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Bayesian inference for the seismic moment tensor using regional waveforms and a data-derived distribution of velocity models

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Bayesian inference for the seismic moment tensor using regional waveforms and a data-derived distribution of velocity models

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Abstract

The largest source of uncertainty in any source inversion is the velocity model used to construct the transfer function employed in the forward model that relates observed ground motion to the seismic moment tensor. However, standard inverse procedures often does not quantify uncertainty in the seismic moment tensor due to error in the Green's functions from uncertain event location and Earth structure. We attempt to incorporate this uncertainty into an estimation of the seismic moment tensor using a distribution of velocity models calculated in a prior effort based on different and complementary data sets. The posterior distribution of velocity models is then used to construct Green's functions for use in Bayesian inference of an unknown seismic moment tensor using regional waveform data. The combined likelihood is estimated using data-specific error models and the posterior of the seismic moment tensor is estimated and can be interpreted in terms of most-probable source-type.

1 Introduction

The seismic moment tensor (MT) has long been used in earthquake and explosion source analysis, and there has been a renewed interest in how it can inform us about the seismic source particularly in the geophysical monitoring community due to its application in event identification and yield analysis (Ford et al., 2020; Pasyanos & Chiang, 2022). Some of this interest derives from our recent ability to routinely determine six-component MT that takes advantage of the full description of the MT to characterize isotropic and non-isotropic radiation of seismic sources. The elements of the tensor are used to derive the source-type and subsequently tested against theoretical mechanisms such as explosion, earthquake and collapse. MTs and their source-types have been shown to be valuable geophysical monitoring tools in identifying explosions (Alvizuri et al., 2018; Chiang et al., 2018; Mustać et al., 2020) and other nuisance signals, as well as discriminating explosions from earthquakes when varied-data type inversion is applied to the analysis. The MTs can be used to augment traditional semi-empirical based methods that utilize surface-to-body-wave magnitude ratios (Selby et al., 2012) and regional phase amplitude ratios (Bottone et al., 2002; Walter et al., 2018). The effort to develop and improve the MT discriminant for monitoring and enforcement of nuclear test-ban treaties continue to be an area of active research.

However, parameter uncertainties in seismic MT inversion are rarely available. The inverse procedure often does not quantify MT model errors such as event location, data noise and Earth model that are essential for estimating solution robustness. Here we propose to adopt the Bayesian probabilistic framework to incorporate uncertainties in MT inversions. The probabilistic formulation described by Tarantola and Valette (1982) casts the inverse problem in a Bayesian framework where information on the model parameters is represented in probabilistic term. With this approach the solution is given as the complete posterior probability density function of the data and model parameters, instead of a single best-fit solution.

2 Method

The seismic MT is a 3×3 matrix consists of nine force couples that represent the equivalent body forces for seismic sources of different geometries, which due to conservation of angular momentum reduce to six independent couples and dipoles. In the Bayesian framework, an appropriate choice of a priori moment tensor probability is important, but what constitutes as a random moment tensor is not straightforward and depends on the coordinate domain of the parameterized MT. Tape and Tape (2015) have shown that uniformly distributed MTs have uniformly distributed orientations (eigenvectors) but not uniformly distributed source-types (eigenvalues). In fact uniformly distributed MTs favors double-couples in the source-type space.

Tape and Tape (2015) have provided two approaches to generate uniformly distributed moment tensors based on the 5-D space of all MTs of unit norm, in which one of the approaches uses a parameterization of the MT that is closely related to the MT orientations and source types. The Tape parameterization has five parameters and have finite upper and lower bounds: κ (strike), h (dip cosine), σ (slip), u (similar to lune colatitude) and v (similar to lune longitude), where the pair (u, v) determines the normalized eigenvalues of MT (source type). We can now construct the uniform priors using the five Tape parameterization and the seismic moment.

Observed data is related to the seismic MT via a transfer function that is the Green's function for the path from source to receiver. We employ the principle of reciprocity (e.g. Dahlen & Tromp, 1998) to produce Green's functions for many source locations and one station location. Zhao et al. (2006) shows that the component n of displacement u_n at receiver location $\boldsymbol{x}$ from source location $\boldsymbol{x'}$ is related to the spatial gradient of the Green's tensor $\partial_i G_{nj}(\boldsymbol{x}, \boldsymbol{x'}; t)$ where $i, j = 1, 2, 3$ are the cartesian spatial components. Betti's theorem (i.e., reciprocity) states that the spatial gradient of the Green's tensor from source to receiver is equal to the strain Green's tensor from receiver to source $(\partial_i G_{jn}(\boldsymbol{x'}, \boldsymbol{x}; t) + \partial_j G_{in}(\boldsymbol{x'}, \boldsymbol{x}; t))/2$, which is composed of spatial gradients of the Green's tensor from receiver to source.

