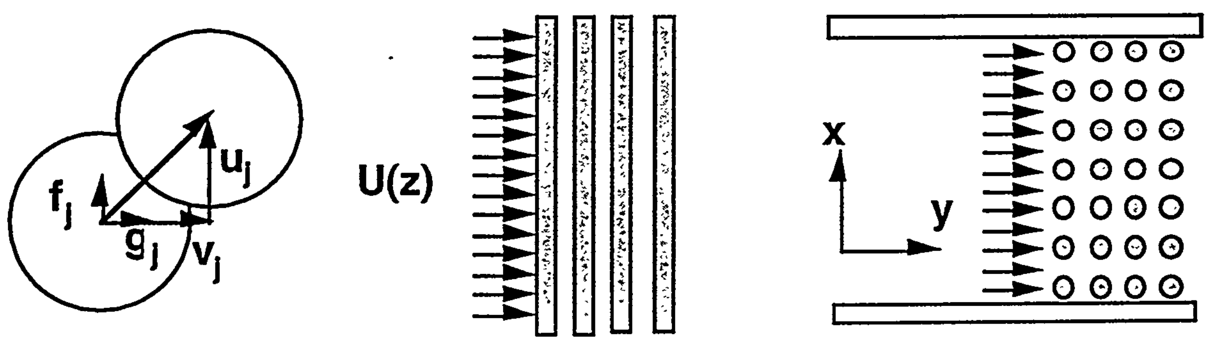

Fig. 1. Tube array in crossflow.

where ρ is fluid density; t is time; ω is circular frequency of tube oscillations; α_{jk}, β_{jk}, σ_{jk}, and τ_{jk} are added mass coefficients; α'_{jk}, β'_{jk}, σ'_{jk}, and τ'_{jk} are fluid-damping coefficients; and α''_{jk}, β''_{jk}, σ''_{jk}, and τ''_{jk} are fluid-stiffness coefficients.

Fluid-force coefficients can be determined by measuring the fluid forces acting on the tubes that are due to oscillations of a particular tube. For example, if tube k is excited in the y direction, its displacement in the y direction is given by

$$v_k = v \cos \omega t. \tag{3}$$

The fluid force acting on tube j in the x direction can be written

$$f_j = \frac{1}{2}\rho U^2 c_{jk} \cos\left(\omega t + \phi_{jk}\right) v, \tag{4}$$

where c_{jk} is the fluid-force amplitude and ϕ_{jk} is the phase angle by which the fluid force acting on tube j leads the displacement of tube k.

With Eqs. 1 and 3, we can also write the fluid-force component as

$$f_j = \left(\rho \pi R^2 \omega^2 \sigma_{jk} + \rho U^2 \sigma''_{jk}\right) v \cos \omega t - \rho U^2 \sigma'_{jk} v \sin \omega t. \tag{5}$$

Combining Eqs. 4 and 5 yields

$$\sigma''_{jk} = \frac{1}{2} c_{jk} \cos \phi_{jk} - \frac{\pi^3}{U_r^2} \sigma_{jk}, \qquad \sigma'_{jk} = \frac{1}{2} c_{jk} \sin \phi_{jk}, \tag{6}$$

where U_r is the reduced flow velocity ($U_r = \pi U/\omega R$).

The added mass coefficient σ_{jk} in Eq. 6 can be calculated by applying the potential-flow theory (Chen 1975, 1987). Then σ'_{jk} and σ''_{jk} can be calculated from Eq. 6, when the force amplitude c_{jk} and phase angle ϕ_{jk} are measured. Other fluid-force coefficients can be obtained in the same manner.

Experiment

Measurements of fluid-force coefficients were performed in a water channel. Force transducers to measure the motion-dependent fluid forces consist of relatively rigid main bodies of the tubes supported by relatively flexible tube. Two sets of strain gauges are placed on the outer surface of the supporting tube to measure the forces in the two perpendicular directions (Chen, Zhu, and Cai 1993).

