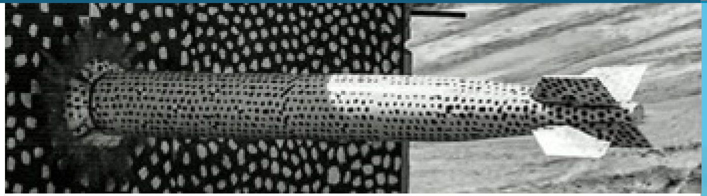
SAND2022-15511C

Sandia National Laboratories

# Estimating Pixels of Displacement for Modal Digital Image Correlation Test Planning



*Experimental Structural Dynamics Department*

Dan Rohe and Bryan Witt

U.S. DEPARTMENT OF ENERGY NNSA



Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

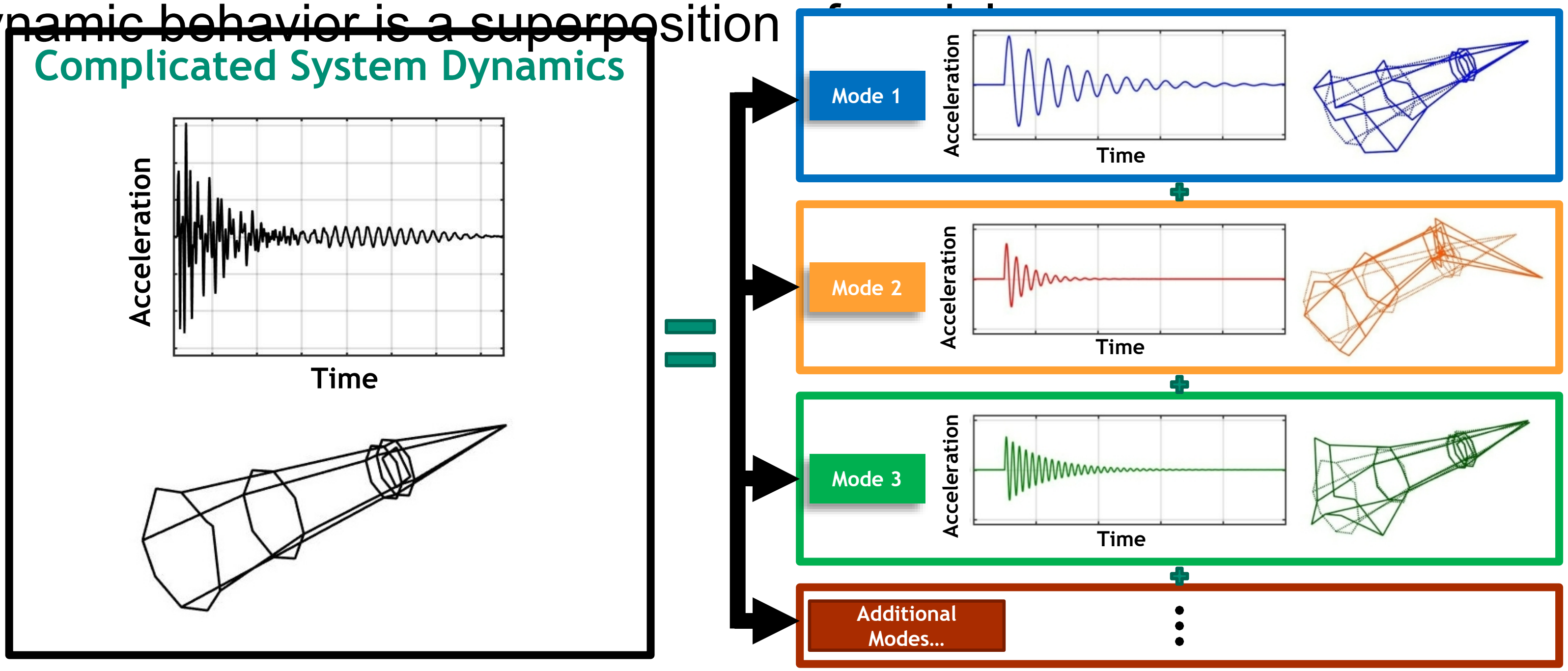
# Overview of Modal Testing with DIC

# Overview of Experimental Modal Analysis (EMA)

- **What are modes?**
  - Modes are inherent properties of all objects
  - Describe how objects naturally respond to stimulation at different frequencies
  - **The fundamental building blocks of all complex dynamic response**

Natural frequency
Damping ratio
Deformation shape

- All complex dynamic behavior is a superposition of modal responses

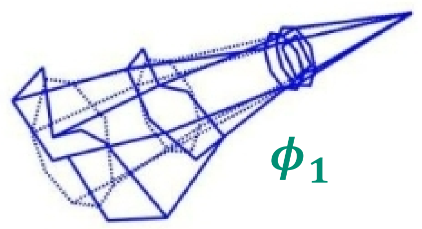


# Overview of Experimental Modal Analysis (EMA)

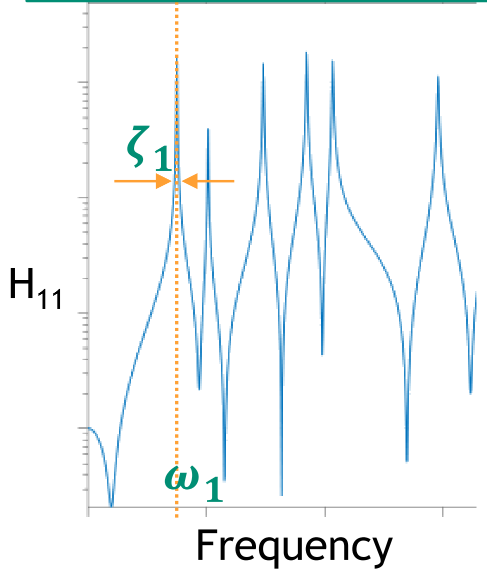
- **How do you measure modes?**
  - Excite the system and measure the input force → $F(t)$
  - Measure the system output response → $A(t)$
  - Compute Frequency Response Functions (FRF) → $H(\omega) = \frac{A(\omega)}{F(\omega)}$
  - Curve fit the FRF → $H_{ij}(\omega) = \sum_{k=1}^{m} \frac{-\omega^2 \phi_{ik}\phi_{jk}}{\omega_k^2 - \omega^2 + 2j\zeta_k \omega \omega_k}$
    - Modal parameters are extracted from the fit FRF

