Institute for Fundamental Theory Preprint UFIFT – HEP – 95 – 31

hep-th/9512107

Universality and Clustering in 1 + 1 Dimensional Superstring-Bit Models*

OREN BERGMAN† AND CHARLES B. THORN‡

Institute for Fundamental Theory
Department of Physics, University of Florida
Gainesville, FL 32611

ABSTRACT

We construct a 1+1 dimensional superstring-bit model for D=3 Type IIB superstring. This low dimension model escapes the problems encountered in higher dimension models: (1) It possesses full Galilean supersymmetry; (2) For noninteracting polymers of bits, the exactly soluble linear superpotential describing bit interactions is in a large universality class of superpotentials which includes ones bounded at spatial infinity; (3) The latter are used to construct a superstring-bit model with the clustering properties needed to define an S-matrix for closed polymers of superstring-bits.

* Work supported in part by the Department of Energy under grant number DE–FG05–86ER–40272 and in part by the Institute for Fundamental Theory.
† E-mail address: oren@phys.ufl.edu
‡ E-mail address: thorn@quark.phys.ufl.edu

String-bit models are attempts to reformulate string theories in $D-1$ space dimensions as field theories of point-like string-bits moving in $d \equiv D-2$ space dimensions, in which string is composite, not fundamental [1–5]. The bits bind together to form long closed polymer chains, which in the continuum limit have precisely the properties of closed relativistic string. String-bits must carry an internal "color" degree of freedom which defines ordering around the chain. This can be achieved if the bits transform in the adjoint representation of the unitary group $U(N_c)$, with $N_c \geq 2$. A crucial feature of our light-cone approach to string-bit models is that they possess Galilean invariance in d space dimensions, not Poincaré invariance in $d+1$ space dimensions: the bits enjoy a **non-relativistic** dynamics.

Second-quantization of string-bits employs a string-bit creation operator $\phi^\dagger(x)^\beta_\alpha$, where α and β run over the N_c colors and x denotes the d space coordinates. Denote the zero-bit state by $|0\rangle$. A bare closed chain of M bits is then described by:

$$|\Psi(x_1,\ldots,x_M)\rangle = \int dx_1 \cdots dx_M \times \mathrm{Tr}[\phi^\dagger(x_1)\cdots\phi^\dagger(x_M)]|0\rangle\Psi(x_1,\ldots,x_M)\ , \tag{1}$$

implying that $\Psi(x_1,\ldots,x_M)$ is cyclically symmetric. To describe closed chains and their interactions, the Hamiltonian governing string-bit dynamics must allow for their formation and assure their stability.

Chain formation requires that string-bits have an attractive interaction between nearest neighbors on a chain, with non-nearest neighbors interacting much more weakly. It is well-known [6,3,5] that this pattern of interactions arises in a many body system of particles described by $N_c \times N_c$ matrix creation operators using 't Hooft's $N_c \to \infty$ limit [7]. For an interaction Hamiltonian of the form

$$H_{int} = \frac{1}{N_c}\int dxdy V(y-x)\mathrm{Tr}[\phi^\dagger(x)\phi^\dagger(y)\phi(y)\phi(x)], \tag{2}$$

the limit $N_c \to \infty$ leads to nearest-neighbor interactions in a bare closed chain of string-bits. For N_c finite but large, $O(1/N_c)$ effects allow a single bare closed chain to break into two bare closed chains. Thus $1/N_c$ serves as a chain coupling constant, and 't Hooft's $1/N_c$ expansion produces chain perturbation theory.

Free light-cone string is recovered for $N_c \to \infty$ in the continuum limit given by $M \to \infty$, $m \to 0$, with mM kept fixed, or equivalently in the low energy limit given by $E \ll T_0/m$, where T_0 is the string tension and m is the Newtonian mass of a bit. The total Newtonian mass of a chain becomes an effectively continuous P^+ of a string. The x^- coordinate of string thus emerges dynamically in string-bit models as the conjugate to Newtonian mass. The other light-cone coordinate x^+ is identified as time, and its conjugate P^- as the bit Hamiltonian. The $O(1/N_c)$ chain interactions become string interactions in the continuum limit.

