



Dynamic collision frequencies, scattering, stopping, and spectra

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Overarching goal: Develop a complete and consistent atomic model for constitutive and observable data



- 1) Start with existing average-atom model based on density functional theory: **DFT-AA**
- 2) Use *ab-initio* multi-center DFT calculations to constrain DFT-AA model choices
- 3) Extend DFT-AA for dynamic properties (XRTS & line shapes)
- 4) Extend DFT-AA for multiconfiguration electronic structure (opacities)
- 5) Extend DFT- AA to non-equilibrium plasmas

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Improving Predictive Capability in REHEDS Simulations with Fast, Accurate, and Consistent Non-Equilibrium Material Properties

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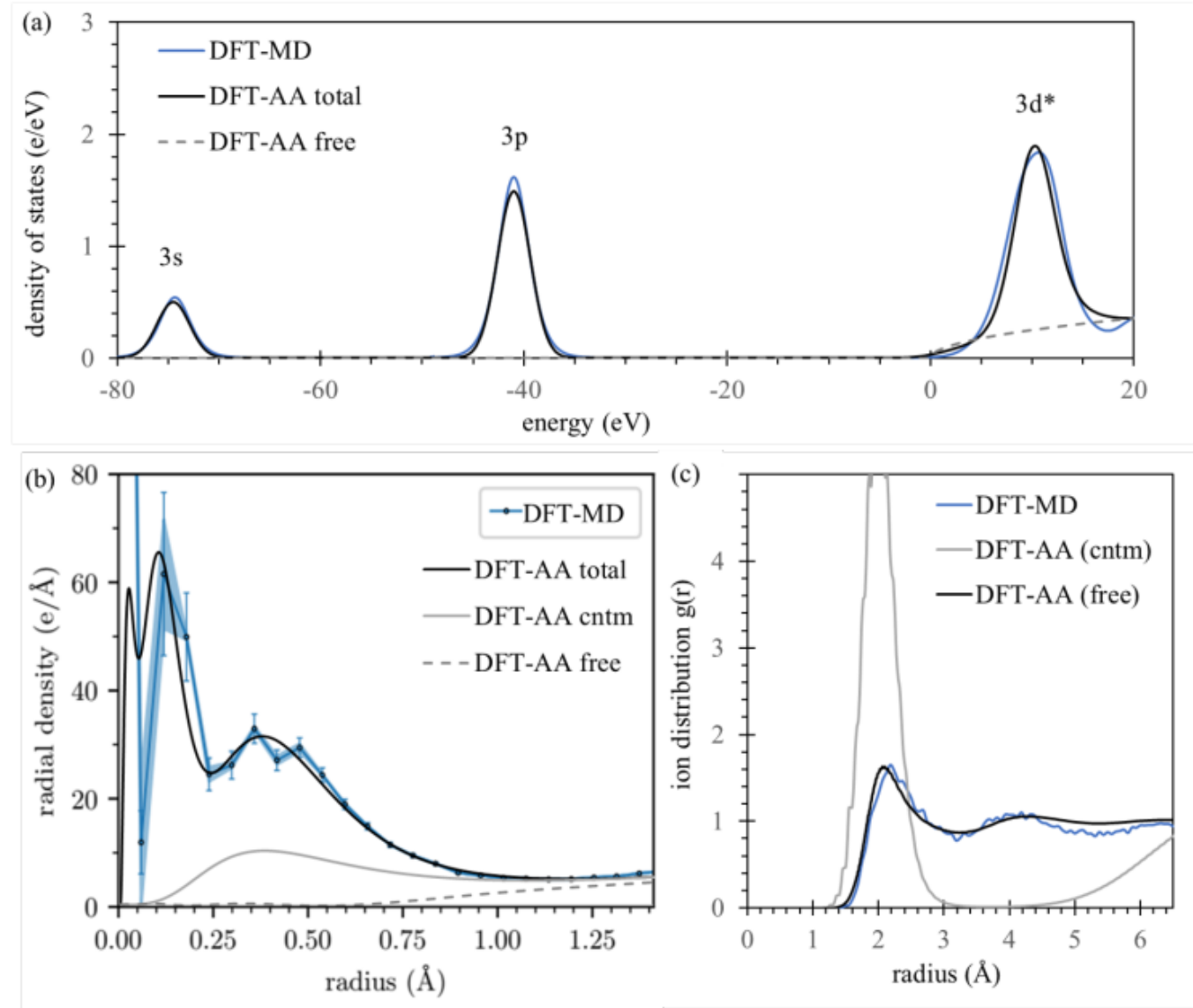
Rigorously validate against experimental data and reference models for XRTS, transport, EOS, non-LTE, and line shapes

Overarching goal: Develop a complete and consistent atomic model for constitutive and observable data



- 1) Start with existing average-atom model based on density functional theory: **DFT-AA**
→ self-consistent electronic and ionic structure, EOS, static transport coeff.
- 2) Use *ab-initio* multi-center DFT calculations to constrain choices
→ boundary conditions, Z^*

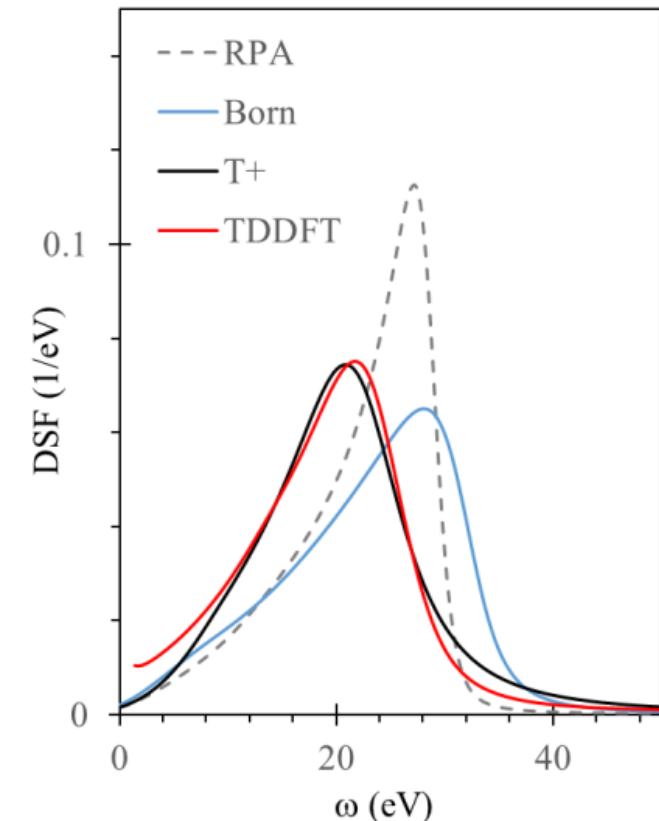
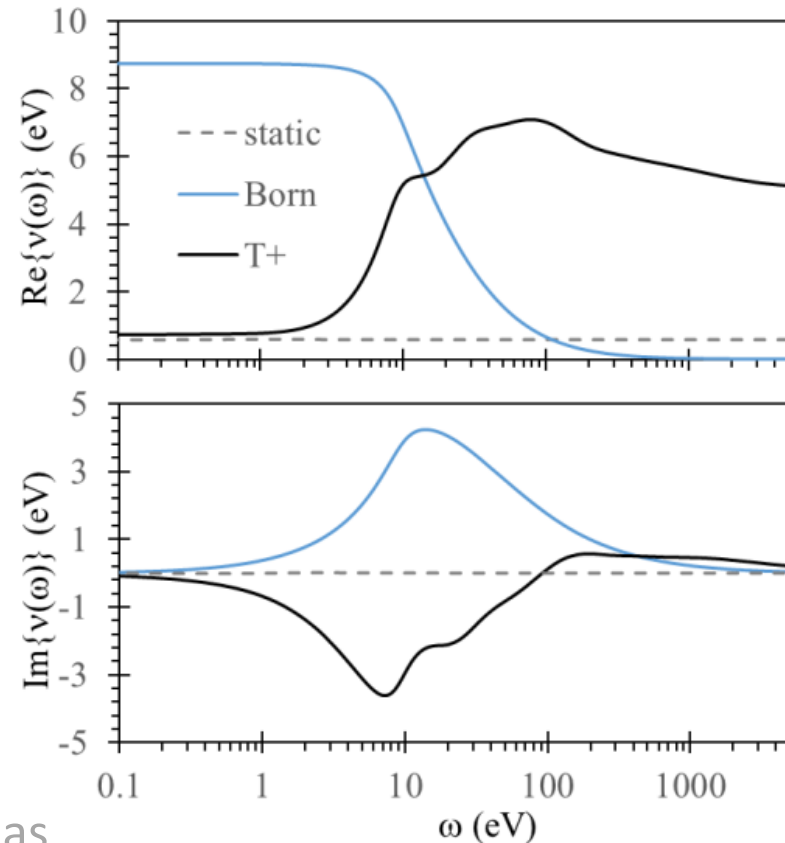
Example: Solid iron at $T = 1\text{eV}$:
electronic density of states (top)
and radial distribution functions



