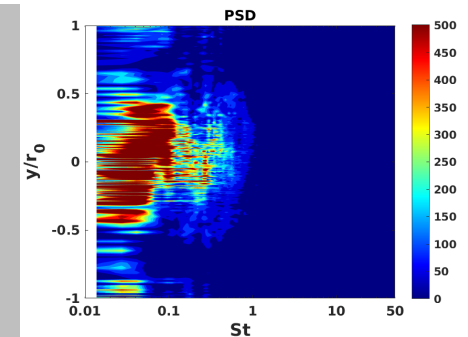
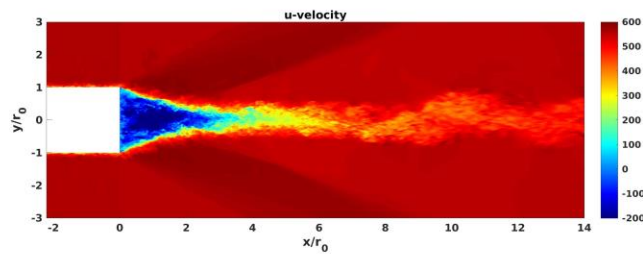


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Base Pressure Fluctuation Modeling: Theory, Simulation and Measurement

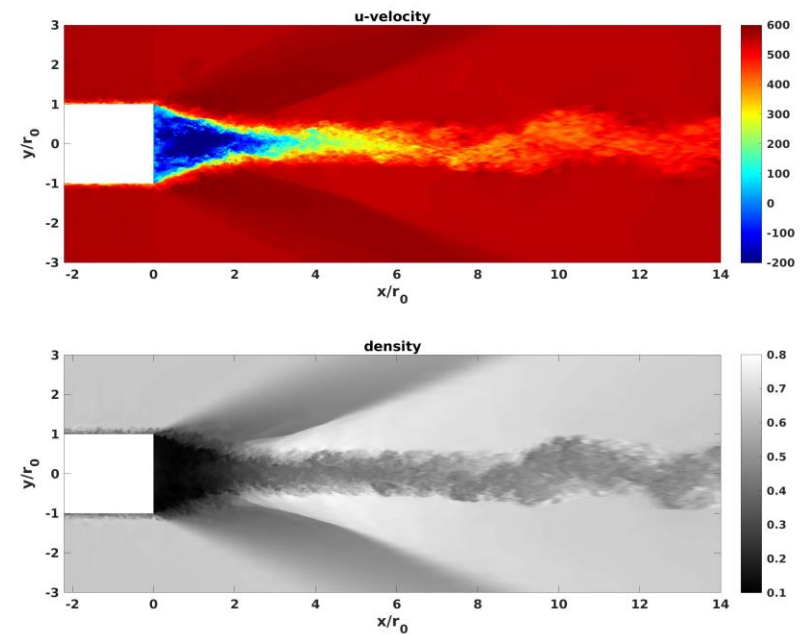
Lawrence DeChant, Brian Robbins, Cory Stack, Ashley Saltzman



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Base Pressure Fluctuations...

- Base pressure fluctuation
- Can induce significant system vibratory input
- Potentially important for some FSI problem classes

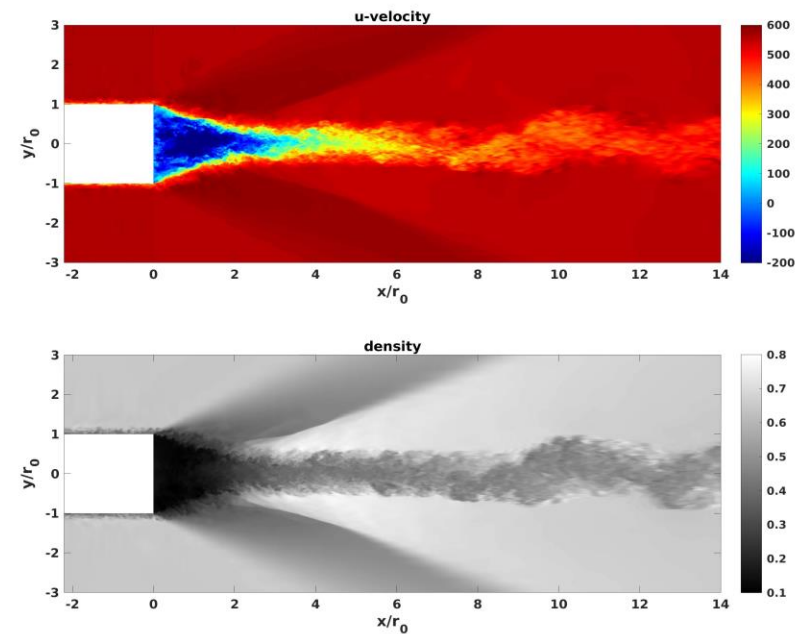


Outline

- **Introduction**
- **PSD Model DeChant and Smith (2011)**
- **Critical Strouhal Number**
 - Janssen and Dutton
 - Offset sting
 - Shear layer instability
- **Loading Amplification Theory**
- **RMS behavior**
- **Mean Base Pressure Estimates**
- **Conclusions**
- **Future Work**

Base Pressure Fluctuations...

