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# Dynamic collision frequencies, scattering, stopping, and spectra

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Radiative Properties of Hot Dense Matter  
Santa Fe, NM  
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# Overarching goal: Develop a complete and consistent atomic model for constitutive and observable data



- 1) Start with existing average-atom model based on density functional theory: **DFT-AA**
- 2) Use *ab-initio* multi-center DFT calculations to constrain DFT-AA model choices
- 3) Extend DFT-AA for dynamic properties (XRTS & line shapes)
- 4) Extend DFT-AA for multiconfiguration electronic structure (opacities)
- 5) Extend DFT- AA to non-equilibrium plasmas

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**Improving Predictive Capability in REHEDS Simulations with Fast, Accurate, and Consistent Non-Equilibrium Material Properties**

S.B. Hansen, A.D. Baczewski, T. Gomez, T.W. Hentschel, C.A. Jennings, A. Kononov, T. Nagayama, K. Adler, A. Cangi, K. Cochrane, B. Robinson, and A. Schleife

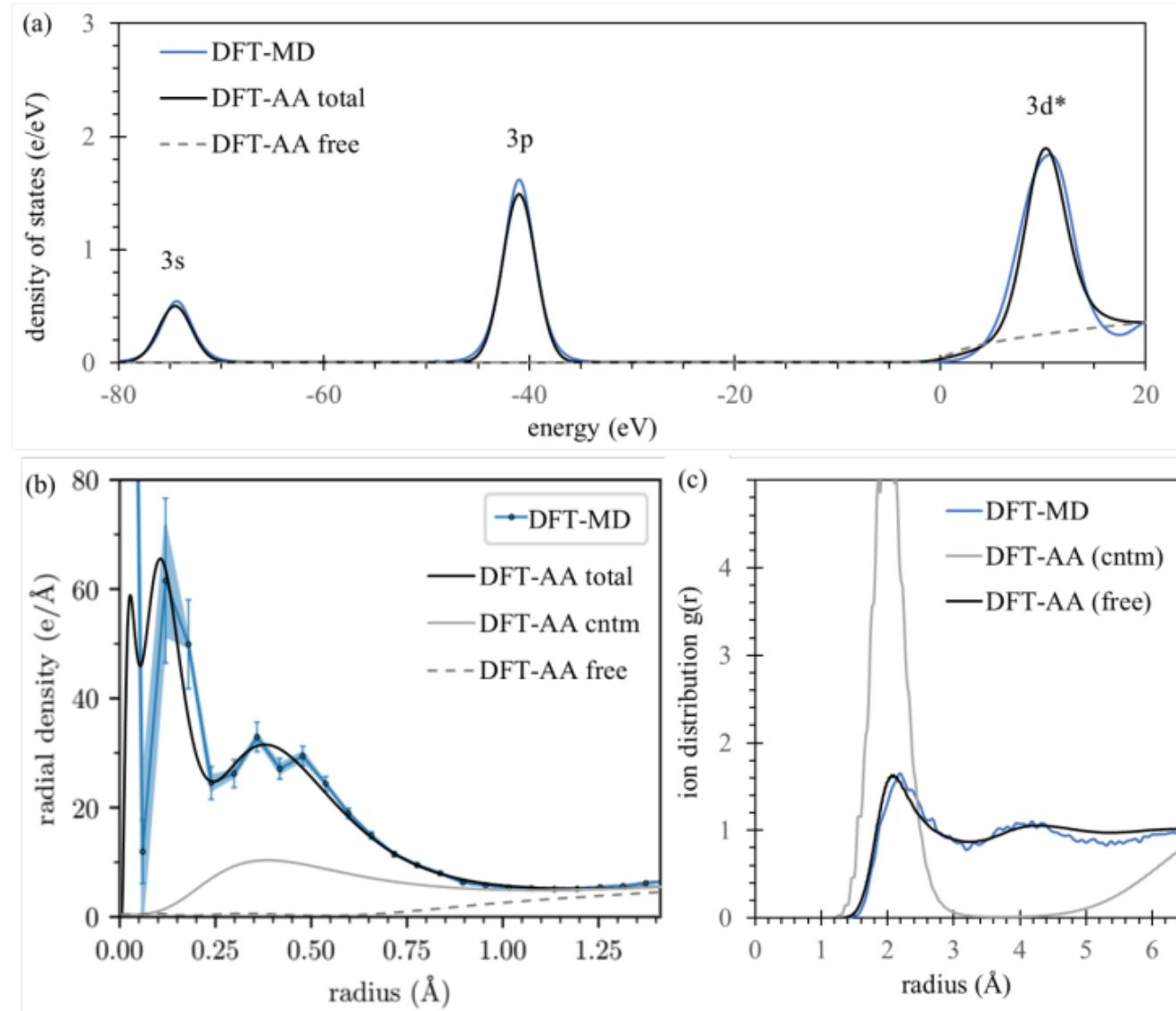
Rigorously validate against experimental data and reference models for XRTS, transport, EOS, non-LTE, and line shapes

# Overarching goal: Develop a complete and consistent atomic model for constitutive and observable data



- 1) Start with existing average-atom model based on density functional theory: **DFT-AA**  
→ self-consistent electronic and ionic structure, EOS, static transport coeff.
- 2) Use *ab-initio* multi-center DFT calculations to constrain choices  
→ boundary conditions,  $Z^*$

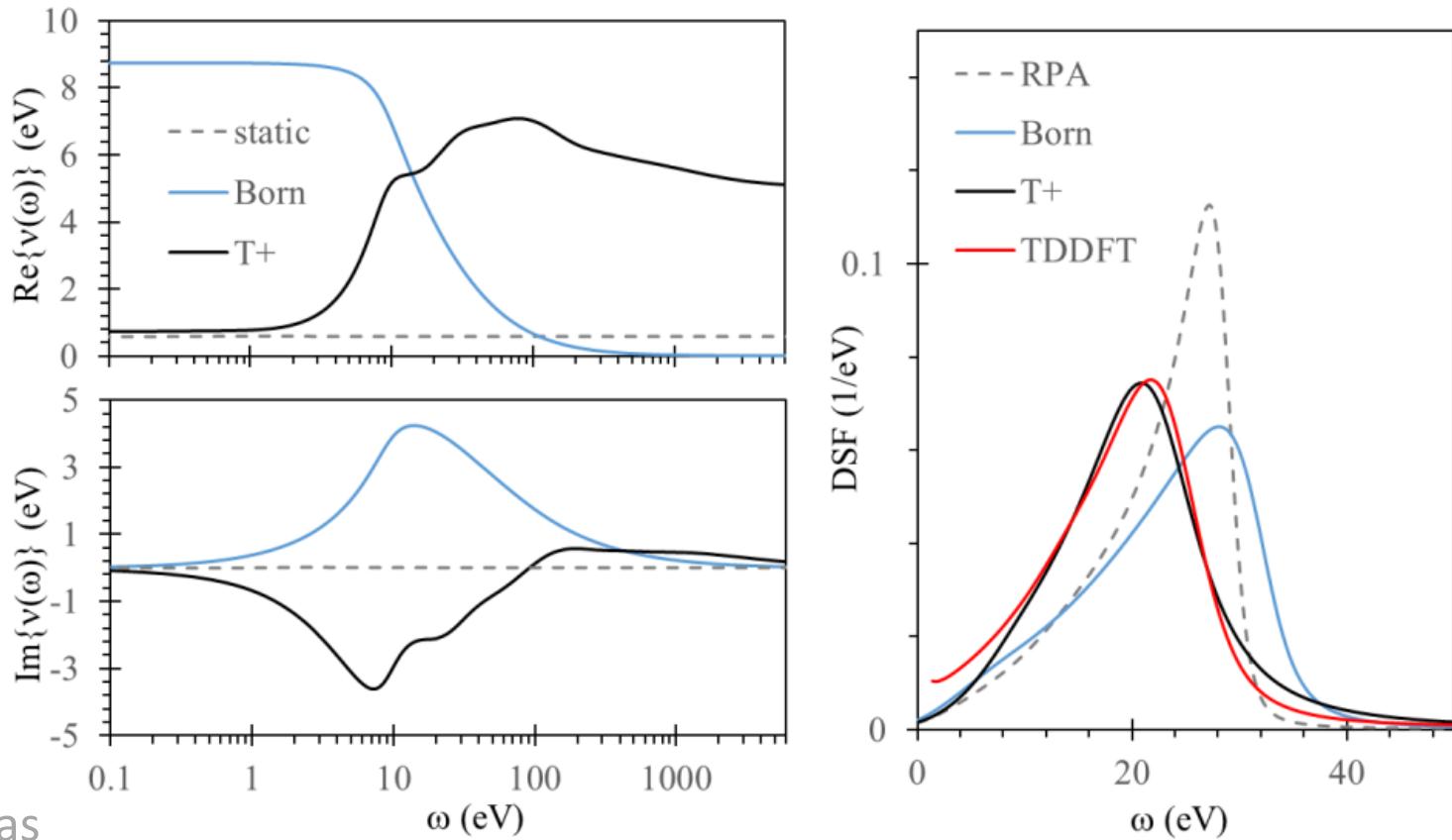
Example: Solid iron at  $T = 1\text{eV}$ :  
electronic density of states (top)  
and radial distribution functions