Modal Superposition

Mode shapes




Natural Frequencies & Damping ratios



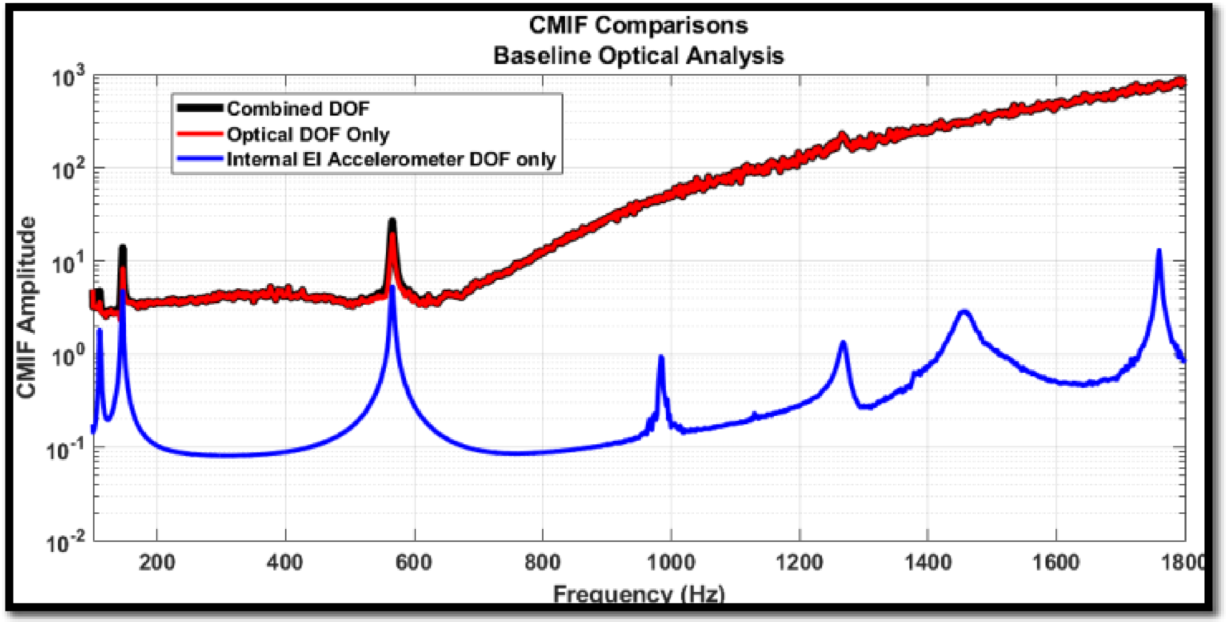

- Keys to success:
  - Measure input and output accurately
  - Excite in locations that activate the modes of interest
  - Place sensors in appropriate locations to spatially resolve modes of interest

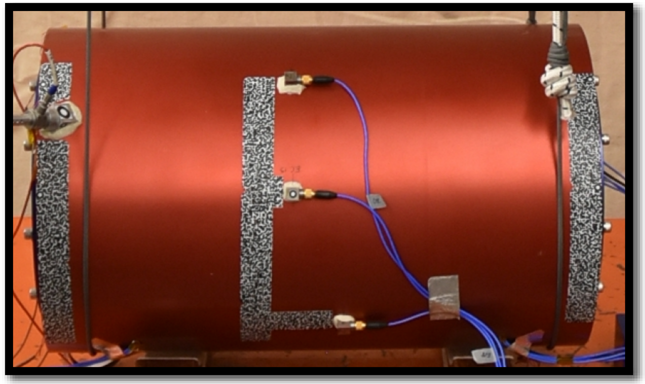
**Correlated models are used to predict dynamic behavior in complex environment simulations.**

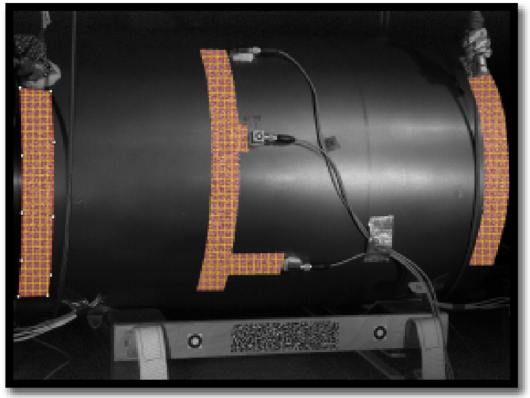
# Combining DIC with Modal Testing

- **Optical Techniques have advantages over discrete sensors**
  - Full-field data is obtained rather than single-point measurements
  - Large number of measurements can distinguish modes more readily
  - Strains can be computed

- **Modal Testing using DIC can be challenging**
  - Synchronization of cameras with other data acquisition systems
  - Must manually process data into frequency response functions, Modal DIC software packages are in their infancy
  - Limited to external visible surfaces
  - Displacements are generally very small, especially over 1 kHz.





# Evaluating Test Feasibility – Theory

# Evaluating Feasibility – Estimating Pixels of Displacement

## Requirements

### Things we need:

- FEM mode shapes
- Undeformed node positions
- Intended input force signal
- Camera matrices from pose estimation

### Assumptions:

- Small displacements
- Negligible rigid body rotations
- Intended force signal is realizable

## Theory

Test article has natural frequencies $\omega_n$, damping $\zeta_n$, and mode shapes $\Phi$

We will partition the mode shape matrix to the input and output degrees of freedom $\Phi_i$ and $\Phi_o$, respectively.

The admittance (disp) FRF can be expressed at each frequency line $\omega$ as:

$$H_{XF}(\omega) = \frac{\Phi_o \Phi_i^T}{-\omega^2 + \omega_n^2 + 2j\zeta_n \omega_n \omega}$$ (o,i)

This gives a relationship between input force $f$ and displacement response $x$, or their FFTs as:

$$X = H_{XF} F$$ (o,1)=(o,i)(i,1)

From modal theory, the responses are related to modal coordinates $Q$:

$$X = \Phi_o Q$$ (o,1)=(o,m)(m,1)

From the above we can see that a transfer function between modal coordinates and force is:

$$H_{QF}(\omega) = \frac{\Phi_i^T}{-\omega^2 + \omega_n^2 + 2j\zeta_n \omega_n \omega}$$ (m,i)

# Evaluating Feasibility – Estimating Pixels of Displacement

## Theory (cont)

We're now thinking in terms of modal coordinates $q$.

Each mode will contribute to the response differently, and will have a different maximum values of $q$ and thus different maximum pixels of displacement associated with that mode. We can define a new transfer function between modal coordinate response $q$ and pixels of displacement:

$$H_{PQ} = \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ 0 & S_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_m \end{bmatrix} \quad (m,m)$$

where each $S_m$ is the scaling of $\Delta px/q$ for each mode $m$. We can calculate the $S_m$ terms as follows:

1. Multiply mode shapes by a small scaling factor (e.g. 1e-4)
2. Project the undeformed node positions through our estimated camera matrices K[R|T] to get <u>undeformed</u> pixel positions
3. Add the scaled shape values to the undeformed node positions to get deformed node positions
4. Project the deformed node positions through our estimated camera matrices K[R|T] to get <u>deformed</u> pixel positions
5. For each mode, calculate the pixel displacements at each node $(\Delta u, \Delta v)$ as difference between deformed and undeformed positions
6. For each mode, take the $\max(\text{norm}(\Delta u, \Delta v))$ across all nodes
7. Divide these values by the small scaling factor. These are the diagonal $S_m$ terms, and are constant over frequency

