



Sandia  
National  
Laboratories

# Varied Delays in Spiking Neural Networks Support Learning with Conjunctive Temporal Features

Presented by

Felix Wang, Corinne Teeter

---

11/9/2022

SNUFA 2022

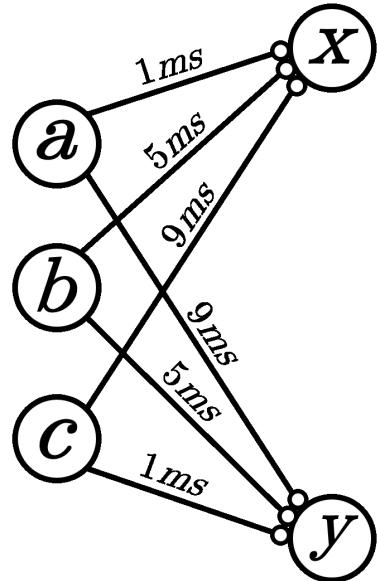


Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

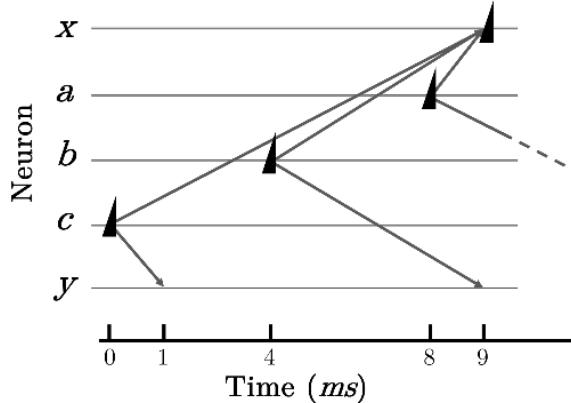
# The importance of spike timing



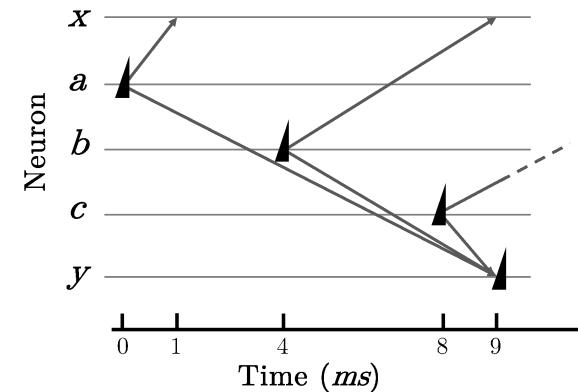
Structure in network connectivity drives spike-timing patterns



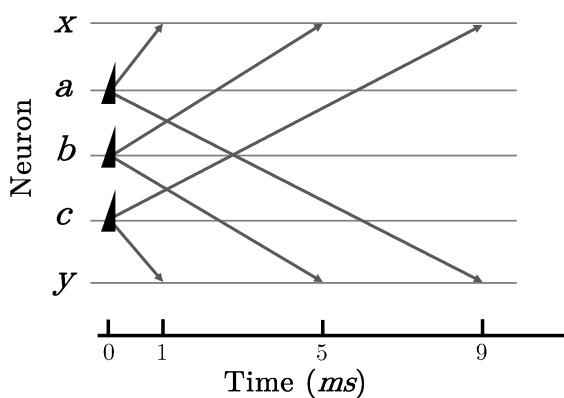
Simple network with timing-dependent connectivity