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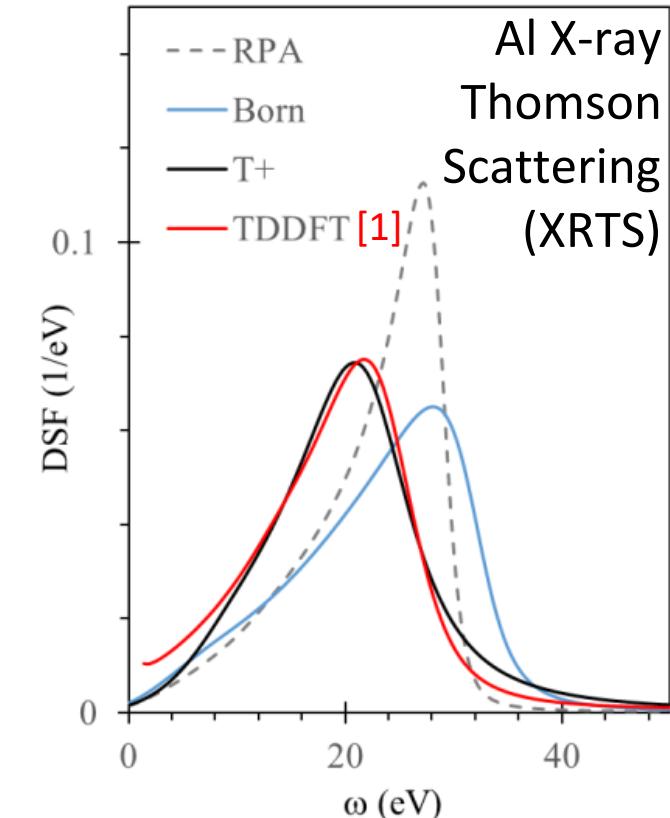
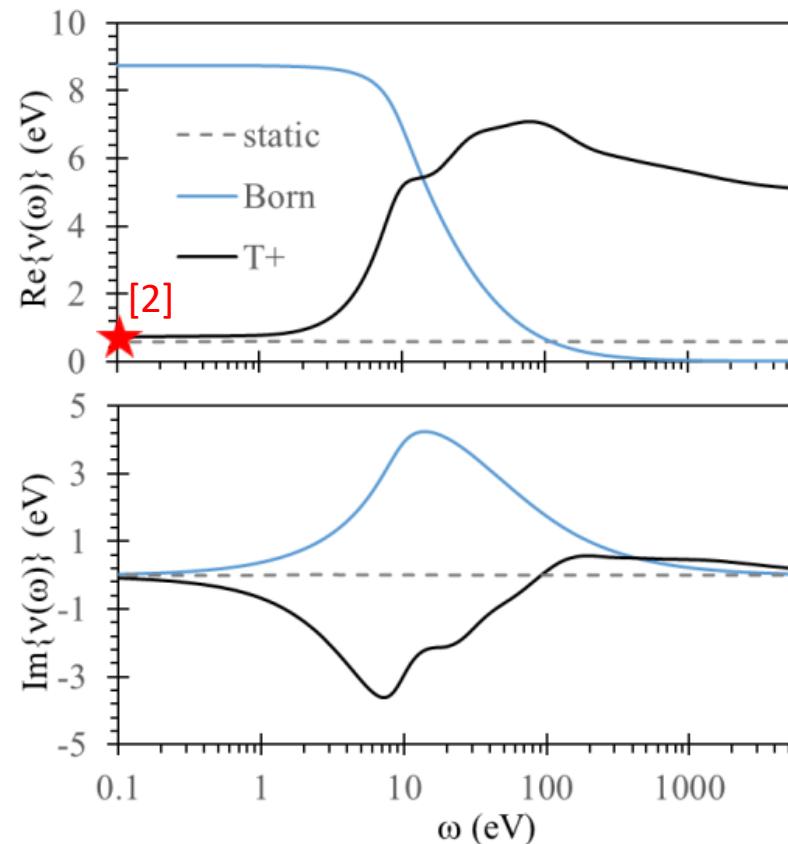


We will show that strong collisions and inelastic collisions improve DFT-AA agreement with TDDFT [1], static conductivities [2], and XRTS experiments [3]

# The dynamic collision frequency modifies response functions and observables



- In a direct (zero-frequency) electric field, the static collision frequency  $\nu(0)$  represents collisions that control DC conductivity  $\sigma(0) = \frac{n_e}{\nu(0)}$
- In an oscillating field, the dynamic collision frequency  $\nu(\omega)$  represents the damping of collective plasma oscillations with frequency  $\omega$
- The dynamic collision frequency informs the Mermin dielectric function  $\epsilon^M(q, \omega)$  and observables like the dynamic structure factor (DSF)  $\sim |\frac{1}{\epsilon(q, \omega)}|$   
→ XRTS and stopping powers



Al X-ray  
Thomson  
Scattering  
(XRTS)

# The Born approximation for cross sections is commonly used in calculations of $v(\omega)$



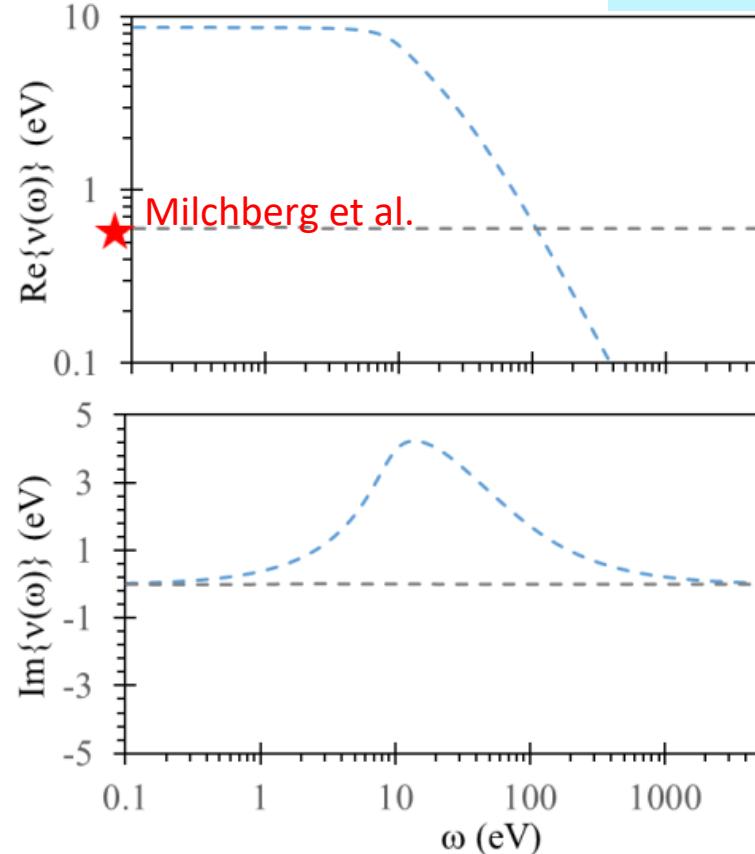
$$v(\omega) \sim$$

$$\int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

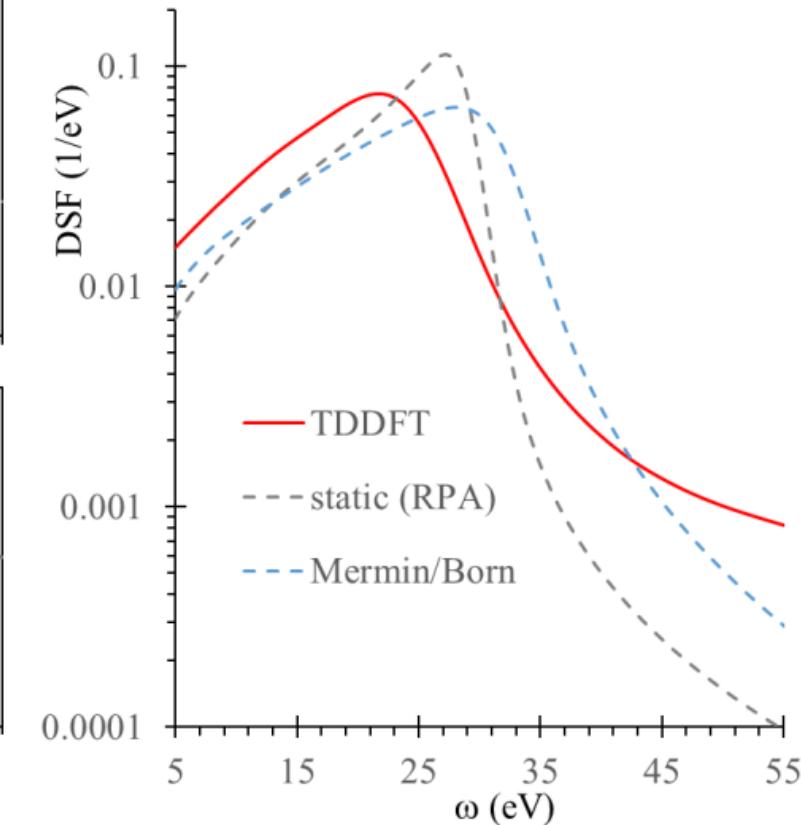
The Born cross section with a screened Yukawa potential is:

$$\frac{\partial \sigma^{tr}}{\partial \theta} = \frac{1}{4\pi^2} \left| \frac{4\pi Z^*}{q^2 + q_s^2} \right|^2$$