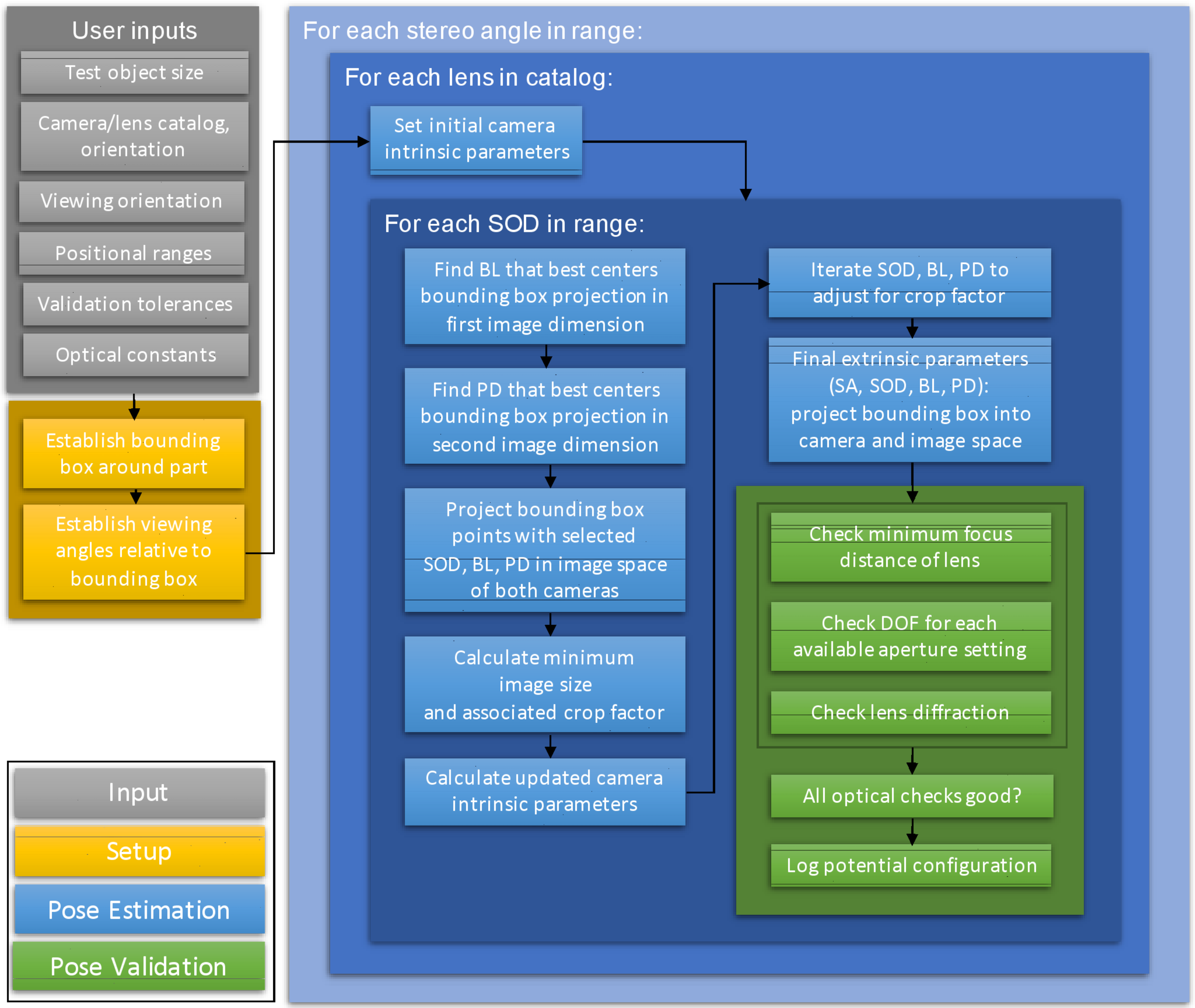
# Camera Pose Optimization

## We can use info about our part and equipment to down-select to a short list of viable camera poses.

- Iterative approach
- Only considers what equipment and space you have available
- Easily coded in Matlab, Python, etc.

## Output for each lens:

- Optimal camera positions
- Valid lens aperture settings
- Recommended speckle feature size
- **Nominal resolution (px/mm)**
  - **Makes it simple to pick a good setup!**




For more information: B. Witt, J. Wilbanks, B. Owens, D. Rohe, Stereo Photogrammetry Camera Pose Optimization, Proceedings of the 39th International Modal Analysis Conference (2021)

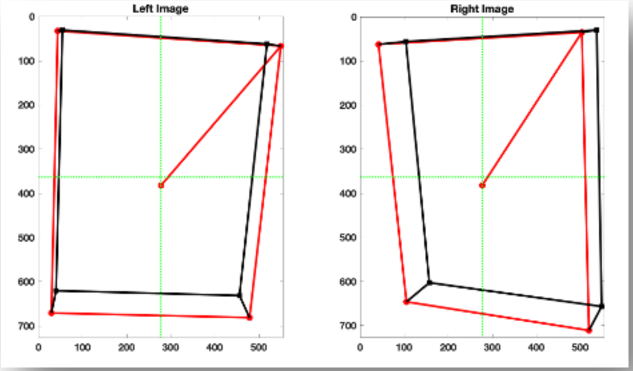
# Camera Pose Optimization – Behind the Scenes

- Once you have camera positions, stereo angle, and intrinsic parameters we can project bounding box coordinates into image space!

$$\mathbf{u}_i = \mathbf{K}_i \mathbf{R}_{iv} [\mathbf{I}|\mathbf{C}]_i \mathbf{R}_{\mathbf{vb}} \mathbf{X}_{\mathbf{b}}$$

$$\begin{bmatrix} au \\ av \\ a \end{bmatrix}_i = \begin{bmatrix} f_x & q & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} \\ r_{yx} & r_{yy} & r_{yz} \\ r_{zx} & r_{zy} & r_{zz} \end{bmatrix}_{iv} \begin{bmatrix} 1 & 0 & 0 & C_x \\ 0 & 1 & 0 & C_y \\ 0 & 0 & 1 & C_z \end{bmatrix}_i \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} & 0 \\ r_{yx} & r_{yy} & r_{yz} & 0 \\ r_{zx} & r_{zy} & r_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{vb} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}_b$$

Box pixel positions



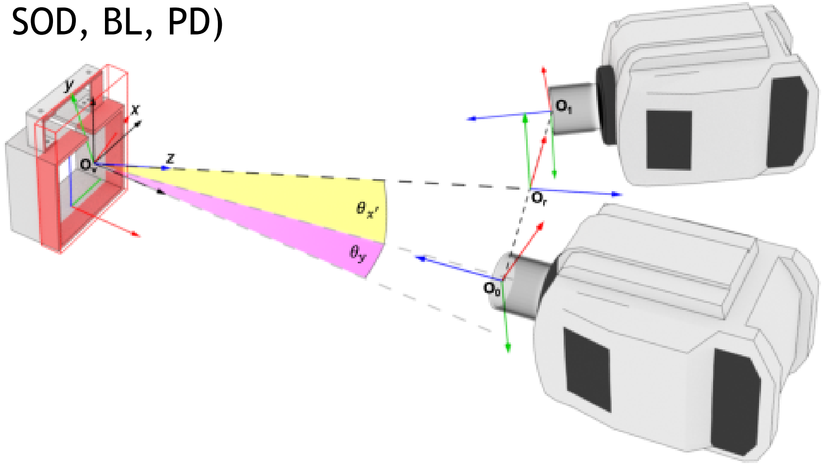

Transform into image space (estimated camera intrinsic parameters)