A series of tube arrays has been tested: two tubes normal to flow, two tubes in tandem, tube row, triangular arrays, and square tube arrays. In each case, one of the instrumented tubes is connected to the exciter which provides sinusoidal displacement at a frequency varying from about 0.02 Hz to 2 Hz. Displacement and force signals are filtered by band-pass filters to eliminate low- and high-frequency noises. These signals are analyzed to obtain the oscillation displacement of the tube, the magnitudes of the forces acting on the active tubes, and the phase between the motion-dependent fluid force and tube displacement.

Figures 2 show a typical example of fluid damping and fluid stiffness as a function of reduced flow velocity of two tubes in tandem with the pitch to diameter ratio of 1.35. From Figs. 2 and other data which cannot be included in this paper, the following general characteristics are noted:

- At low reduced flow velocity, fluid damping and fluid stiffness coefficients are function of reduced flow velocity. At high reduced flow velocity, they are almost independent of reduced flow velocity.
- At low reduced flow velocity, they are function of Reynolds number and excitation amplitude.
- At low reduced flow velocity, fluid damping is dominant in many cases while at high reduced flow velocity, fluid stiffness becomes more important. In any case, both fluid damping and fluid stiffness have to be considered.

MATHEMATICAL MODEL BASED ON UNSTEADY FLOW THEORY

Consider a group of n identical tubes as shown in Fig. 1 subject to excitations f'_i and g'_i, in the x and y directions. The variables associated with the tube motion in the x and y directions are flexural rigidity EI, tube mass per unit length m, and structural damping coefficient C_S. The equations of motion for tube j in the x and y directions are (Chen 1987)