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- 3) **Extend DFT-AA for dynamic properties (XRTS & line shapes)**
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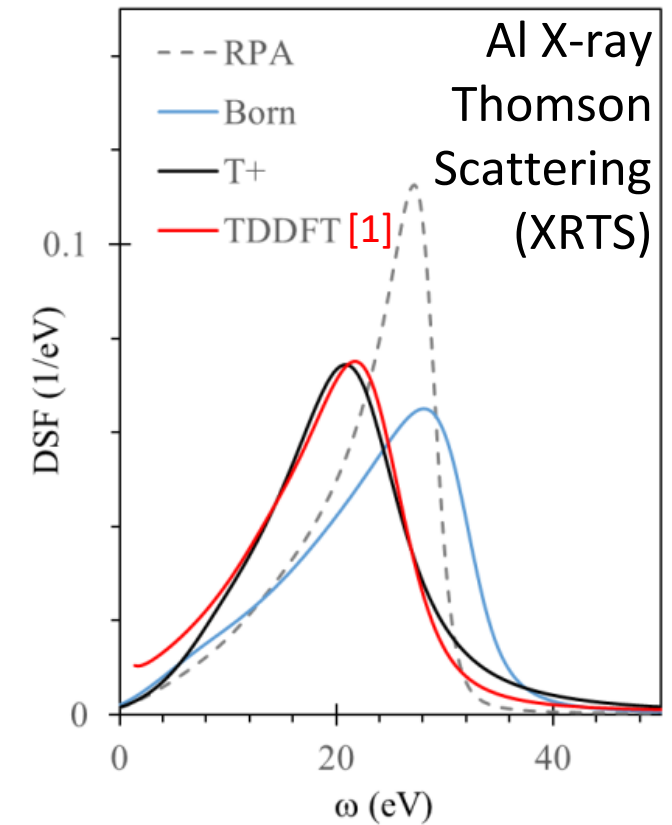
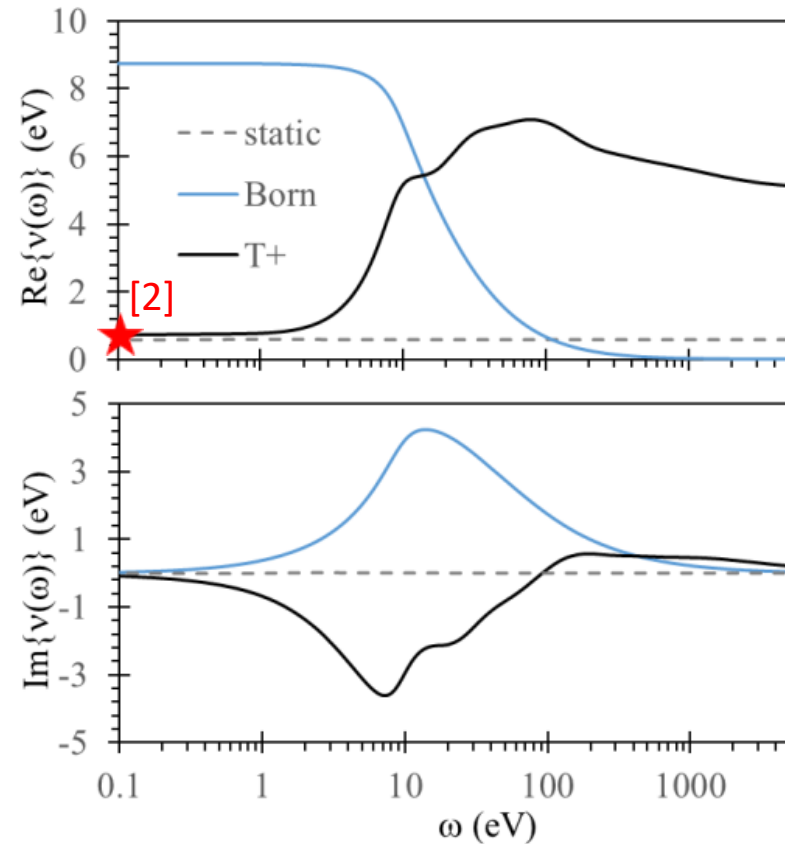


We will show that strong collisions and inelastic collisions improve DFT-AA agreement with TDDFT [1], static conductivities [2], and XRTS experiments [3]

The dynamic collision frequency modifies response functions and observables



- In a direct (zero-frequency) electric field, the static collision frequency $\nu(0)$ represents collisions that control DC conductivity $\sigma(0) = \frac{n_e}{\nu(0)}$
- In an oscillating field, the dynamic collision frequency $\nu(\omega)$ represents the damping of collective plasma oscillations with frequency ω
- The dynamic collision frequency informs the Mermin dielectric function $\epsilon^M(q, \omega)$ and observables like the dynamic structure factor (DSF) $\sim \left| \frac{1}{\epsilon(q, \omega)} \right|$
→ XRTS and stopping powers



The Born approximation for cross sections is commonly used in calculations of $\nu(\omega)$



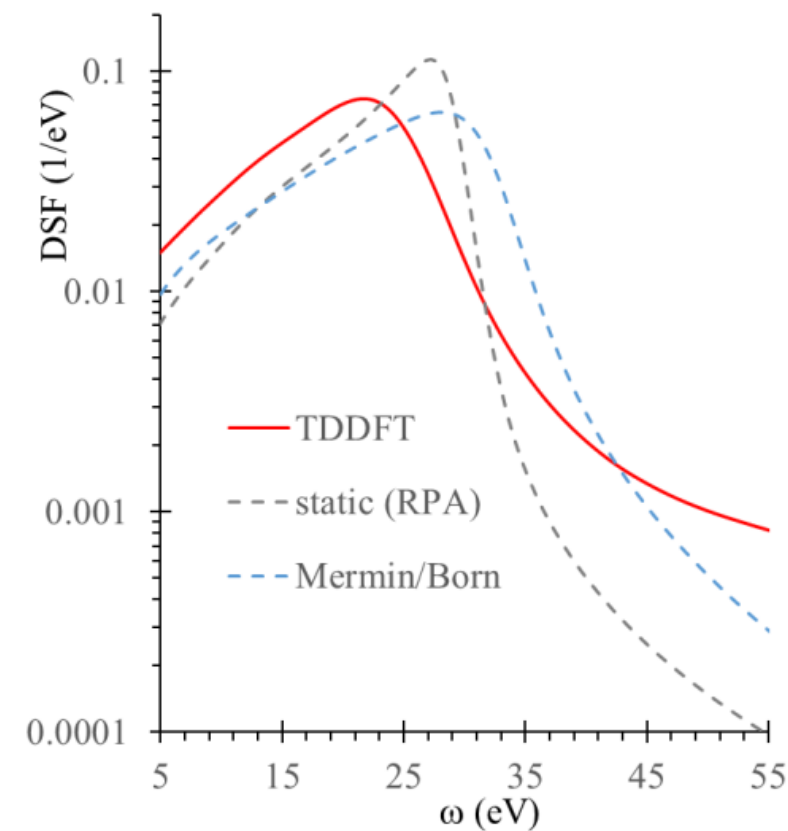
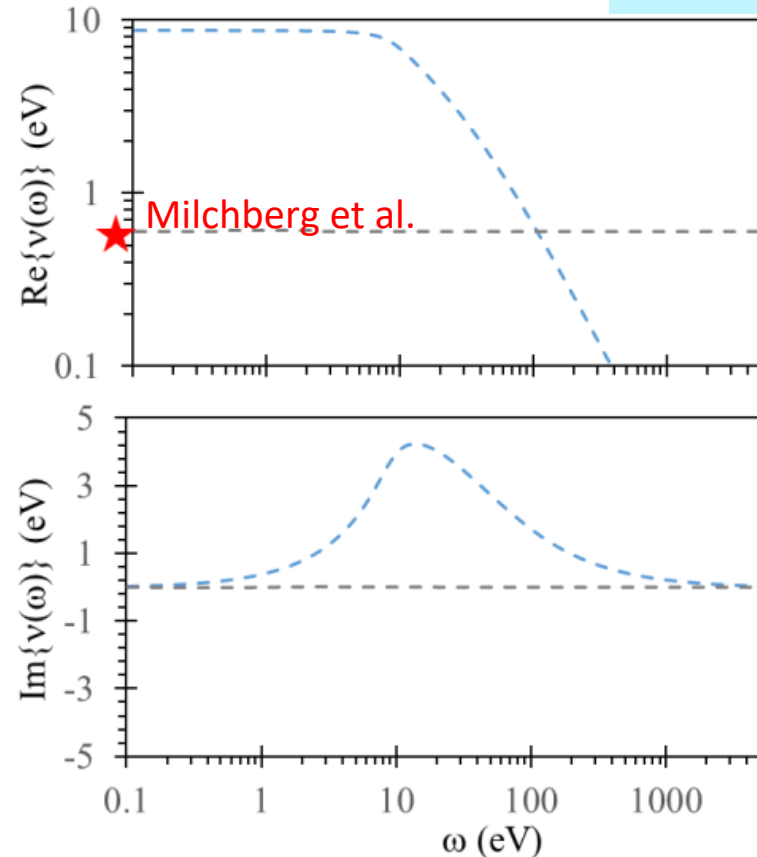
Example: Solid Al at T = 1eV

$$\nu(\omega) \sim \int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

The Born cross section with a screened Yukawa potential is:

$$\frac{\partial \sigma^{tr}}{\partial \theta} = \frac{1}{4\pi^2} \left| \frac{4\pi Z^*}{q^2 + q_s^2} \right|^2$$

Whatever our choices for Z^* and q_s , the real and imaginary parts of $\nu(\omega)$ have a fixed functional form that moves $\epsilon^M(q, \omega)$ away from TDDFT



