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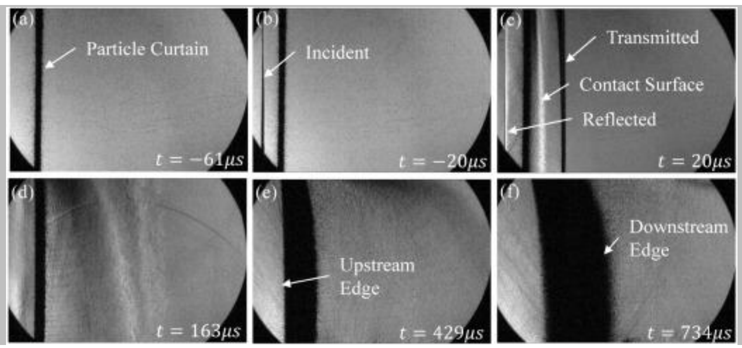

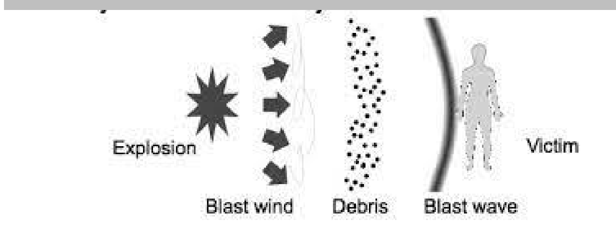

Shock Induced Particle Curtain Dispersion: Asymptotic Drag Law Scaling Formulations and Relationship to Streamwise Pressure Difference Models

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U.S. DEPARTMENT OF ENERGY

National Nuclear Security Administration

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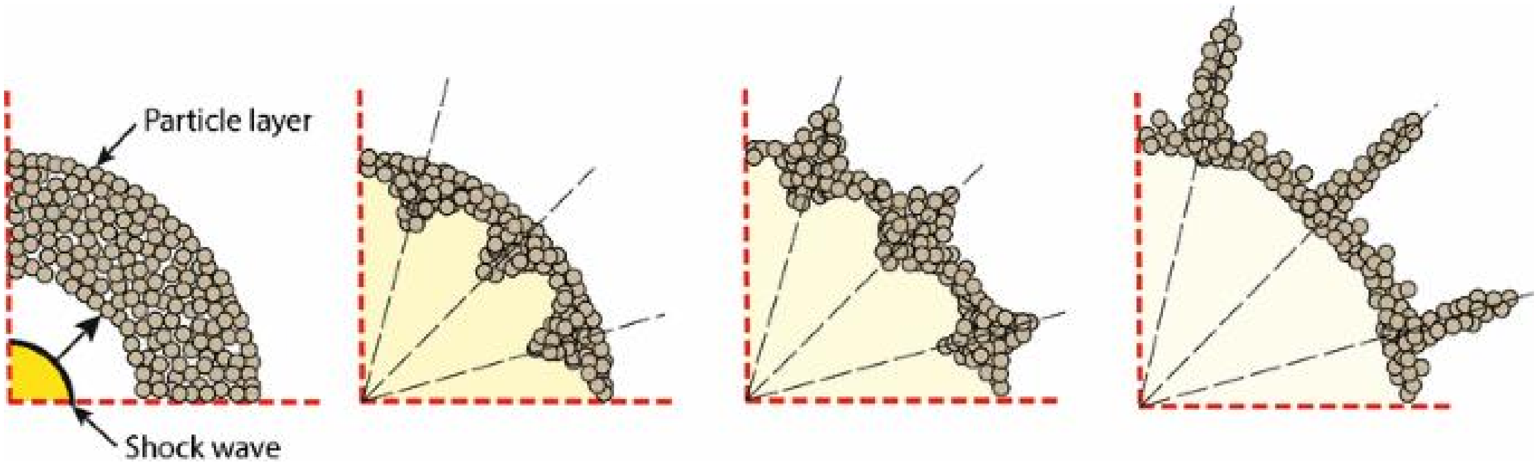
Outline