- Base pressure fluctuation
- Can induce significant system vibratory input
- Potentially important for some FSI problem classes



- Base pressure fluctuation PSD essential for FSI problem
- Theory-based model Dechant and Smith (2011)
- Leverages Ahlborn et. al. (2002)

$$\Phi_{pp}(\omega) = \Phi_{mag} \frac{(a^2 + c^2 + \omega^2)}{(a^2 + c^2 - 2a\omega + \omega^2)(a^2 + c^2 + 2a\omega + \omega^2)}$$

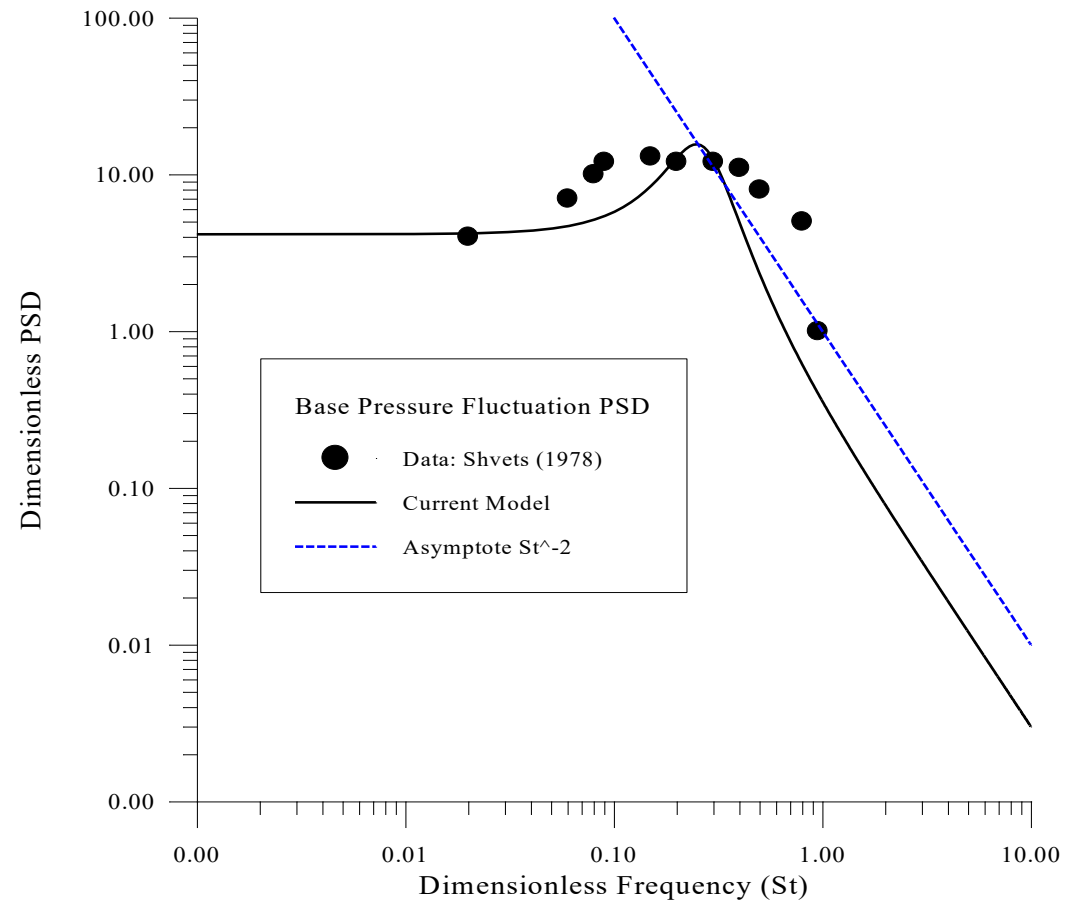
- Where

$$a = \left(\frac{1.53 S_t (1 + C_D)}{2\pi^2 C_D} \right)^{1/2} \quad \Phi_{mag} = \frac{D}{U} \frac{(1.75(2)(K_C) a q_e)^2}{(1 + M_e)^4}$$