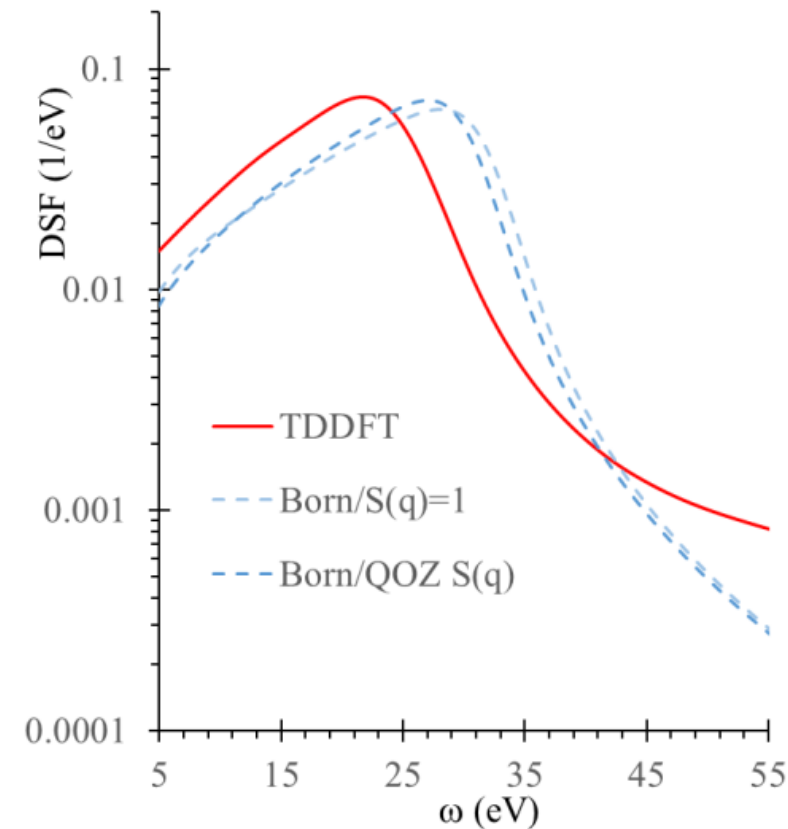
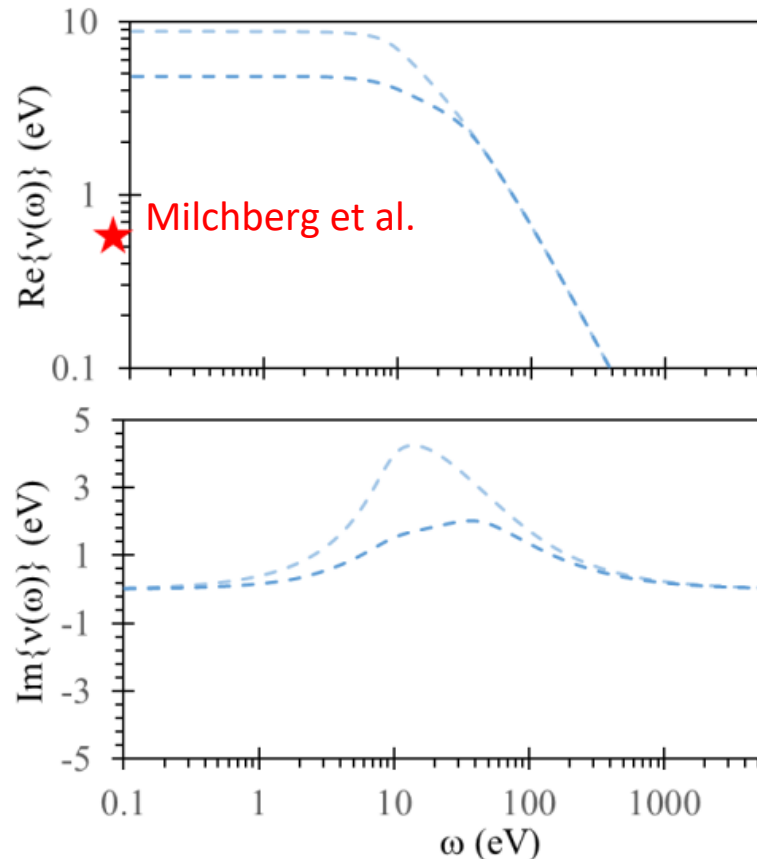
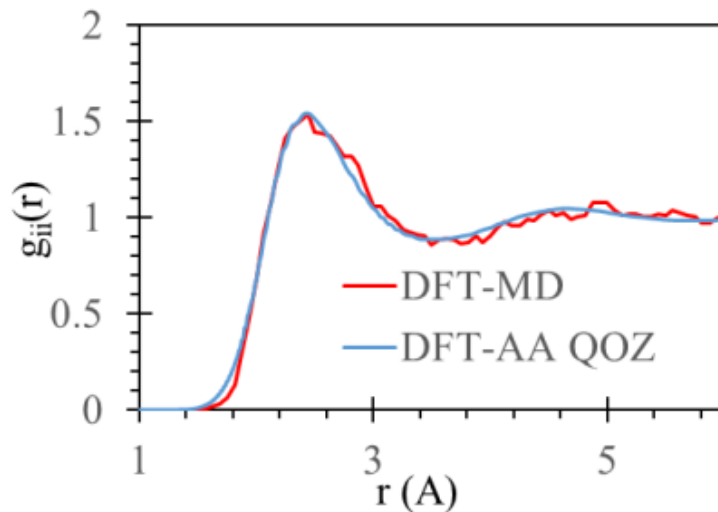
Weak scattering (Born) gives unreliable $\nu(0)$ and inevitable $\text{Im}\{\nu(\omega)\} > 0$ shifts the RPA plasmon peak to higher energies

Modifying the ion-ion structure factor doesn't help much



$$v(\omega) \sim \int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

We can use a self-consistent $S_{ii}(q)$ from $g_{ii}(r)$ following Starrett & Saumon



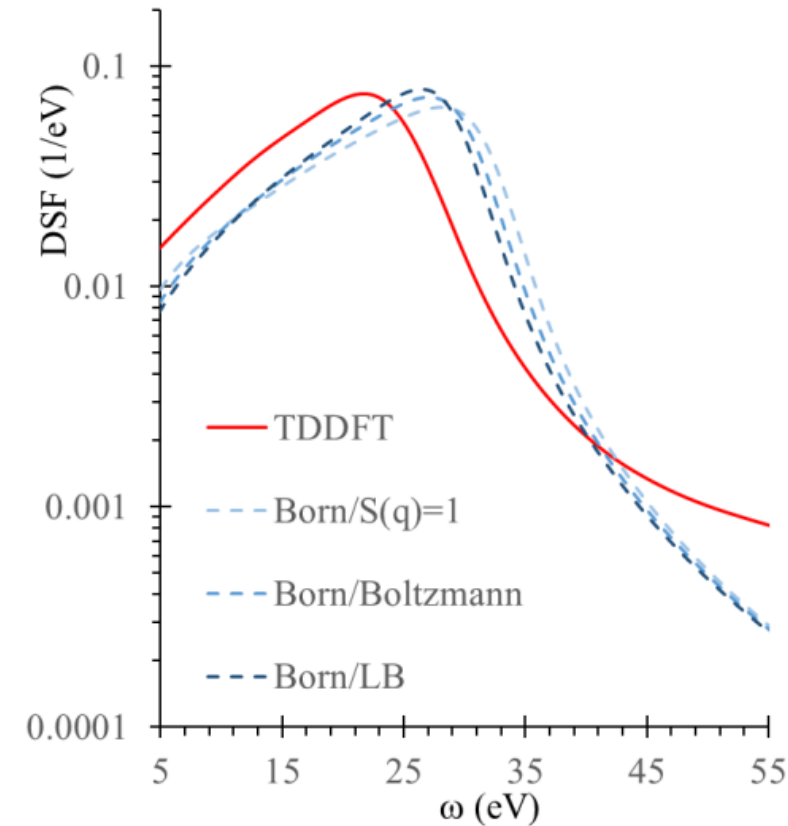
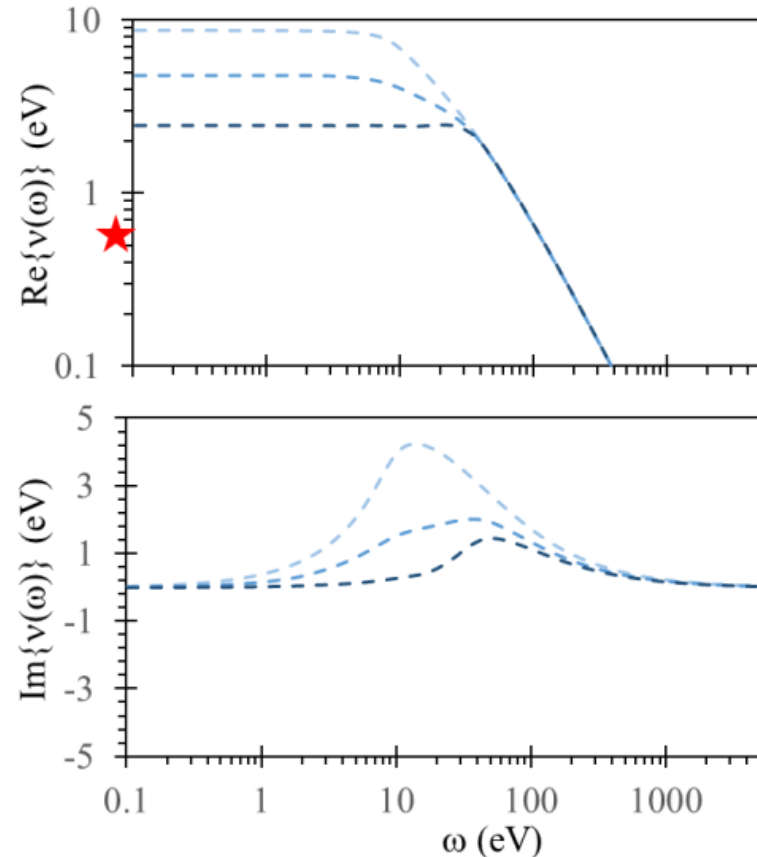
While we find good agreement with DFT-MD ionic structure, changing $S_{ii}(q)$ doesn't change the functional form of $v(\omega)$