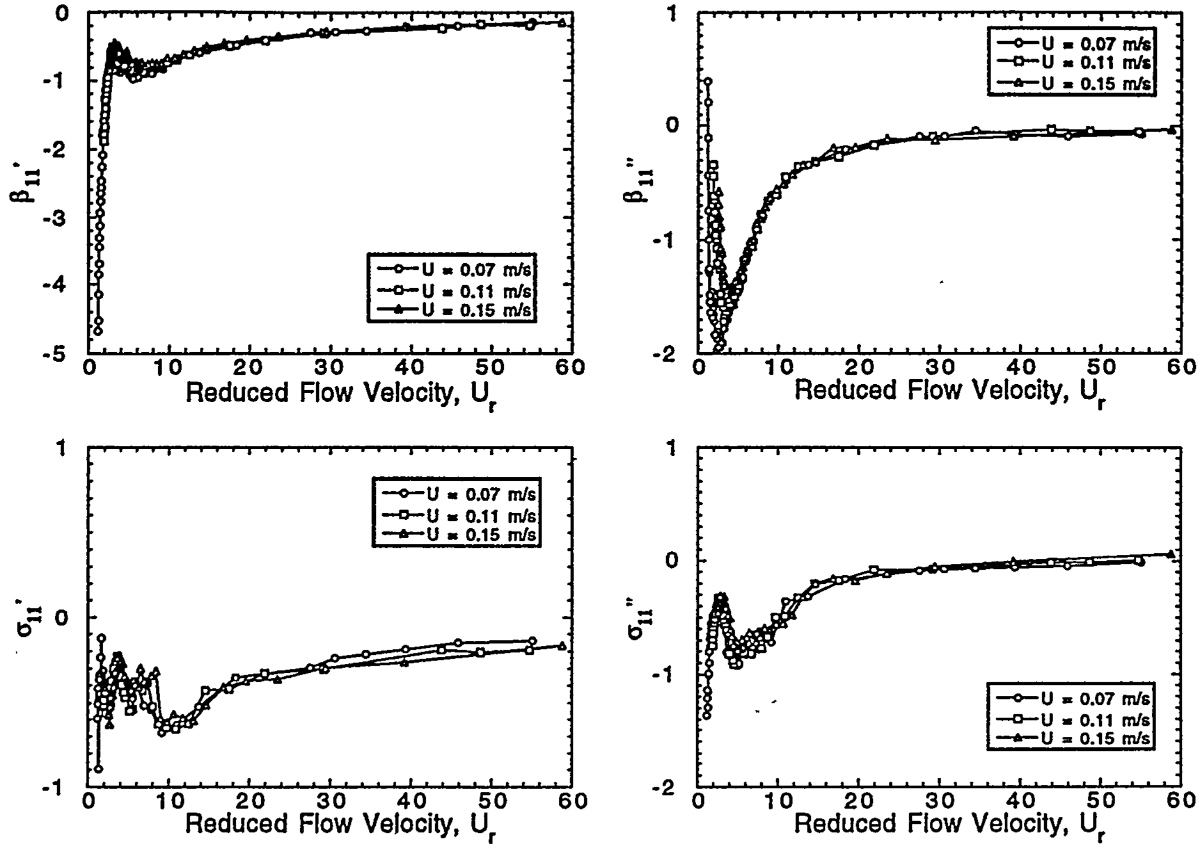

Fig. 2a. Fluid damping and fluid stiffness for two tubes in tandem.

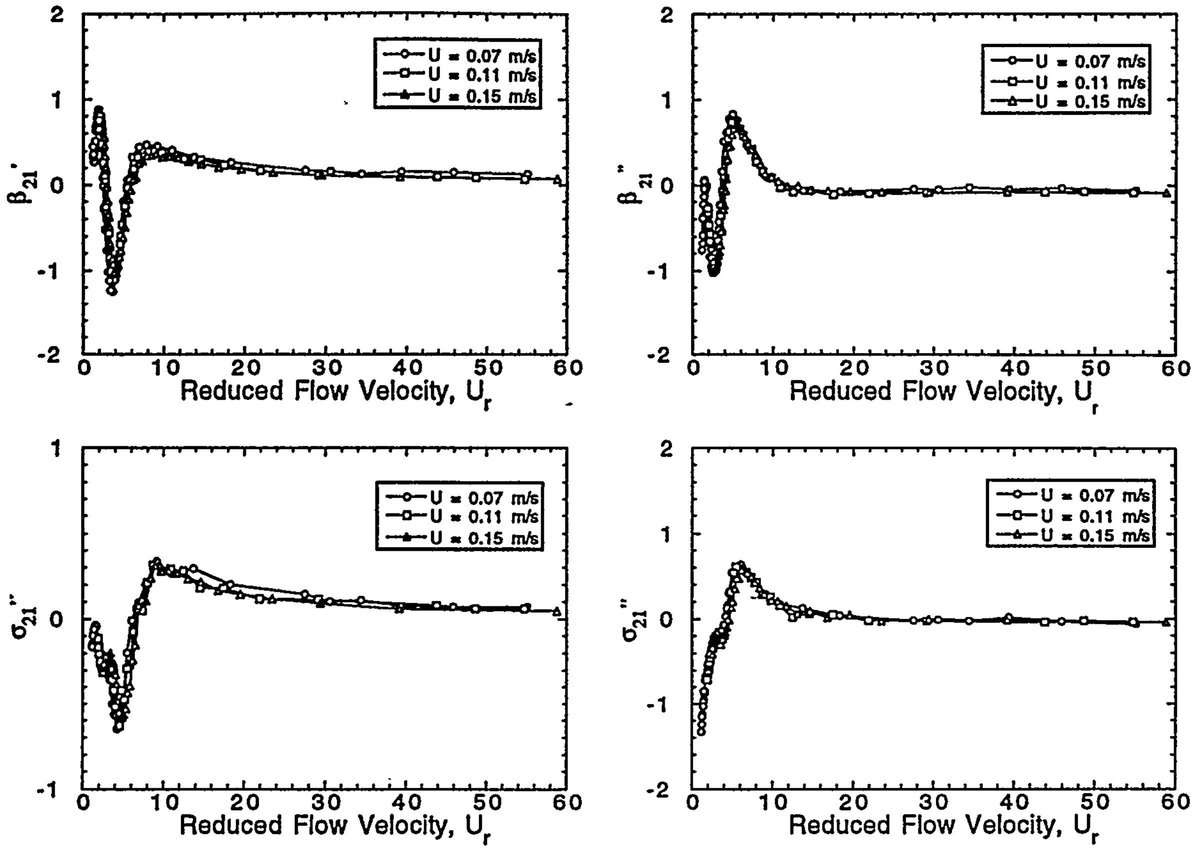

Fig. 2b. Fluid damping and fluid stiffness for two tubes in tandem.