We use reciprocity as described by Eisner and Clayton (2001) to calculate the displacement gradients in SW4 (Petersson & Sjögreen, 2017) for a grid of event epicenters, depths, and origin times centered on the USGS solution. The epicentral and depth range are ±3 km and 600 to 2400 m in 600 m steps, and the time range is ±0.2 s in 0.1 s steps, which results in 2420 origins. The Green's functions for these origins are calculated over many velocity model realizations from the posterior calculated in Pasyanos et al. (2006). Figure 1 is a reproduction from that work showing the mean upper mantle velocity and corresponding uncertainty.

Graves and Wald (2001) lays out the forward model for component n of displacement u at the receiver

$$u_n = a_1 \times mxx_n + a_2 \times myy_n + a_3 \times mzz_n + a_4 \times mxy_n + a_5 \times mxz_n + a_6 \times myz_n. \tag{1}$$

The coefficients $(a_1, ..., a_6)$ are the moment tensor elements and the predictor variables $(mxx_n, ..., myz_n)$ are components of the strain Green's tensor for a single force in direction n imposed at the receiver recorded at

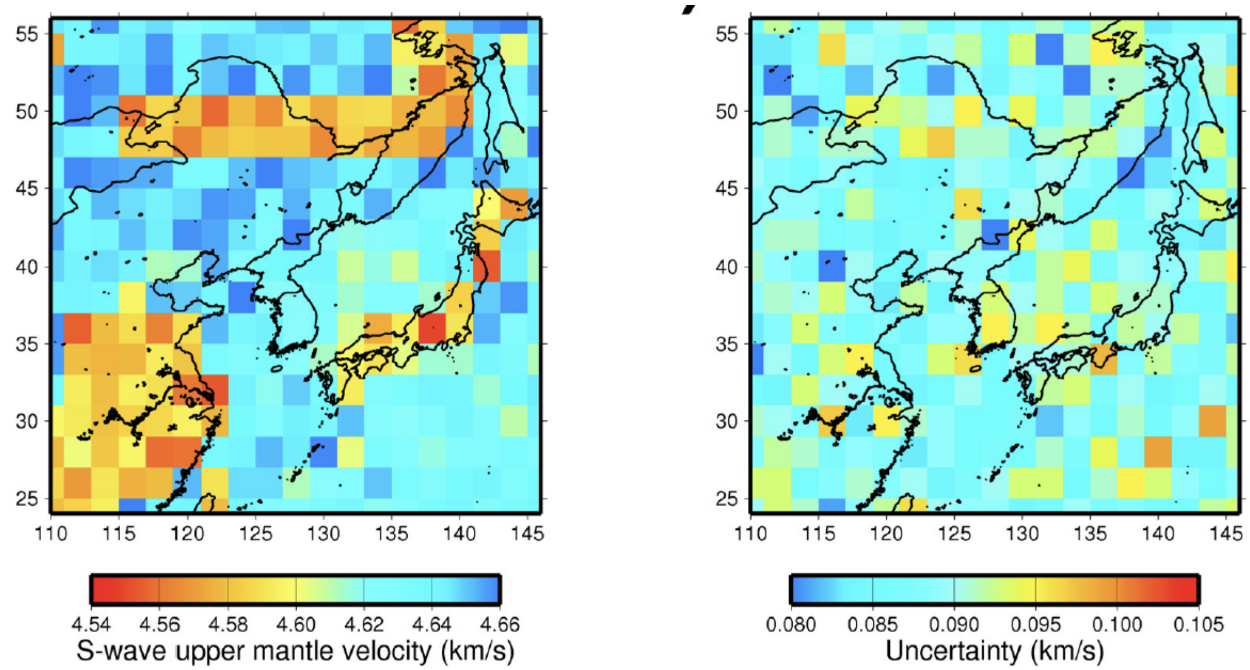

Figure 1: Figure 13 from Pasyanos et al., 2006. Map of upper mantle S wave velocities and corresponding uncertainties for the YSKP region.

the source. For each of the 3 components $n = x, y, z$ we have 6 elements of the strain Green's tensor,

$$\begin{aligned}
mxx &= \partial u_x / \partial x \\
myy &= \partial u_y / \partial y \\
mzz &= \partial u_z / \partial z \\
mxy &= \partial u_y / \partial x + \partial u_x / \partial y \\
mxz &= \partial u_z / \partial x + \partial u_x / \partial z \\
myz &= \partial u_z / \partial y + \partial u_y / \partial z
\end{aligned} \tag{2}$$

In order to output the mij waveforms for each source location in SW4 we make three calculations; one for each direction of a single force applied at the receiver location and recorded at the source locations using the `strains` output. The off-diagonal mij waveforms are multiplied by two to account for the moment tensor source symmetry.