Optimal activation of neuron x



Optimal activation of neuron y

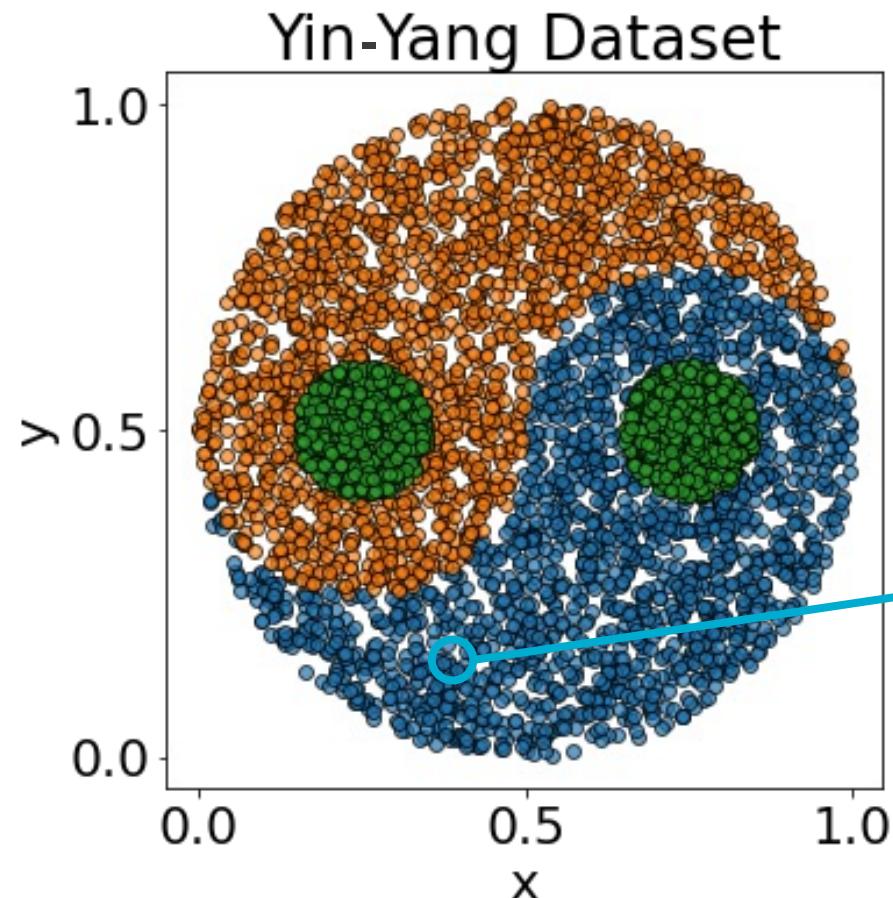


Suboptimal activation of neurons x, y

# Yin-Yang Dataset

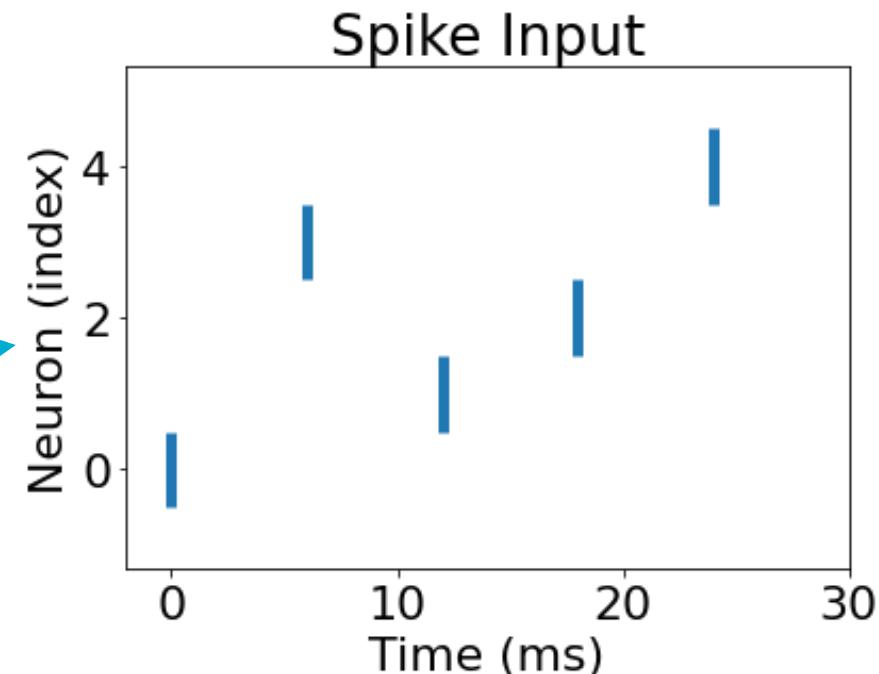


Spatio-temporally encoded, 3-class dataset developed for research on biologically plausible error backpropagation



**Spike Encoding:**

- Bias Neuron : 0
- x-coordinate :  $x t_{max}, (1 - x) t_{max}$