- Notice that the critical frequency “a” depends on body drag Cd

- PSD model Dechant and Smith (2011)
- Note maximum amplitude near critical Strouhal number

$$St = \frac{\frac{1}{2\pi^2} K(1 + C_D)}{C_D + \frac{4NK}{Re}}$$



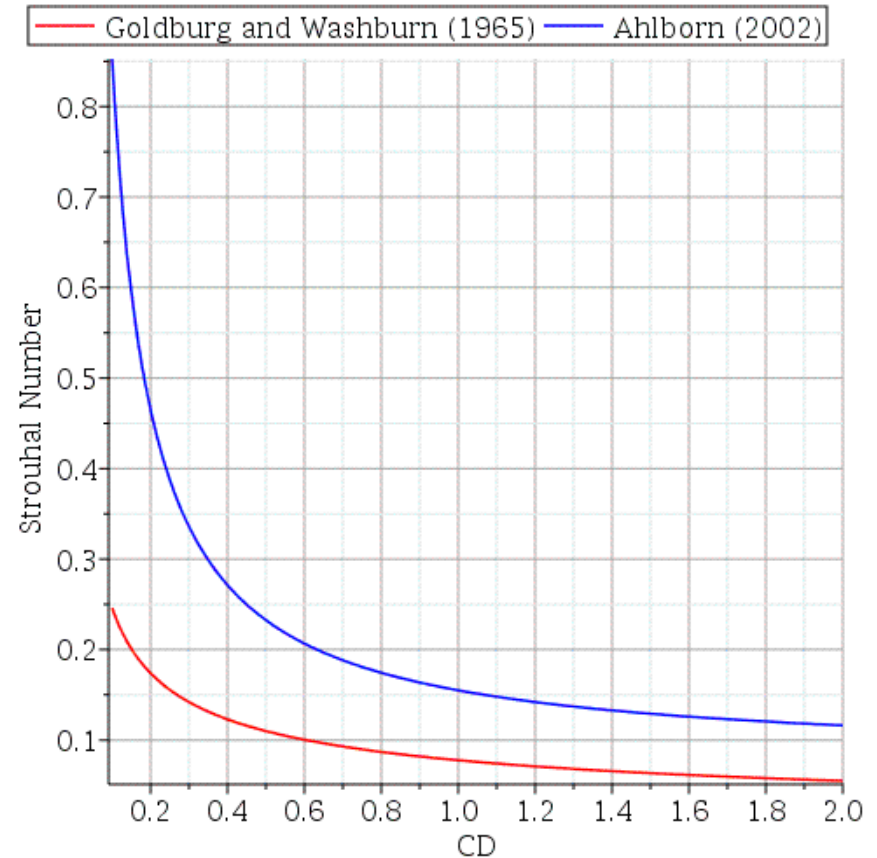
Critical Strouhal Number

- Critical Strouhal number
- Notice the dependence on C_D

$$St = \frac{\frac{1}{2\pi^2} K(1 + C_D)}{C_D + \frac{4NK}{Re}}$$

- Low frequency requires large drag
- Other models known

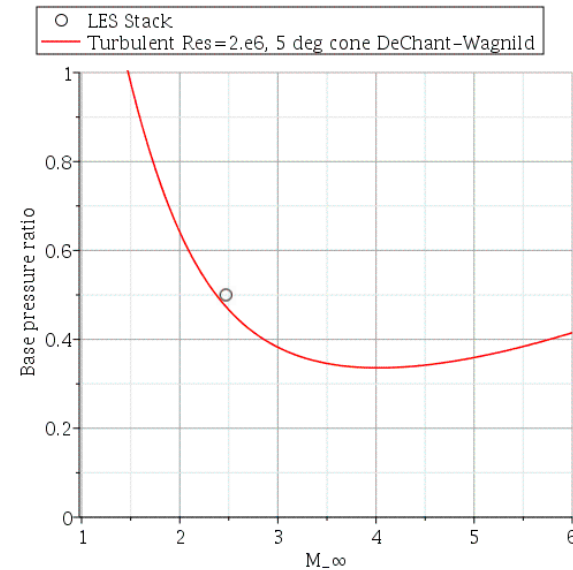
$$St_D = \frac{fD}{U} \approx \frac{\sqrt{2}}{4} (0.22) C_D^{-1/2}$$