$$EI\frac{\partial^4 u_j}{\partial z^4}+C_s\frac{\partial u_j}{\partial t}+m\frac{\partial^2 u_j}{\partial t^2}+\sum_{k=1}^{n}\rho\pi R^2\left(\alpha_{jk}\frac{\partial^2 u_k}{\partial t^2}+\sigma_{jk}\frac{\partial^2 v_k}{\partial t^2}\right)$$
$$-\sum_{k=1}^{n}\left(\frac{\rho U^2}{\omega}\alpha'_{jk}\frac{\partial u_k}{\partial t}+\frac{\rho U^2}{\omega}\sigma'_{jk}\frac{\partial v_k}{\partial t}\right)-\sum_{k=1}^{n}\rho U^2\left(\alpha''_{jk}u_k+\sigma''_{jk}v_k\right)=f_i, \tag{7}$$

$$EI\frac{\partial^4 v_j}{\partial z^4}+C_s\frac{\partial v_j}{\partial t}+m\frac{\partial^2 v_i}{\partial t^2}+\sum_{k=1}^{n}\rho\pi R^2\left(\tau_{jk}\frac{\partial^2 u_k}{\partial t^2}+\beta_{jk}\frac{\partial^2 v_k}{\partial t^2}\right)$$
$$-\sum_{k=1}^{n}\left(\frac{\rho U^2}{\omega}\tau'_{jk}\frac{\partial u_k}{\partial t}+\frac{\rho U^2}{\omega}\beta'_{jk}\frac{\partial v_k}{\partial t}\right)-\sum_{k=1}^{n}\rho U^2\left(\tau''_{jk}u_k+\beta''_{jk}\right)=g_i. \tag{8}$$

The in-vacuum variables are mass per unit length m, modal damping ratio ζ_v, and natural frequency f_v (= $\omega_v/2\pi$). The modal function of a tube vibrating in vacuum and in fluid is $\psi(z)$;

$$\frac{1}{\ell}\int_0^1\psi^2(z)dz=1, \tag{9}$$

where ℓ is the length of the tubes. Let

$$u_j(z,t)=a_j(t)\psi(z) \qquad \text{and} \qquad v_j(z,t)=b_j(t)\psi(z), \tag{10}$$

where $a_j(t)$ and $b_j(t)$ are functions of time only. Calculation of Eqs. 7 and 8 yields

$$\ddot{a}_j+\gamma\sum_{k=1}^{n}\left(\alpha_{jk}\ddot{a}_k+\sigma_{jk}\ddot{b}_k\right)+2\zeta_v\omega_v\dot{a}_j-\frac{\gamma}{\pi^3}U_v^2\left(\frac{\omega_v^2}{\omega}\right)\sum_{k=1}^{n}\left(\alpha'_{jk}\dot{a}_k+\sigma'_{jk}\dot{b}_k\right)$$
$$+\omega_v a_j-\frac{\gamma}{\pi^3}U_v^2\omega_v^2\sum_{k=1}^{n}\left(\alpha''_{jk}a_k+\sigma''_{jk}b_k\right)=p_i \tag{11}$$

and

$$\ddot{b}_j+\gamma\sum_{k=1}^{n}\left(\tau_{jk}\ddot{a}_k+\beta_{jk}\ddot{b}_k\right)+2\zeta_v\omega_v\dot{b}_j-\frac{\gamma}{\pi^3}U_v^2\left(\frac{\omega_v^2}{\omega}\right)\sum_{k=1}^{n}\left(\tau'_{jk}\dot{a}_k+\beta'_{jk}\dot{b}_k\right)$$
$$+\omega_v^2 b_j-\frac{\gamma}{\pi^3}U_v^2\omega_v^2\sum_{k=1}^{n}\left(\tau''_{jk}a_k+\beta''_{jk}b_k\right)=q_i, \tag{12}$$