Whatever our choices for  $Z^*$  and  $q_s$ , the real and imaginary parts of  $v(\omega)$  have a fixed functional form that moves  $\epsilon^M(q, \omega)$  away from TDDFT



Example: Solid Al at  $T = 1\text{eV}$



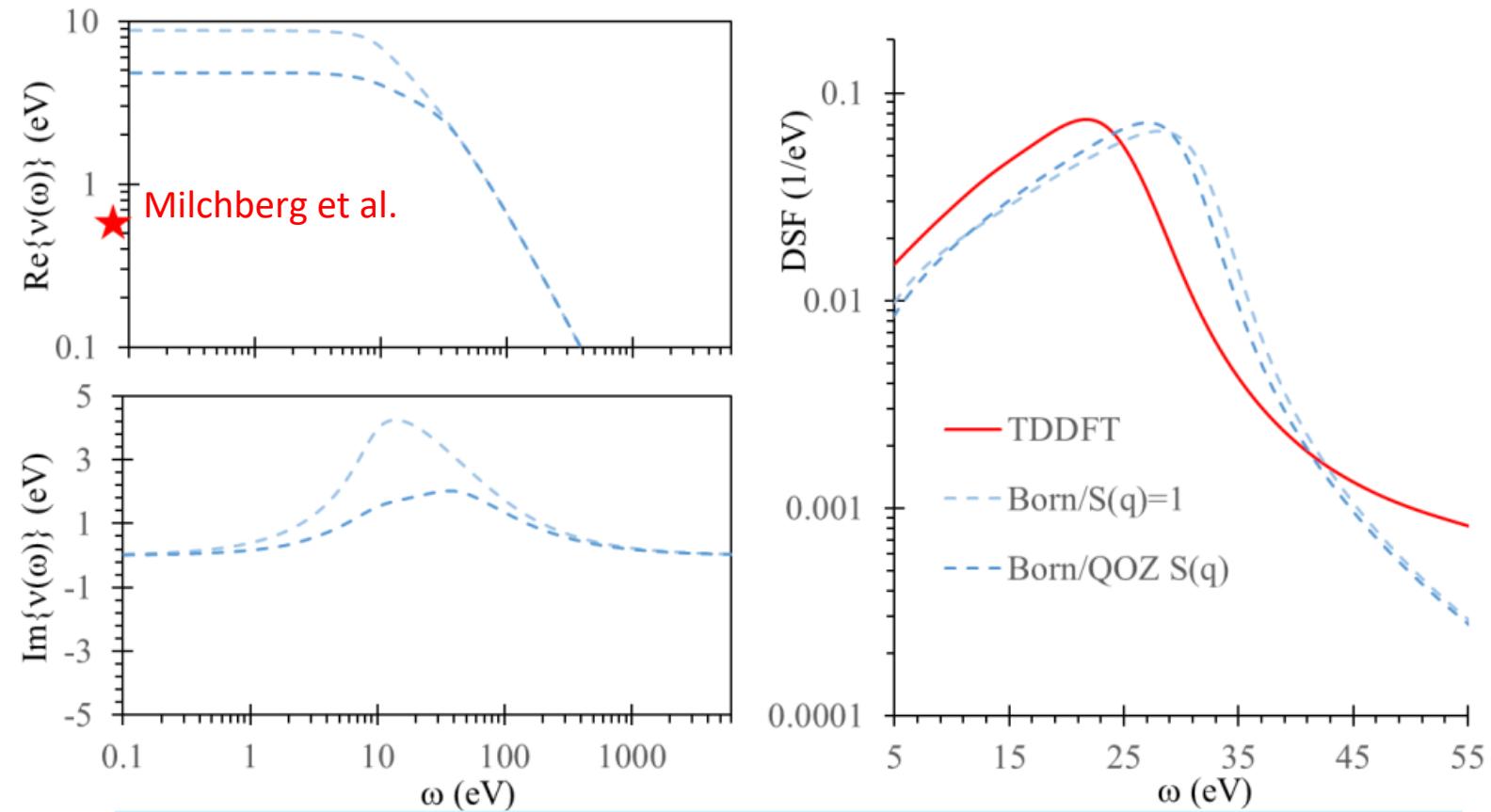
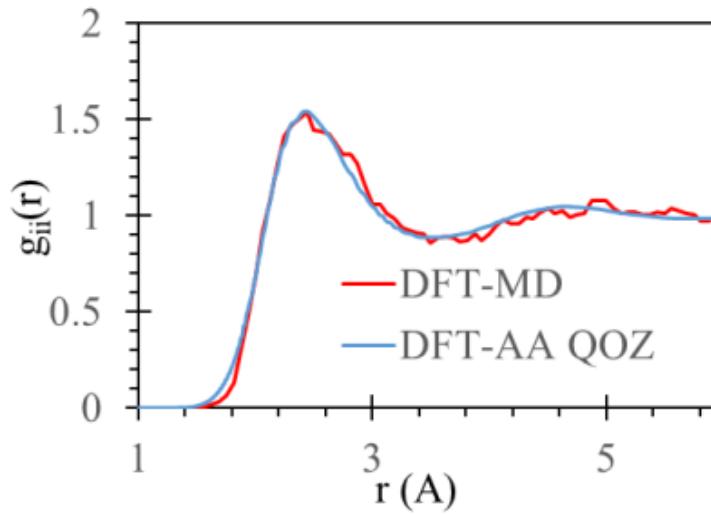
Weak scattering (Born) gives unreliable  $v(0)$  and inevitable  $\text{Im}\{v(\omega)\} > 0$  shifts the RPA plasmon peak to higher energies

# Modifying the ion-ion structure factor doesn't help much



$$v(\omega) \sim \int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

We can use a self-consistent  $S_{ii}(q)$  from  $g_{ii}(r)$  following Starrett & Saumon

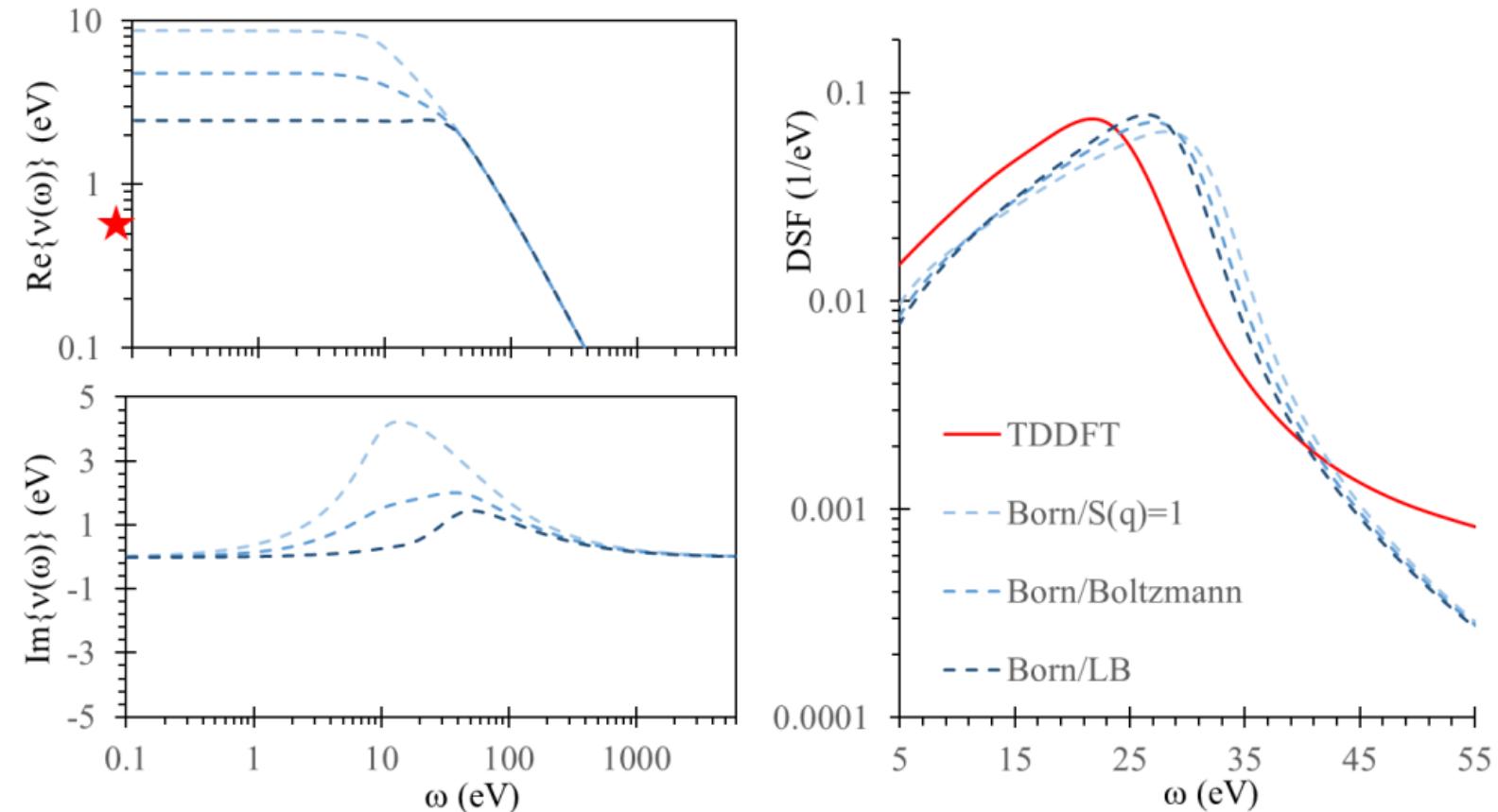


While we find good agreement with DFT-MD ionic structure, changing  $S_{ii}(q)$  doesn't change the functional form of  $v(\omega)$