Rotate into camera CS (using stereo angle)

Translate to camera positions (defined with SOD, BL, PD)

Transform from box to viewing angle CS (rotation matrix from Euler angles)

Box 3D coordinates



# Camera Pose Optimization using Blender

## Alternatively, you can use rendering tools such as Blender

- Set up approximate cameras in a GUI
- Extract corresponding camera matrices to transform world coordinates into pixels
- Gives excellent physical context to the results for test planning
- Fairly steep learning curve, but incredibly powerful tool

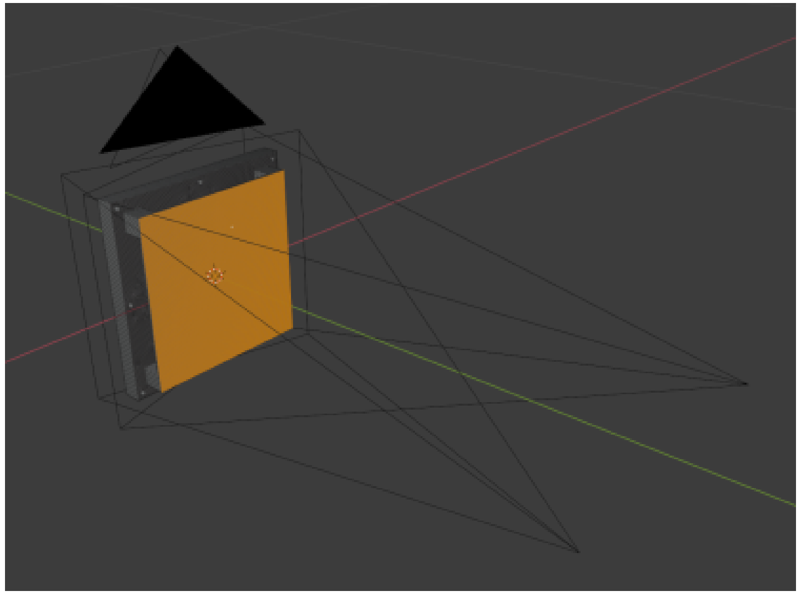
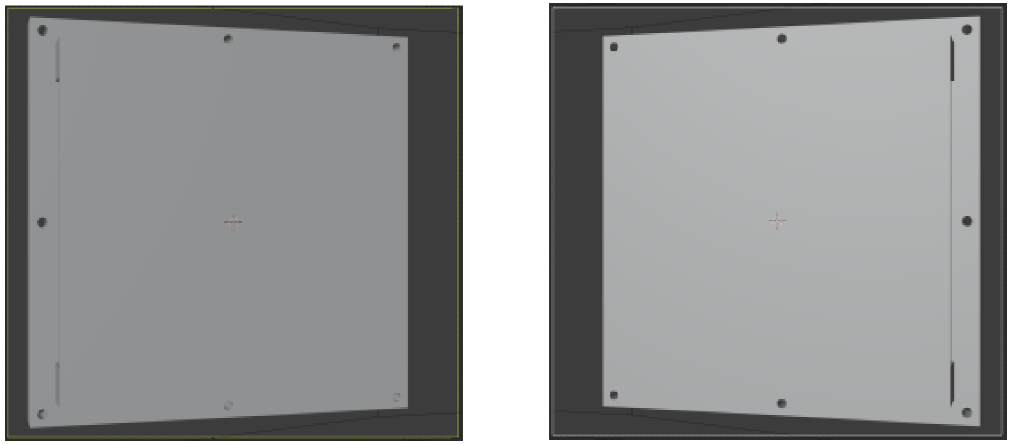


Plate imported into Blender with two cameras set up

$$\begin{bmatrix} cu \\ cv \\ c \end{bmatrix} = [K][R|t]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad [K] = \begin{bmatrix} \frac{p_u f_{mm}}{s s_{u,mm}} & s & (1/2 - s_u)p_u \\ 0 & \frac{p_v f_{mm}}{s s_{v,mm}} & (1/2 - s_v)p_v \\ 0 & 0 & 1 \end{bmatrix}$$

Camera transformation between world space and image space



Approximate camera views from Blender

For more information: Rohe, D.P., Jones, E.M.C. Generation of Synthetic Digital Image Correlation Images Using the Open-Source Blender Software. *Exp Tech* (2021). https://doi.org/10.1007/s40799-021-00491-z

# Evaluating Feasibility – Estimating Pixels of Displacement

## Theory (cont)

Now we have all the transformations we need to go from an input force to pixels of displacement mode-by-mode:

$$H_{PF} = H_{PQ}H_{QF} = \frac{[\ \ddots S \ddots ]\Phi_i^T}{-\omega^2+\omega_n^2+2j\zeta_n\omega_n\omega}$$

(m,i)=(m,m)(m,i)

We can estimate pixels of displacement for any given input force $f$ by taking its FFT ($F$) and complex conjugate transpose ($F^*$) and calculating the cross power spectrum

$$G_{FF} = FF^*$$

(i,i)=(i,1)(1,i)

Finally, the cross power spectra of the pixel displacements are:

$$G_{PP} = H_{PF}G_{FF}H_{PF}^*$$

(m,m)=(m,i)(i,i)(i,m)

from which we can calculate the RMS pixels of displacement per mode from the autospectra:

$$RMS_{px}(m) = \sqrt{\sum G_{PP(m,m)}\, df}$$

(m,1)

where $df$ is the frequency spacing.