- y-coordinate :  $y t_{max}, (1 - y) t_{max}$



# EventProp for Training SNNs

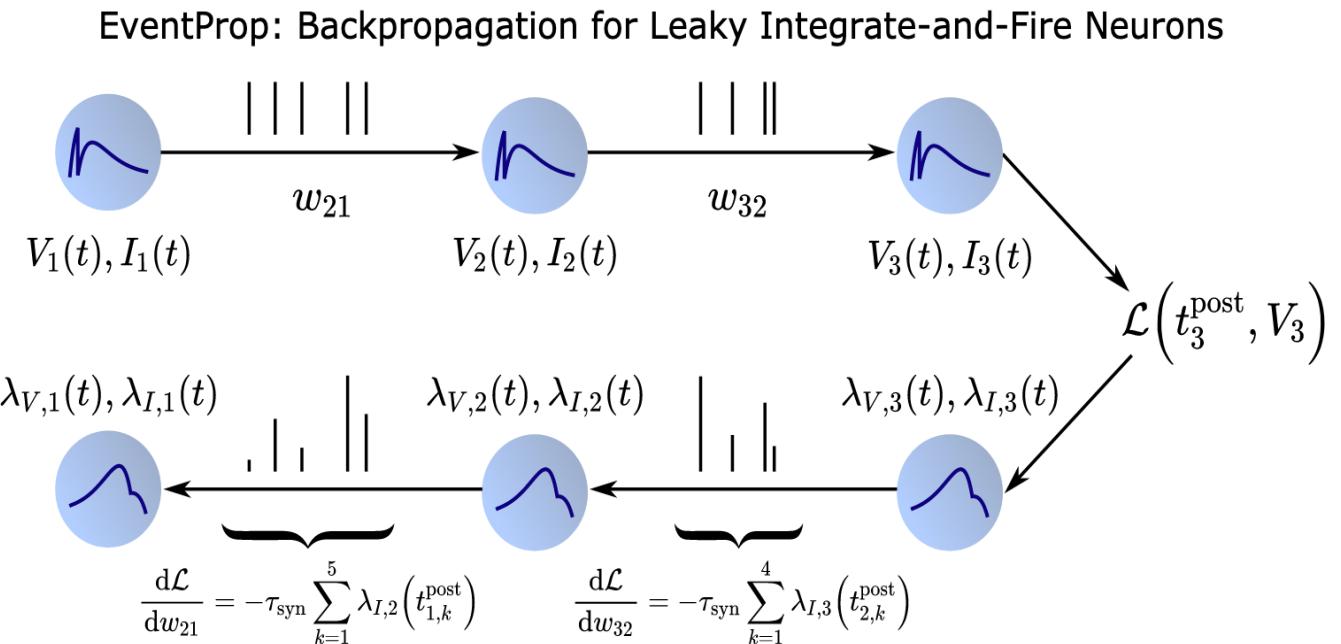
Derives an exact gradient across event-based neuron models using the adjoint method and partial-derivative jumps

For spike-based loss (e.g. time-to-spike), this is computed as:

$$\begin{aligned}\tau_{mem} \lambda'_V &= -\lambda_V - \frac{\partial l_V}{\partial V} \\ \tau_{syn} \lambda'_I &= -\lambda_I + \lambda_V\end{aligned}$$