# Including dynamic screening via Lenard-Balescu integration [1,2] also has only a modest effect

$$v(\omega) \sim \int \frac{\epsilon^{-1}(q, \omega) - \epsilon^{-1}(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

Replacing  $\epsilon(q, \omega)$  with  $\epsilon^{-1}(q, \omega)$  gives a better approximation for the collisional term in the generalized Boltzmann equation

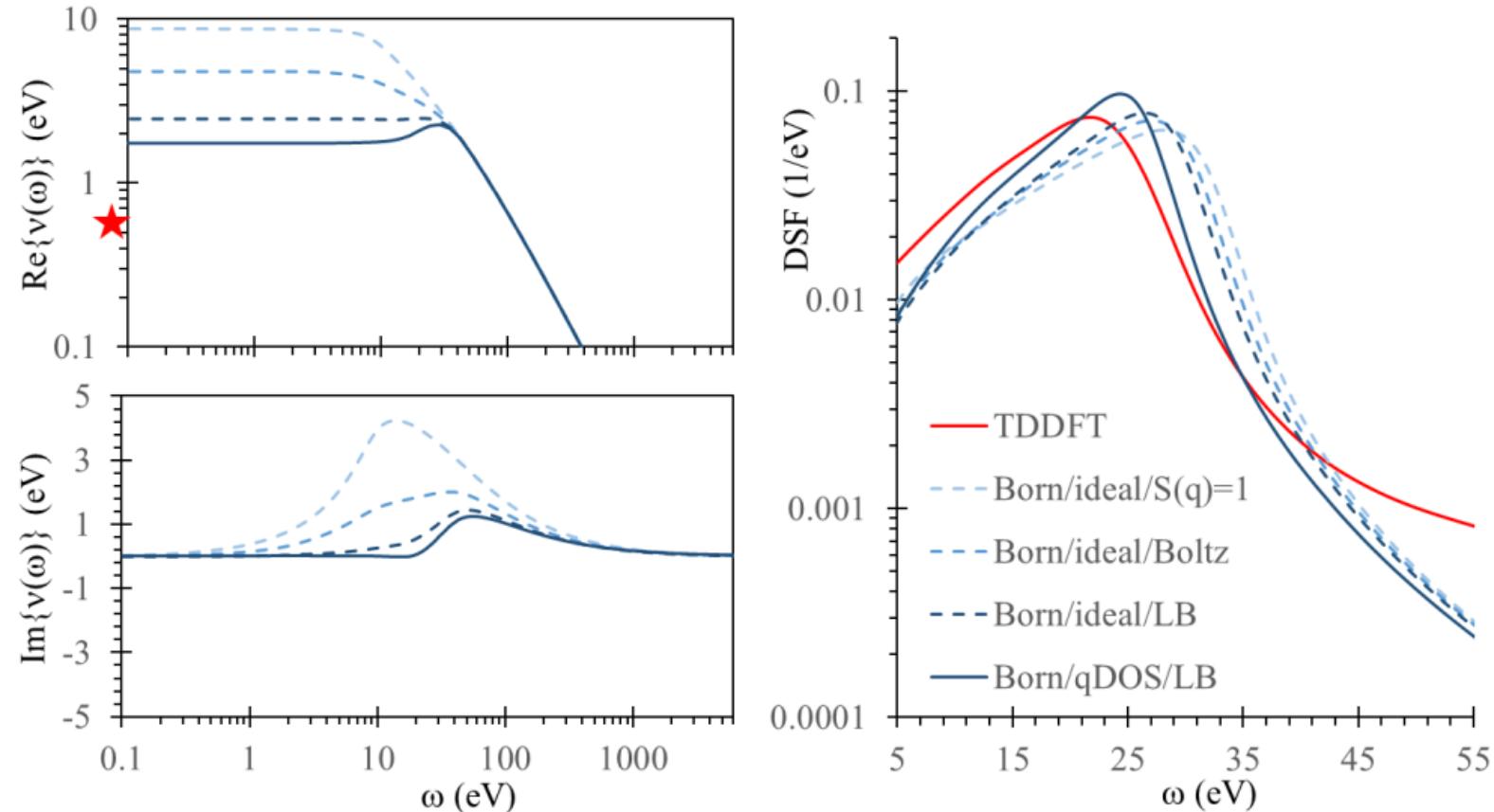
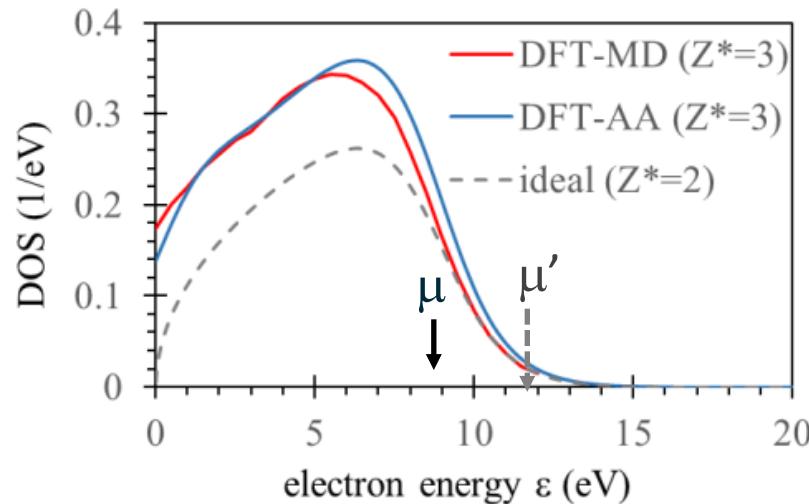


While this modification further decreases  $v(0)$  and the plasmon shift, both remain far from trusted values

# Using the quantum density of states gives a more reasonable DSF but still does not capture $v(0)$

$$v(\omega) \sim \int \frac{\epsilon_q^{-1}(q, \omega) - \epsilon_q^{-1}(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{tr}}{\partial \theta} dq$$

Using the quantum DOS modifies the integrals in  $\epsilon(q, \omega)$  and makes  $Z^*$  consistent with  $\mu$