# Evaluating Test Feasibility – Demonstration

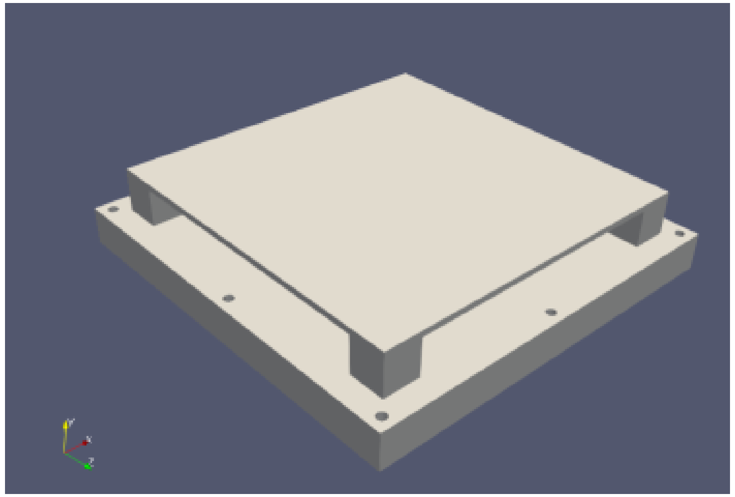
# Test Article of Interest

## Plate on Fixture

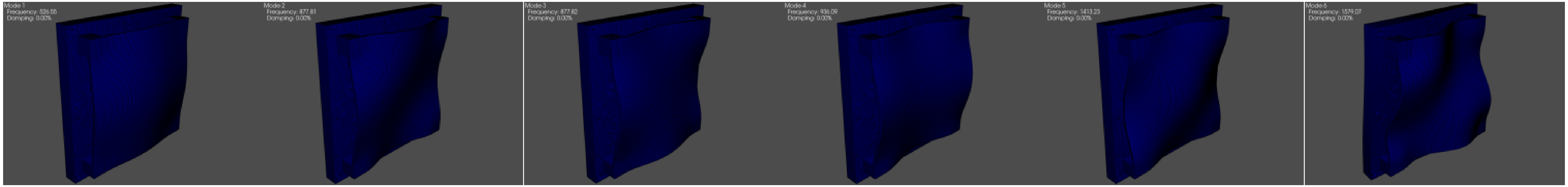
- 7 in (177.8 mm) square aluminum plate
- 3/8 in (9.5 mm) thick
- 3/4 in (19.05 mm) square posts at corners
- No joints, monolithic part

## Interested in response up to 1,600 Hz

- 6 elastic shapes in this regime
- 1 set repeated modes
- 1 mode not well-excited by back plate, likely will miss this mode.



Finite element model



Repeated Modes

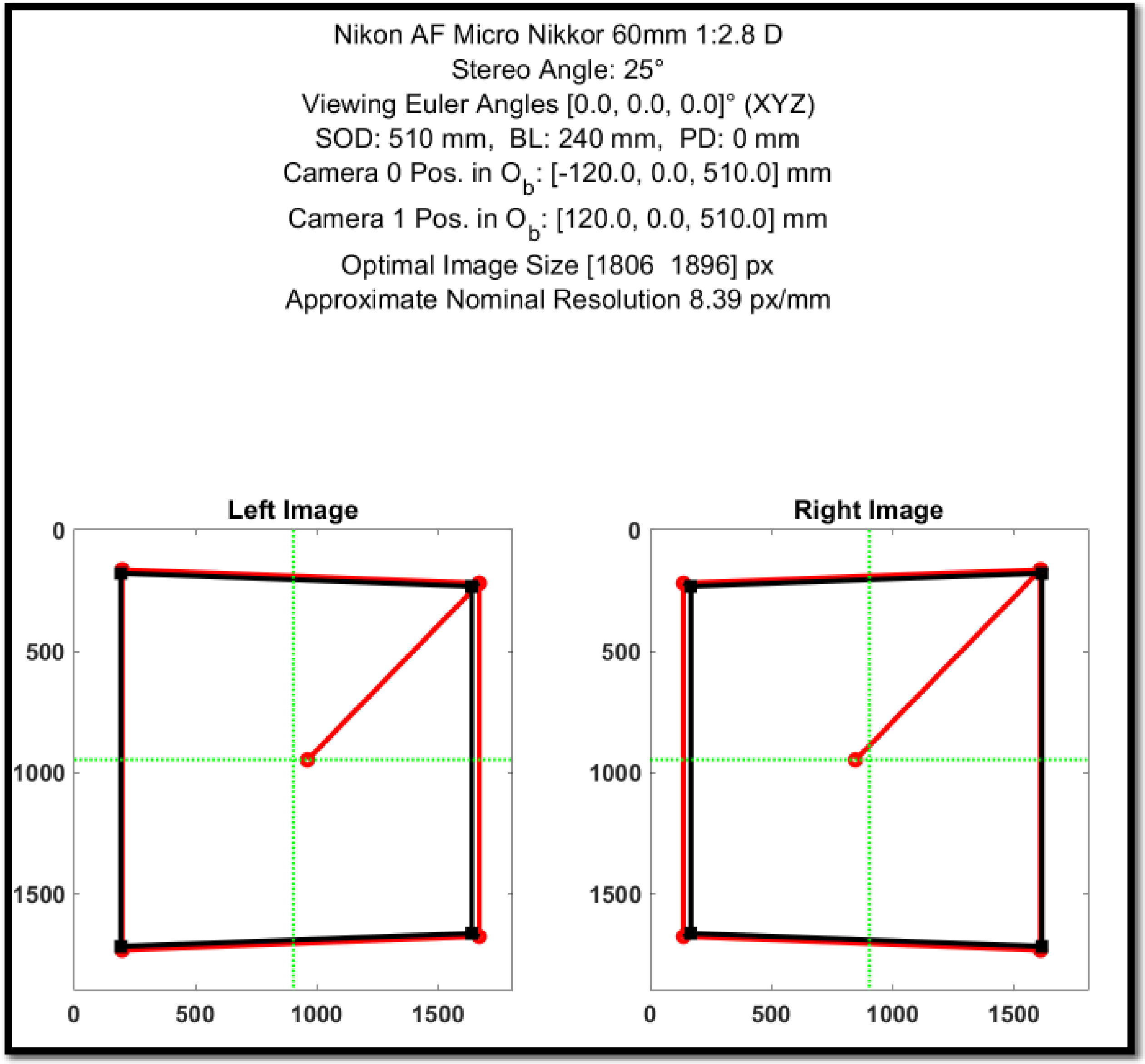
Not excited by drive point.

# Camera Pose Estimation

- **Using the pose optimization tool**
  - Selected the Nikon 60mm lenses

| | |
|---|---|
| **Cameras** | **Phantom v2640 UHS**<br>-Sensor Size = [2048,1920] px<br>-Pixel Size = 0.0135 mm |
| **Lenses** | **Nikon AF Micro Nikkor 60mm 1:2.8 D**<br>-Min Focal Distance = 220 mm<br>-Aperture Range = [32, 2.8] |
| **Camera Orientation** | Horizontal |
| **Camera Stand Orientation** | Horizontal |
| **Stereo Angle (degrees)** | 25 |
| **Bounding Box [bx,by,bz] (mm)** | [178,178,5] |
| **Bounding Box Margin [x,y,z]%** | [0.2, 0.2,0.2] |
| **Viewing Euler Angles (degrees; y,x',z'')** | [0,0,0] |
| **SOD Range (mm)** | 10:10:2000 |
| **BL Range (mm)** | 10:10:2000 |
| **PD Range (mm)** | -1000:10:1000 |
| **Minimum Focus Check Tol** | 0.95 |
| **DOF Check Tol** | 0.95 |
| **Lens Diffraction Check Tol** | 0.95 |
| **Acceptable Circle of Confusion (mm)** | 0.0135 (one pixel) |
| **Lighting Wavelength (mm)** | 600/1E6 |

Est resolution = 8.44 px/mm
Est speckle size = 0.47 mm (for 4 px)
And we get the camera matrices.