Transitions (at spikes  $t_k^{post}$ ):

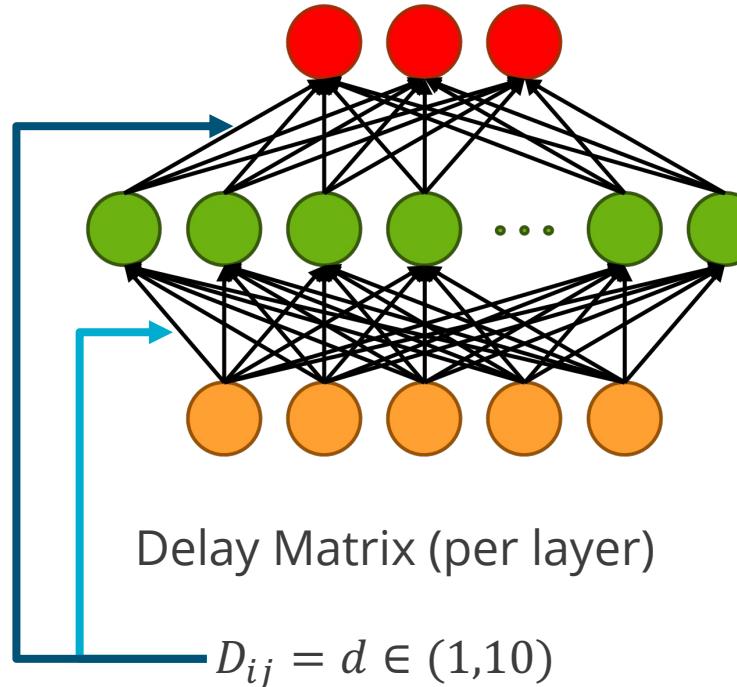
$$(\lambda_V^-)_{n(k)} = (\lambda_V^+)_{n(k)} + \frac{1}{\tau_{mem}(\dot{V}^-)_{n(k)}} \left[ v(\lambda_V^+)_{n(k)} + (W^T(\lambda_V^+ - \lambda_I))_{n(k)} + \frac{\partial l_p}{\partial t_k^{post}} \right]$$



# Simple Feedforward Network with Delays



We implement a delay matrix for indexing during the forward and backward passes



As contrasted with typical DL-based SNN formulations which have uniform, single-time-step delays between layers

Forward and backward temporal indices

$$T_{f,ij} = \min(0, t - D_{ij})$$

$$T_{b,ij} = \max(T_{\text{max}}, t + D_{ij})$$

Updates to the forward time-evolution:

$$I_j^+ = I_j^- + W_j e_{n,T_{f,j}}$$

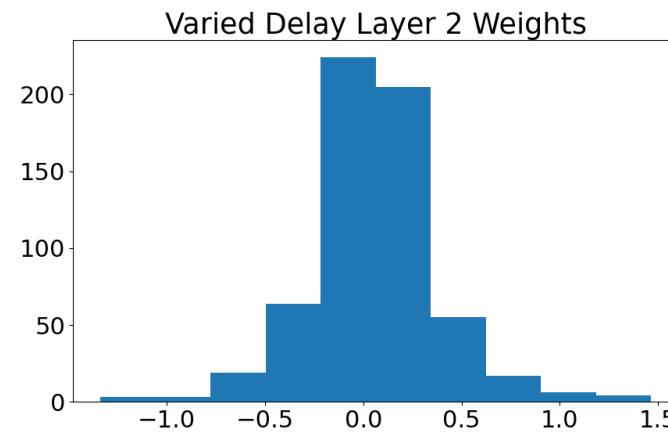
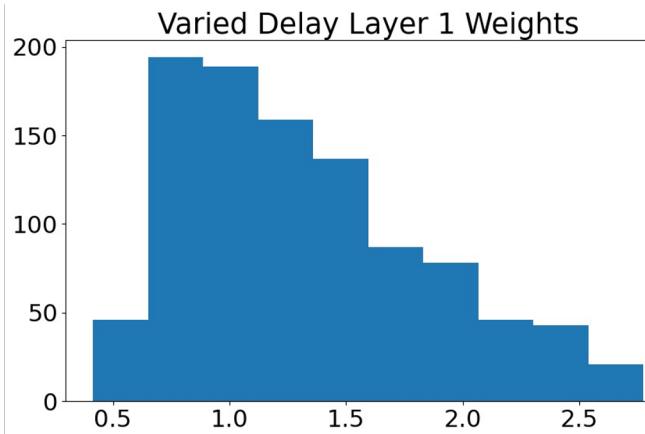
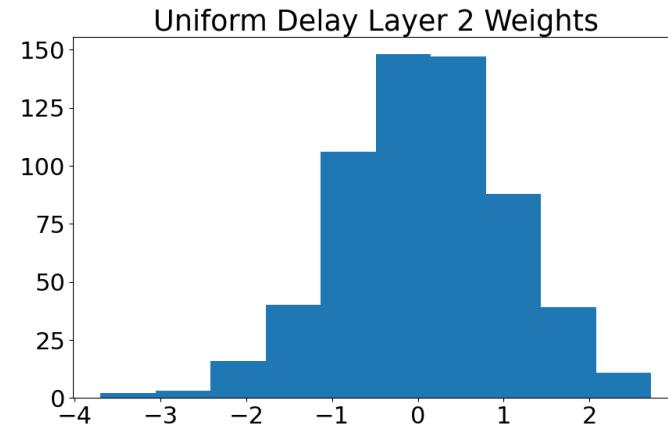
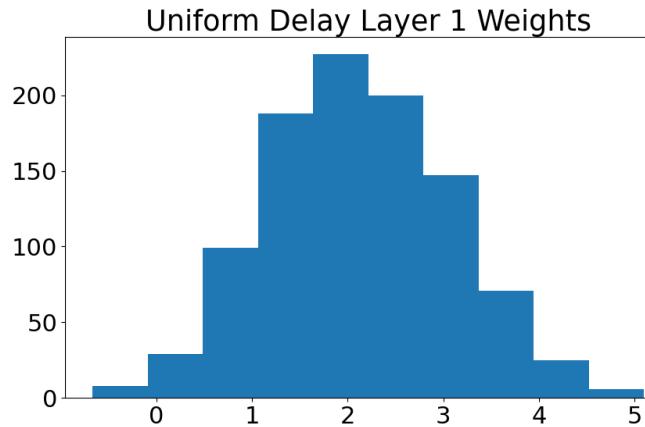
Updates to the backward gradient jump:

$$(\lambda_{V_i}^-)_{n(k)} = \dots \left( W_i^T (\lambda_{V,T_{b,i}}^+ - \lambda_{I,T_{b,i}}^-) \right)_{n(k)} \dots$$

