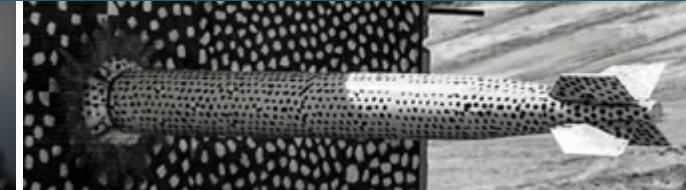
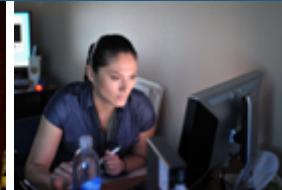




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# Estimating seismic source attributes using seismoacoustic models and acoustic, seismic, and rotational motion data



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# Goal of our present work



## 1) Estimate effective seismic source attributes of underground explosions

- Based on linear inversions
- Assume:
  - Time variable seismic source functions
  - Arbitrary numbers and locations of sources
  - Jointly invert multiple datatypes

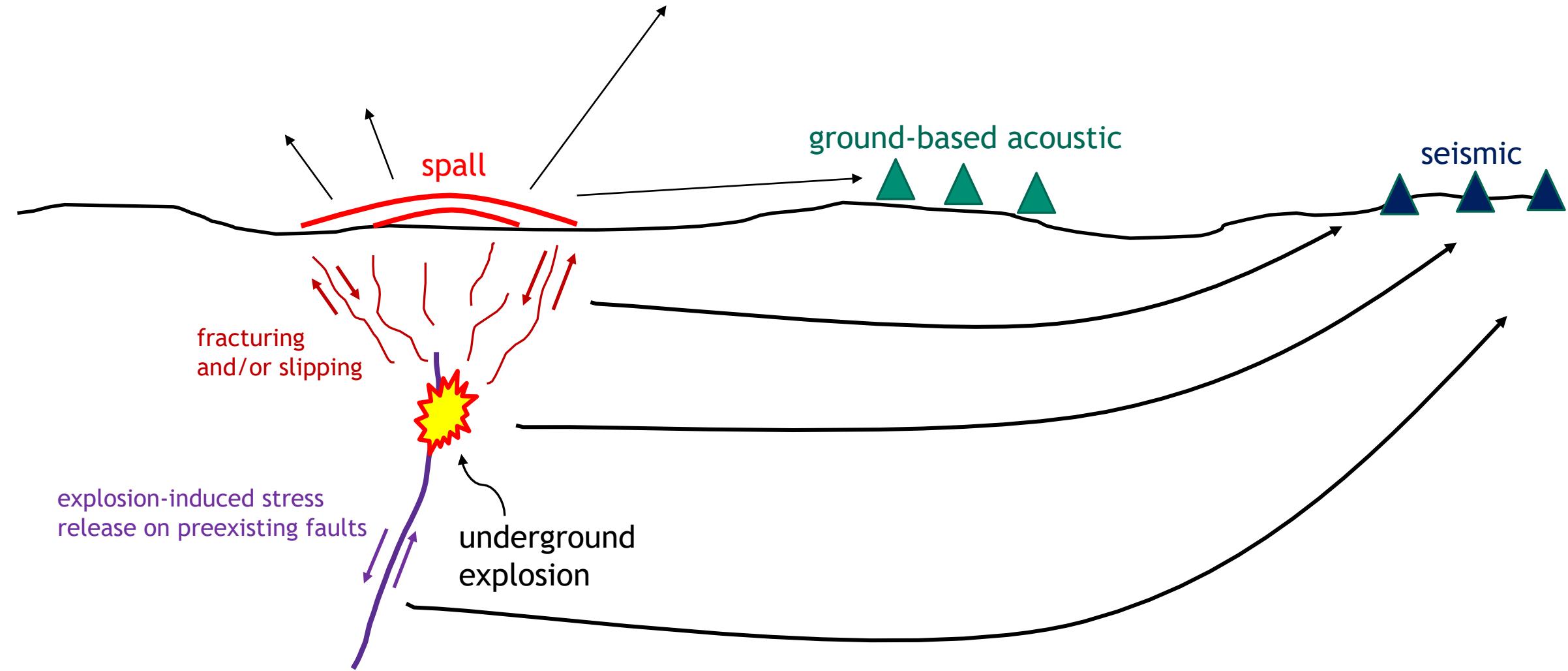
## 2) Questions we are asking

- Are elastic models strictly necessary?
- Do more data types increase accuracy or mitigate the effects of noise and/or model uncertainty?
- Can we resolve time-varying source mechanisms?

3 Conceptual model(s)



balloon-borne acoustic



# Classical linear inversion: Moment Tensors

4

$$u_i(t) = \sum_{j=1}^6 \widehat{G}_{ij}(t) m_j \xrightarrow{\text{pseudo-Green's functions scaled by } m_j} \mathbf{u} = \mathbf{G}\mathbf{m} \xrightarrow{\text{solve for } \mathbf{m}} \mathbf{m}_j = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix} \rightarrow M_{ij} \rightarrow \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yy} & m_{yz} & m_{zz} \\ m_{zz} & & \end{bmatrix}$$

“absorb” source function into the forward model by convolving it with the Green’s function

$$\widehat{G}_{ij}(t) = \int_{-T}^T G_{ij}(t-\tau) s(\tau) d\tau$$

$s(t)$  is the source function; assumed, *a-priori*

## Assumptions

- linear
- single point source
- source function identical for all components of the MT
- low frequency wavefield insensitive to Earth heterogeneities

→ works pretty good for low frequency teleseismic and/or global scale data ←

## Relax assumptions:



1) Multiple sources such as explosion, fault release, spall, etc.