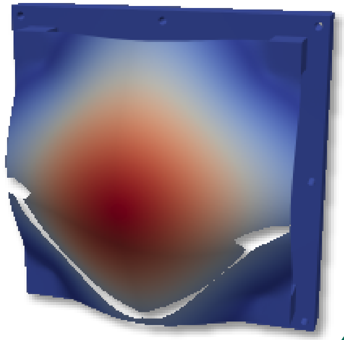
# Evaluating Feasibility – Calculating Pixels of Displacement

## Scale the mode shapes to be very small

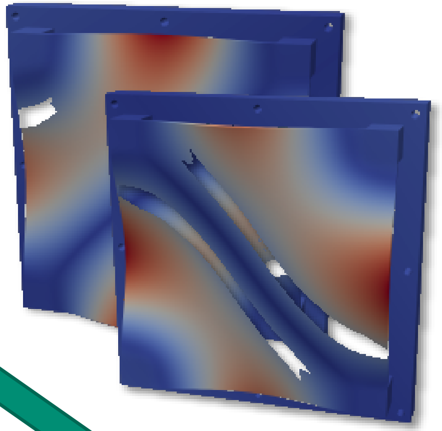
**Camera equations are nonlinear, but we can linearize about the small displacements anticipated in a modal test**

- 1e-4 is a good value
- Remember to partition to only nodes within the ROI

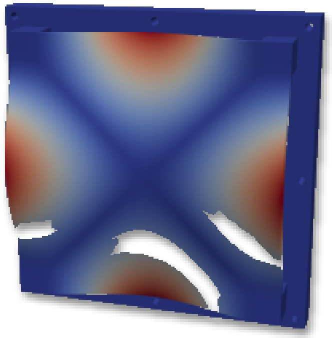
Mode 1



Modes 2/3



Mode 4 …



## Calculate pixel positions of undeformed and deformed nodes in ROI

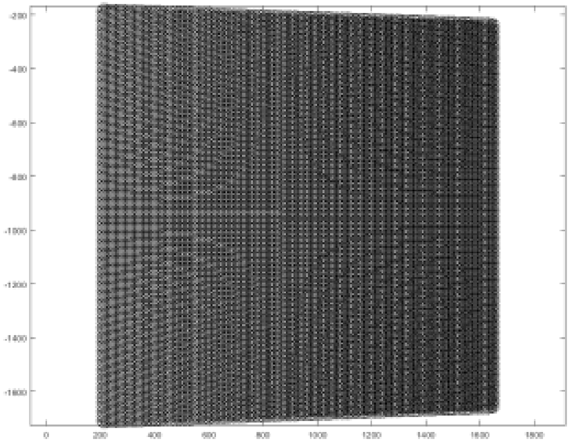
$$\begin{bmatrix} au \\ av \\ a \end{bmatrix}_i = \begin{bmatrix} f_x & q & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} & t_x \\ r_{yx} & r_{yy} & r_{yz} & t_y \\ r_{zx} & r_{zy} & r_{zz} & t_z \end{bmatrix}_{ir} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}_r$$

From Pose Estimation

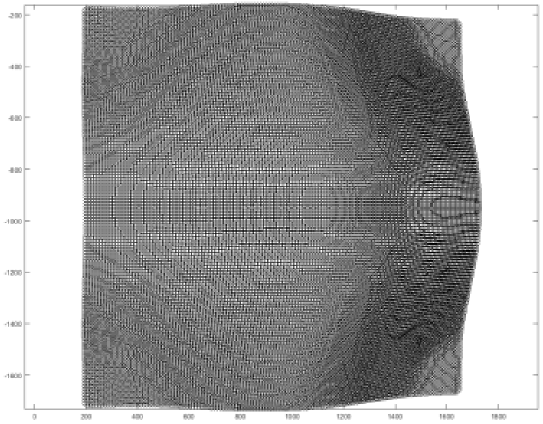
Undeformed from FEM

Deformed from scaled shapes

Undeformed pixel positions



Deformed pixel positions (highly exaggerated)



Left Camera

# Evaluating Feasibility – Calculating Pixels of Displacement

## Calculate $H_{PQ}$

- Take difference of deformed/undeformed pixel positions $(\Delta u, \Delta v)$
- For each mode, find $\max(\mathrm{norm}(\Delta u, \Delta v))$ across all nodes
- Divide these by the scale factor, and populate the diagonal terms of $H_{PQ}$

## Synthesize $H_{QF}$ from FEM data

$$H_{QF}(\omega) = \frac{\Phi_i^T}{-\omega^2 + \omega_n^2 + 2j\zeta_n\omega_n\omega}$$

Shapes & frequencies, from FEM. May have to estimate damping…

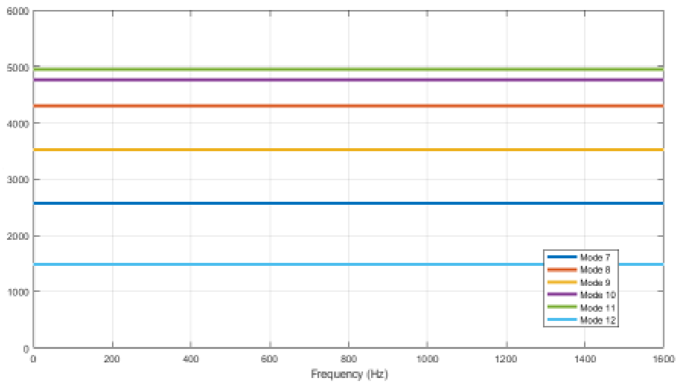
- Will tell you if there are poorly excited modes

## Calculate transfer function between force and pixel of displacement per mode

$$H_{PF} = H_{PQ} H_{QF}$$

- Will tell you if there are modes that will not generate a lot of pixel displacements. These will be difficult to extract with DIC.

$H_{PQ}$

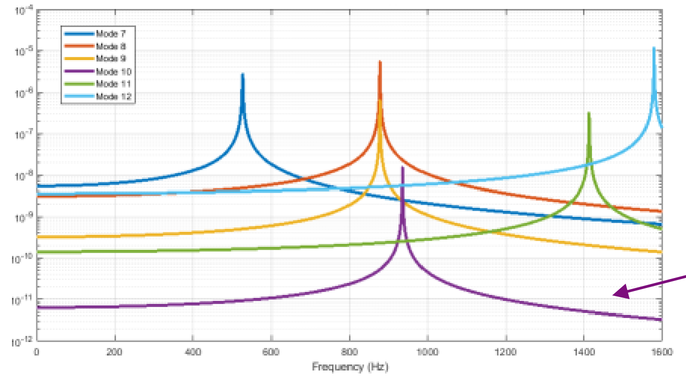



Constant over frequency

Relates to how shapes move in the image

*

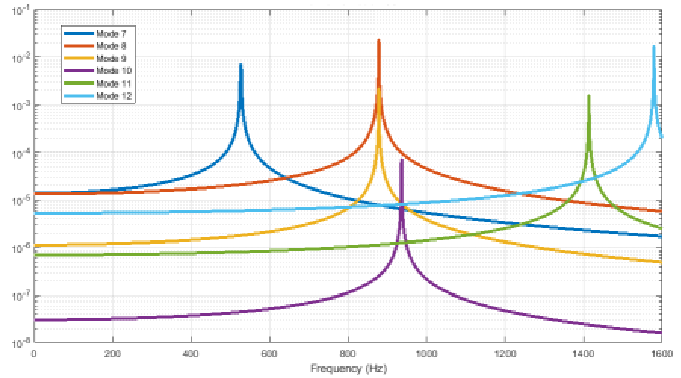
$H_{QF}$




Check excitation strategy if you have poorly excited modes.