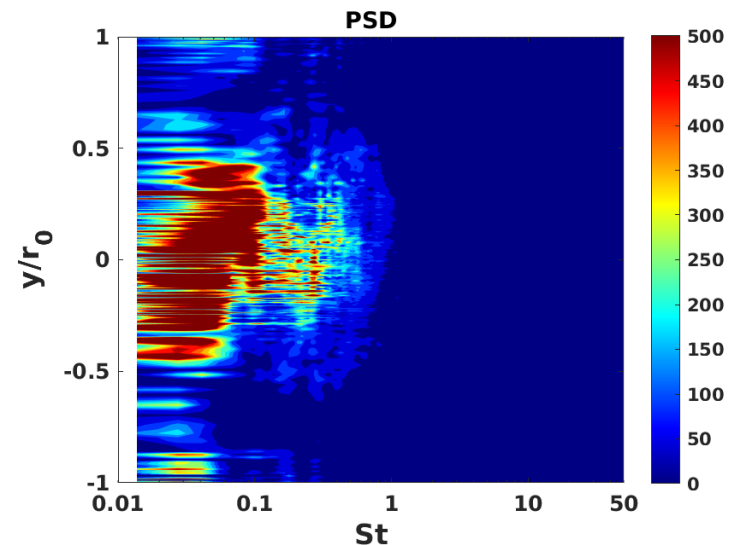
Critical Strouhal... Janssen and Dutton/Simulation

- Bluff body base unsteadiness
- Janssen and Dutton (2004)
- Base pressure ratio correlated with drag

$$C_{D_base} = 2 \left(1 - \frac{p_b}{p_\infty} \right) \left(\frac{q_\infty}{p_\infty} \right)^{-1} = \frac{2}{\gamma M_\infty^2} \left(1 - \frac{p_b}{p_\infty} \right)$$



- Theory... $St=0.09$
- Reasonable agreement with computation



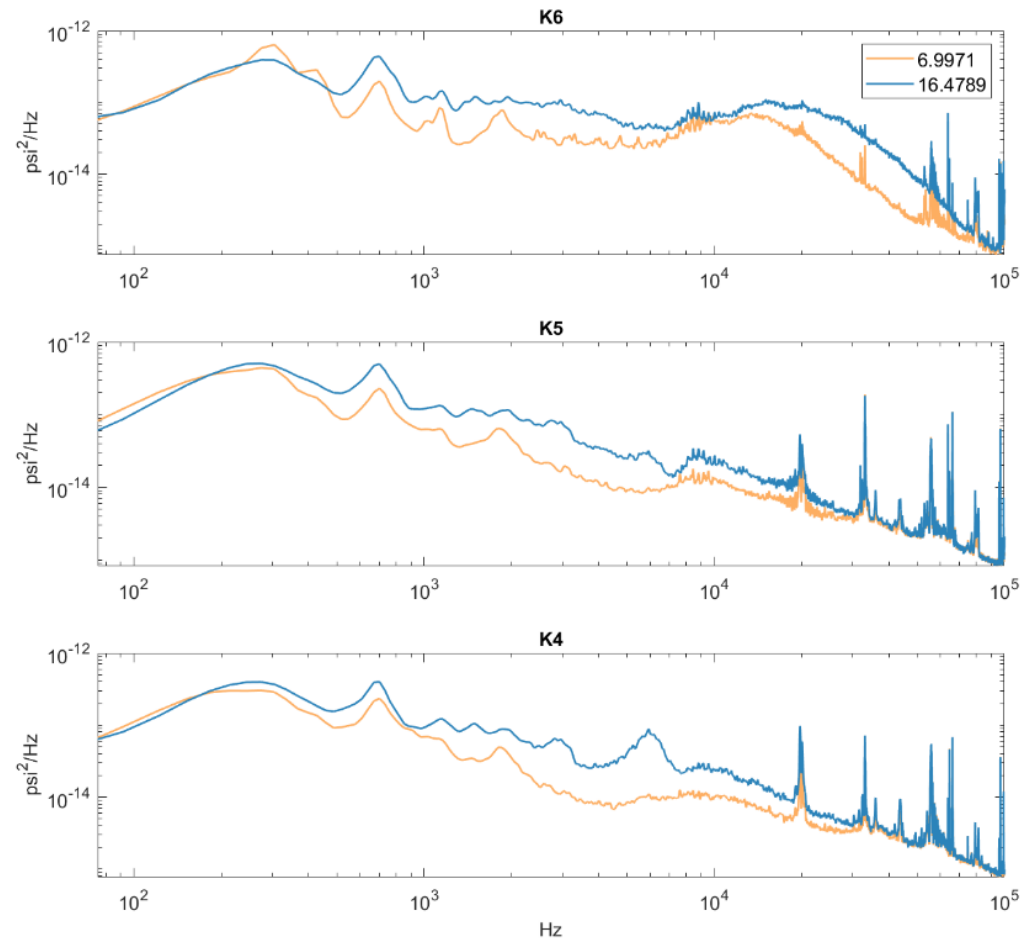
Critical Strouhal number... Offset Sting

