

CONF-951006-35

Paper to be presented at the ANS 1995 Winter Meeting, San Francisco, California,  
October 29-November 2, 1995.

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**COMPARISON OF SIMPLIFIED AND STANDARD SPHERICAL HARMONICS  
IN THE VARIATIONAL NODAL METHOD\***

by

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\*Work supported by the U.S. Department of Energy, Nuclear Energy Programs under Contract W-31-109-ENG-38.

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# Comparison of Simplified and Standard Spherical Harmonics in the Variational Nodal Method

E. E. Lewis & G. Palmiotti

Recently, the variational nodal method has been extended through the use of the Rumyantsev interface conditions<sup>1</sup> to solve the spherical harmonics ( $P_N$ ) equations of arbitrary odd order.<sup>2</sup> Here, we generalize earlier x-y geometry work<sup>3</sup> to fit the corresponding simplified spherical harmonics ( $SP_N$ ) equations into the variational nodal framework. Both  $P_N$  and  $SP_N$  approximations are implemented in the multigroup VARIANT code at Argonne National Laboratory in two- and three- dimensional Cartesian and hexagonal geometries. The availability of angular approximations through  $P_5$  and  $SP_5$ , and of flat, linear and quadratic spatial interface approximations allows investigation of both spatial truncation and angular approximation errors. Moreover, the  $SP_3$  approximation offers a cost-effective method for reducing transport errors.

The even-parity  $SP_N$  approximations are derived by first writing the slab geometry  $P_N$  approximation for odd order  $N$ . Let  $\psi$  and  $\chi$  be vectors of length  $(N+1)/2$  of the even and odd parity flux moments. Then

$$E \frac{\partial}{\partial x} \chi + \sigma \psi = b[\sigma_s \phi + S]$$

and

$$O \frac{\partial}{\partial x} \psi + \sigma \chi = 0,$$

where  $b_i = \delta_{1i}$  and  $E$  and  $O$  are two-striped lower and upper triangular matrices, respectively. The even parity equation obtained by eliminating  $\chi$  is then

$$- \frac{\partial}{\partial x} \frac{1}{\sigma} H \frac{\partial}{\partial x} \psi + \sigma \psi = b[\sigma_s \phi + S],$$

where  $H = E O$ , and  $\psi$  and  $\chi$  are related by

$$- \frac{1}{\sigma} H \frac{\partial}{\partial x} \psi = E \chi.$$

The  $SP_N$  equations are obtained simply by letting  $\frac{\partial}{\partial x} \rightarrow \vec{\nabla}$  and allowing  $\psi$  and  $\chi$  to become functions of the x, y and z. Thus

$$- \vec{\nabla} \frac{1}{\sigma} H \vec{\nabla} \psi + \sigma \psi = b[\sigma_s \phi + S] \quad (1)$$

and

$$- \frac{1}{\sigma} H \vec{n} \cdot \vec{\nabla} \psi = E \chi. \quad (2)$$

The following functional may be shown to have Eq. 1 as its Euler Lagrange equations within the node and Eq. 2 as an interface condition

$$F_v[\psi, \chi] = \int_v dV \left[ \vec{\nabla} \psi^T \frac{1}{\sigma} H \vec{\nabla} \psi + \sigma \psi^T \psi - \sigma_s \phi^2 - 2\phi S \right] + 2 \sum_{\gamma} \int_{\gamma} d\Gamma \psi^T E \chi_{\gamma}$$

From here on, the procedure is the same as published previously.<sup>4</sup> Spatial polynomial approximations are used for  $\psi$  and  $\chi$ ; a Ritz procedure is applied, and the resulting equations are cast in response matrix form.

Studies have been undertaken to compare the relative performance of  $SP_N$  and  $P_N$  approximations in two and three dimensions. In model fixed-source problems  $SP_N$  closely mimic the corresponding  $P_N$  solutions where large numbers of interfaces are not present. In criticality problems, the results shown in Fig 1 for the "rods-in" Takada Benchmark II in x-y-z geometry<sup>5</sup> are indicative of the eigenvalue errors which are found. In all cases studied the spatial truncation errors - which may be isolated by comparing flat, linear and quadratic interface conditions with the same angular approximation - are found to be positive. Errors attributable to the angular approximations - which may be isolated by comparing the spatially converged quadratic approximations - are negative. Thus, in some configurations, going from a lower to a higher order space or angular approximation may produce an accuracy loss as a result of the decreased error cancellations.