# SNN Training Results



Layer weights are distributed qualitatively differently

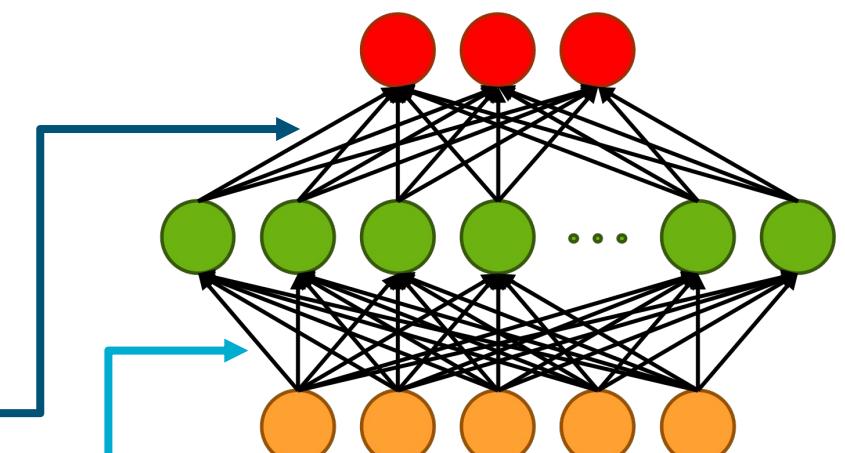


We sampled the Yin-Yang dataset:

- 5000 times for training
- 1000 times for testing

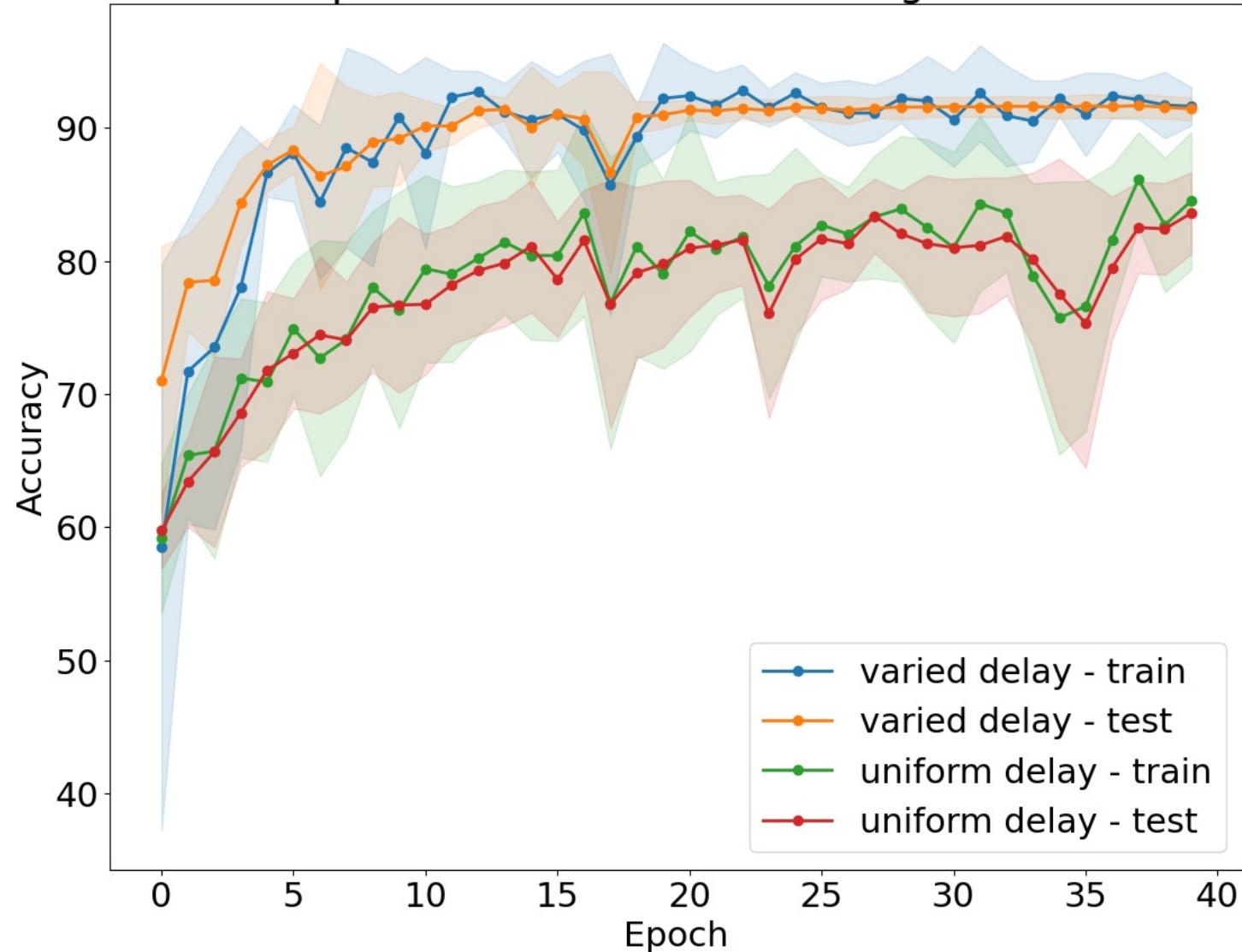
We used stochastic gradient descent (SGD) optimization with a simple learning rate scheduler