- 5 degree Mach 8 cone experiments

- Zhang et. al. (2019)

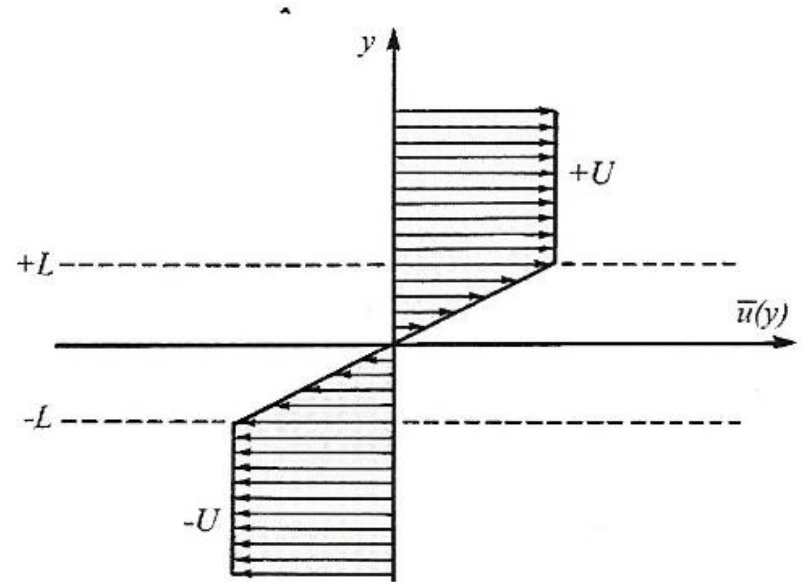
$$St = \frac{fD}{U} = \frac{(700)(0.077)}{1160} = 0.046$$

- Presence of sting significantly modifies base pressure region and changes critical Strouhal behavior



Critical Strouhal number... Shear Layer Instability..

- Shear layer instability introduces a shear layer shedding frequency
- “Most dangerous” fastest growing frequency offers estimate for low frequency behavior.
- Probably best suited to blocked base problem



$$f = \frac{\omega}{k} = \frac{\omega}{2\pi} = \frac{(0.20)(0.40)}{2\pi} \left(\frac{U}{L} \right) = 0.051 \left(\frac{U}{D} \right)$$

Loading Amplification Factor....

- The PSD responds with maximum magnitude at the critical frequency
- Estimation of the magnitude is essential
- Consider these simple expressions for pressure fluctuation

$$p' \sim \rho u' u' \propto \rho (U_2 - U_1)^2 \exp(-(\sigma \frac{y}{x})^2)$$

- One can estimate a pressure correlation function and a coherence function. One can demand a relationship between correlation and coherence so as to estimate parameters (focusing on the damping function) β

$$f(y, \alpha, \beta, B, C) = \exp(-2\sigma y) - C \int_0^\infty \exp(-\frac{2}{\pi} \frac{\beta}{(\alpha^2 + \beta^2)} \omega) \exp(-B\omega y) d\omega = 0$$

- Damping... notice damping is a function of critical frequency α

$$\beta = \frac{2\sigma - \sqrt{4\sigma^2 - \pi^2 \alpha^2 B^2}}{\pi B} \approx \frac{\sqrt{2\pi}}{4} \frac{\alpha^2}{\sigma}$$