Including dynamic screening via Lenard-Balescu integration [1,2] also has only a modest effect



$$v(\omega) \sim \int \frac{\epsilon^{-1}(q, \omega) - \epsilon^{-1}(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

Replacing $\epsilon(q, \omega)$ with $\epsilon^{-1}(q, \omega)$ gives a better approximation for the collisional term in the generalized Boltzmann equation



While this modification further decreases $v(0)$ and the plasmon shift, both remain far from trusted values

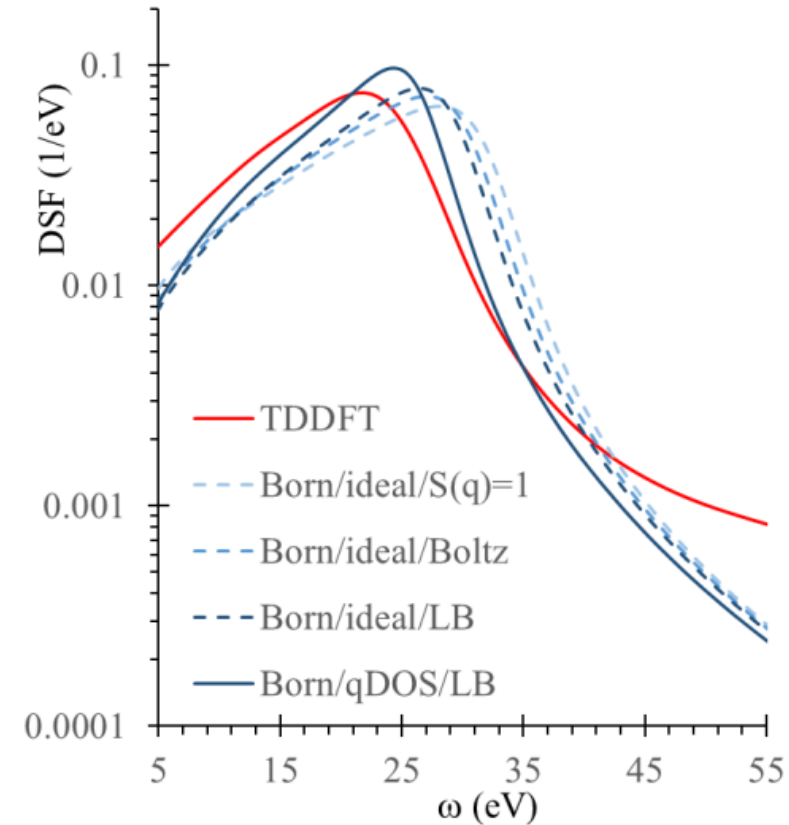
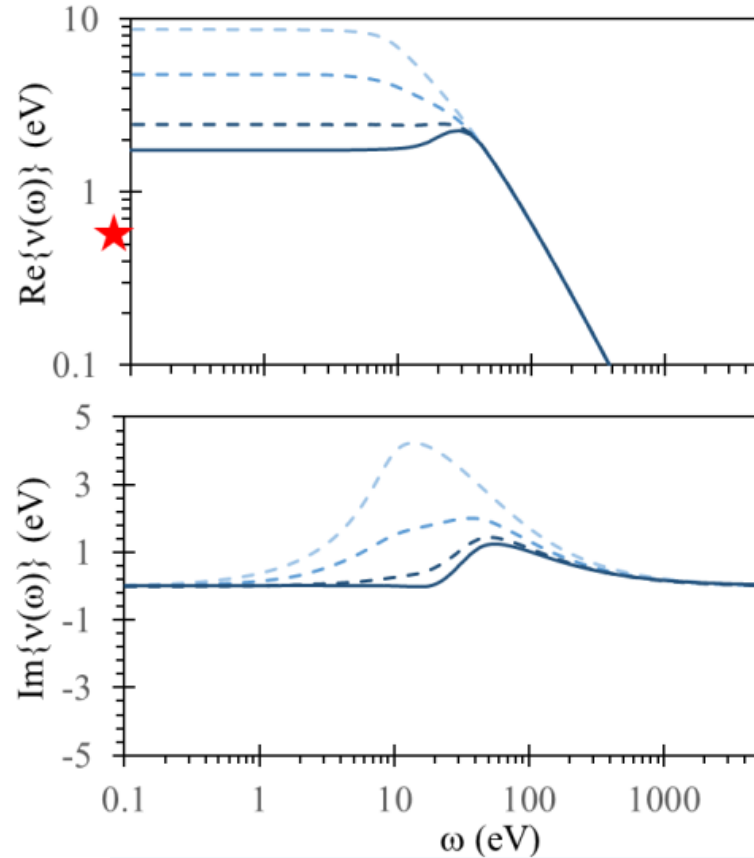
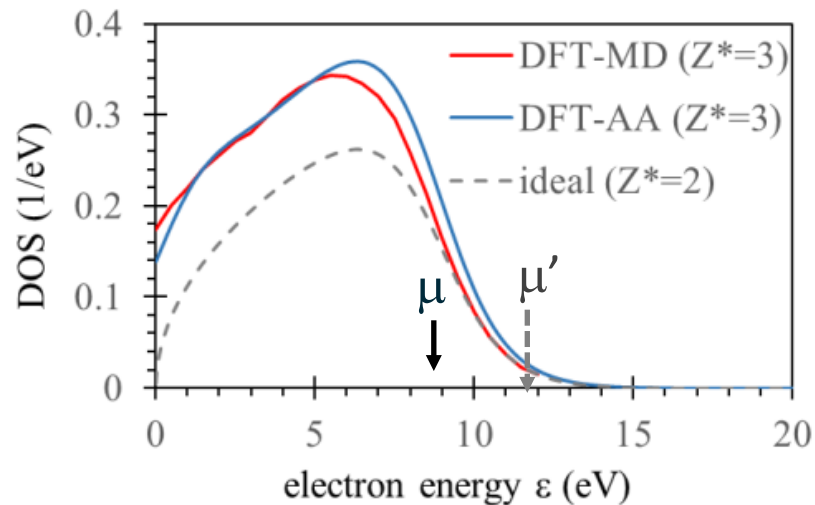
Using the quantum density of states gives a more reasonable DSF but still does not capture $v(0)$



$$v(\omega) \sim$$

$$\int \frac{\epsilon_q^{-1}(q, \omega) - \epsilon_q^{-1}(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

Using the quantum DOS modifies the integrals in $\epsilon(q, \omega)$ and makes Z^* consistent with μ