=

$H_{PF}$




Notice changes in peaks relative to each other.

This takes into account excitation strategy & how shapes move in the image.

Mode 4 will be difficult…

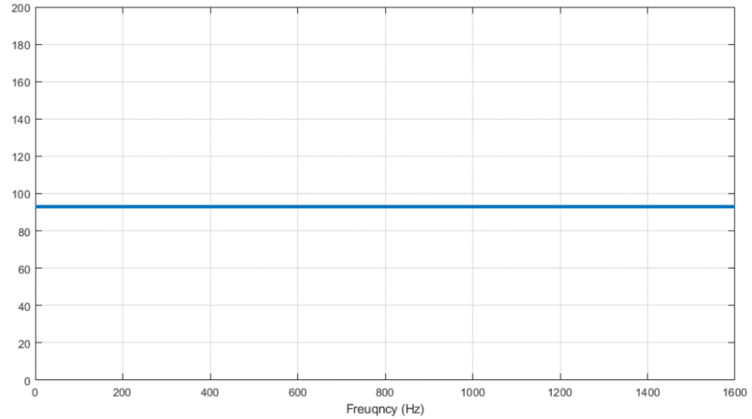
# Evaluating Feasibility – Calculating Pixels of Displacement

## Calculate cross spectra of input force

$G_{FF} = FF^*$ } User specifies intended force signal

- Make sure your force units are consistent with the FEM
- Make sure your intended signal is realizable with your equipment (shaker power, armature dynamics, frequency range, etc)

$G_{FF}$ (ASD)



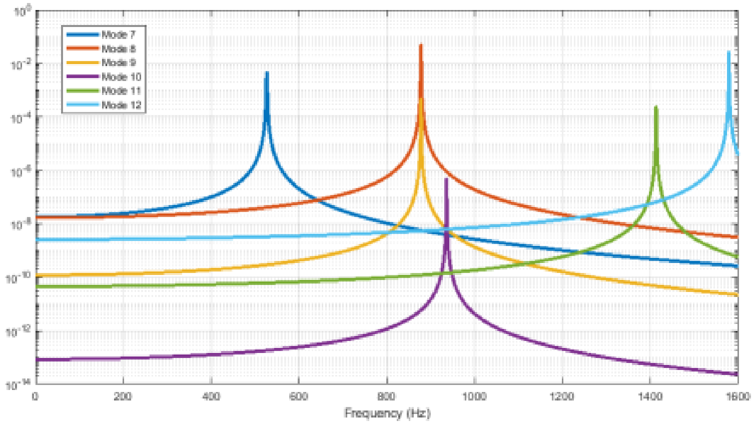

Flat Force Signal

Scaled to give 1 $lbf_{RMS}$

## Calculate cross spectra of pixel displacements per mode

$$G_{PP} = H_{PF} G_{FF} H_{PF}^*$$

$G_{PP}$ (ASD)




Looks like what we anticipated…

# Evaluating Feasibility – Calculating Pixels of Displacement

## Calculate RMS of pixels of displacement for each mode of interest

$$RMS_{px}(m) = \sqrt{\sum G_{PP(m,m)}\, df}$$

Sqrt of area under autospectra curves

- Tells you the "energy" of the pixel shifts for each mode

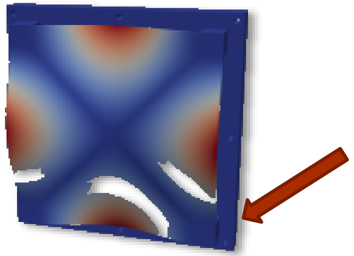
## Determine if your test is feasible

- Experience indicates $\boldsymbol{RMS_{px}} = \mathbf{0.01}$ is typically enough to be able to extract that mode.
- In some optimal testing scenarios (very low noise), we have extracted modes at $\boldsymbol{RMS_{px}} = \mathbf{0.001}$

## What if it is not??

- Try scaling the input force higher
- Change excitation location or add additional input forces
- Change camera pose
- Narrow the FOV to increase (px/mm) resolution
- Consider taking data in multiple views

| Mode | Frequency (Hz) | RMS Displacement (px) |
|---|---|---|
| 1 | 526.5 | 0.125 |
| 2 | 877.8 | 0.259 |
| 3 | 877.8 | 0.047 |
| 4 | 936.1 | 0.00002 |
| 5 | 1413.2 | 0.023 |
| 6 | 1579.1 | 0.240 |



One poorly excited mode,
but overall test should be feasible!

# Test Execution and Results

# Test Setup and Execution

Test article excited via shaker

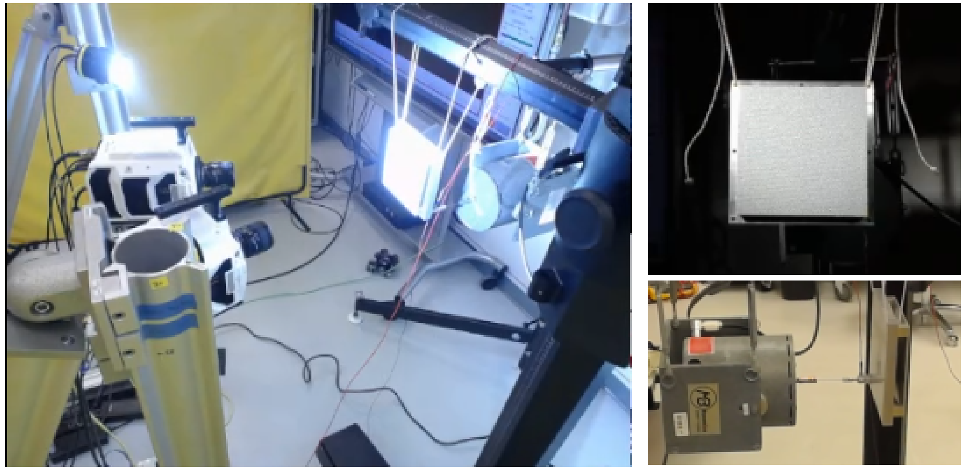
- Flat force achieved by specifying shaped voltage spectrum
- Excitation force was periodic every 4096 samples