$$u = (\hat{G}m_{explosion}) + (\hat{G}m_{spall}) + \dots$$

2) source functions can be independent and time variable

$$u = (G \otimes m_{ij}(x_1, t_{ij})_{explosion}) + (G \otimes m_{ij}(x_2, t_{ij})_{spall}) + \dots$$

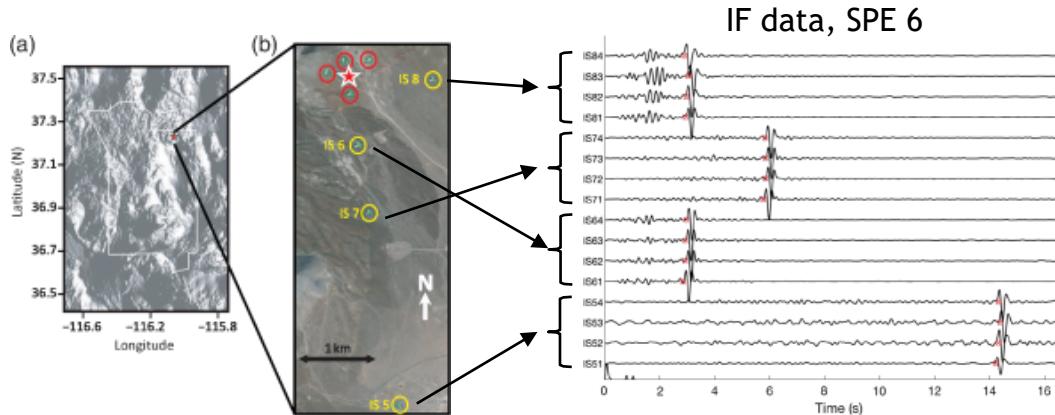
3) several wavefield types (seismic, acoustic, rotational, etc. )

$$\begin{bmatrix} u_{seismic} \\ u_{acoustic} \\ \vdots \\ u_{rotational} \end{bmatrix} = \begin{bmatrix} (G_{seismic} \otimes m_{expl}) + (G_{seismic} \otimes m_{spall}) + (G_{seismic} \otimes m_{EQ}) + \dots \\ (G_{seismic} \otimes m_{expl}) + (G_{seismic} \otimes m_{spall}) + \dots \\ \vdots \\ (G_{rot} \otimes m_{expl}) + (G_{rot} \otimes m_{EQ}) + \dots \end{bmatrix}$$

# 1) Multiple source types

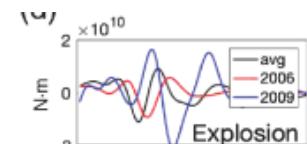


- Experiment: Source Physics Experiment Phase I
- Data: locally (<5km) recorded Infrasound produced by buried chemical explosion
- Source model: (buried) isotropic explosion and (surface) spall
- Estimate Green's functions using finite differences
  - constant wavespeed ACOUSTIC earth model
  - atmospheric model based on meteorological observations
  - wave propagation is fully ACOUSTIC
- Invert for source time functions

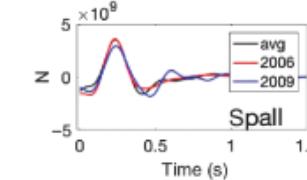


$$u_{\text{infrasound}} = (Gm_{\text{explosion}}) + (Gm_{\text{spall}})$$

estimated STFs (SPE 6)



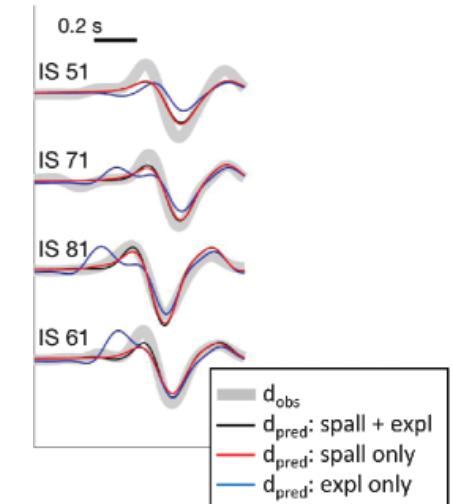
estimated isotropic source function



estimated spall source function

different colors correspond to different atmospheric models

fit to data (SPE 6)