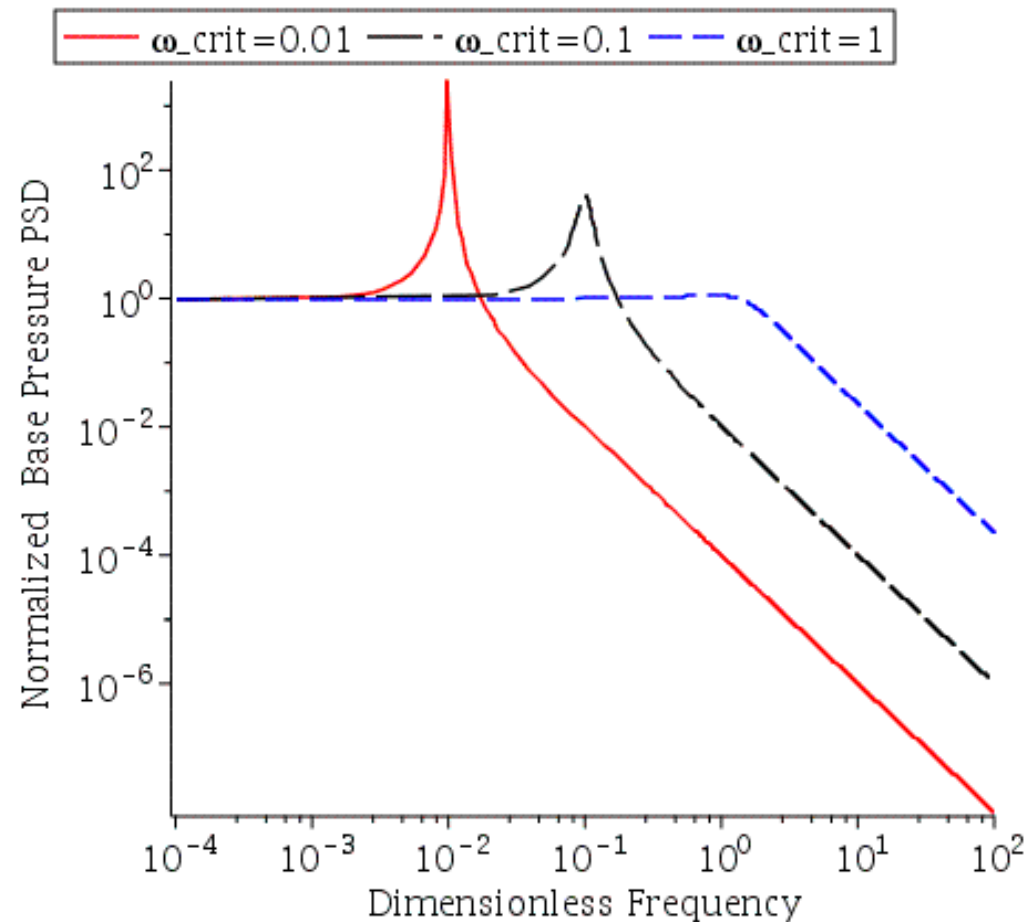
Loading Amplification Factor....

- Use of this damping expression in the PSD yields..
- Notice that lower frequency behavior induces increased amplitude at the critical frequency

$$\beta = \frac{\sqrt{2}\pi}{4} \frac{(0.257)^2}{1.4} \approx 0.052$$

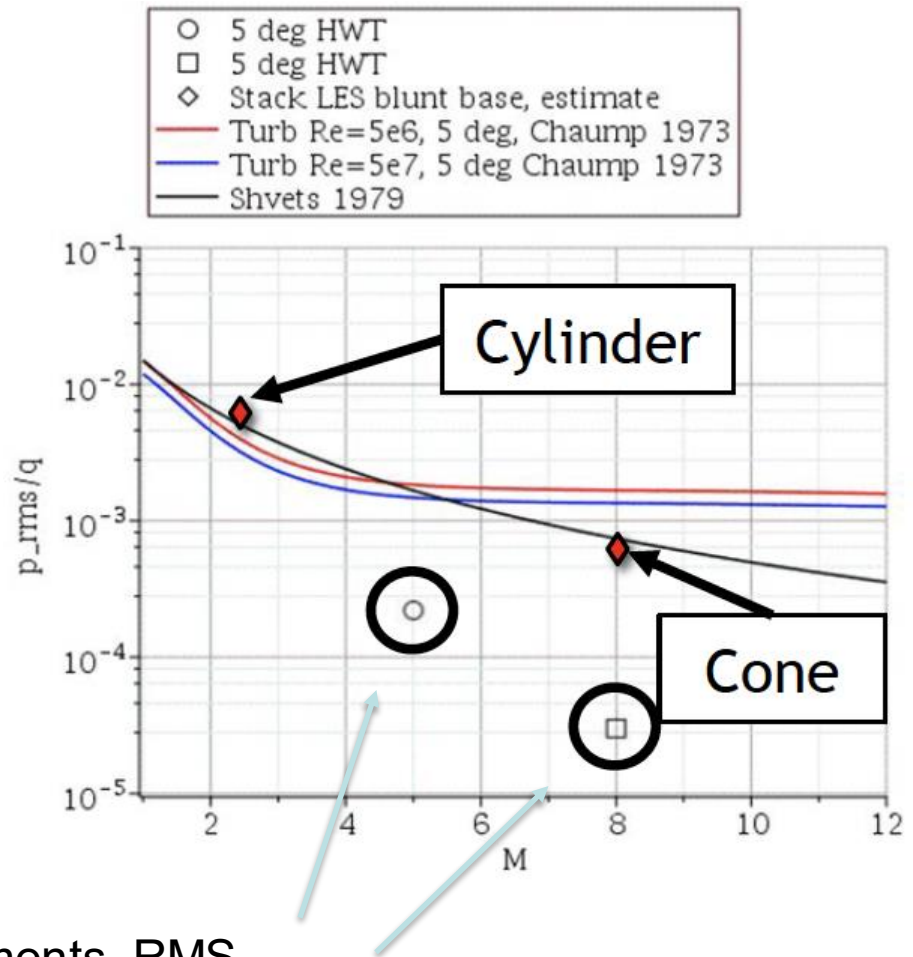
- Compares well with empirical closure...