Stability of string depends on the ground state energy of a long closed chain, generically given by:

$$E_{0,M} = \frac{1}{m}\left[aM + \frac{b}{M} + O(\frac{1}{M^2})\right]. \tag{3}$$

The first term is the same for a single chain of M bits and two chains of M_1 and M_2 bits, with $M_1 + M_2 = M$. For long chains, the nature of the true ground state then depends on the second term. If $b > 0$ then $E_{0,M} < E_{0,M_1} + E_{0,M_2}$, and a long chain is stable. If $b < 0$ then $E_{0,M} > E_{0,M_1} + E_{0,M_2}$, the chain is unstable to decay into two smaller chains, and it will through the $O(1/N_c)$ terms alluded to before. Consider a chain of bosonic bits interacting via the nearest-neighbor harmonic potential $V(x) = (\omega^2/2m)x^2$. The ground state energy for an M-bit chain in d space dimensions is found to be [1,5]

$$E_{0,M} = \frac{\omega d}{2m}\sum_{n=1}^{M-1}\sin\frac{n\pi}{M} = \frac{\pi\omega d}{m}\left[2M - \frac{1}{6M} + O(\frac{1}{M^2})\right], \tag{4}$$

and so a long chain of bosonic bits is unstable against decay into two smaller chains. This is just the string-bit manifestation of the tachyonic instability of bosonic string. The negative coefficient of $1/2mM$ is the mass-squared of the tachyon.

This instability is absent in superstring theory which requires the addition of fermionic modes on string in a supersymmetric fashion. For string-bit models the bits are in super-multiplets with a "statistics" degree of freedom distinguishing bosons from fermions. This degree of freedom gives rise to "statistics waves" on long chains, similar to spin waves. Supersymmetrizing the harmonic string-bit model leads to a cancellation of the contribution

to the ground state energy of the coordinate "phonon" waves with that of the "statistics" waves. In fact, the ground state energy is exactly zero for any M [5]. We shall see later that this is a universal property of supersymmetric string-bit models, and not special to the harmonic interaction. Note that recovering the free superstring mass spectrum at $M = \infty$ requires only $b = 0$, whereas in this model **all** the terms in (3) vanish. If this were not so, the sign of the first nonvanishing term would determine the stability of finite long chains. If these were unstable except when $M = \infty$, the bit model could not provide a **fundamental** basis for superstring theory, and at best would only make sense in the continuum limit.

We have constructed superstring-bit models in $2+1$ and $8+1$ dimensions that underlie $D = 4$ and $D = 10$ type IIB superstring theory respectively [5]. In the $D = 4$ case we could include extra degrees of freedom either as real compactified dimensions, or as additional internal bit degrees of freedom [8] which, on long chains, would produce "flavor waves" playing the role of extra dimensions. The $N = 2$ spacetime Poincaré supersymmetry of the D-dimensional type IIB superstring requires the corresponding $(D-2)+1$-dimensional superstring-bit model to possess an $N = 1$ Galilean supersymmetry. The symmetry is Galilean because light-cone variables break the manifest $SO(D-1,1)$ to $SO(D-2) \times SO(1,1)$. Discretization of P^+ breaks this $SO(1,1)$ and also mixes the right- and left-moving supercharges, leaving only an $N = 1$ supersymmetry generated by the right + left combinations. The Galilean supercharges $\mathcal{Q}$ and $\mathcal{R}$ transform as spinors of the transverse $SO(D-2)$ subgroup of $SO(D-1,1)$, and have opposite chirality in the $SO(1,1)$ subgroup. Together they build a single supercharge transforming as a spinor of the Lorentz group $SO(D-1,1)$, and generating the super-Poincaré algebra.

For general d the Galilean supercharges $\mathcal{Q}^A, \mathcal{R}^{\dot{A}}$ must each have d components for a satisfactory superstring limit of the string-bit model. The corresponding super-Galilei algebra then reads:

$$\{\mathcal{Q}^A, \mathcal{Q}^B\} = mM\delta^{AB} \quad , \quad \{\mathcal{Q}^A, \mathcal{R}^{\dot{B}}\} = \frac{1}{2}\alpha^{A\dot{B}} \cdot \mathbf{P} \, ,$$
$$\{\mathcal{R}^{\dot{A}}, \mathcal{R}^{\dot{B}}\} = \delta^{\dot{A}\dot{B}} H/2 \, , \tag{5}$$

where M is the total number of bits, and $\mathbf{P}$ is their total momentum. The superstring-bit models of [5] implement all but the last of these relations: There are additional terms not proportional to $\delta^{\dot{A}\dot{B}}$. For the supersymmetric harmonic model these terms are sub-leading in the $1/N_c$ expansion, so the $\mathcal{R}$-supersymmetry is broken by chain interactions. For other superstring-bit models this happens already at the level of the free chain spectrum. In either case, it might still be that the full Poincaré supersymmetry can be recovered in the continuum stringy physics.