different colors correspond to different source model assumptions

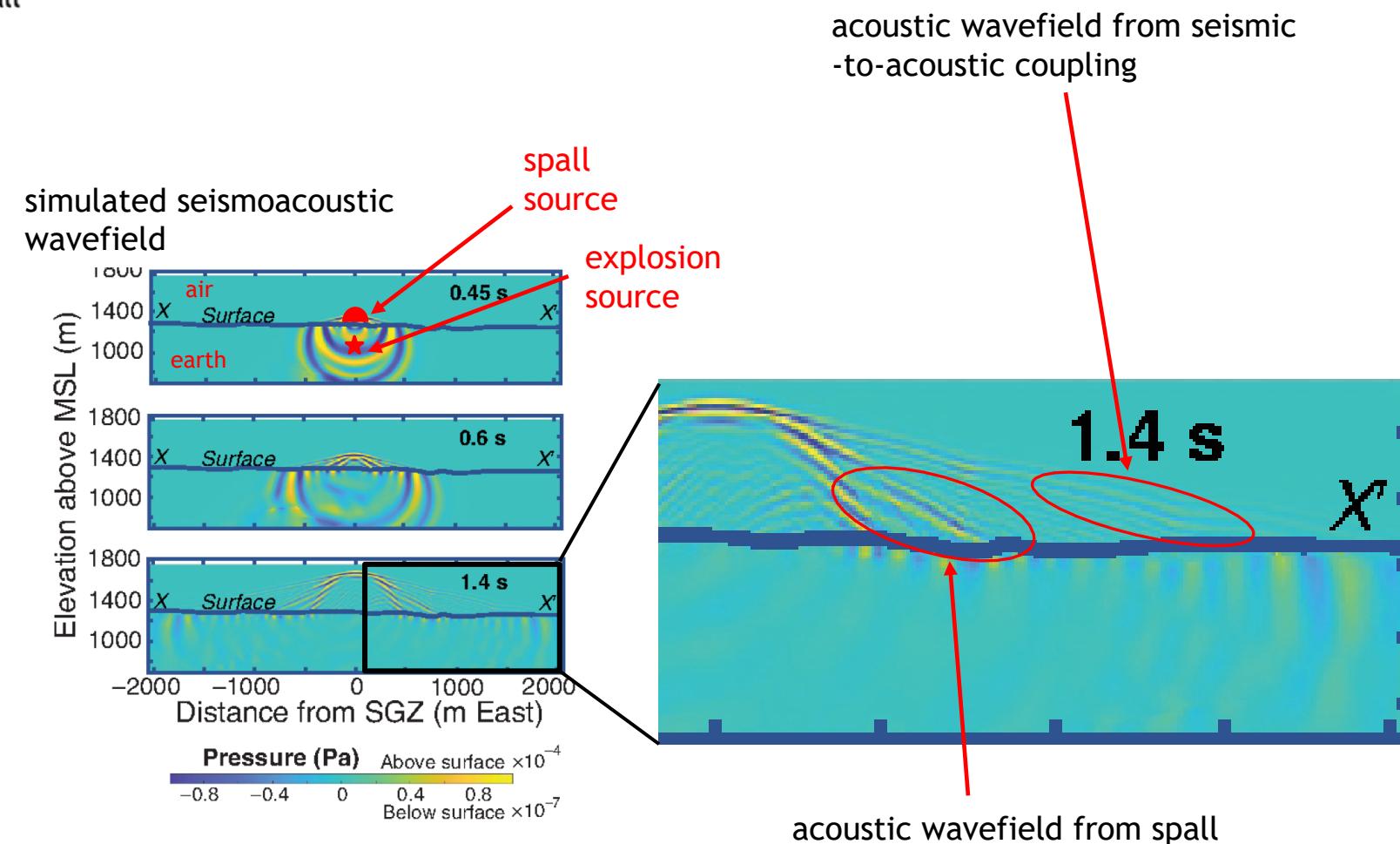
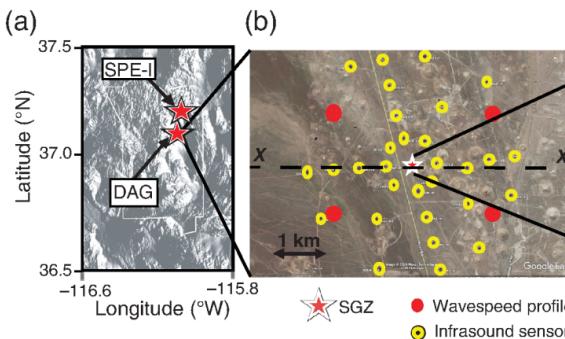
## Results / conclusions

- 1) Spall source “well behaved”, relatively insensitive to atmospheric model
- 2) Explosion source is a mess
- 3) Predicated data is almost totally controlled by spall term (suggests buried source term lives in the null space for this setup)
  - Question: is this because spall is the primary driver of the infrasound **OR** is it because our forward model doesn't do a good job predicting the elastic part of the wavefield?