$$\beta = \frac{1}{20} = 0.05$$



Base Pressure Fluctuation.... RMS

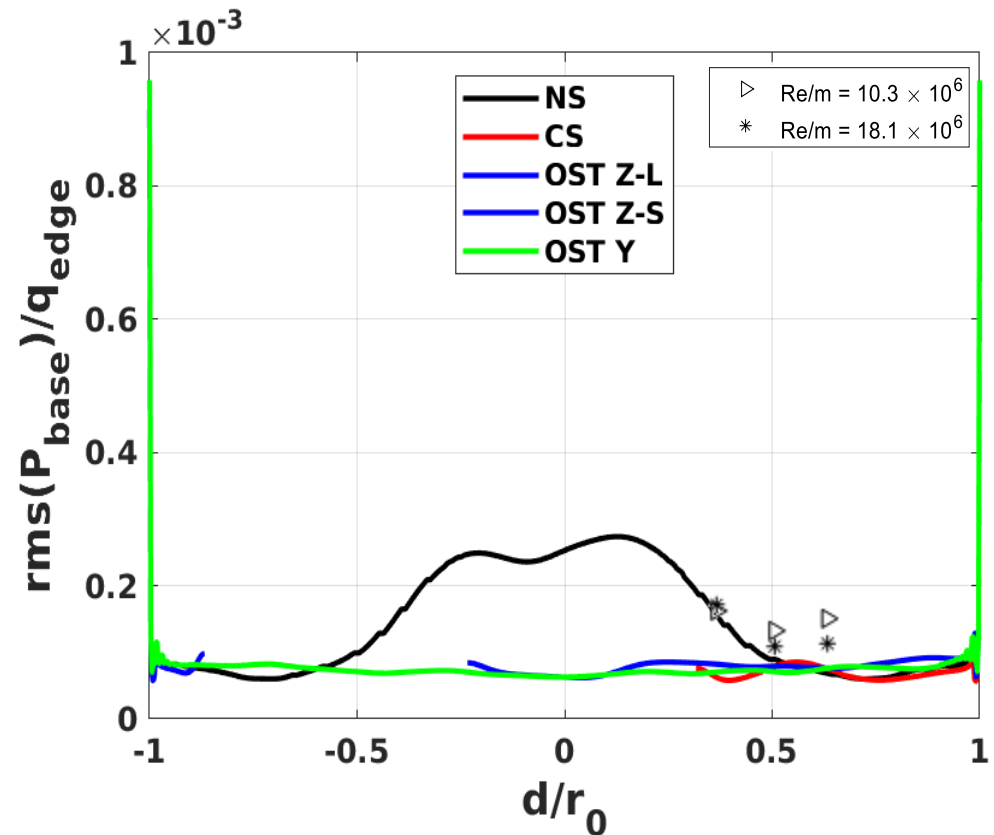
- Amplification applies at critical frequency, but magnitude requires base pressure RMS
- Pressure fluctuation RMS is required
- Traditionally a simple Mach number correction is used
$$\frac{p_{rms}}{q_{\infty}} = \frac{0.06}{(1 + M_{\infty})^2}$$
- “Cylinder” and “Cone” simulations trend well



HWT measurements, RMS
suppressed by presence of sting

Base Pressure Fluctuation.... RMS

- The modification of base pressure field due to the presence of sting is apparent in the previous measurements
- Recent measurements (Saltzmann) mounted model on an upstream blade suggests different behavior
- NS=no sting
- Base pressure has been indeed suppressed by the presence of the sting