- **Particle Shock Interaction**
- **Early-Late Time Asymptotic Drag-Based Models**
- **Streamwise Pressure Difference Shock Particle Curtain Dispersion Models**
- **Appendix: Compressible Rapid Expansion Pressure Loss Model**
- **Conclusions**
- **Future Work**

Particle Shock Interaction…

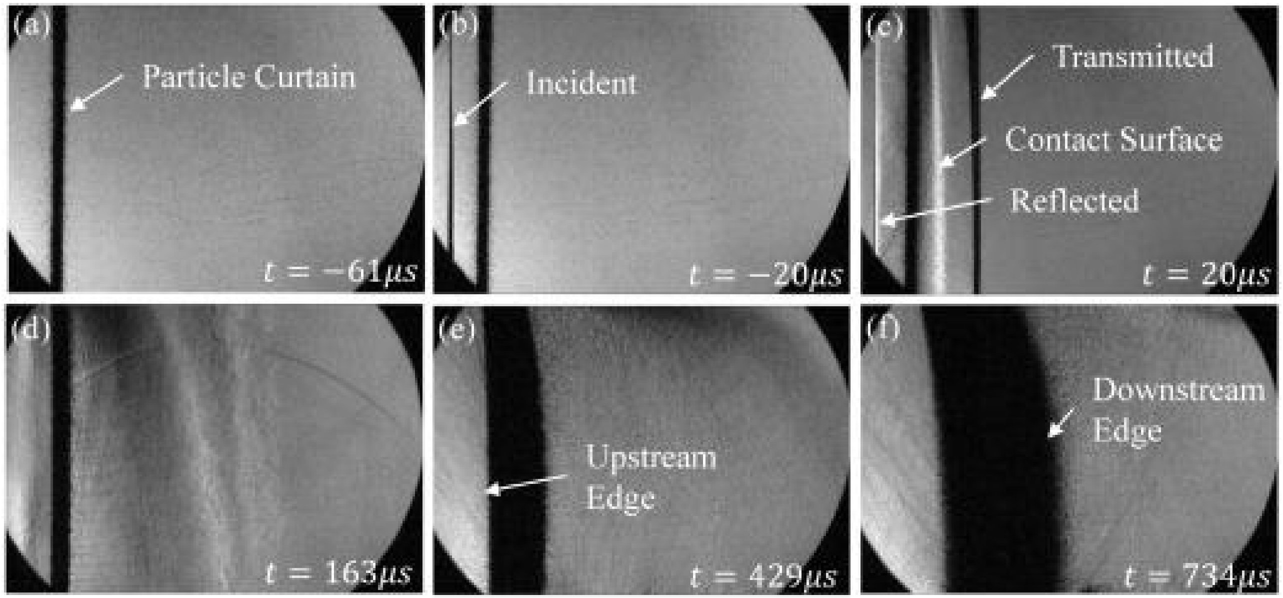
- **Shock interaction with a planar particle curtain is a fundamental physical problem, with application for blast/fragmentation, shrapnel dispersion etc.**



(Following Frost 2018)

Particle Shock Interaction

- Shock interaction with a planar particle curtain measurement and development
- DeMauro et. al. (2019)…
- Daniel and Wagner (2022)…
- Provides a canonical, geometrically least complex problem…



Following (Ling, Wagner, Beresh, Kearney and Balachandar (2021)

Early-Late Time Asymptotic Drag-Based Models

- Determination of particle curtain dynamics provides insight into local drag-dispersion behavior.
- The particle curtain width x(t) is described by:

$$\varphi \rho_p \delta_0 \frac{d^2 x}{dt^2} = \rho \left(\alpha_1 \frac{\nu}{d} + \alpha_2 (U - \frac{dx}{dt}) \right) (U - \frac{dx}{dt})$$