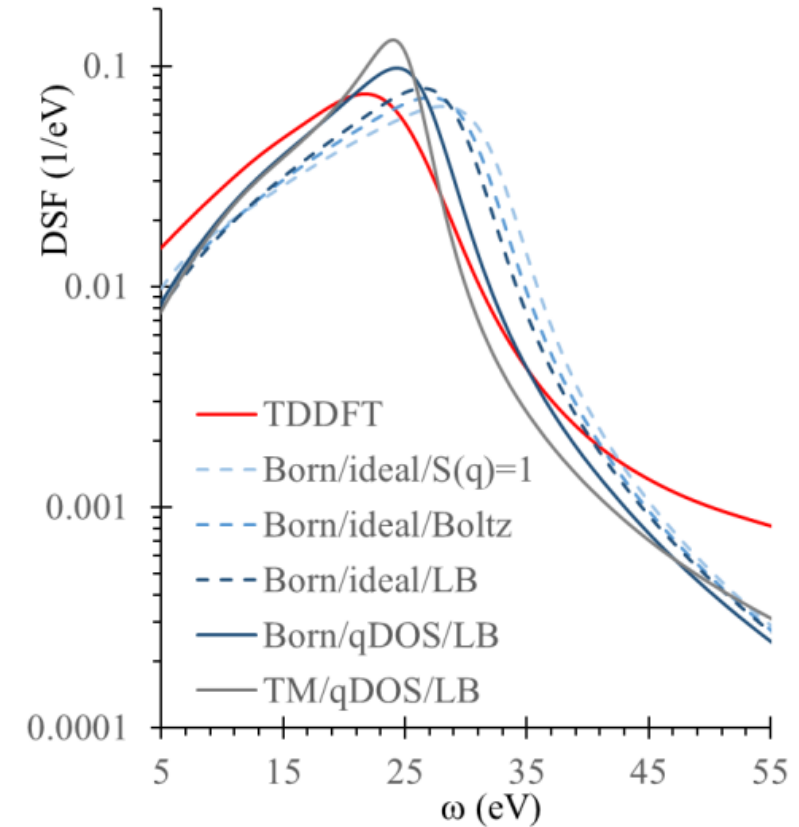
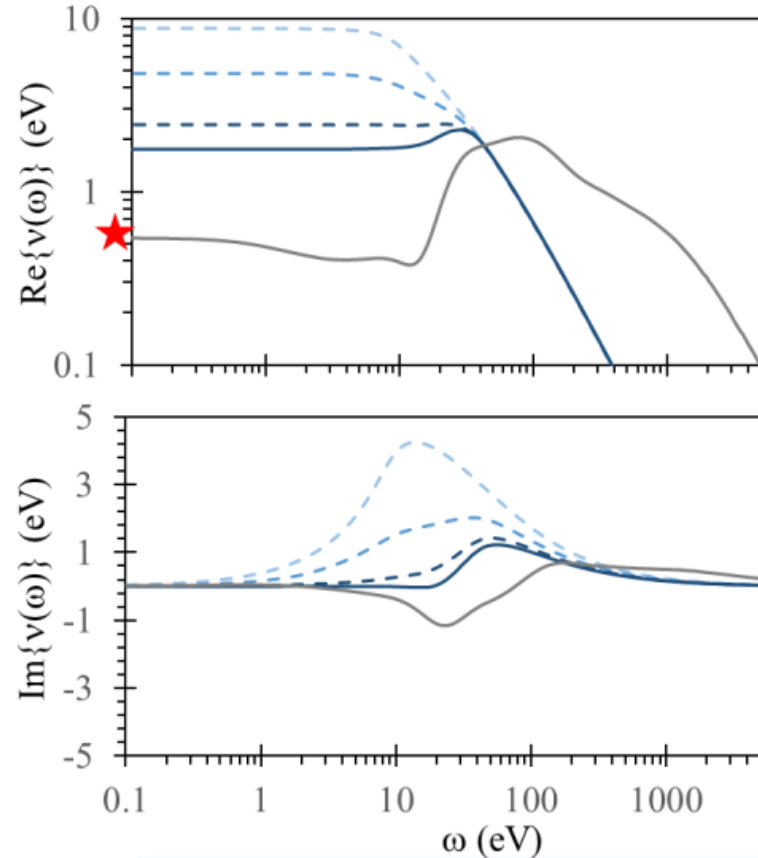
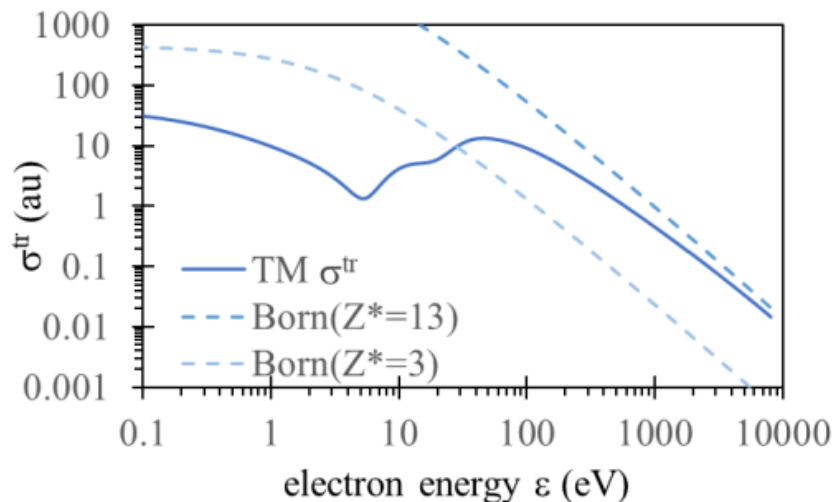
The full quantum density of states better represents the continuum electron structure and eliminates ambiguity in Z^* , but disagreement in $v(0)$ persists

T-matrix cross sections capture strong collisions, significantly improving the $v(0)$ limit



$$v(\omega) \sim \int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

The T-matrix cross sections use phase shifts from the self-consistent continuum orbitals



Strong scattering significantly improves $v(0)$ and departs significantly from the Born form, giving $\text{Im}\{v(\omega)\} < 0$

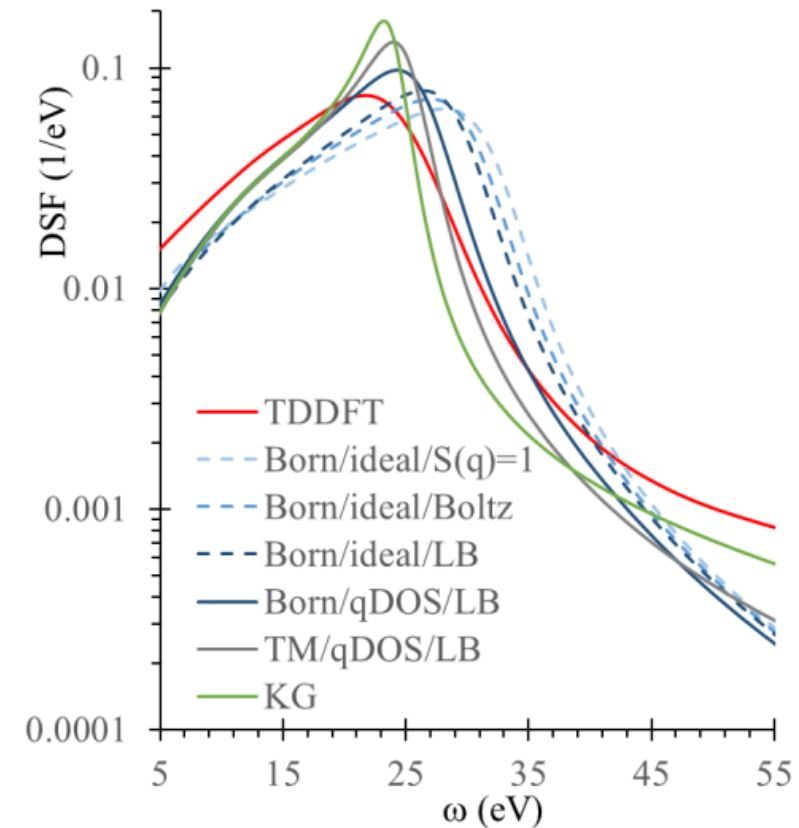
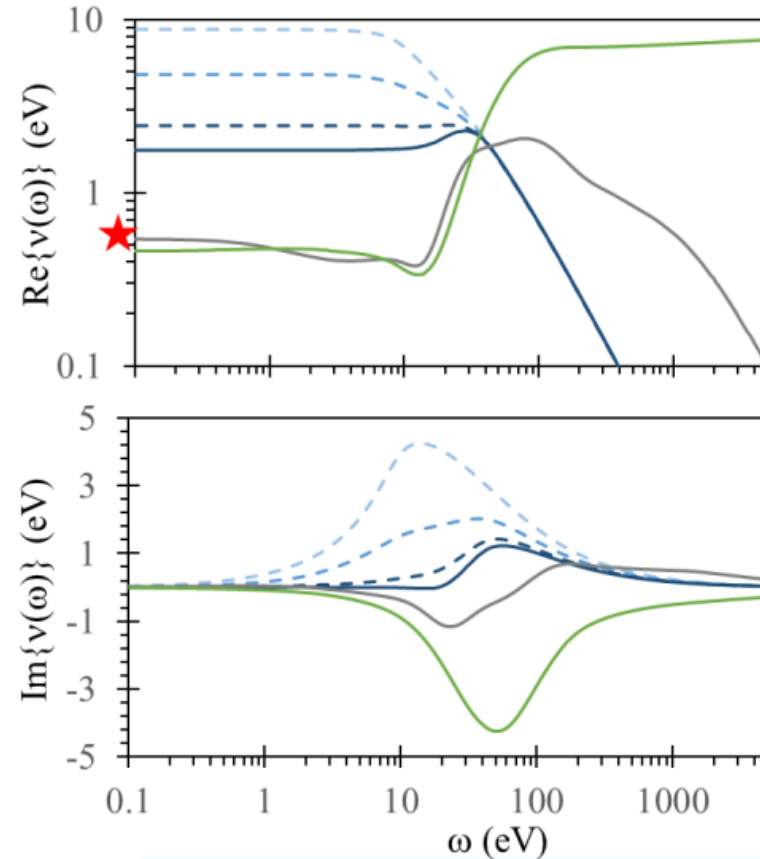
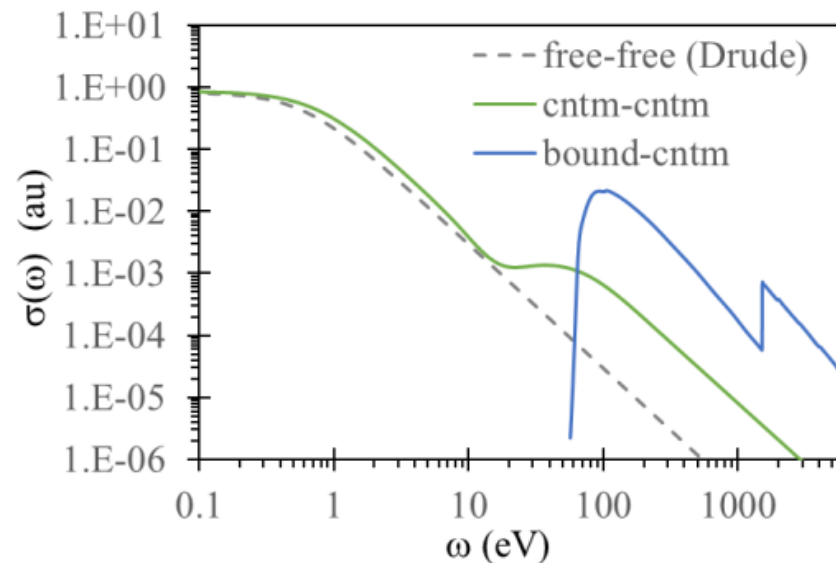
Independent Kubo-Greenwood calculations capture a similar $v(0)$ limit and frequency dependence



$$v(\omega) = \frac{n_e}{\sigma(\omega)} + i\omega$$

following Reinholz et al. [1]