For $d = 1$, the full superalgebra closes by default, since $\mathcal{R}$ and $\mathcal{Q}$ then have only one component each. A $1+1$-dimensional superstring-bit model would underlie $D = 3$ superstring, the lowest dimensional superstring possible [9]. Specializing the supercharges of [5] to this case gives

$$\begin{aligned}\mathcal{Q} &= \sqrt{\frac{m}{2}} \int dx \mathrm{Tr}[e^{i\pi/4}\phi^\dagger(x)\psi(x) + \mathrm{h.c.}] \\ \mathcal{R} &= -\frac{1}{2\sqrt{2m}} \int dx \mathrm{Tr}[e^{-i\pi/4}\phi^\dagger(x)\psi'(x) + \mathrm{h.c.}] \\ &\quad + \frac{1}{2N_c\sqrt{2m}} \int dx dy W(y-x) \\ &\qquad \times \mathrm{Tr}[e^{-i\pi/4}\phi^\dagger(x)\rho(y)\psi(x) + \mathrm{h.c.}]\,, \end{aligned} \tag{6}$$

where $\phi^\dagger(x)^\beta_\alpha$ is the bosonic creation operator, $\psi^\dagger(x)^\beta_\alpha$ is the fermionic creation operator, and $\rho^\beta_\alpha = [\phi^\dagger\phi + \psi^\dagger\psi]^\beta_\alpha$. The superalgebra is given by:

$$\{\mathcal{Q}, \mathcal{Q}\} = mM,\ \{\mathcal{Q}, \mathcal{R}\} = -P/2,\ \{\mathcal{R}, \mathcal{R}\} = H/2. \tag{7}$$

The last equation can be taken as the **definition** of the Hamiltonian for this system, which in turn implies that the model is invariant under both $\mathcal{Q}$ and $\mathcal{R}$.

If the function $W(x)$ is taken to be odd, the two-bit sector is equivalent to Witten's supersymmetric quantum mechanics [10], where $W(x)$ is the superpotential. In that case the ground state energy of a two-bit closed chain vanishes. For understanding superstring theory, we are interested in a class of superpotentials for which the ground state energy of **any** length chain vanishes, and for which the gap to excite the chain is finite. Exploring this issue for noninteracting chains, we consider the first-quantized system obtained by acting

with $\mathcal{R}$ on a bare chain and taking the limit $N_c \to \infty$. The first-quantized supercharge is then given by

$$R = \frac{1}{2\sqrt{2m}} \sum_{k=1}^{M} \Big\{ (e^{i\pi/4}\theta_k + e^{-i\pi/4}\pi_k)p_k \\ -(e^{-i\pi/4}\theta_k + e^{i\pi/4}\pi_k)W(x_{k+1} - x_k) \Big\} , \tag{8}$$

where $p_k = -i\partial/\partial x_k$ and $\pi_k = \partial/\partial\theta_k$. The summation is understood to be cyclic, i.e. $k = M+1$ is equivalent to $k = 1$. The first-quantized M-bit Hamiltonian is then given by

$$H_M = \frac{1}{2m} \sum_{k=1}^{M} \Big\{ p_k^2 + W^2(x_{k+1} - x_k) \\ + W'(x_{k+1} - x_k)[\theta_k\pi_k - \pi_k\theta_k \\ + \pi_{k+1}\theta_k - \theta_{k+1}\pi_k - i(\theta_k\theta_{k+1} + \pi_k\pi_{k+1})] \Big\}. \tag{9}$$

We denote states in the first-quantized Hilbert space of an M-bit chain by $|\cdots)$ to distinguish them from states in the second-quantized bit Fock space, denoted $|\cdots\rangle$. The ground state of the chain is then denoted by $|0)$.

Consider the linear superpotential

$$W(x) = T_0 x \, , \tag{10}$$

which defines the supersymmetric harmonic model. The ground state is annihilated by R implying it belongs to a "small" representation of the Galilei superalgebra and has zero energy. In addition its spectrum approaches that of a relativistic superstring with tension T_0 in the continuum limit. Consider a deformation of the superpotential

$$W(x) \to W(x) + \delta W(x) \, , \tag{11}$$

where $\delta W(x)$ is small in the interval $|x| < L$. If $L \gg 1/\sqrt{T_0}$ this deformation can be treated perturbatively, since for $|x| > L$ the unperturbed wavefunctions are exponentially small. Due to the Galilei superalgebra (7), the exact Hamiltonian can be written as the square of the new supercharge $R + \delta R$. The change in the energy of the ground state to first order is then:

$$\delta E_{0,M} = 4(0|\{R, \delta R\}|0) \,, \tag{12}$$

which vanishes since $R|0) = (0|R = 0$. We stress that this holds for **any** length chain.