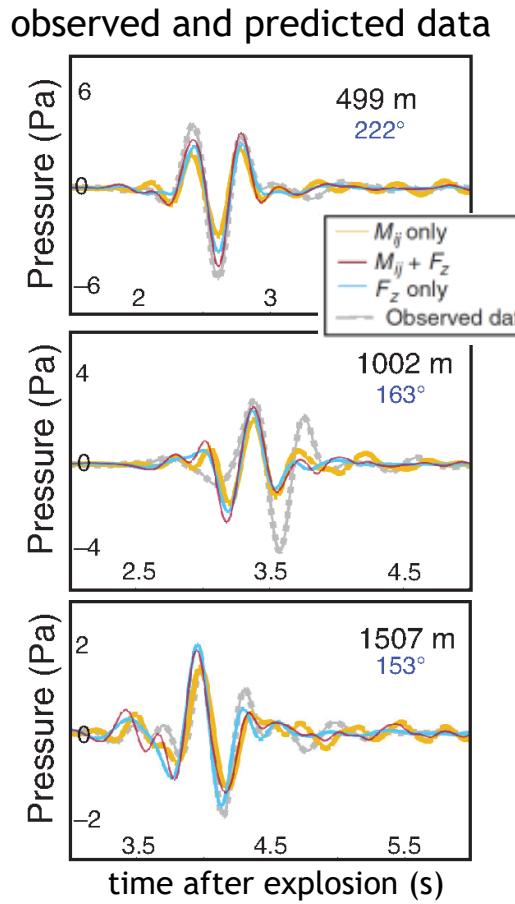
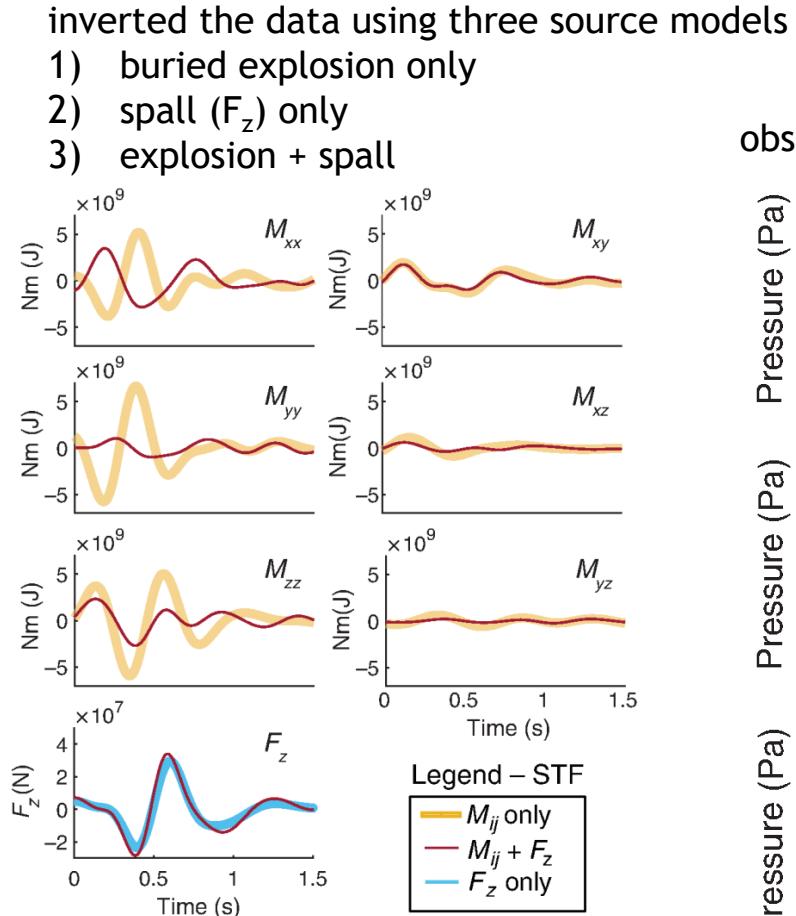
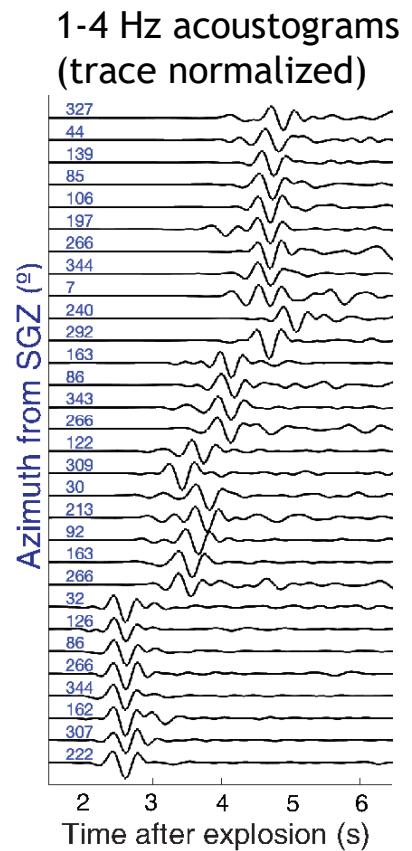
## 1) Multiple source types, fully elastic model

- Experiment: Source Physics Experiment Dry Alluvium Geology (DAG)
- Data: locally (<3km) recorded Infrasound produced by buried chemical explosion
- Source model: (buried) explosion and (surface) spall
- Estimate Green's functions using finite differences
  - Coupled elastic/acoustic model
  - wave propagation is seismoacoustic
- Invert for source time functions ( $M_{ij}$  and  $F_z$ )

recorded infrasound produced by a buried ( $z=-52\text{m}$ ) explosion: 10.4 metric tons (TNT equivalent)



# 1) Multiple source types, fully elastic model



## Results:

- 1) Buried, full-tensor source is MUCH better resolved
- 2) explosion + spall model fits the data the best, but...
- 3) Spall-only model still describes most of the data, however...
- 4) Without spall source, the isotropic portions of  $M_{ij}$  attempt to compensate
  - $M_{ij}$  looks very CLVD-like

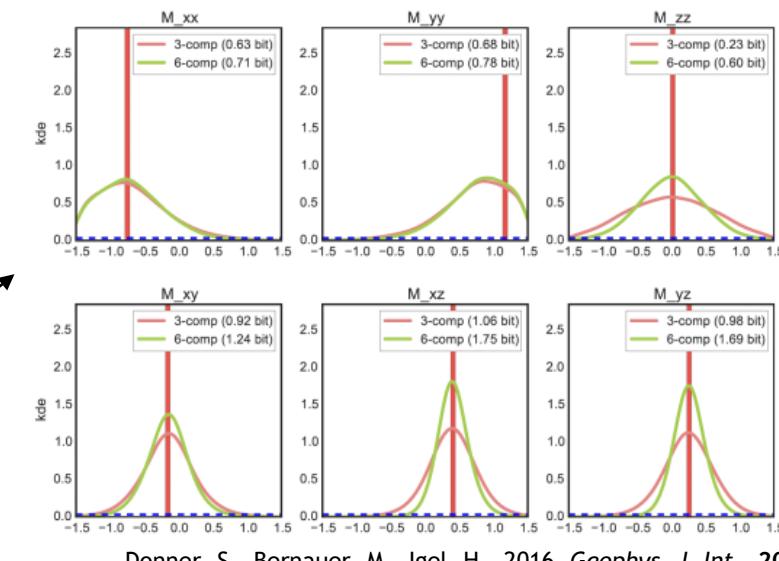
## Conclusion:

There is A LOT of value in using the full seismoacoustic model when inverting IF data produced by buried explosions