The quantum Kubo-Greenwood dynamic conductivity $\sigma(\omega)$ adds a distinct feature to the Drude $\sigma(\omega)$



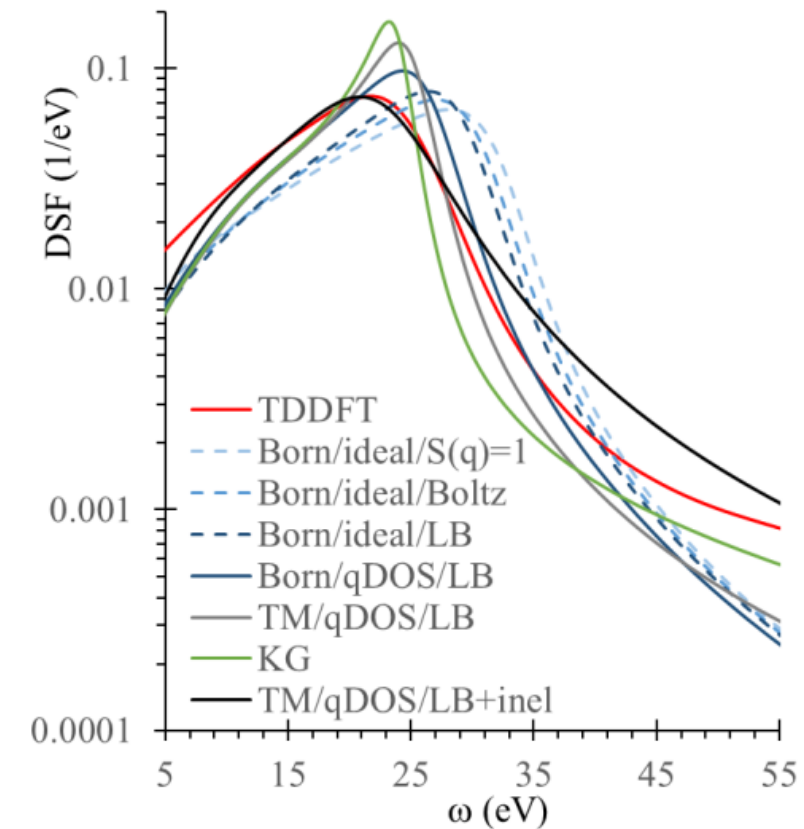
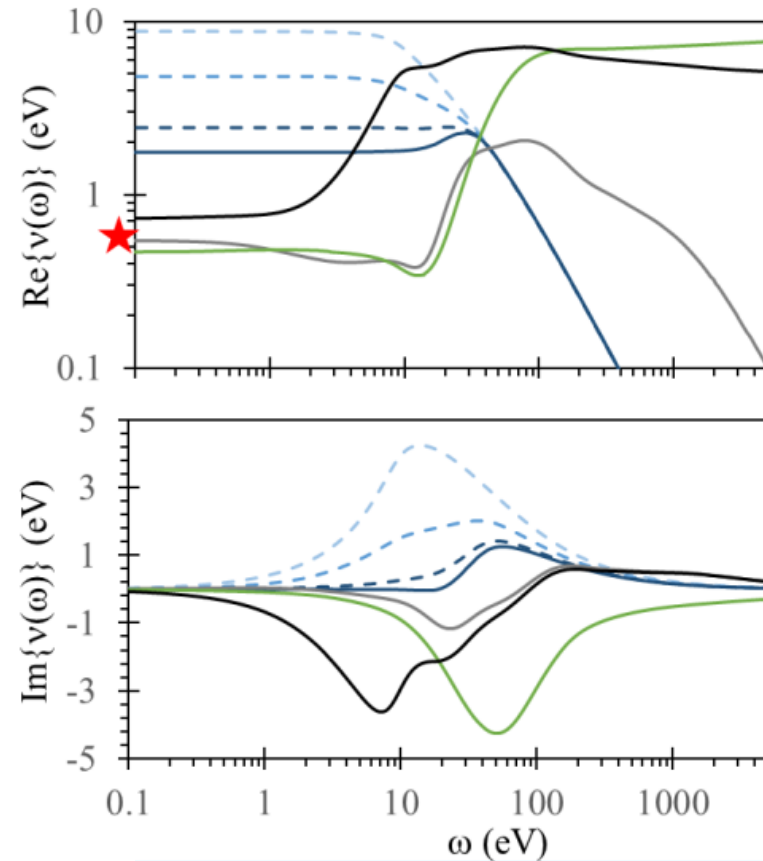
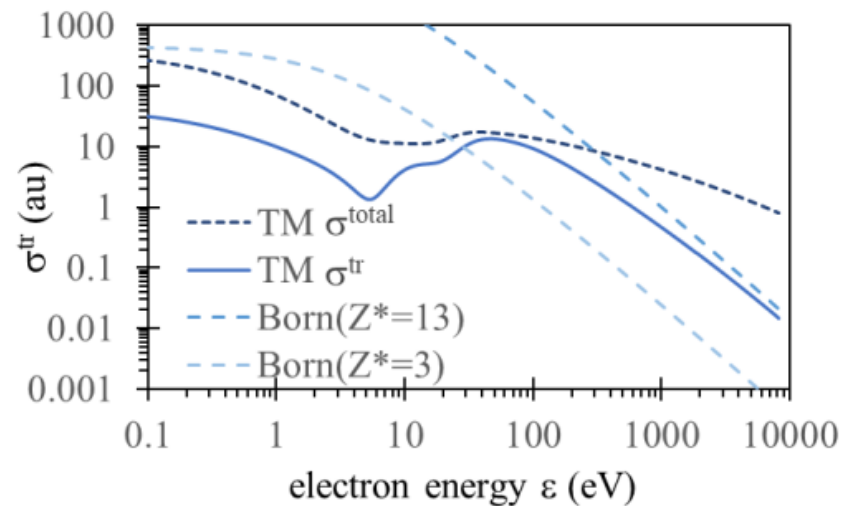
The K-G $v(0)$ limit enforces the conductivity sum rule; $\sigma(\omega)$ shows the non-Drude feature observed by Witte et al. [2] – but now the XRTS plasmon peak is too narrow

A trial estimate of inelastic processes brings our DFT-AA calculations into good agreement with reference data



$$v^{inel}(\omega) \sim n_e \iint (\sigma^{tot} - \sigma^{tr}) d\varepsilon_1 d\varepsilon_2 \times e^{(\mu - \varepsilon_f)/\tau}$$

Inelastic collisions include Pauli blocking and represent the relaxation time associated with a given excitation from equilibrium



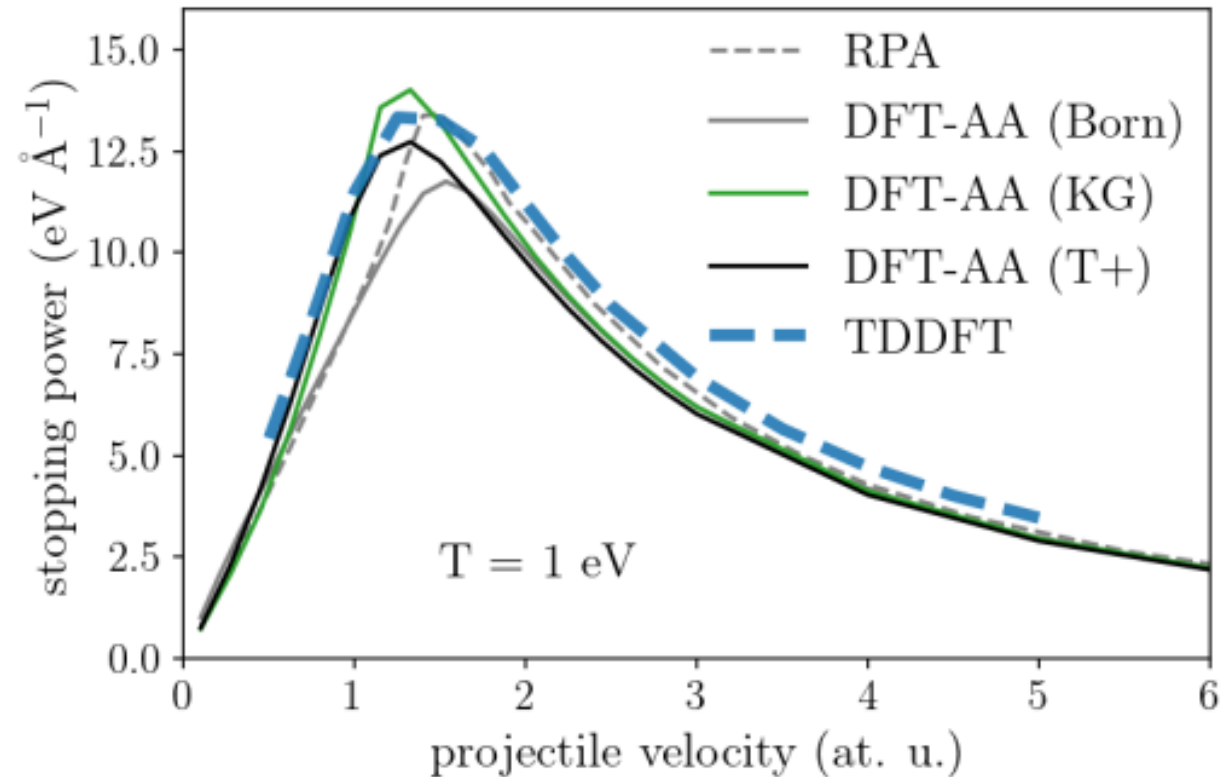
The impact of inelastic processes on the $v(0)$ limit and σ^{DC} will be investigated in future work

Our dynamic collision frequencies also improve DFT-AA stopping powers



$$\frac{dE}{dx} \sim \iint \left| \frac{1}{\epsilon(q, \omega)} \right| \omega d\omega dk$$

Modifications to the dielectric function impact charged-particle stopping powers.