The spectrum of excitations of the chain is generated by acting on the above ground state with mode raising operators. For the $2+1$-dimensional harmonic model these were derived in [5]. Dropping a dimension and with it the spinor indices gives:

$$A_n^\dagger = \frac{(\hat{p}_n + i\omega_n \hat{x}_n)}{\sqrt{2\omega_n}} \,,\; A_n = \frac{(\hat{p}_n - i\omega_n \hat{x}_n)}{\sqrt{2\omega_n}} \,, \tag{13}$$

where $\omega_n = 2T_0 \sin n\pi/M$, for the coordinate modes raising and lowering operators, and

$$B_n^\dagger = \xi_n \hat{\theta}_n + \eta_n \hat{\pi}_n \,,\; B_n = \eta_n \hat{\theta}_n + \xi_n \hat{\pi}_n \,, \tag{14}$$

where $\xi_n = (1/\sqrt{2})(\sin n\pi/2M + \cos n\pi/2M)$ and $\eta_n = (1/\sqrt{2})(\sin n\pi/2M - \cos n\pi/2M)$, for the "statistics" modes raising and lowering operators. In the above $\hat{x}_n, \hat{\theta}_n, \hat{p}_n, \hat{\pi}_n$ are the Fourier transforms of $x_k, \theta_k, p_k, \pi_k$, respectively. Consider an excitation with a single raising operator $A_n^\dagger|0)$. The zero'th order energy of this state is given by

$$E_{n,M}^{(0)} = (0|A_n H A_n^\dagger|0) = \frac{2T_0}{m} \sin \frac{n\pi}{M} \,. \tag{15}$$

The shift in the energy due to the deformation (11) is given to first order by

$$\delta E_{n,M} = 4(0|A_n\{R, \delta R\}A_n^\dagger|0) \,. \tag{16}$$

Using the commutation relations

$$\begin{aligned} [A_n, R] &= -\frac{i}{2} e^{i\pi n/2M} \sqrt{\frac{\omega_n}{m}} B_n \\ [A_n, \delta R] &= \frac{-\sin n\pi/M}{2\sqrt{\omega_n m M}} \sum_k \Big\{ (e^{-i\pi/4}\theta_k + e^{i\pi/4}\pi_k) \\ &\qquad \times e^{i\pi n(2k+1)/M} \delta W'(x_{k+1} - x_k) \Big\} \,, \end{aligned} \tag{17}$$

and the fact that $(0|\delta W'(x_{k+1} - x_k)|0)$ is cyclically invariant we find

$$\begin{aligned} \delta E_{n,M} &= -2ie^{i\pi n/2M} \sqrt{\frac{\omega_n}{m}} (0|B_n \delta R A_n^\dagger|0) + \text{h.c.} \\ &= \frac{2}{m} (0|\delta W'(x_2 - x_1)|0) \sin \frac{n\pi}{M} \,. \end{aligned} \tag{18}$$

The net effect is then just to shift the string tension

$$T_0 \to T_0 + (0|\delta W'(x_2 - x_1)|0) \,, \tag{19}$$

so the gap remains finite for all M. For excitations of the form $\prod_i A^\dagger_{n_i}|0)$, with more than one raising operator, we have

$$\begin{aligned} \delta E &= 4(0|\prod_i A_{n_i}\{R, \delta R\}\prod_i A^\dagger_{n_i}|0) \\ &= -2i\sum_j e^{i\pi n_j/2M}\sqrt{\frac{\omega_{n_j}}{m}}(0|B_{n_j}\prod_{i\neq j} A_{n_i}\delta R\prod_k A^\dagger_{n_k}|0) \\ &\quad +\text{h.c.}\,. \end{aligned} \tag{20}$$

As we commute the $A^\dagger_{n_k}$'s to the left we pick up an additional derivative of δW and a factor of $\sqrt{\omega_{n_k}/M} = O(1/M)$ for each $A^\dagger_{n_k}$ that contracts with δR. The rest contract with some of the A_{n_i}'s. The remaining A_{n_i}'s are then commuted to the right, all contracting with δR to produce additional derivatives and additional powers of $1/M$. Finally, B_{n_j} is contracted with what's left. The result is a sum of terms of increasing odd number of derivatives of δW multiplied by increasing powers of $1/M$, the coefficient of $\delta W^{(2l+1)}$ being proportional to M^{-2l-1}. In the limit $M \to \infty$ only the the $l = 0$ term contributes, so the analogue of (18) holds for any excitation of the form $\prod_i A^\dagger_{n_i}|0)$. The argument for excitations created by products of $B^\dagger_n$'s or products of both $A^\dagger_n$'s and $B^\dagger_n$'s is similar. Thus the only effect of the deformation (11) on long chains is to renormalize the string tension as in Eq.(19).