- Nondimensionalize:

$$t^{**} = \frac{U}{\delta_0} t \quad ; \quad x^* = \frac{x}{\delta_0}$$

- Expected (non-dimensional) differential equation

$$\frac{d^2 x^*}{dt^{**2}} = \varphi^{1/2} \left(\frac{\rho}{\rho_p} \right) \left(C_1 \mathrm{Re}_d^{-1} + C_2 (1 - \frac{dx^*}{dt^{**}}) \right) (1 - \frac{dx^*}{dt^{**}})$$

- "laminar" low Re drag; "turbulent" high Re drag

Early-Late Time Asymptotic Drag-Based Models

- **Flow field is broadly modified by the presence of the particles, freestream (velocity) is no longer constant/attainable… modify relative velocity terms as (a=1/4):**

$$(1-\frac{dx^*}{dt^{**}}) \rightarrow (1-\alpha\varphi^{-a}\frac{dx^*}{dt^{**}})$$

- **Yields ODE**

$$\frac{d^2x^*}{dt^{*2}} = \left(c_1 + c_2(1-\alpha^{-1}\frac{dx^*}{dt^{**}})\right)(1-\alpha^{-1}\frac{dx^*}{dt^{**}})$$

- **Where:**

$$c_1 \equiv \left(\frac{\rho}{\rho_p}\right)\mathrm{Re}_d^{-1} C_1 \qquad c_2 \equiv \left(\frac{\rho}{\rho_p}\right) C_2 \qquad t^* = \varphi^{1/4} t^{**} = \varphi^{1/4}\frac{U}{\delta_0} t$$

- **Exact solution**

$$x^* = \alpha t^* - \frac{\alpha}{c_2}\ln\left(1 - \frac{c_2}{c_1}(1-\exp(-\frac{c_1}{\alpha}t^*))\right)$$

Early-Late Time Asymptotic Drag-Based Models

- **Solution has early and late attributes; leverage**

$$\frac{d^2 x^*_{early}}{dt^{*2}} \approx \left(c_1 + c_2\right)$$

$$x^*_{late} = \alpha t^* + b$$

- **Use a matching argument: function value, slope and curvature we can get an estimate parameter:**

$$\alpha = 9(c_1 + c_2)$$

- **The constants c1 and c2 are easily estimated empirically…**

$$c_1 = 0.032 \qquad c_2 = 0.041$$

- **A traditional Galerkin/collocation procedure model is shown to provide similar estimates c1 and c1… detail in manuscript**

