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Identifying Recurrent Causal Activity Patterns in Spiking Neural Networks

Bradley H. Theilman, Felix Wang, and James B. Aimone (bhtheil@sandia.gov, felwang@sandia.gov, jbaimon@sandia.gov)

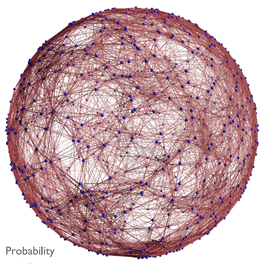
Albuquerque, NM

Introduction

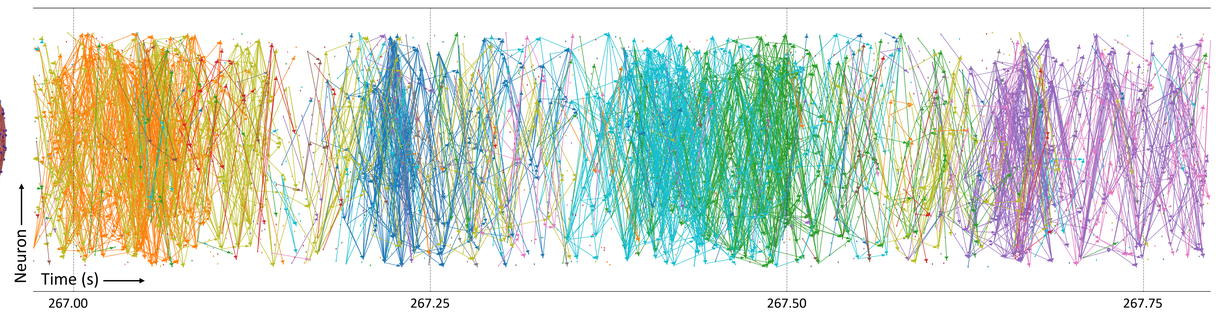
Spiking neural networks display flexible computational capacities. Extracting computations from spiking neural networks requires identifying useful abstractions of the combinatorically explosive number of possible activity patterns. Modern high-performance computing allows us to explore these patterns and their relationships using new approaches for decomposing network activity.

Neural activity is shaped by network connectivity. By combining activity with connectivity in a single mathematical structure, we extract computational "threads" the emerge from synaptic plasticity in simulated spiking networks. These threads are extended and overlap in time. We introduce a method to identify recurrences of causally-similar threads. We propose identifying these threads with elementary computations performed by the network.

Graphical Neural Activity Threads (GNATs)

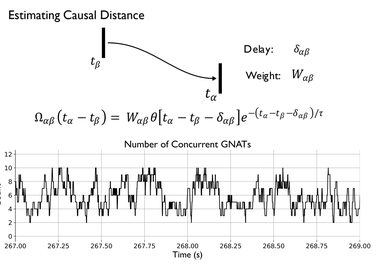


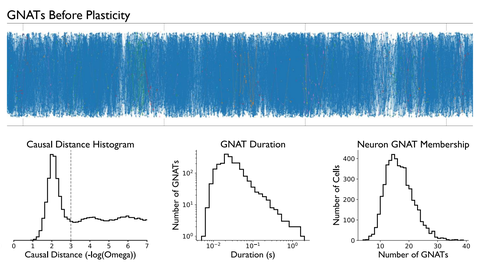
The network consists of 6000 Izhikevich model neurons distributed on the surface of a sphere (subset, top). The E/I ratio is 80/20. Neurons connect with probability given by a sigmoid function of distance (bottom). Synapses were assigned random conduction delays between 1-20 ms. The network was evolved with random input and STDP for 10 simulated minutes before analysis.



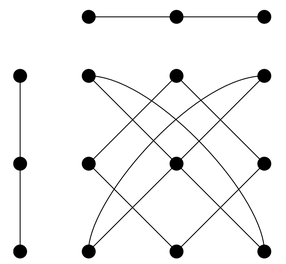
We construct a directed graph on the spikes themselves by connecting two spikes if the presynaptic spike causally contributed to the postsynaptic spike according to $\Omega_{\alpha\beta}(t_\alpha - t_\beta)$. This graph decomposes into weakly connected components we call Graphical Neural Activity Threads (GNATs).

GNATs are causally disjoint – spikes may belong to the same time bin but not contribute to the same causal chains. Do these spikes belong to the same "computation," implied by a state vector approach?



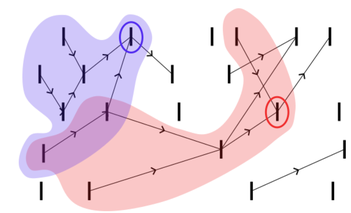


Modular Graph Product



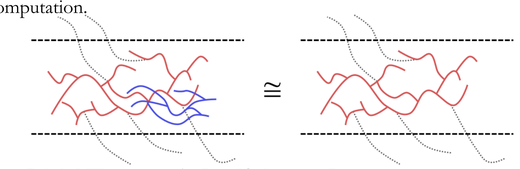
The modular graph product constructs a new graph from two smaller graphs. The modular vertex set is the cartesian product of the vertex sets of the source graphs. Edges in the modular product correspond to shared edges or non-edges in the source graphs (XNOR). Cliques in the modular product correspond to isomorphic induced subgraphs in the source graphs [3]. Cliques are hard to enumerate, so we compute a "reduced" modular product by only including shared edges. Weakly connected components in the reduced modular product include isomorphic sub-threads.