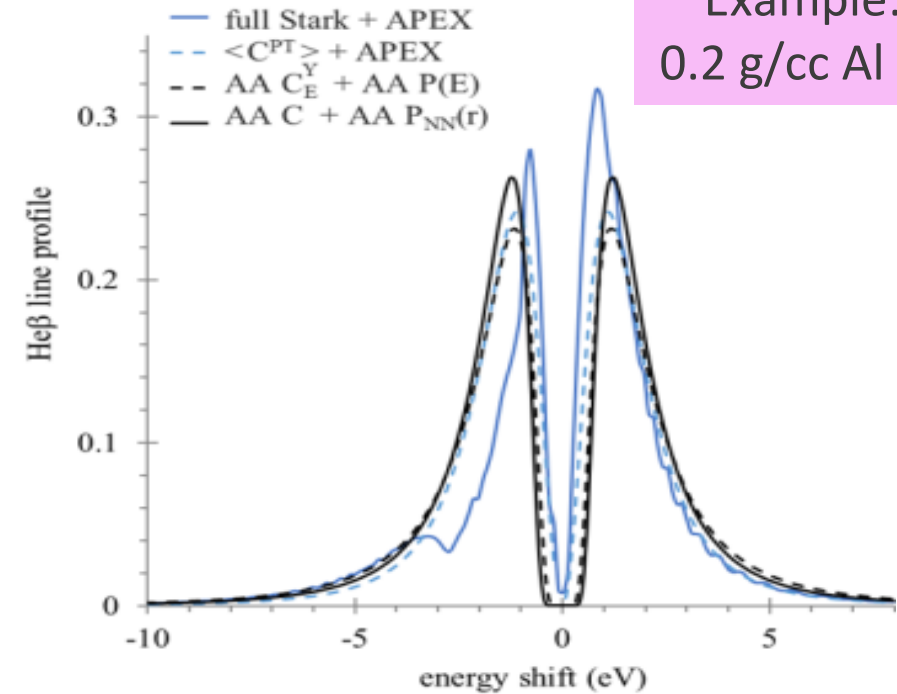


Using $v(\omega)$ from both Kubo-Greenwood and T-matrix + inelastic collisions significantly improve low-velocity dE/dx

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Example: He β of
0.2 g/cc Al at 250 eV

We have shown how DFT-AA can provide reasonable estimates for ion-Stark broadening
(see K. Adler's presentation)

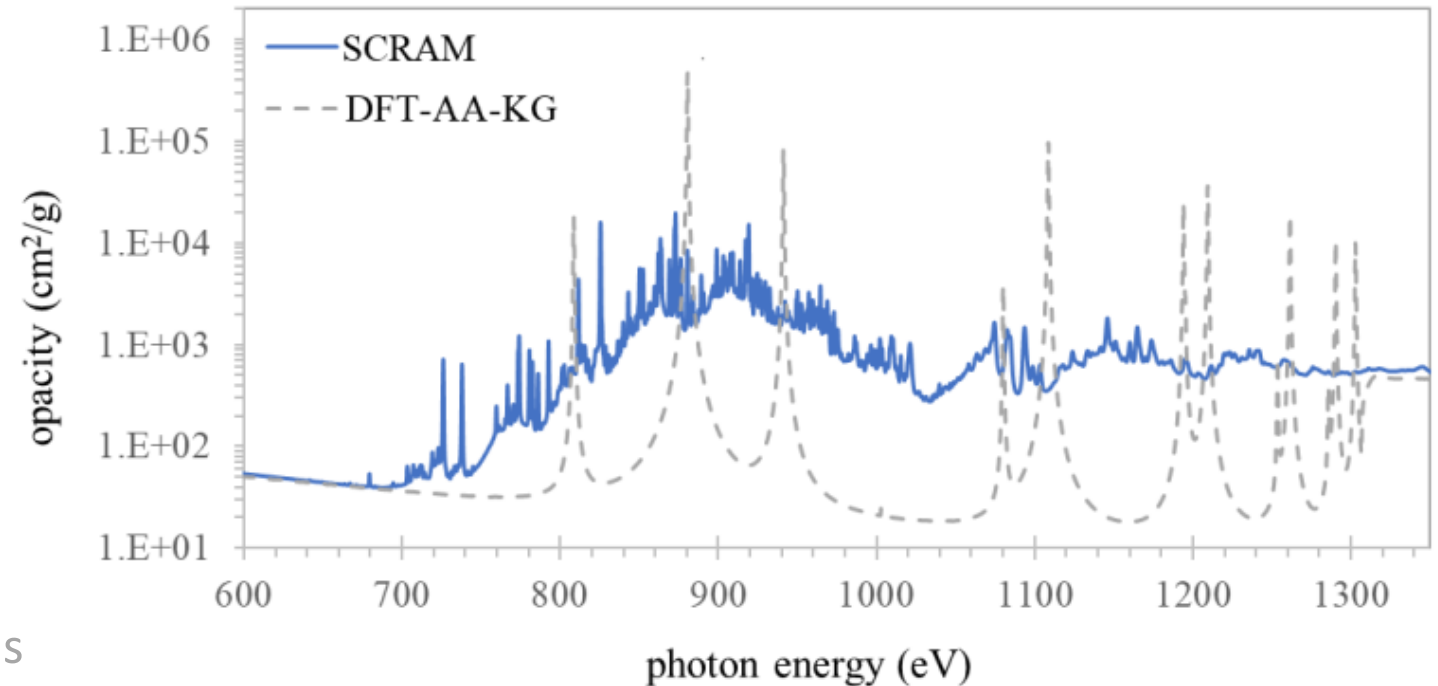
With T. Gomez and C Iglesias, we are exploring whether dynamic collision frequencies with inelastic collisions can provide electron broadening terms

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Example: 0.2 g/cc Fe at 180 eV

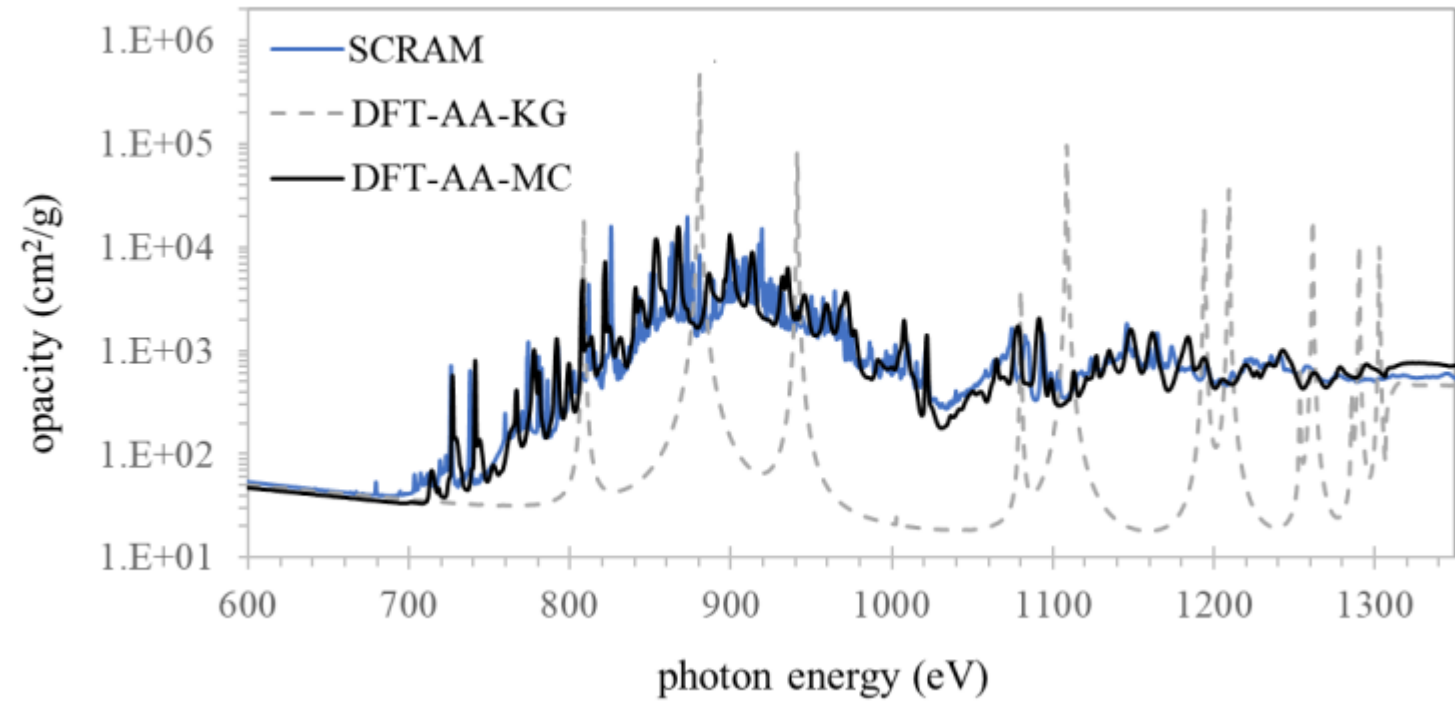


Good line shapes are only useful when combined with detailed multiconfiguration electronic structure and spectra