Other general observations are that space and angular approximations interact more strongly in  $P_N$  approximations, necessitating the refinement of the spatial approximation in tandem with increased  $P_N$  order. Conversely the accuracy of the  $SP_N$  approximations saturate as a result of the angular moments which are not included. The  $SP_3$  approximation frequently offers substantial increases in accuracy at roughly double the cost of a corresponding nodal diffusion calculation, while full  $P_N$  calculations are substantially more expensive. On an IBM rs6000 the CPU times for the results in Fig. 1 were 78, 148 and 916 sec. for the  $P_1$ ,  $SP_3$  and  $P_3$  calculations with linear interface conditions.

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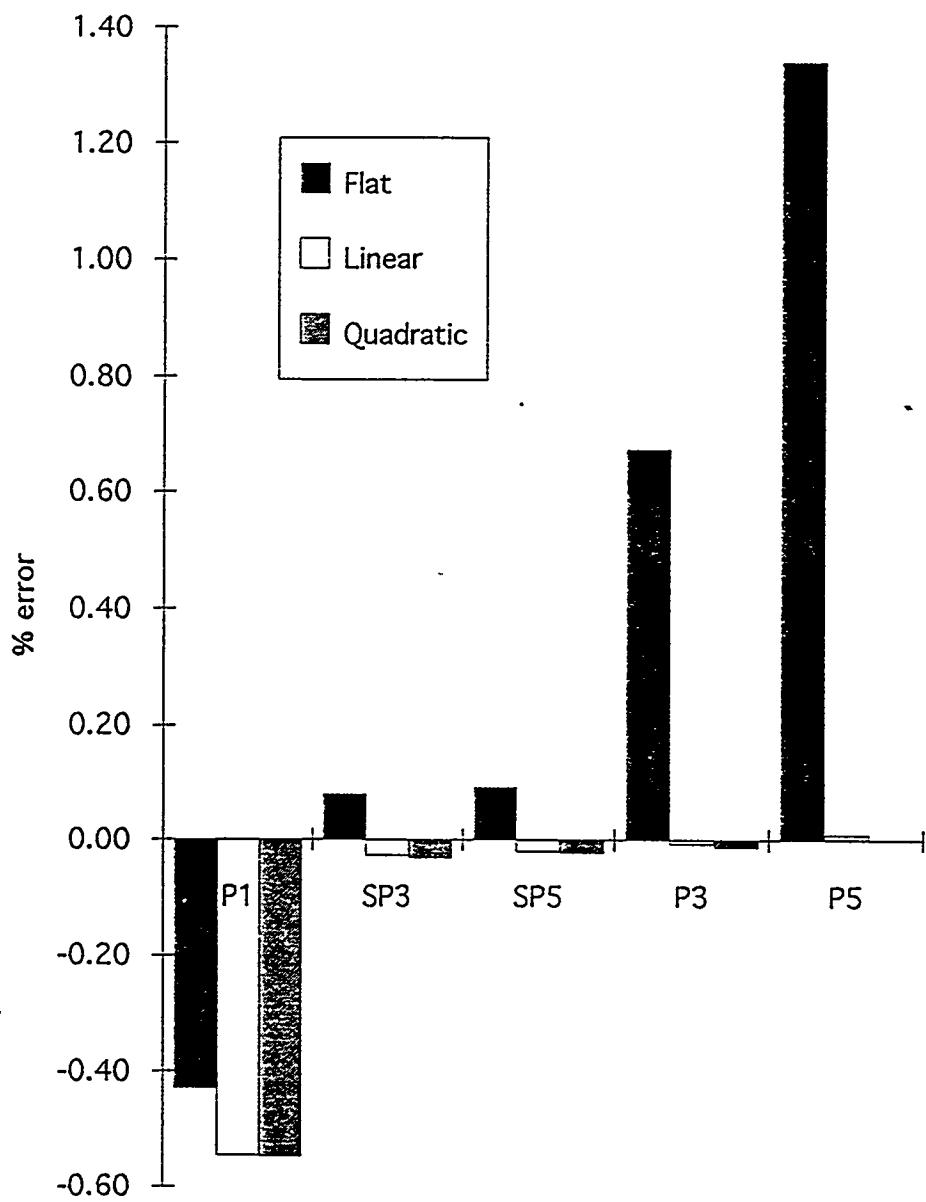


Figure 1. Eigenvalue Errors for the "Rods In" Takada Benchmark II  
(reference  $k = 0.95954$ )