GNATs and Topology

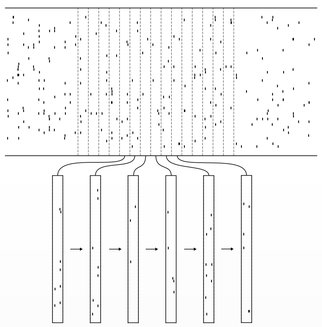


The sets of causal ancestors of each spike form a basis of open sets for a topology on the network activity. This connects GNATs to the rich theory of finite topological spaces [1, 2]. Under this topology, continuous maps are precisely those maps that preserve the causal structure.

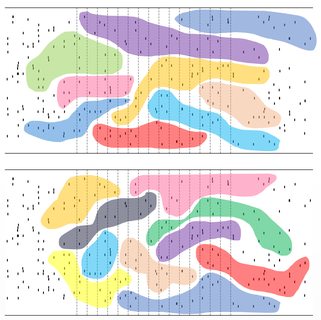
Topological concepts such as **homeomorphism** and **homotopy equivalence** translate to equivalences between population spike trains. Homeomorphic or homotopic spike trains may instantiate the same computation.



Decomposing Neural Activity

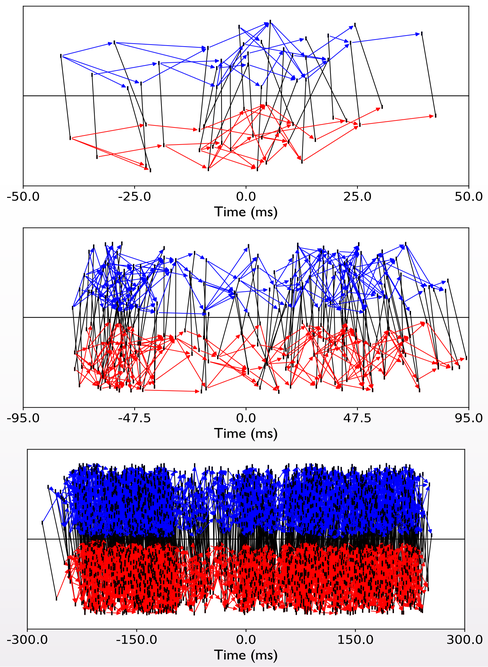


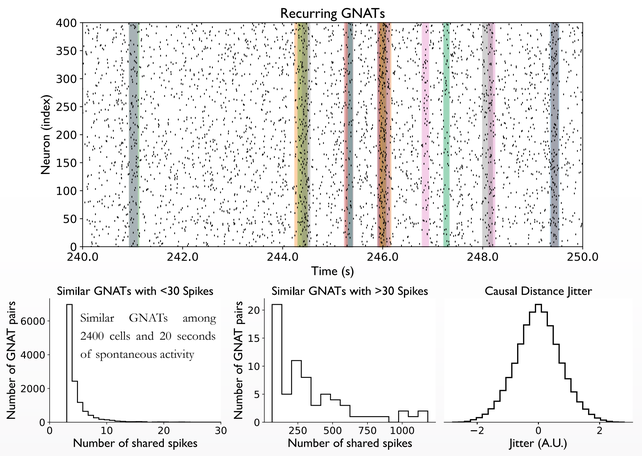
- Neural state vectors are defined by time bins *relative to the experimenter's clock*.
- Computations are defined by the sequence of state vectors.
- Variability requires a probabilistic description of dynamics linking successive state vectors.



- Spikes are related by the causal action of presynaptic spikes on postsynaptic spikes.
- The causal relation is local in space and time and *independent of the experimenter*.
- This partitions the network activity into disjoint, temporally-extended regions.
- Distinct underlying causal processes could produce similar temporal sequences. Which one matters for computation?

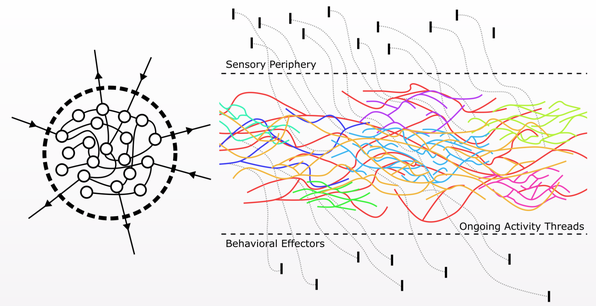
Identifying Isomorphic GNATs





Using the modular graph product (above right), we compute 2nd-order GNATs by tracing the causal relations between pairs of pairs of spikes. Weakly connected components of this 2nd order activity graph correspond to maps between threads that respect the causality relation. This allows us to extract causally-similar GNATs (left). The spikes in each pair of matching GNATs show related, but not identical, temporal relationships. They would likely not be detected through statistical methods operating solely on spike times. The combination of activity and structure embodied by the GNATs provides a network-centric definition of analogous computations.

GNATs and Spiking Computation



External spikes interact with ongoing activity threads to actualize computations. GNATs extract the traces of computations performed on input "sensory" spike patterns and reveal how they are transformed into "motor" outputs.

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