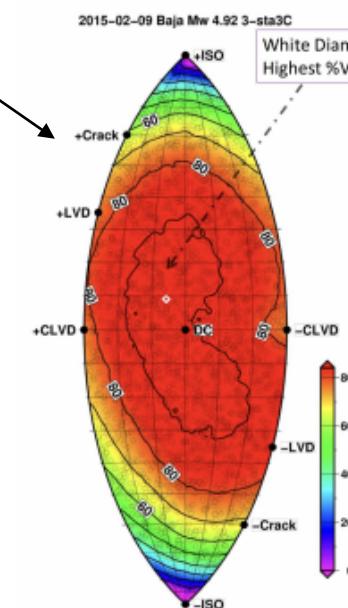
# Several Wavefield types



- translational seismograms + rotational motions
- Previous methods estimated
  - scalar MT components (e.g. “beachball”)
  - PDFs of MT components via Bayesian methods (e.g. Donner et al., 2016, 2020; Bernauer et al., 2014)
  - misfit of all source mechanisms (e.g. Ichinose et al., 2019)
- Previous work suggested that adding rotational motions helps to decrease uncertainty in scalar MTs (and hence seismic mechanism) FOR LOW FREQUENCY TELESEISMIC DATA
- We extend this idea to estimate time-variable source functions:
  - translation + rotational data
  - high frequency explosion source data
  - effects of model uncertainty



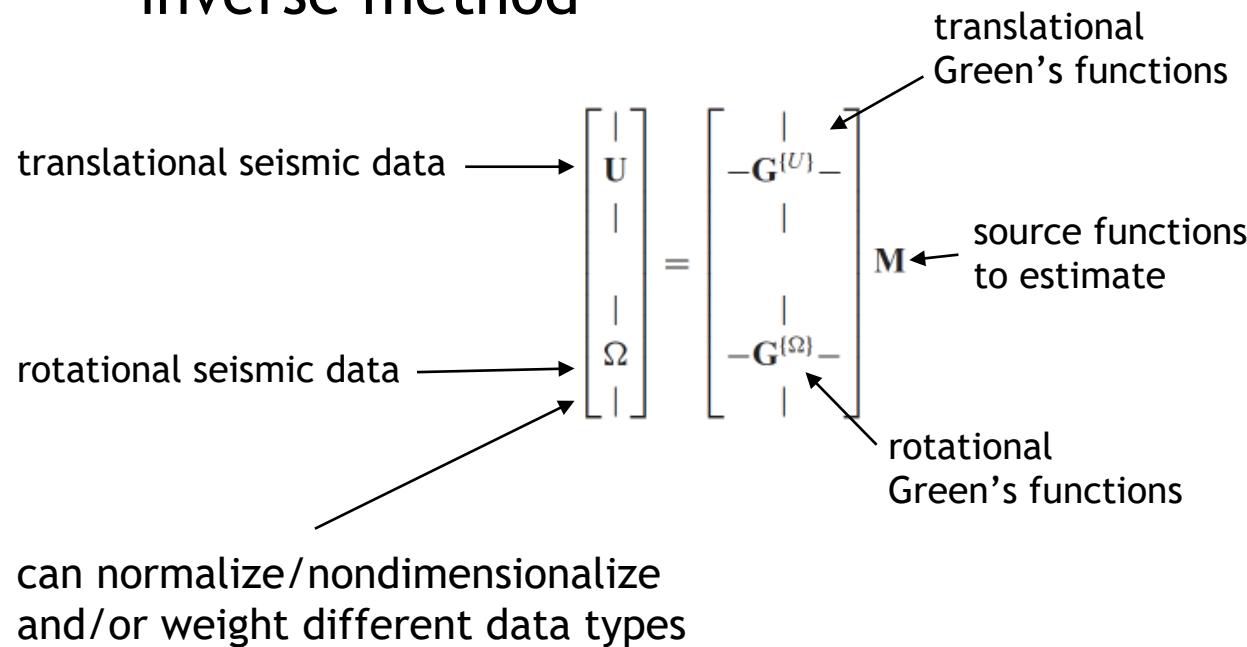
Donner, S., Bernauer, M., Igel, H., 2016, *Geophys. J. Int.*, 207, 562-570