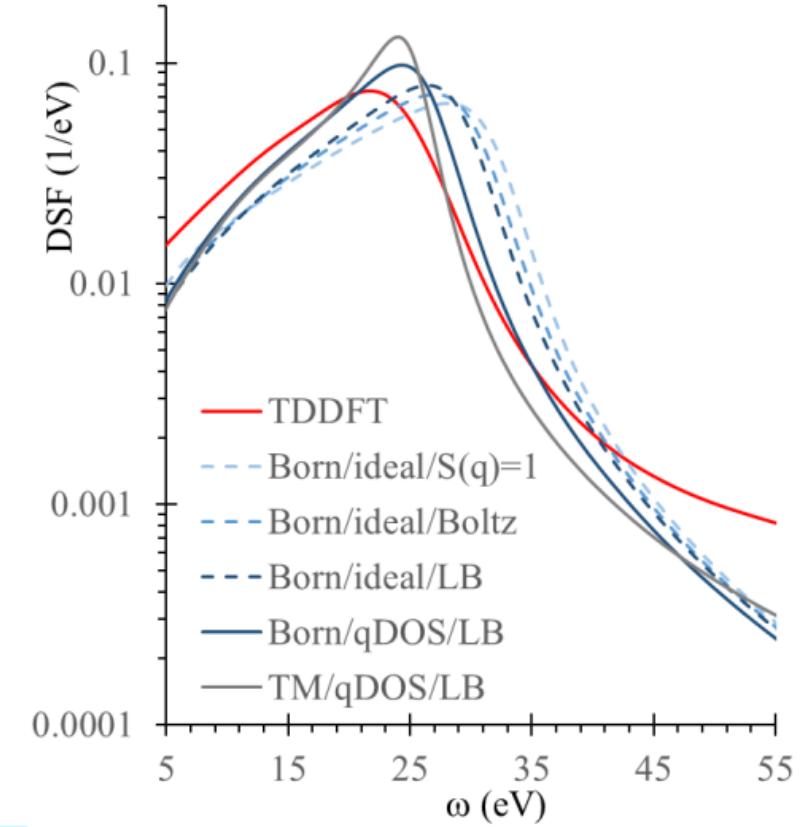
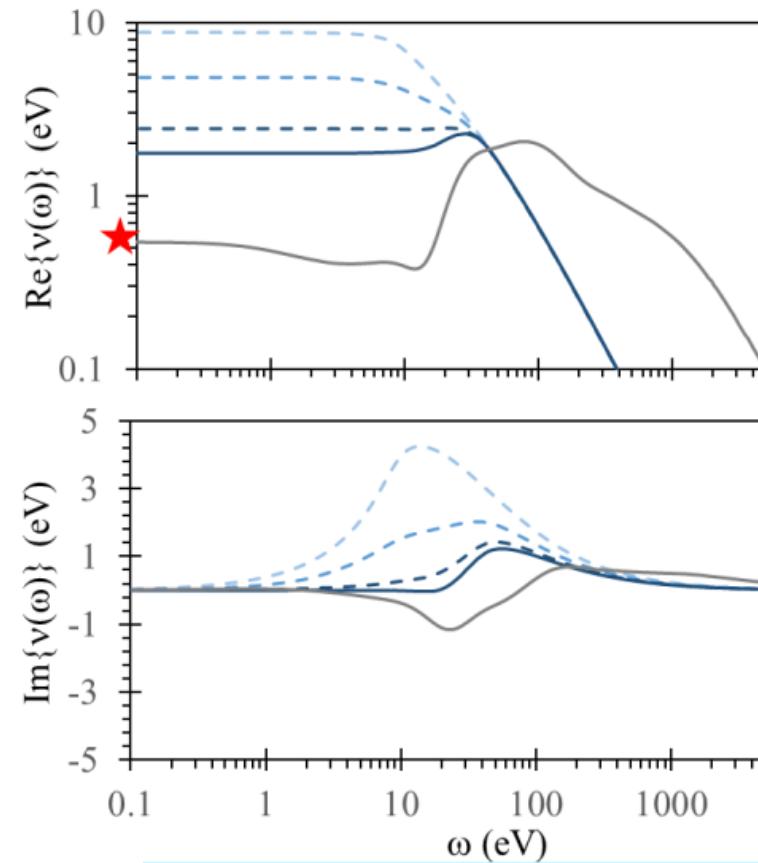
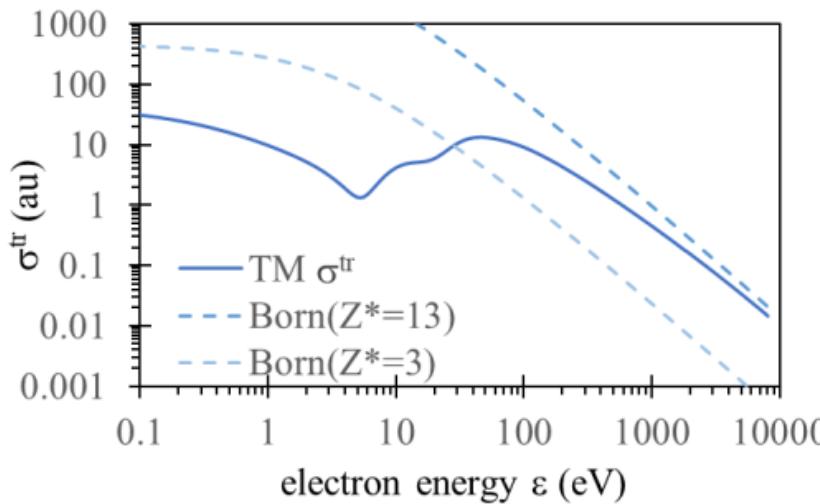
The full quantum density of states better represents the continuum electron structure and eliminates ambiguity in  $Z^*$ , but disagreement in  $v(0)$  persists

# T-matrix cross sections capture strong collisions, significantly improving the $\nu(0)$ limit



$$\nu(\omega) \sim \int \frac{\epsilon(q, \omega) - \epsilon(q, 0)}{\omega} q^6 S_{ii}(q) \frac{\partial \sigma^{\text{tr}}}{\partial \theta} dq$$

The T-matrix cross sections use phase shifts from the self-consistent continuum orbitals



Strong scattering significantly improves  $\nu(0)$  and departs significantly from the Born form, giving  $\text{Im}\{\nu(\omega)\} < 0$

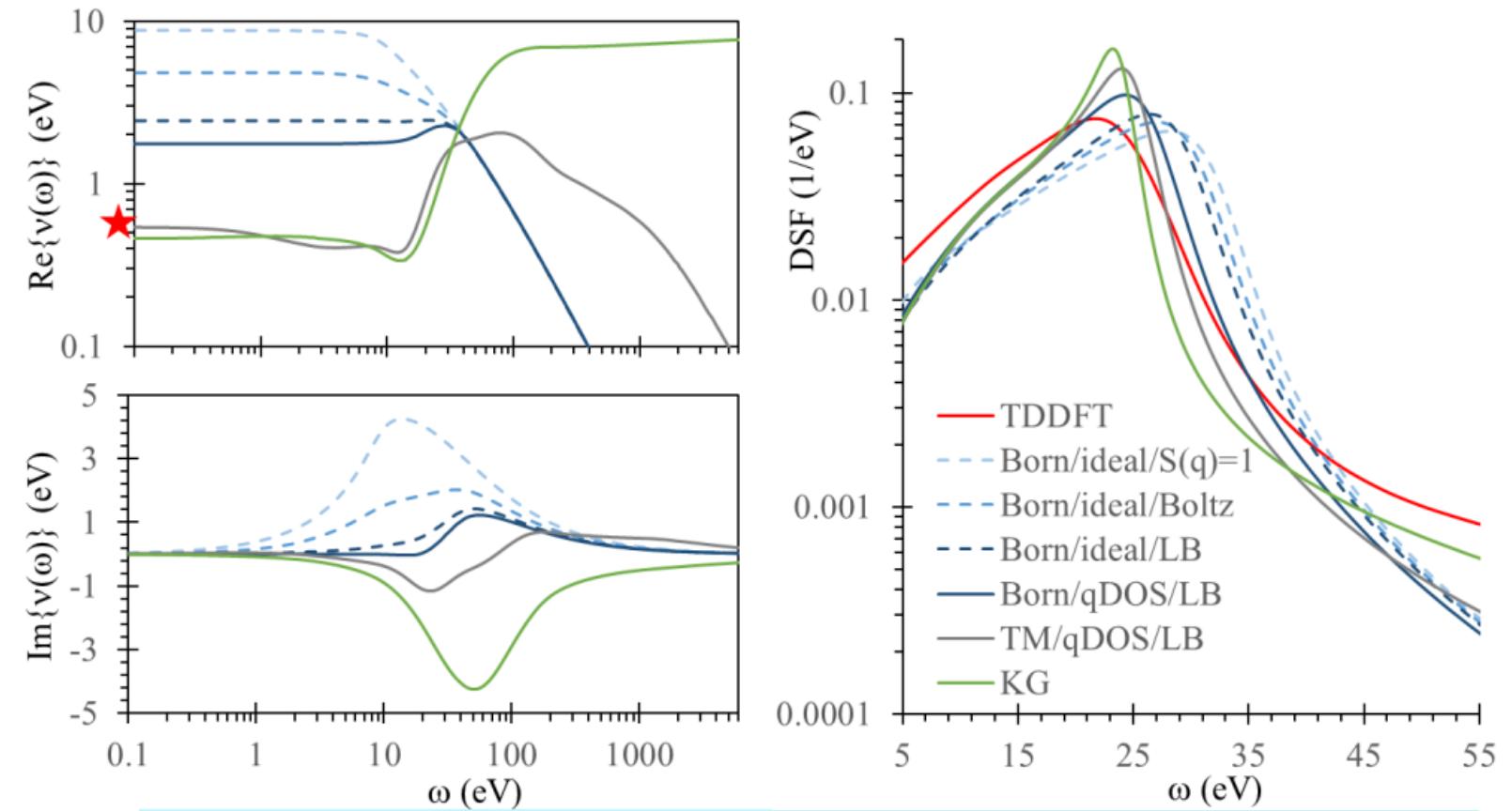
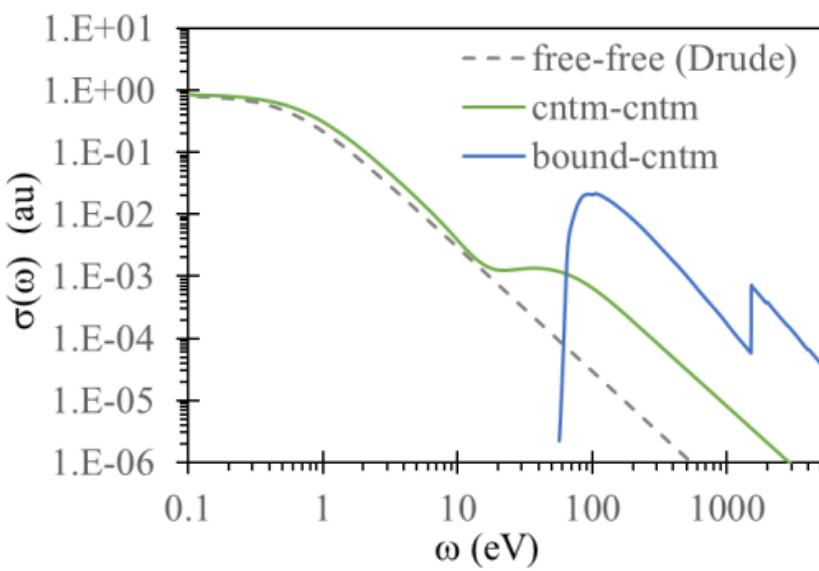
# Independent Kubo-Greenwood calculations capture a similar $\nu(0)$ limit and frequency dependence



$$\nu(\omega) = \frac{n_e}{\sigma(\omega)} + i\omega$$

following Reinholz et al. [1]