Early-Late Time Asymptotic Drag-Based Models

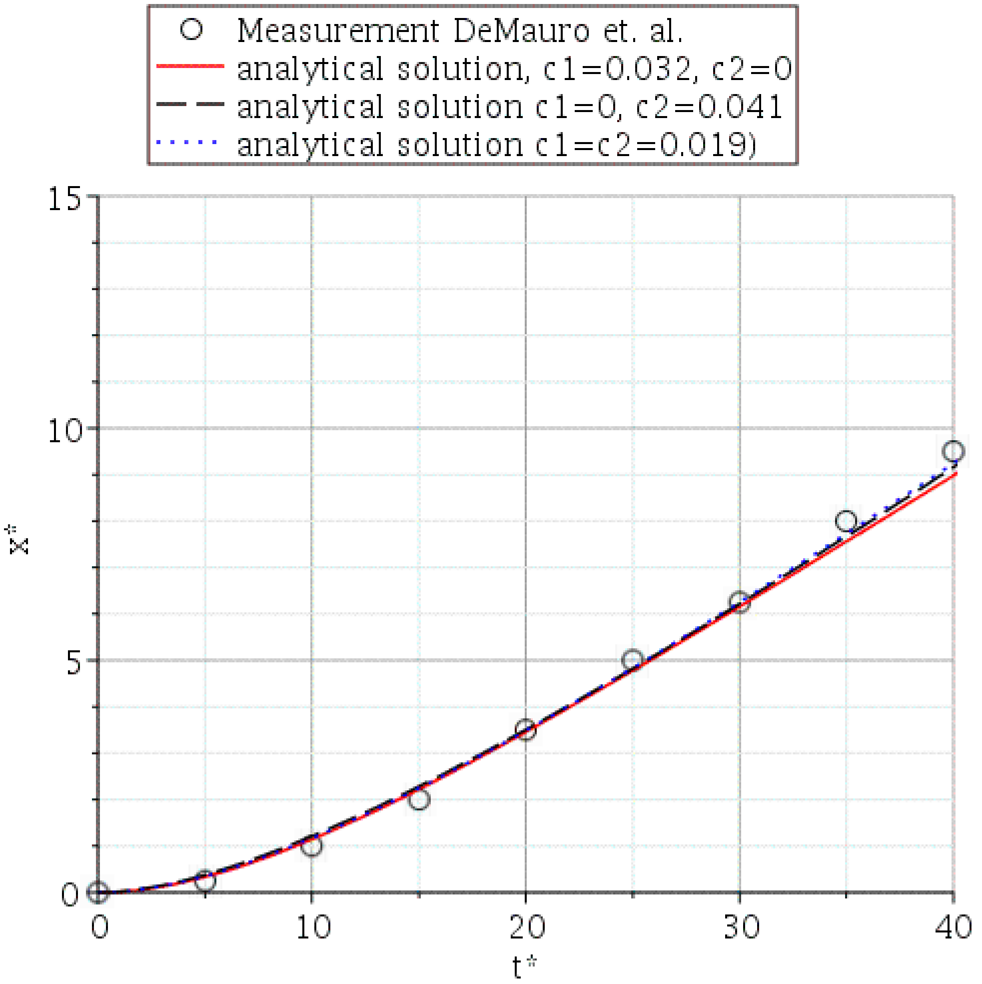
- **Fit of data with measurements De Mauro et. al. 2019 shows good agreement for a wide range of possible c1 and c2 closures… It's not very sensitive..**
- **The key behavior is not so much the particular values of c1 and c2 , but the functional form to change the asymptotic behavior via:**

$$(1-\frac{dx^*}{dt^{**}}) \to (1-\alpha\varphi^{-a}\frac{dx^*}{dt^{**}})$$

- **The matching argument provides information for α… Combined with the scaling..**

$$t^* = \varphi^{1/4} t^{**} = \varphi^{1/4}\frac{U}{\delta_0} t$$

- **Supports the flow behavior**



Streamwise Pressure Difference Shock Particle Curtain Dispersion Models

- **A simple force balance based on the pressure difference across the particle curtain obviates the use of the drag closure…**

$$\varphi \rho_p \delta_0 A \frac{d^2 x}{dt^2} = F_P \propto \varphi A (p_u - p_d)$$

- **To use this model… we need** $(p_u - p_d)$ **Daniel and Wagner (2022)… provide just such a model…**

$$p_u - p_d = C_{meas} \varphi^{1/2} \rho_1 u_2^2 \quad ; \quad C_{meas} = 9.6$$

- **Estimation of the closure constant** C_{meas} **is our goal.**

- **Approach… leverage classical wind tunnel screen/mesh pressure loss: Pinker and Herbert (1967)/ Huang (1991)**

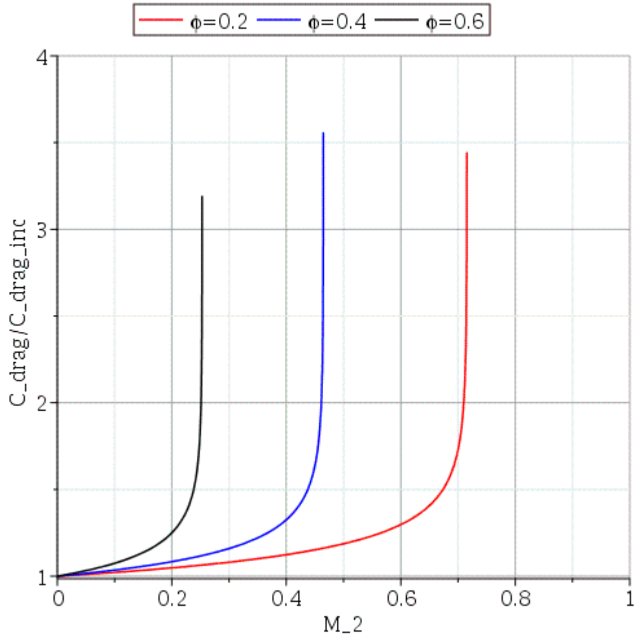
Streamwise Pressure Difference Shock Particle Curtain Dispersion Models