We now argue that these results from first order perturbation theory hold to all orders, and indeed should extend to a large universality class of superpotentials. The ground state energy must remain zero as long as the ground state is in a "small" representation of the superalgebra. Clearly the representation can't change in perturbation theory, but more generally it will remain "small" unless the first excited state becomes degenerate with the ground state, i.e. unless the gap closes. Moreover, the properties of the $O(T_0/mM)$ excitations of long chains have a universal character determined by phonon and statistics waves, which are inevitable collective excitations of stable long chains [4]. Formation of long bare

chains is ensured by a large bond-breaking energy and does not require the infinite range harmonic force.

Finally we turn to the issue of interactions between closed chains. By exploiting the $N_c \to \infty$ limit, we have been able to define a **bare** closed chain, whose interactions with other closed chains is negligible. Taking N_c to be finite will "dress" the bare chains, and will give rise to interactions between different chains. The ionization energy required to break a bond in the closed chain is $O(T_0/m)$. As long as the scattering energy is below threshold for such ionization, the only way for interactions to occur is through bond rearrangement. A necessary condition for defining a closed chain S-matrix is that the model satisfy a **clustering** property: An initial state of two spatially separated closed chains must evolve as two noninteracting chains until enough time has elapsed for them to get close to one another. With a linearly growing superpotential as in (10), this is impossible at finite N_c. The supersymmetric harmonic model is thus unsatisfactory for describing chain scattering. Asymptotically free chains require a bounded superpotential. In order to keep the ionization energy $O(T_0/m)$, however, we must still insist that $W \to \pm W_\infty \neq 0$ at spatial infinity. The restricted universality established above indeed allows us to deform the linear superpotential into

$$W = T_0[x + (L-x)\theta(x-L) - (L+x)\theta(-L-x)], \tag{21}$$

with no change in the continuum properties of free chains.

With the bounded superpotential (21) there can still be large ($O(T_0/m)$) correlation energies between spatially separated clumps of bits. If the clumps correspond to closed chains, or more generally to singlet states, these correlation energies must vanish. This can be achieved by first noting that the generators for color rotations given by

$$G^\beta_\alpha = \int dx[\rho(x) - \sigma(x)]^\beta_\alpha \,, \tag{22}$$

where $\sigma^\beta_\alpha =: [\phi\phi^\dagger - \psi\psi^\dagger]^\beta_\alpha :$, annihilate singlet states. This motivates replacing $\rho(y)$ with $\rho(y) - \sigma(y)$ in Eq. (6) for $\mathcal{R}$. It is easy to check that such a change does not disturb the

superalgebra. To see that it gives the desired clustering property, consider the action of the new $\mathcal{R}$ on a state we denote by $S_1S_2|0\rangle$, where S_1 and S_2 are any color singlet functions of creation operators, such that the locations of all creation operators in S_1 are more than a distance L from all the locations in S_2. Let $(rS)_1$ and $(rS)_2$ be the singlet functions of creation operators defined by

$$\mathcal{R}S_1|0\rangle = (rS)_1|0\rangle \ , \ \mathcal{R}S_2|0\rangle = (rS)_2|0\rangle \ . \tag{23}$$

Then

$$\mathcal{R}S_1S_2|0\rangle = (rS)_1S_2|0\rangle + (-)^{S_1}S_1(rS)_2|0\rangle + \mathcal{R}_IS_1S_2|0\rangle \tag{24}$$

where the action of $\mathcal{R}_I$ is defined by requiring one of the two annihilation operators in the two-body term of $\mathcal{R}$ to contract with a creation operator in S_1, and the other annihilation operator to contract with a creation operator in S_2. Because of the spatial separation of the coordinates in S_1 and the coordinates in S_2, the superpotential $W(x)$ can be replaced with its asymptotic value and taken out of the integral. Consequently one is left with the color rotation operator $\int(\rho - \sigma)$ acting on S_1 or S_2, either of which gives zero. Thus we conclude that

$$\mathcal{R}S_1S_2|0\rangle = (rS)_1S_2|0\rangle + (-)^{S_1}S_1(rS)_2|0\rangle \ . \tag{25}$$

Applying $\mathcal{R}$ once again, using the supersymmetry algebra, and remembering that the supercharge is odd, we infer:

$$HS_1S_2|0\rangle = 4[(r^2S)_1S_2|0\rangle + S_1(r^2S)_2|0\rangle], \tag{26}$$

implying that the hamiltonian acts independently on the two singlets. Therefore as long as they remain spatially well separated the two singlets propagate without mutual interaction.

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