40960 images obtained

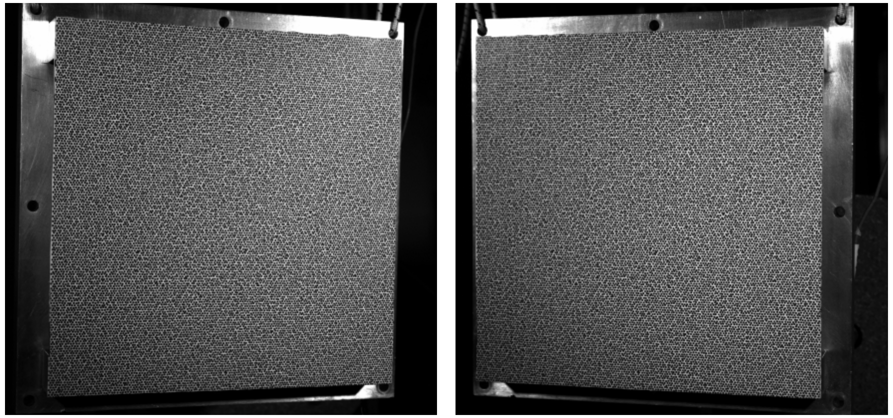
- 10 measurement frames that could be averaged due to the periodic excitation

Averaged images analyzed using DIC

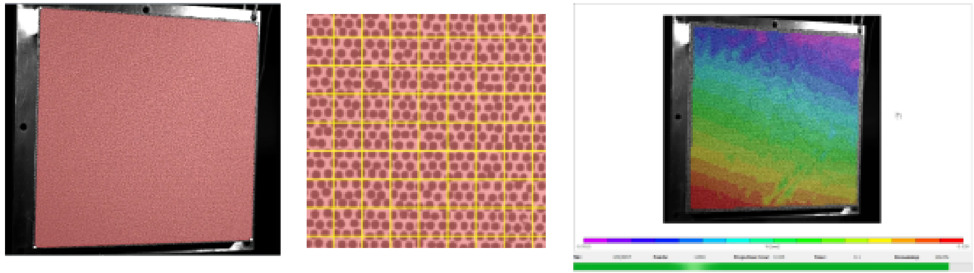
- Only 4096 images analyzed due to averaging
- Displacements extracted and frequency response functions computed
- Modes fit using Synthesize Modes and Correlate (SMAC) algorithm



Test setup and shaker configuration



Example test images



AOI specification and DIC analysis

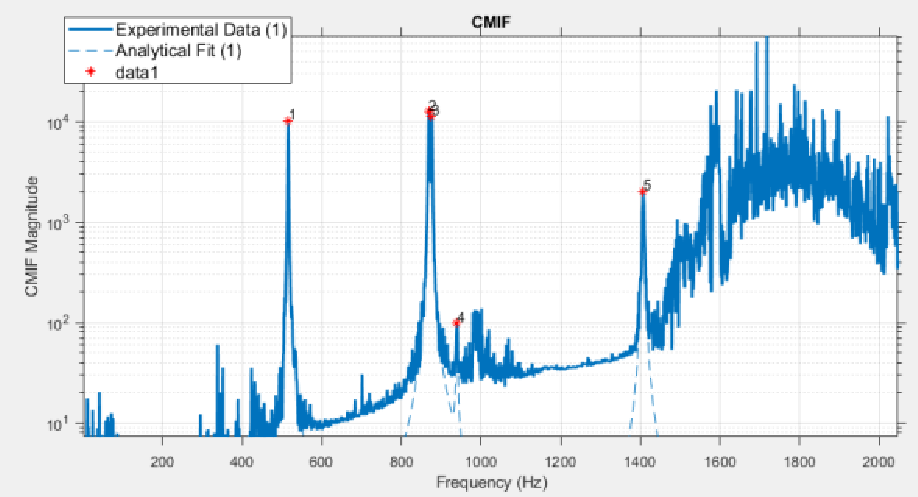
# Test Results

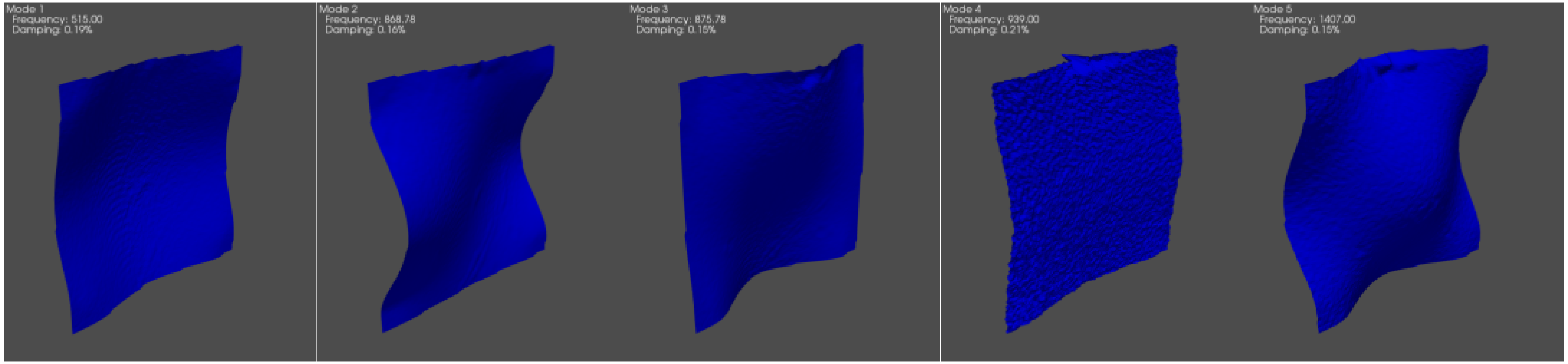
FRFs imported into Synthesize Modes and Correlate (SMAC) curve fitter.

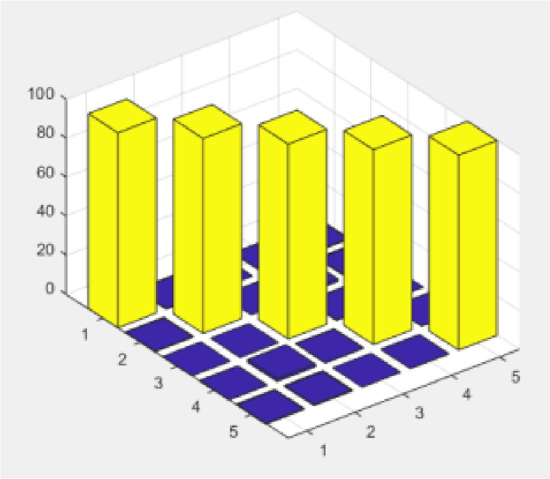
5 modes fit in the bandwidth of interest

- Mode 4 not fit very well due to limited excitation

| Mode | Frequency (Hz) | Damping |
|---|---|---|
| 1 | 515.0 | 0.19% |
| 2 | 868.8 | 0.16% |
| 3 | 875.8 | 0.15% |
| 4 | 939.0 | 0.21% |
| 5 | 1407.0 | 0.15% |



# Summary

# Summary

**DIC is a viable technique for modal testing**

- Modal testing generally uses very small displacements, so we need to ensure that our test will have large enough displacements for our cameras to see
- We can perform quantitative feasibility calculations if we have a finite element model or test data that allows us to predict responses for a given force
- Transforming mode shapes onto the camera image can help us compute pixels of displacement per modal displacement
- We can set up transfer functions between force and pixels of displacement to help us compute the required force for a given test

# Questions?