- **leverage classical wind tunnel screen/mesh pressure loss: Pinker and Herbert (1967)/ Huang (1991)**
- **Simple Incompressible… power law approximation**

$$\Delta p = \frac{1}{2}\lambda_0 \rho_2 u_2^2 \quad ; \quad \lambda_0 = \lambda_0(\varphi) \qquad \lambda_0(\varphi) \approx 3.5\varphi^{3/2}$$

- **Must be extended to compressible flow, to honor inter-particle choking; Empirically..**

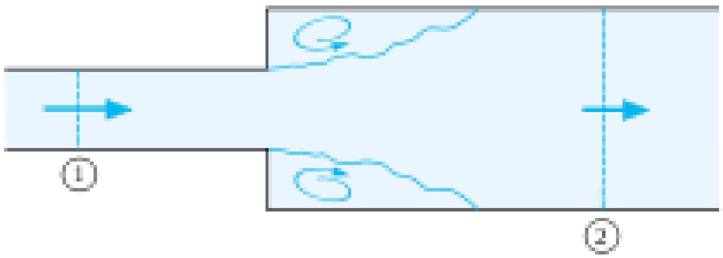
$$\frac{C_{drag}}{C_{drag\_inc}} = \left(\frac{M^*}{M^* - M_0}\right)^b \qquad b \approx 1/7$$



- **But this expression is not bounded for choking.. We need another model**

Streamwise Pressure Difference Shock Particle Curtain Dispersion Models

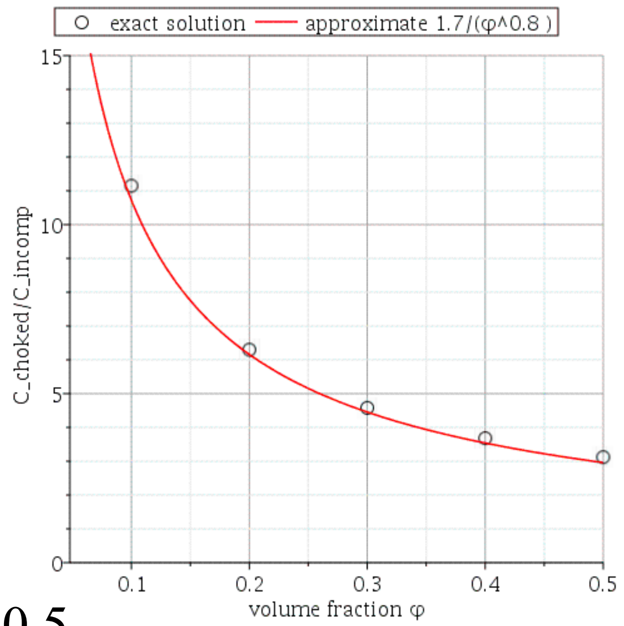
- **Compressible sudden expansion…**
- **Simple balance of mass, momentum and energy balance in compressible form..**
- **Two basic equations…**



$$\frac{p_2 - p_1}{\frac{1}{2}\rho_0 u_0^2} = 2\frac{M_1}{M_0}\left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{1/2}\left(1-\frac{M_2}{M_1}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2}\right)$$

$$\frac{(p_0 - p_1)}{\frac{1}{2}\rho u_0^2} = \frac{p_0}{\frac{1}{2}\rho u_0^2}\left(1-\frac{p_1}{p_0}\right) = \frac{2}{\gamma M_0^2}\left(1-\left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}}\right)$$

- **These expressions are solvable and can be mapped to a more convenient closure expression as:**



$$\left.\frac{C_{drag}}{C_{drag\_inc}}\right|_{M^*=1} \approx 1.7\varphi^{-4/5} \quad 0.1 < \varphi < 0.5$$