Ichinose, G.A., Ford, S.R., Mellors, R.J., 2019, AGU Fall 2019 Meeting, S032

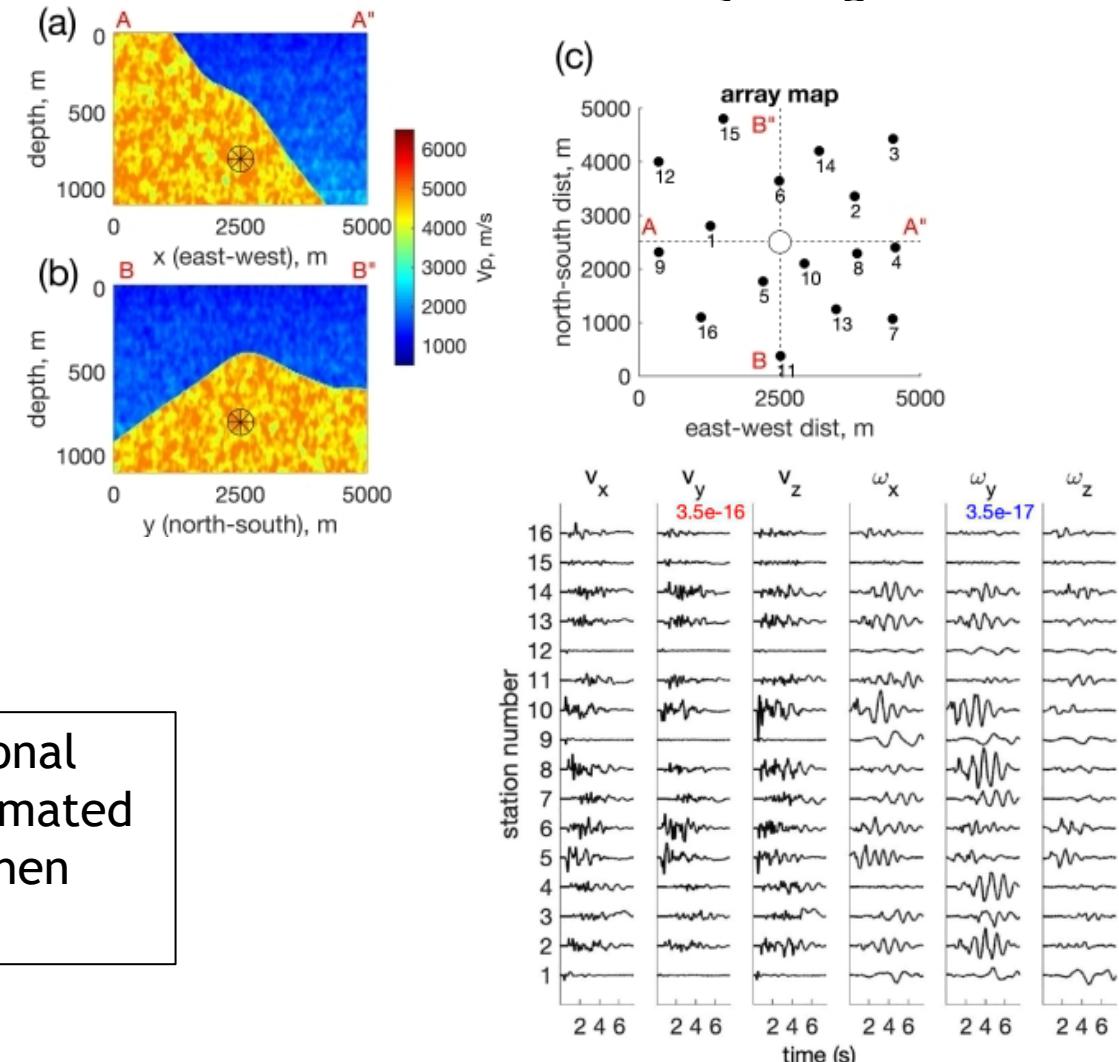
# Several Wavefield (data) types

## inverse method



**Hypothesis we test:** The addition of rotational motions reduces the uncertainty in the estimated source functions for high-frequency data when the earth model is uncertain.

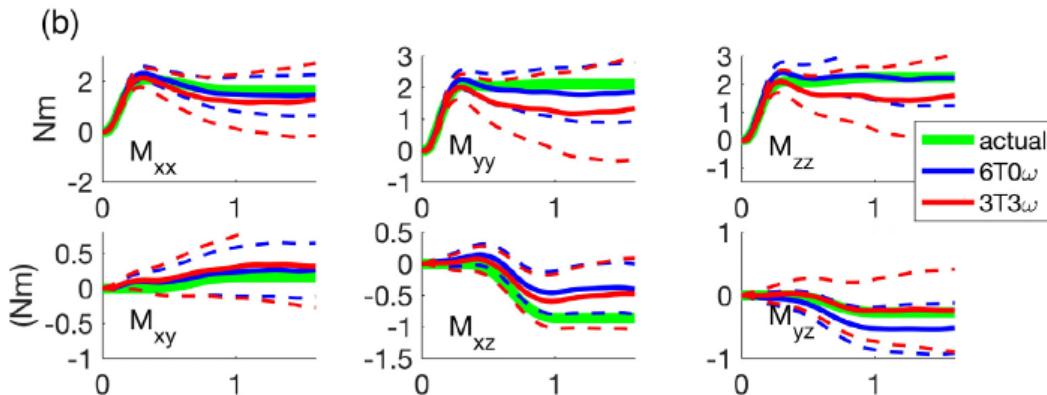
Test with numerical experiments,  
Effects of model uncertainty using Monte Carlo



# Results: mixed

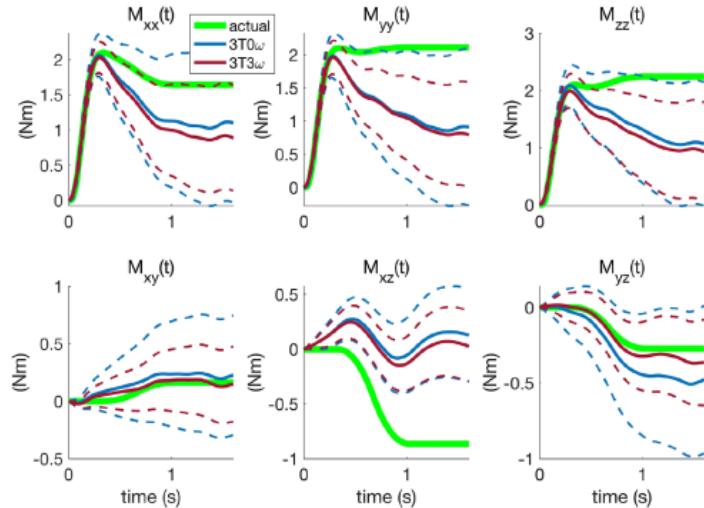


1) For “many” stations/channels, adding rotational motions INcreased uncertainty of the results!

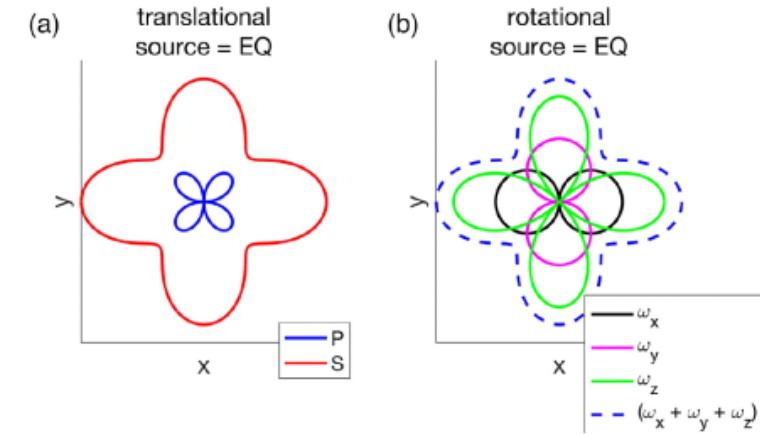


Why??