The data covariance is modeled after the observations. This is a preferred approach because now the residuals are "weighted" by a realistic covariance and not an a priori weighting scheme (e.g., Chen et al., 2015). Figure 2 shows the zero-lag covariance used in the method.

We use 5 million sources to uniformly cover the source space and transform them to moment tensor elements.

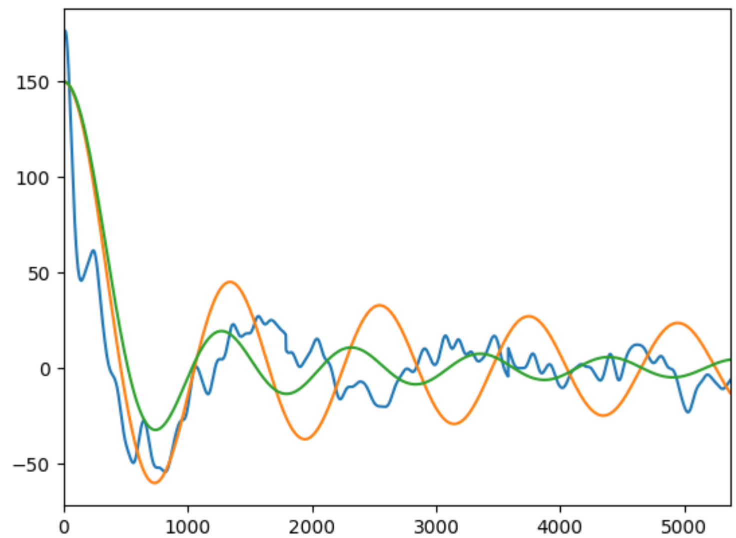

Figure 2: Empirical (blue) and analytical covariance functions. The analytical functions are bessel (orange) and sinc (green).

3 Case Study: DPRK09

In 2009, the Democratic Peoples Republic of Korea (DPRK) announced a nuclear test at the Punggyi-Ri Test Site. Kim and Richards (2007) and Koper et al. (2008) show initial discrimination and the USGS location of the event.

We will employ the intermediate displacement waveforms from stations MDJ and INCN for the moment tensor inference. Figure 3 shows the observation at MDJ and INCN in orange along with predictions using an MT for the explosion from Chiang et al. (2018) for several velocity models and source locations.

Here we represent the MT inversion result in terms of the marginal likelihood on the lune. Figure 4 shows the marginal likelihood for a single station inversion (MDJ) and the combined marginal likelihood from stations MDJ and INCN. The best solutions are predominantly isotropic as indicated by the darker colors near the poles. For the single station inversion, the most probable MT solutions are distributed between the positive and negative isotropic, but incorporating waveform data from INCN significantly improves the uncertainties in the MT solutions by eliminating the implosive sources. The combined marginal likelihood shows the most probable solutions are explosive in nature consistent with an underground explosion event.

4 Future work

Future work will be to produce more likelihood surfaces for the regional waveform data type and incorporate the additional data type of teleseismic-P. We will also sample the posterior using an MCMC approach and compare with the grid-based posterior calculation approach described here.

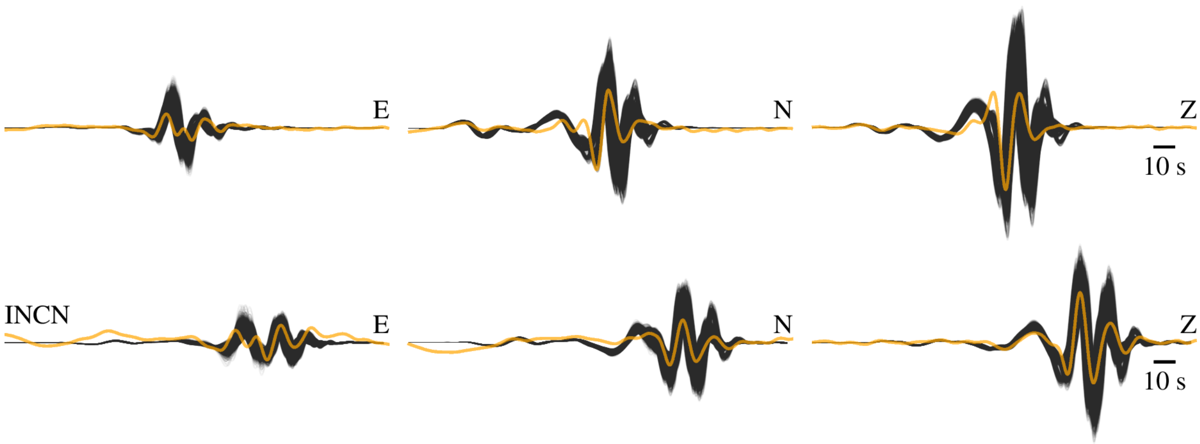

Figure 3: Observation of DPRK09 at MDJ (top) and INCN (orange) compared with predictions for a DPRK09 MT (Chiang et al., 2018) using 8 velocity models, 484 locations, and 5 origin times.