where the dot denotes differentiation with respect to t, and

$$U_v=\frac{U}{f_v D}, \qquad \gamma=\frac{\rho\pi R^2}{m}, \qquad p_i=\frac{1}{m\ell}\int_0^\ell f'_i\psi(z)dz, \qquad q_i=\frac{1}{m\ell}\int_0^\ell g'_i\psi(z)dz. \tag{13}$$

Once the fluid-force coefficients are known, it is straightforward to analyze the stability of a tube array in crossflow.

Fluidelastic instability of tube arrays is caused by high-velocity flow. Various types of dynamic instability can be classified as follows:

Fluid-damping-controlled instability (single-mode flutter). The dominant terms are associated with the symmetric part of fluid-damping matrix. The instability arises because the fluid-dynamic forces create negative damping. Parameters α'_{jj} and β'_{jj} play the most important role in determining the stability-instability boundaries.

Fluid-stiffness-controlled instability (coupled-mode flutter). The dominant terms are associated with the antisymmetric fluid-stiffness matrix. It is called coupled-mode flutter because a minimum of two modes are required to produce it. In this case, a''_{jk} and τ''_{jk} for j = k play the major role in determining the stability characteristics.

In the unsteady flow theory, all fluid forces are included and characterized properly and the theory is applicable in all cases regardless of different flow regimes or system parameters. On the other hand, quasistatic and quasisteady flow theories are applicable in some specific parameter ranges, depending on the case. Therefore, the unsteady flow theory can be used to assess the validity of other approximate flow theories of the fluidelastic instability of tube arrays in crossflow.

NONLINEAR RESPONSE

Tube arrays in heat exchangers, steam generators, condensers, boilers, etc., are frequently restrained by tube support plates (TSP), antivibration bars, and other types of supports. To facilitate manufacture and to allow for thermal expansion of the tubes, small clearances are used between tubes and tube supports. When the tubes oscillate due to subcritical vibration or fluidelastic instability caused by flowing fluid, tube failure can occur through fretting wear, fatigue, and impacting associated with dynamic tube/support interaction. Because a loosely supported tube in crossflow is a nonlinear dynamic system with many interesting nonlinear characteristics. Both analytical and experimental studies have been performed.

The experiments were performed in a rectangular flow channel that is 10.7 cm (4.2 in.) wide and 25.9 cm (10.2 in.) high and is situated in a test chamber connected to a water loop with a maximum flow rate of 0.052 m^3/s (700 gpm). The details of the test chamber are the same as those in earlier experiments for tube arrays in crossflow (Chen, Jendrzejczyk, and Wambsganss 1985).

The arrangement of the tube row is shown in Fig. 3. Each tube element was suspended as a simply supported beam on two O-rings mounted 91.4 cm (36 in.) apart (A to D). The O-rings were seated in compression plates. The tube was submerged in fluid between the two O-ring supports (A to D) but was subjected to flow only in its middle portion B to C (measuring 25.9 cm [10.2 in.]). Two types of tubes were used: (a) Tube 3 was a brass tube with 1.59 cm (5/8 in.) OD, 1.59 mm (1/16 in.) wall thickness, and 126.37 cm (49.75 in.) length; and (b) Tubes 1, 2, 4, and 5 were stainless steel tubes with 1.59 cm (5/8 in.) OD, 0.32 cm (1/8 in.) wall thickness, and 99.06 cm (39.0) length. The overhung portion (the portion outside the tube supports A and D) for Tubes 1, 2, 4, and 5 was 3.81 cm (1.5 in.), and for Tube 3, 3.18 cm (1.25 in.) at one end and 31.75 cm (12.5 in.) at the other end. The overhung portion of the tubes between D and G was in air. To raise their natural frequencies in the lift (i.e., perpendicular-to-flow) direction, Tubes 1-2 and 4-5 were welded together outside the flow region. Tube 3 was also "supported" by a brass baffle TSP, shown in Fig. 3 at Location G. The diametral clearance (gap) between the tube and the TSP was set at various values to study the effect of clearance on tube response.