2) However.... given a small number of stations, adding rotational motions (at same location) decreases uncertainty. Likely due to simply having more channels of data to invert.



- Rotational motions are more sensitive to model heterogeneity, and thus uncertainty.
- Theoretical combined radiation pattern of all rotational motions (Aldridge, 2000) is virtually identical to that of translational shear waves.



- This is at odds with previous work based on rotational radiation patterns given by Cochard et al., (2006), eqn 30.4, which predicts the radiation patterns of these two datatypes to be different: → combining two data types should “fills in” holes in the radiation pattern.
- Who's right??

# Time variable source mechanisms



Current techniques to estimate source mechanism assume that the source is a point in space and time.  
Too restrictive for low-yield, local scale data?

We've already developed methods to account for multiple sources and independence of source functions, so let's simply compute the mechanism as a function of time.

$$u_i(t) = \sum_{j=1}^6 \widehat{G}_{ij}(t) m_j$$

$$\widehat{G}_{ij}(t) = \int_{-T}^T G_{ij}(t-\tau) s(\tau) d\tau$$

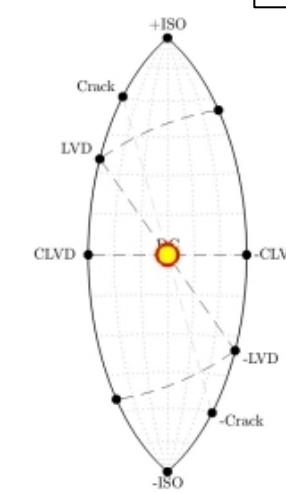
moment tensor

$$m_j \rightarrow M_{ij} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \rightarrow \text{SVD} \rightarrow [\lambda_1 \ \lambda_2 \ \lambda_3]$$

$s(t)$  is the source function;  
assumed, *a-priori*



■ T-axis  
✗ P-axis



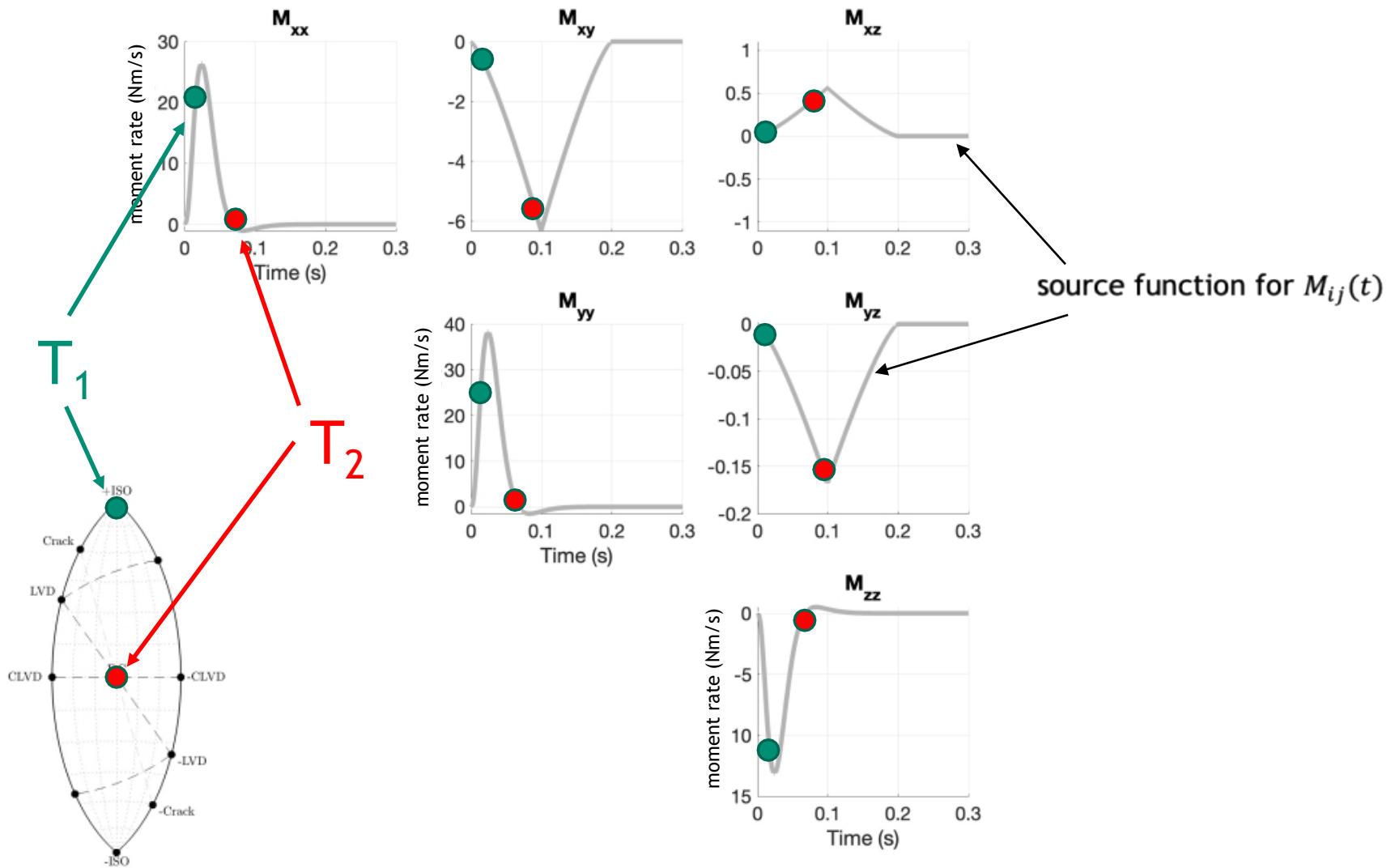
use eigenvalues to calculate spherical coordinates on the fundamental wedge