The quantum Kubo-Greenwood dynamic conductivity  $\sigma(\omega)$  adds a distinct feature to the Drude  $\sigma(\omega)$

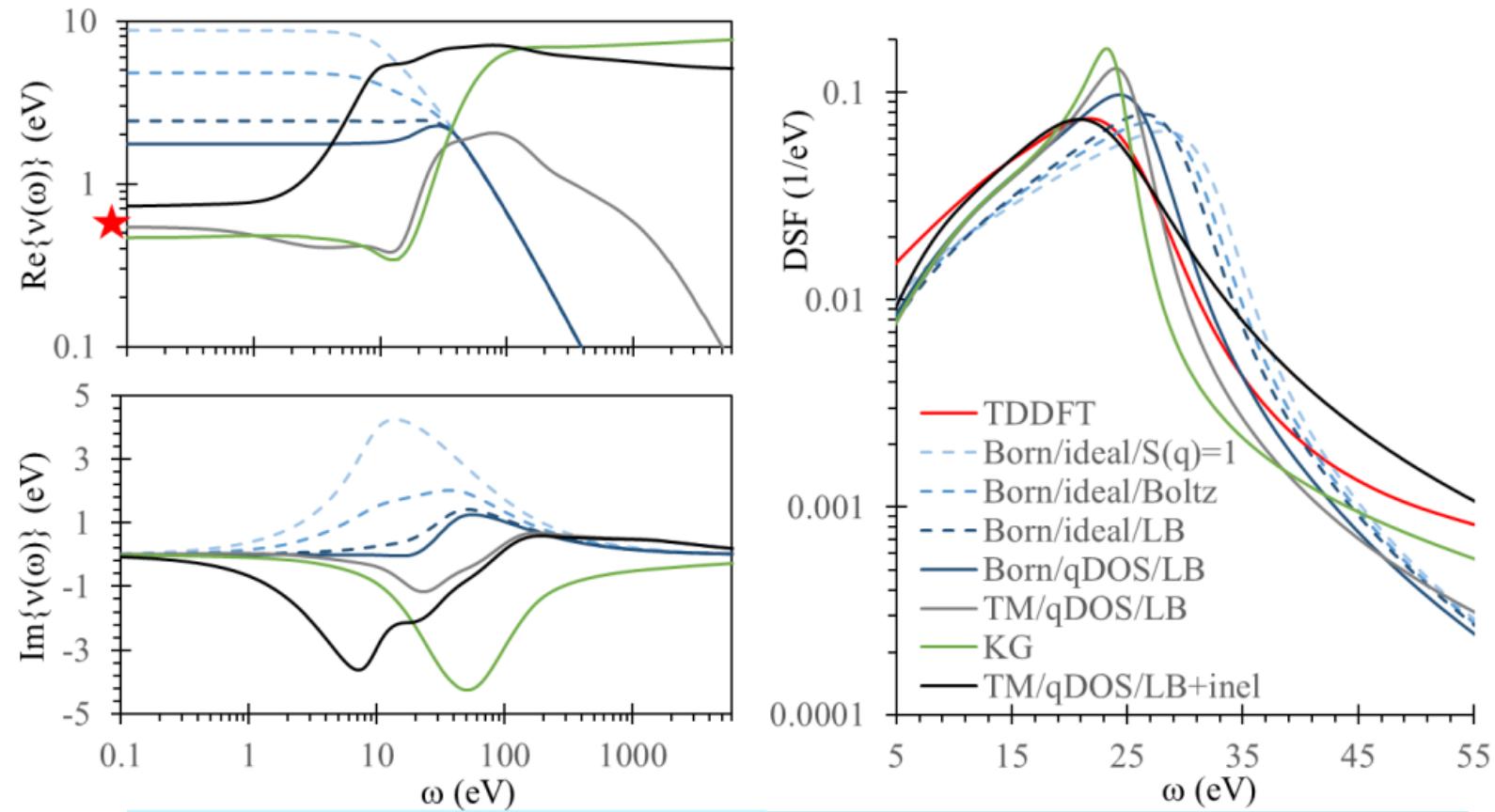
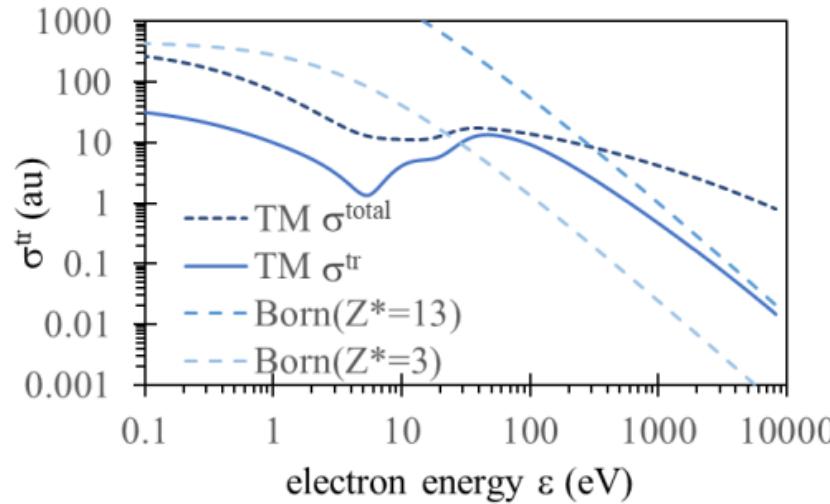


The K-G  $\nu(0)$  limit enforces the conductivity sum rule;  
 $\sigma(\omega)$  shows the non-Drude feature observed by Witte et al. [2]  
– but now the XRTS plasmon peak is too narrow

# A trial estimate of inelastic processes brings our DFT-AA calculations into good agreement with reference data

$$\nu^{inel}(\omega) \sim n_e \iint (\sigma^{tot} - \sigma^{tr}) d\varepsilon_1 d\varepsilon_2 \times e^{(\mu - \varepsilon_f)/\tau}$$

Inelastic collisions include Pauli blocking and represent the relaxation time associated with a given excitation from equilibrium



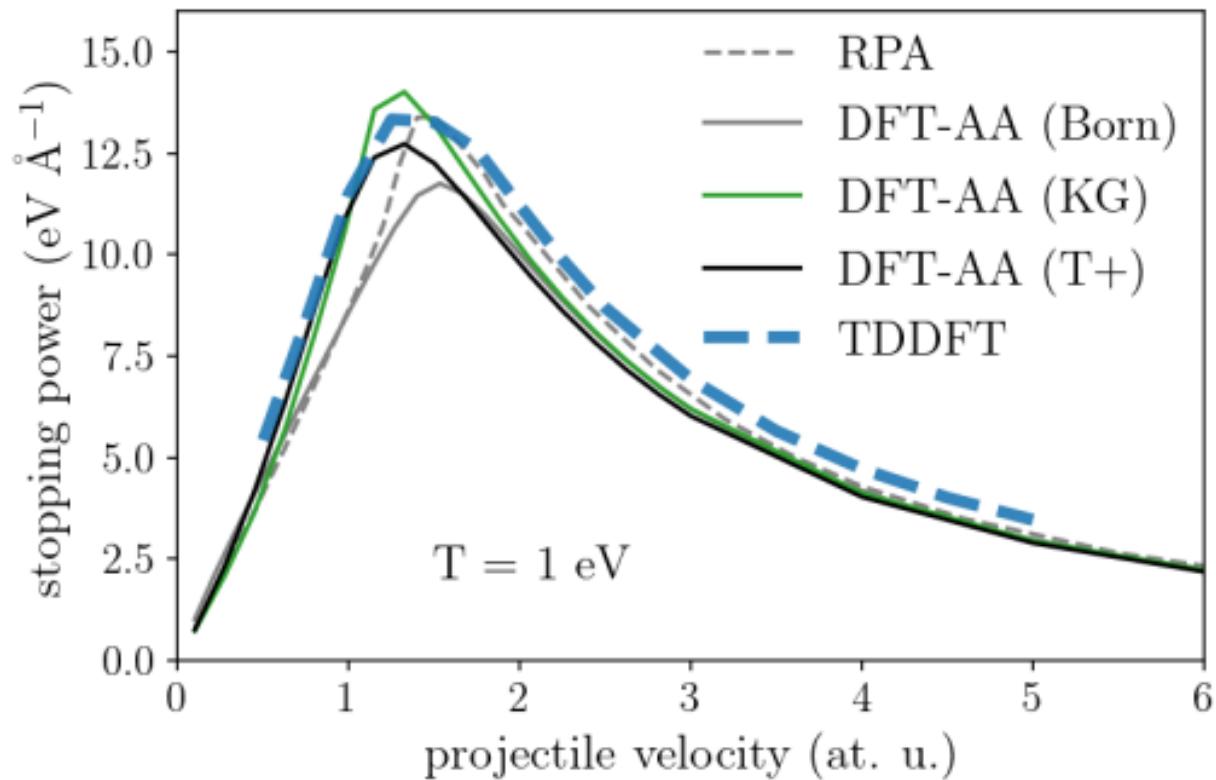
The impact of inelastic processes on the  $\nu(0)$  limit and  $\sigma^{DC}$  will be investigated in future work

# Our dynamic collision frequencies also improve DFT-AA stopping powers



$$\frac{dE}{dx} \sim \iint \left| \frac{1}{\epsilon(q, \omega)} \right| \omega d\omega dk$$

Modifications to the dielectric function impact charged-particle stopping powers.