We have developed an efficient approach to generate multiconfiguration atomic structure



- Using DFT-AA electronic $n\ell$ orbitals as a basis set, we compute Slater coefficients to generate real (integer-occupied) electronic configurations [1]
- Re-optimize all DFT-AA $n'\ell'$ orbitals under small changes in occupations in all $n\ell$ orbitals
→ new Slater coefficients
- Taylor-expand Slater coefficients on occupations to capture orbital relaxation
- Add spin-orbit effects to split $n\ell$ transitions into $n\ell j$



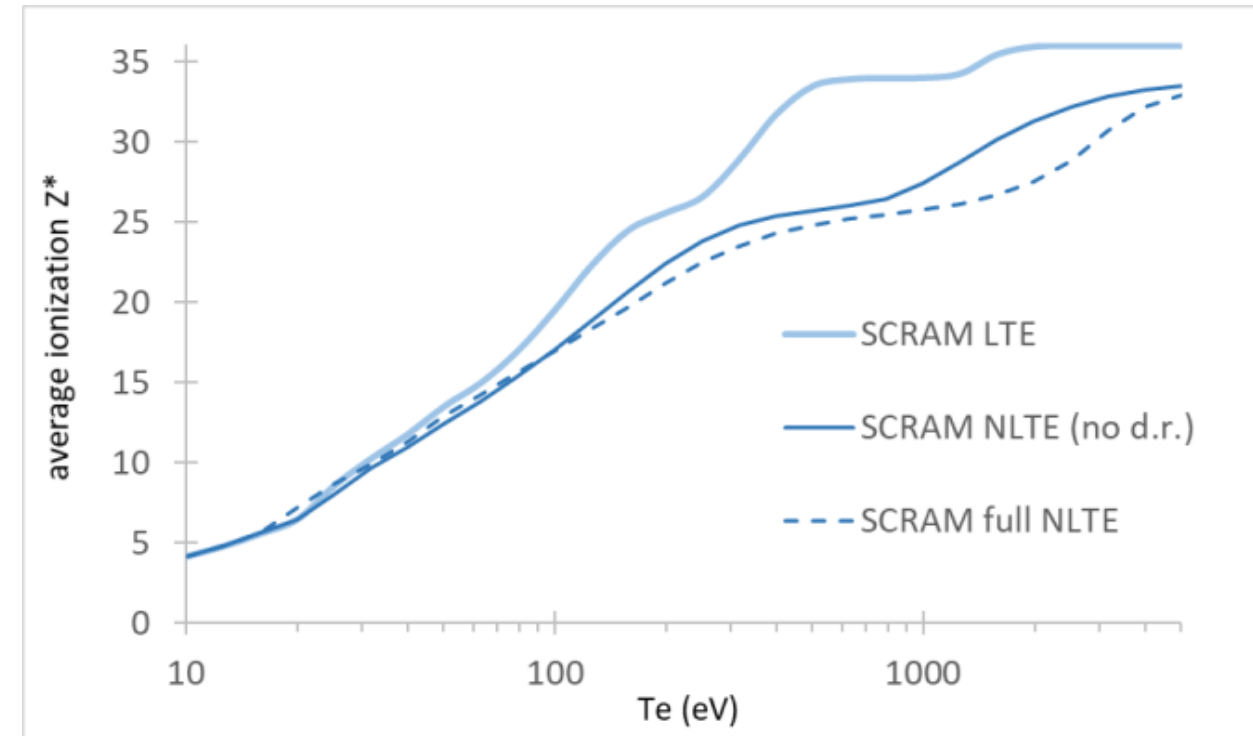
This approach generates very reasonable electronic structure and spectra without re-optimizing orbitals for every configuration

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 $T_r \ll T_e$

Example: 0.01 g/cc Kr



Non-LTE effects have profound impacts on charge state distributions, radiative losses, and emission and absorption spectra, especially for high Z and low densities

We have developed a simple modification to Fermi-Dirac occupation factors for a reasonable non-LTE Z^*



Standard DFT-AA occupations:

$$X_{n\ell} = 2(2\ell+1) / [1 + e^{(E_{n\ell} - m)/T}]$$

New DFT-AA occupations use collisional and photoionization (and recombination) rates

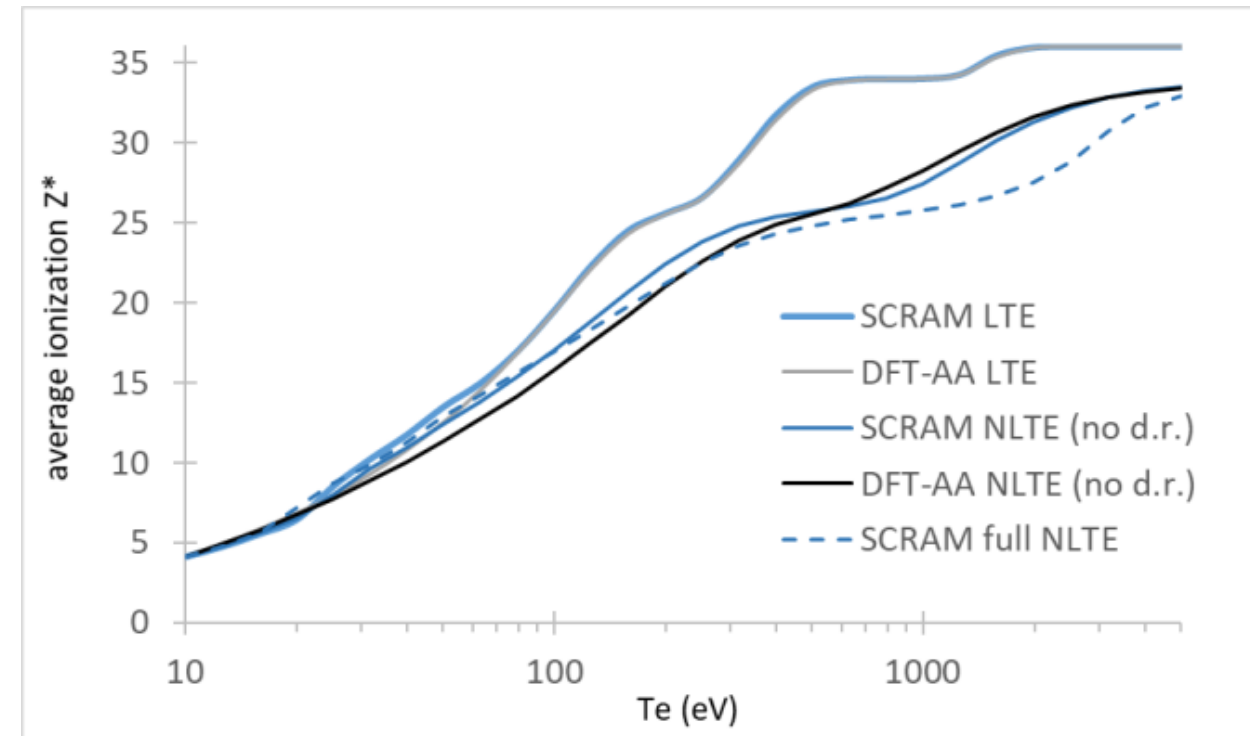
$R_{n\ell} \leftrightarrow_{\text{cntm}}$ that modify the occupations:

$$X_{n\ell} = 2(2\ell+1) / [1 + (R^{\text{ci}} + R^{\text{pi}}) / (R^{\text{cr}} + R^{\text{rr}} + R^{\text{dr}})]$$

→ Recovers LTE

→ Provides reasonable non-LTE Z^*

→ Dielectronic recombination is tricky

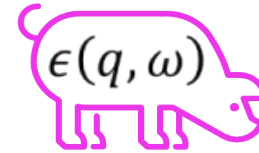


This simple modification gives a first approximation for Z^* that is good enough to provide self-consistent orbitals suitable for full non-LTE collisional-radiative calculations

Conclusion: Extensions of the DFT-AA model can produce reliable constitutive & observable properties



- Dynamic collision frequencies extend DFT-AA model predictions to include average plasma response as well as averaged ionic & electronic structure
 - Conductivities
 - X-ray Thomson Scattering (Hentschel, Baczewski)
 - Stopping powers (Kononov, Hentschel)
 - We can split the DFT-AA electronic structure into real configurations and incorporate non-LTE effects
 - Detailed spectra from self-consistent orbital basis set
 - Self-consistent ion-Stark line shapes (Adler, Gomez)
- All quantities come from a single, internally self-consistent DFT-AA model that agrees with available DFT-MD, TDDFT, CR, & line shape models
- Future: configuration-specific $\epsilon(q, \omega)$
 - when can we average?
 - Self-consistent dynamic electron-collisional broadening



Homer's
magical
animal



Rest well,
Frankenmodels!