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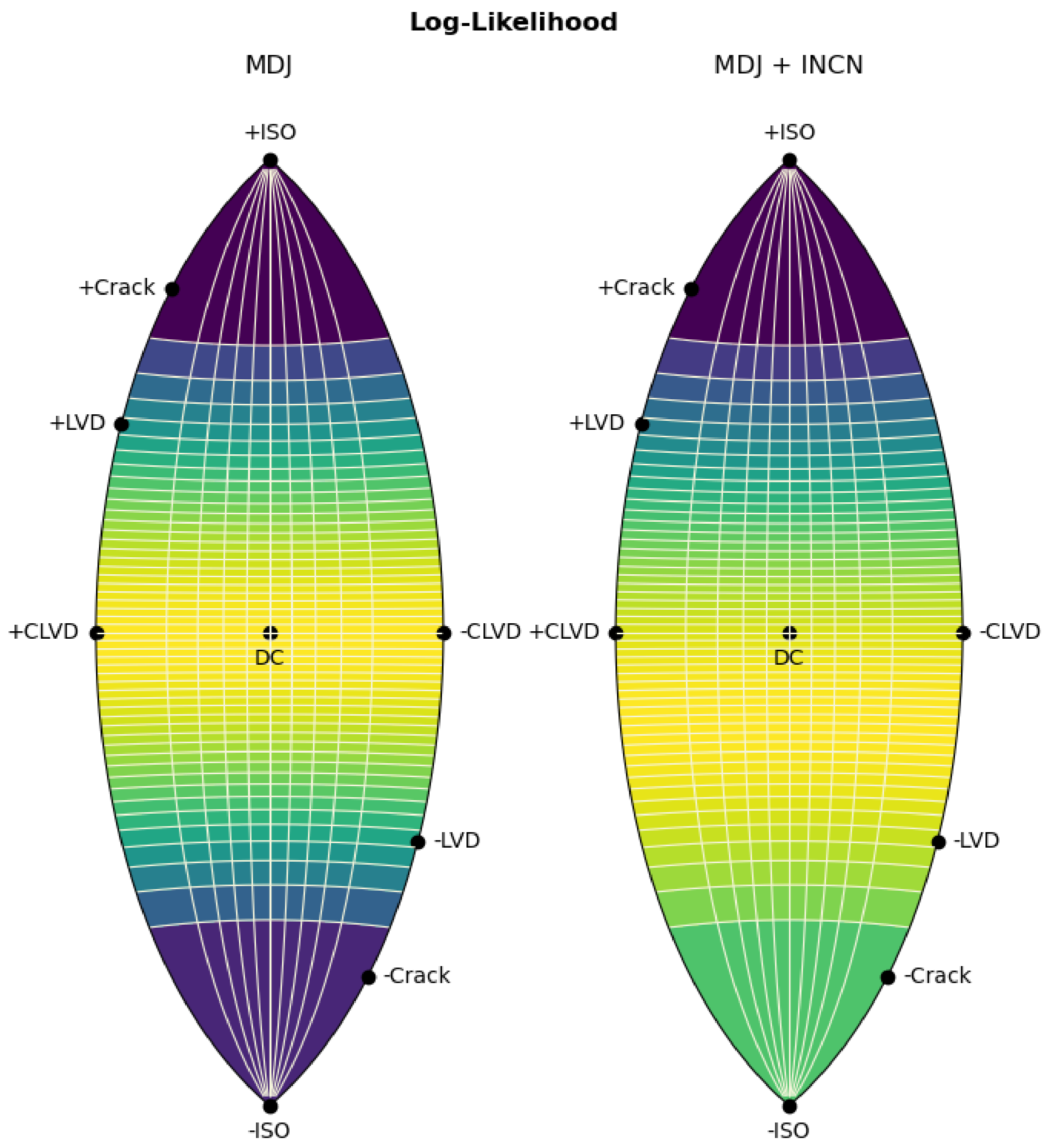

Figure 4: Marginal likelihoods for one velocity model.

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