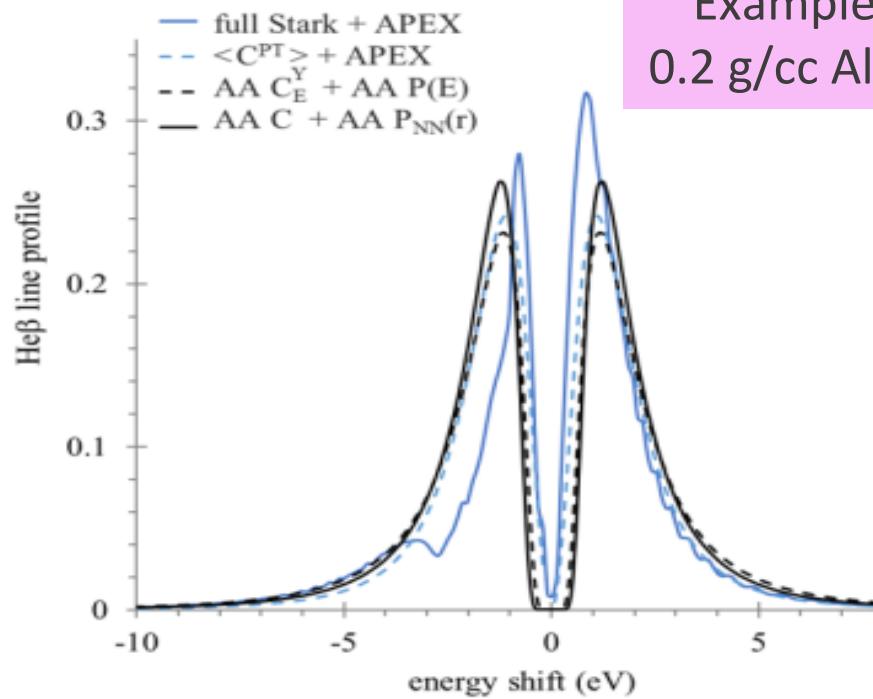


Using  $\nu(\omega)$  from both Kubo-Greenwood and T-matrix + inelastic collisions significantly improve low-velocity  $dE/dx$

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Example: He $\beta$  of 0.2 g/cc Al at 250 eV

We have shown how DFT-AA can provide reasonable estimates for ion-Stark broadening  
**(see K. Adler's presentation)**

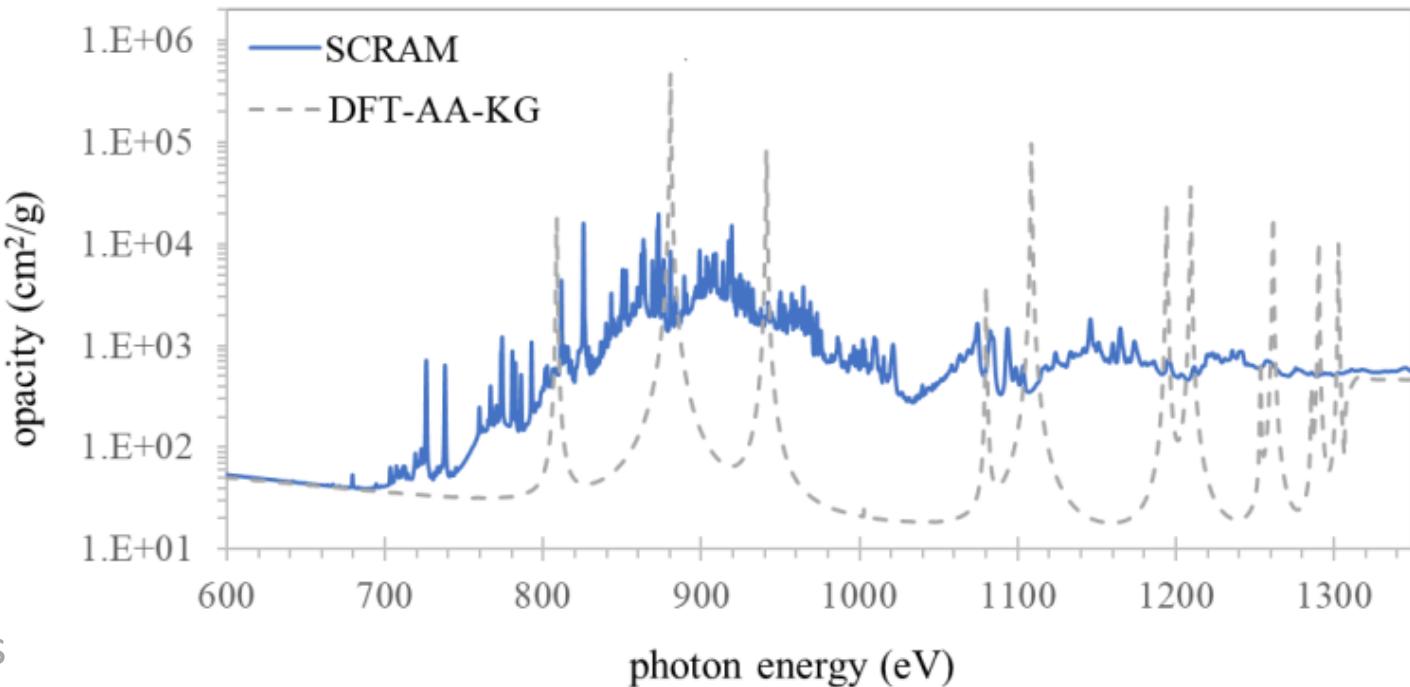
With T. Gomez and C Iglesias, we are exploring whether dynamic collision frequencies with inelastic collisions can provide electron broadening terms

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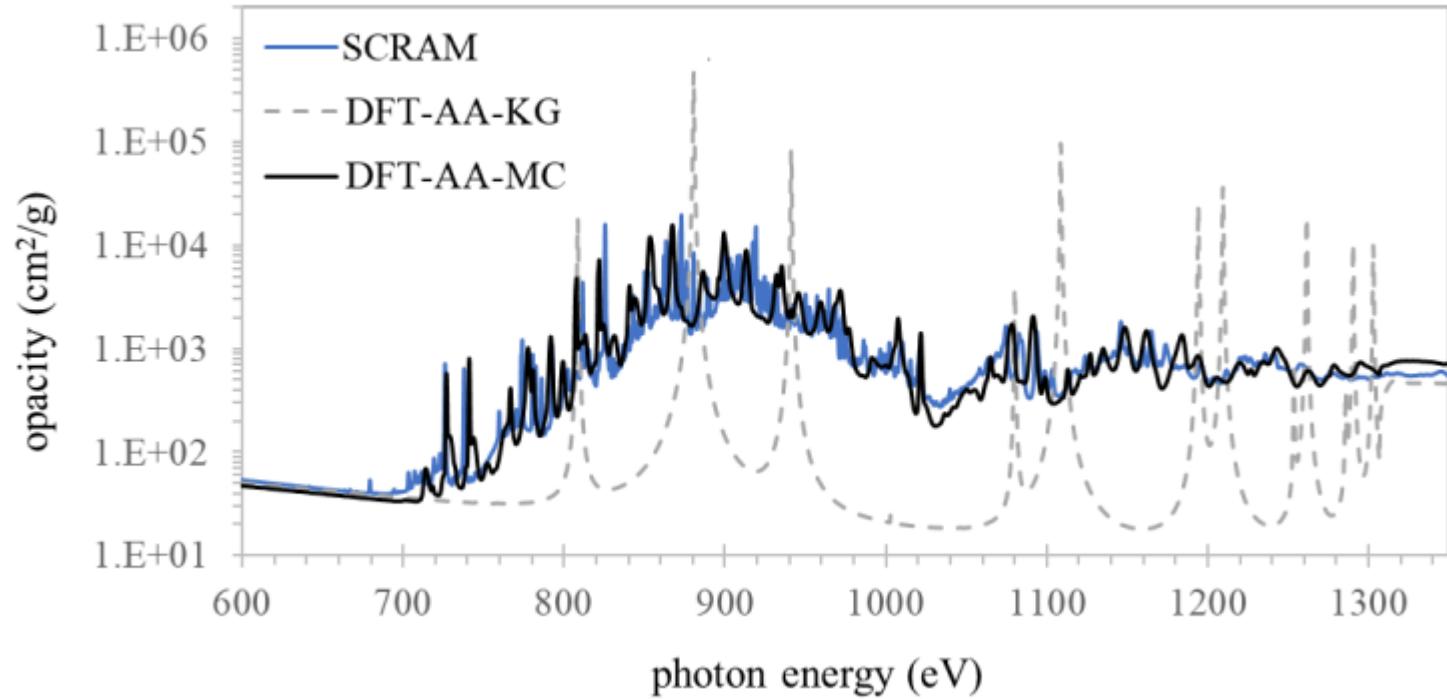
Example: 0.2 g/cc Fe at 180 eV