# Time variable source mechanisms



Concept:

- 1) estimate time-variable source functions  $M_{ij}(t)$ ,
- 2) extract amplitude at discrete time points (or windows)  $M_{ij}(t = T_i)$ ,
- 3) Decompose  $M_{ij}(t = T_i)$  to obtain eigenvalues,
- 4) plot on a lune



# Time variable source mechanisms

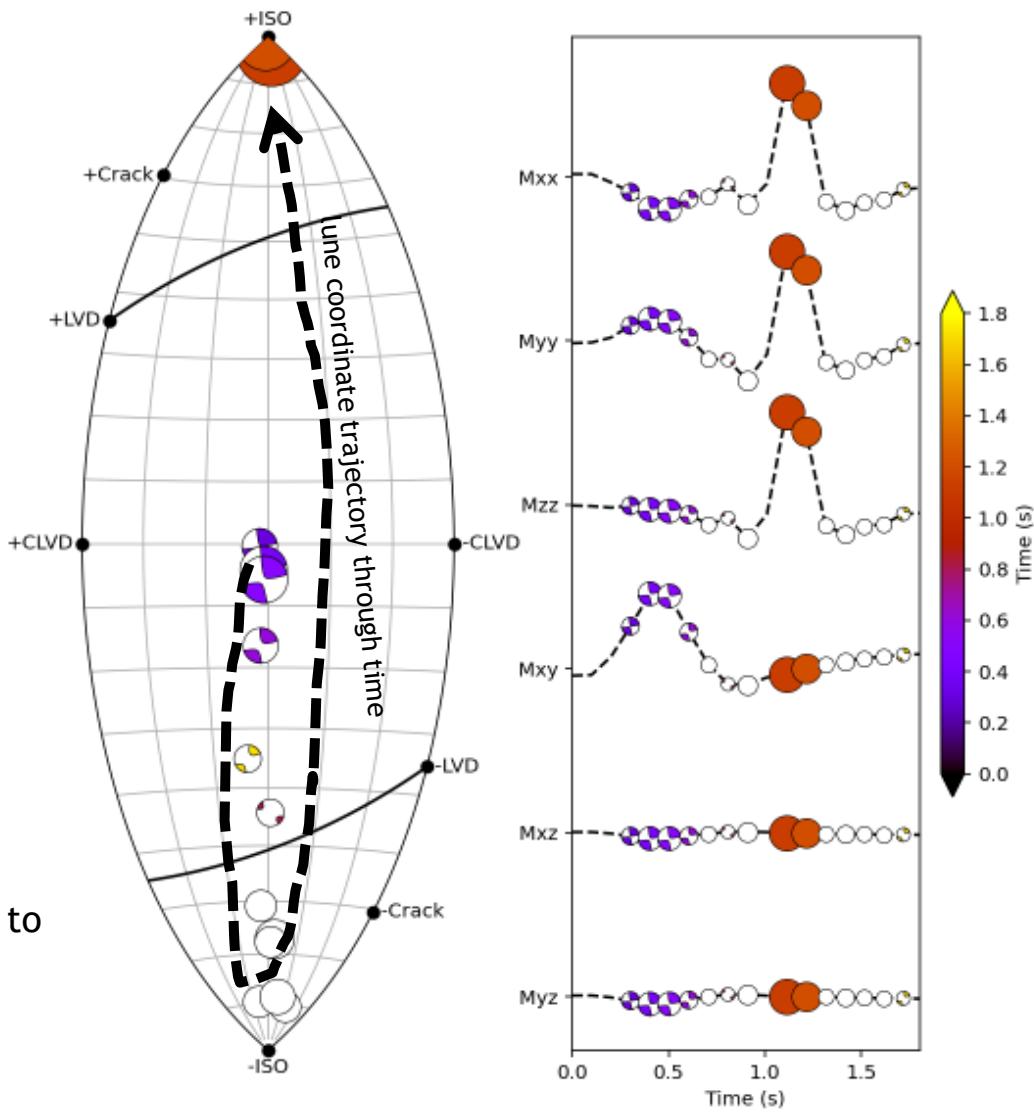


Numerical test: Two sources, offset in time: double couple followed by isotropic.

In concept, we can combine time-variable source function estimates into traditional source mechanism estimates.

This is a work in progress!

- size of beachball corresponds to magnitude
- color corresponds to time





# Concluding remarks

- 1) We've successfully developed and implemented a linear inversion method that estimates time-varying source functions for fully elastic Earth models and multiple source types
- 2) We can now combine multiple data types, such as translational seismic, acoustic, and rotational motions into a joint inversion for source functions
- 3) We've developed a method to estimate time-varying source mechanisms based on the inversion schemes in bullets 1 and 2

## Questions we asked

Are elastic models strictly necessary?

→ no, but they give us more robust source estimates for buried explosions when using acoustic data

Do more data types increase accuracy or mitigate the effects of noise and/or model uncertainty?

→ No and Yes: for many stations and uncertain propagation models, adding rotational motions may increase uncertainty;

→ However, for a small number of stations, the additional rotational channels appears to decrease uncertainty

Can we resolve time-varying source mechanisms?

→ So far, it looks promising!