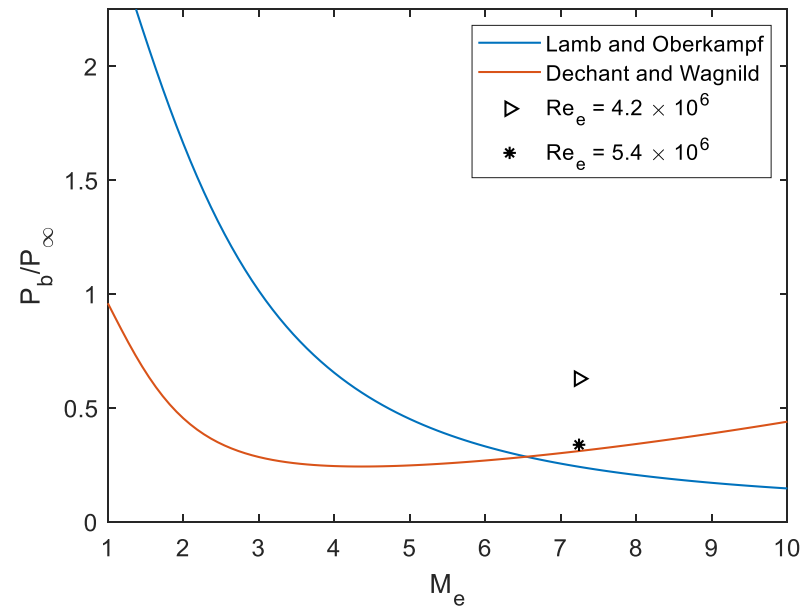
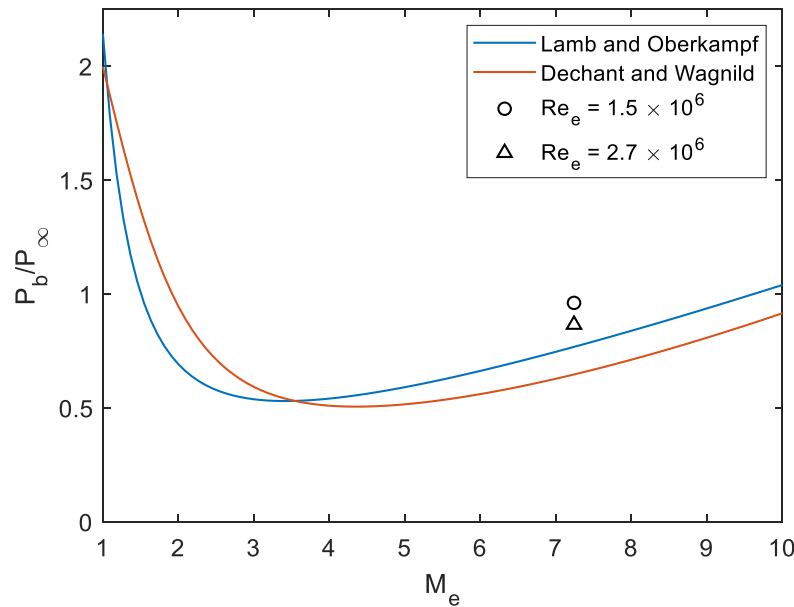
- Part of the base pressure problem involves estimates for the mean base behavior
- A theory-based model (DeChant and Wagnild) uses:
 - Adiabatic expansion (Prandtl-Meyer)
 - Energy conservation (Crocco-Buseman)
 - Simple viscous mixing model describing dividing streamline
 - A stagnation pressure

$$\frac{p_b}{p_\infty} = \frac{p_b}{p_\infty} \bigg|_0 \left(1 + \frac{\frac{\gamma-1}{2} (1 - cM_e^{1.25})^2 \left(\frac{u_D}{u_2} \right)^2 M_e^{2.5}}{1 + \frac{\gamma-1}{2} [1 - (1 - cM_e^{1.25})^2 \left(\frac{u_D}{u_2} \right)^2] M_e^{2.5}} \right)^{\gamma/(1-\gamma)}$$

$$\frac{p_b}{p_\infty} \bigg|_0 = c_0 \left(\frac{u_m}{u_2} \right)^2 = c_0 \left(1 - \frac{1}{2} \left(\operatorname{erf} \left(\frac{\sqrt{2}}{4} \left(\frac{\operatorname{Re}_s}{\operatorname{Re}_{s0}} \right)^{1/2} \right) \right) \right)^2$$

Mean Base Pressure Model

- Compare to 5-degree cone problem, Mach 8
- NS sting/blade mount
- Laminar and turbulent conditions
- Compare with Lamb and Oberkampf (1995) (Classical)
- Both models perform satisfactorily



Conclusions and Future Work

- **Simplified base pressure fluctuation PSD model (DeChant and Smith (2011)) provides reasonable approach**
 - **Examine critical (maximum amplitude) base region Strouhal models**
 - **RMS models**
 - **Reasonable agreement between theory, computation, and measurement**
 - **Mean base pressure estimate useful**
- **Future work:**
 - **Develop Integrated Approximate Base Pressure Fluctuation PSD Model**

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Questions???