Good line shapes are only useful when combined with detailed multiconfiguration electronic structure and spectra

# We have developed an efficient approach to generate multiconfiguration atomic structure

- Using DFT-AA electronic  $n\ell$  orbitals as a basis set, we compute Slater coefficients to generate real (integer-occupied) electronic configurations [1]
- Re-optimize all DFT-AA  $n'\ell'$  orbitals under small changes in occupations in all  $n\ell$  orbitals → new Slater coefficients
- Taylor-expand Slater coefficients on occupations to capture orbital relaxation
- Add spin-orbit effects to split  $n\ell$  transitions into  $n\ell j$



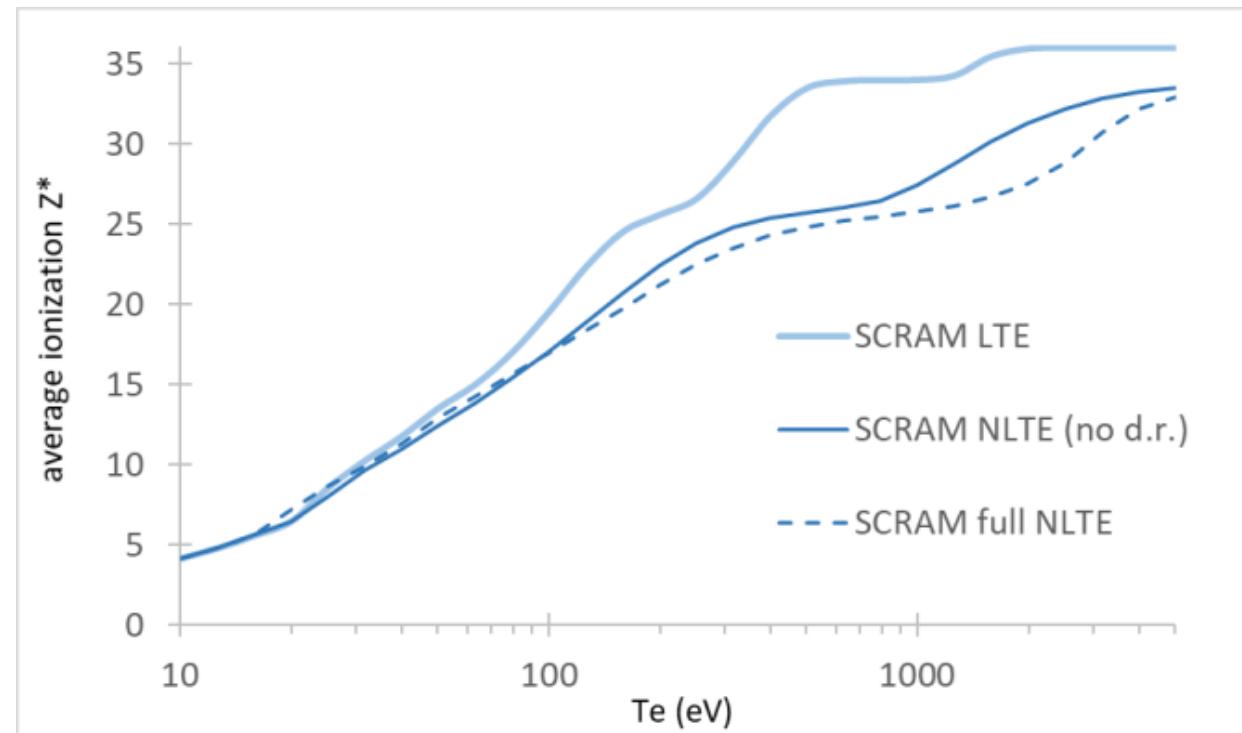
This approach generates very reasonable electronic structure and spectra without re-optimizing orbitals for every configuration

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- 5) **Extend DFT- AA to non-equilibrium plasmas**  
 $T_e \ll T_i$

Example: 0.01 g/cc Kr



Non-LTE effects have profound impacts on charge state distributions, radiative losses, and emission and absorption spectra, especially for high Z and low densities

# We have developed a simple modification to Fermi-Dirac occupation factors for a reasonable non-LTE $Z^*$

Standard DFT-AA occupations:

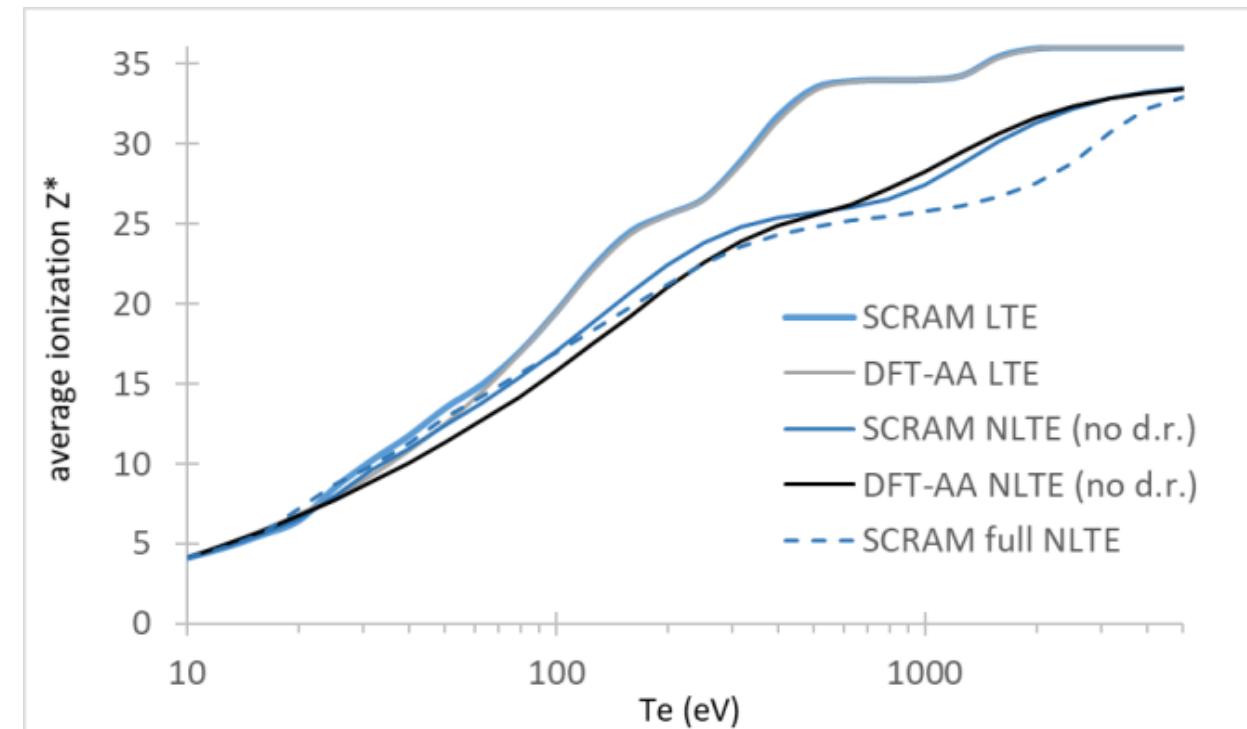
$$X_{n\ell} = 2(2\ell+1) / [1 + e^{(E_{n\ell} - m)/T}]$$

New DFT-AA occupations use collisional and photoionization (and recombination) rates

$R_{n\ell} \leftrightarrow_{\text{cntm}}$  that modify the occupations:

$$X_{n\ell} = 2(2\ell+1) / [1 + (R^{\text{ci}} + R^{\text{pi}}) / (R^{\text{cr}} + R^{\text{rr}} + R^{\text{dr}})]$$

- Recovers LTE
- Provides reasonable non-LTE  $Z^*$
- Dielectronic recombination is tricky



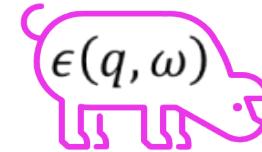
This simple modification gives a first approximation for  $Z^*$  that is good enough to provide self-consistent orbitals suitable for full non-LTE collisional-radiative calculations

# Conclusion: Extensions of the DFT-AA model can produce reliable constitutive & observable properties



- Dynamic collision frequencies extend DFT-AA model predictions to include average plasma response as well as averaged ionic & electronic structure
  - Conductivities
  - X-ray Thomson Scattering (Hentschel, Baczewski)
  - Stopping powers (Kononov, Hentschel)
- We can split the DFT-AA electronic structure into real configurations and incorporate non-LTE effects
  - Detailed spectra from self-consistent orbital basis set
  - Self-consistent ion-Stark line shapes (Adler, Gomez)

All quantities come from a single, internally self-consistent DFT-AA model that agrees with available DFT-MD, TDDFT, CR, & line shape models



Homer's magical animal



Rest well,  
Frankenmodels!

- Future: configuration-specific  $\epsilon(q, \omega)$ 
  - when can we average?
  - Self-consistent dynamic electron-collisional broadening