Two sets of displacement transducers were located at E and F to measure the displacement of Tube 3 (Fig. 3); two transducers at F were close to the TSP, and the other two transducers at a were at approximately the midspan of the overhung portion. The transducers measured the displacements in the lift and drag directions. The objectives were to measure the response characteristics of Tube 3 for various support conditions. Because Tubes 1, 2, 4, and 5 were much stiffer than Tube 3, they could be considered rigid tubes. Three series of tests were performed; the details are given in a report (Chen, Zhu, and Cai 1993).

An analytical model based on the unsteady flow theory (Cai and Chen 1991, 1992) was used to confirm the existence of chaos in the fluidelastic instability of tube arrays in crossflow and to understand the route to chaos. Measurement tools included power spectral densities, phase planes, Poincaré maps, Lyapunov exponents, and fractal dimensions. The detailed analysis method is given in earlier work (Cai and Chen 1991, 1992).

In comparison, the analytical model predicted the general characteristics of tube response reasonably well. Specifically, the following statements can be made.

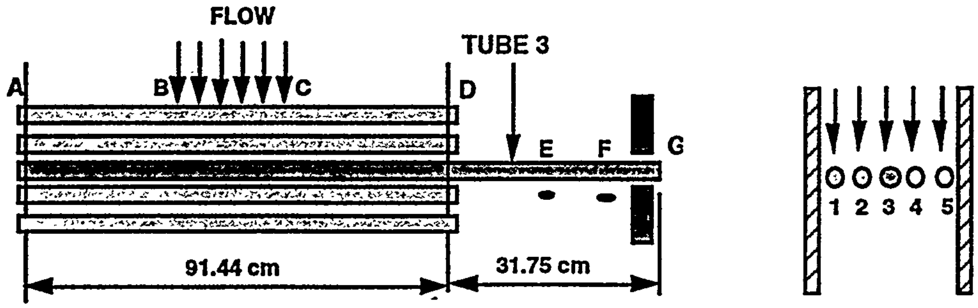

Fig. 3. (a) Row of tubes in crossflow; (b) tube/support relationship.

- Both the analytical model and the experimental data showed chaos in a tube row subject to fluidelastic instability of the TSP-inactive mode.
- When the flow velocity was close to the lower and upper limits of the critical flow velocities, tube motion was chaotic. Between the two limits, the motion was much more regular, with the tube striking the TSP regularly.
- Within the two stability limits, the motions were periodic, quasiperiodic, and chaotic, if the system was well defined. In experiments, minor variations can change the system characteristics drastically; this is a classical chaotic characteristic. The chaotic motion was of the limited-band type.
- The fractal dimension for this system is fairly low; based on the analytical model, it is between 1 and 2.

CONCLUSIONS

In the unsteady flow theory, all fluid force components are included and quantified properly and the theory is applicable in all cases for all system parameters. It can be used to predict the critical flow velocity, tube response, and the effects of various parameters. In addition, it can be used to assess the validity of other simplified theories such as quasi-static and quasi-steady flow theories.

Tests of motion-dependent fluid forces have been performed for a series of tube arrays for various conditions. Fluid damping and stiffness coefficients are found to be functions of Reynolds number, reduced flow velocity, tube arrangement, tube location, tube pitch, and motion amplitude. At this time, only experimental methods can be used to obtain the needed coefficients. Some efforts have been made to develop computational methods for motion-dependent fluid forces.

The nonlinear response based on the unsteady flow theory can predict different motions: random vibration, periodic vibration, quasi-periodic motion, and chaotic vibration. A series of tests have been perform to compare with analytical results.. The analytical model agree well with experimental data. This demonstrate the capability of the unsteady flow theory.

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