Streamwise Pressure Difference Shock Particle Curtain Dispersion Models

- **Build up estimate of pressure difference**
- **Incompressible term:**

$$\lambda_0(\varphi) \approx 7 c_0 \varphi^{3/2} = 3.5\varphi^{3/2} \quad ; \quad c_0 \approx 0.5$$

- **Choked flow correction (integral average)**

$$\left.\frac{C_{drag}}{C_{drag\_inc}}\right|_{ave} \approx 3.5 \qquad \left.\frac{C_{drag}}{C_{drag\_inc}}\right|_{ave} = \frac{1.7}{1-0.1}\int_{0.1}^{1} \varphi^{-4/5}\, d\varphi \approx 3.5$$

- **Density ratio (integral average)**

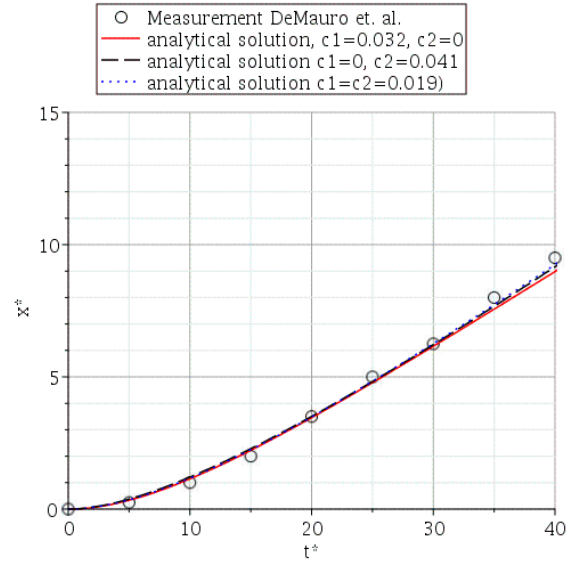
$$\left.\frac{\rho_2}{\rho_1}\right|_{ave} = \frac{1}{1.7-1}\int_{1}^{1.7} \frac{\rho_2}{\rho_1}\, dM_s = 1.6$$

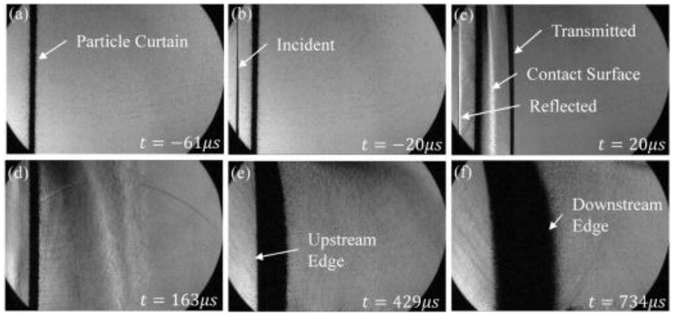
- **Final result…. In good agreement with Daniel and Wagner… C_{meas}=9.6**

$$\Delta p = \frac{1}{2}\lambda_0(\varphi)\left(\left.\frac{C_{drag}}{C_{drag\_inc}}\right|_{ave}\right)\left(\left.\frac{\rho_2}{\rho_1}\right|_{ave}\right)\rho_1 u_2^2 = (3.5\varphi^{3/2})(3.5)(1.6)\rho_1 u_2^2 = 9.8\varphi^{3/2}\rho_1 u_2^2$$

Conclusions and Future Work

- **Early-Late Time Asymptotic Drag-Based Models**
  - **Particle curtain spreading well predicted**
  - **The key behavior=<u>the functional form to change the asymptotic behavior to honor drastic change to flow field</u>**
- **Streamwise Pressure Difference Shock Particle Curtain Dispersion Models**
  - **Pressure difference model (Daniel and Wagner (2022) provide useful description**
  - **closure constants in good agreement with measurement**
- **Future work:**
  - **Apply results to shock-physics framework e.g. CTH**
  - **Support particle shock application space**





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Questions???