We trained for 40 epochs with shuffled mini-batches of 100

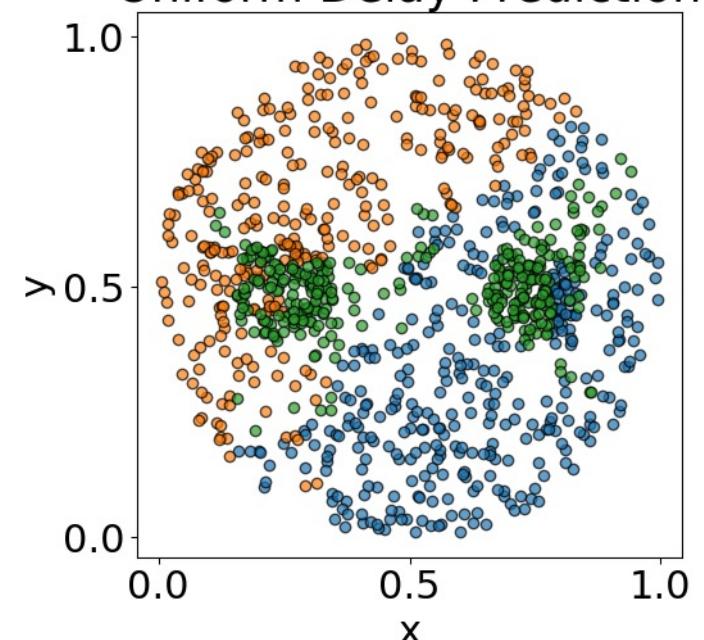


# SNN Classification Results

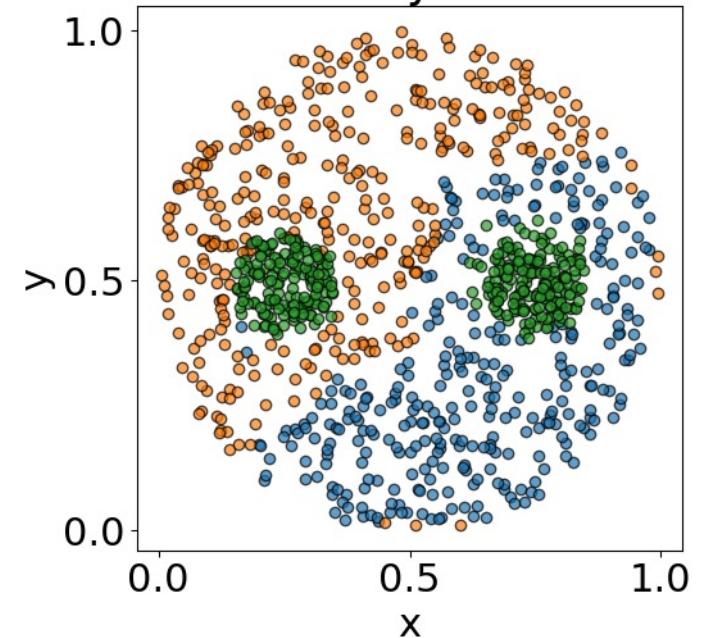
Simple Classification on Yin-Yang Dataset



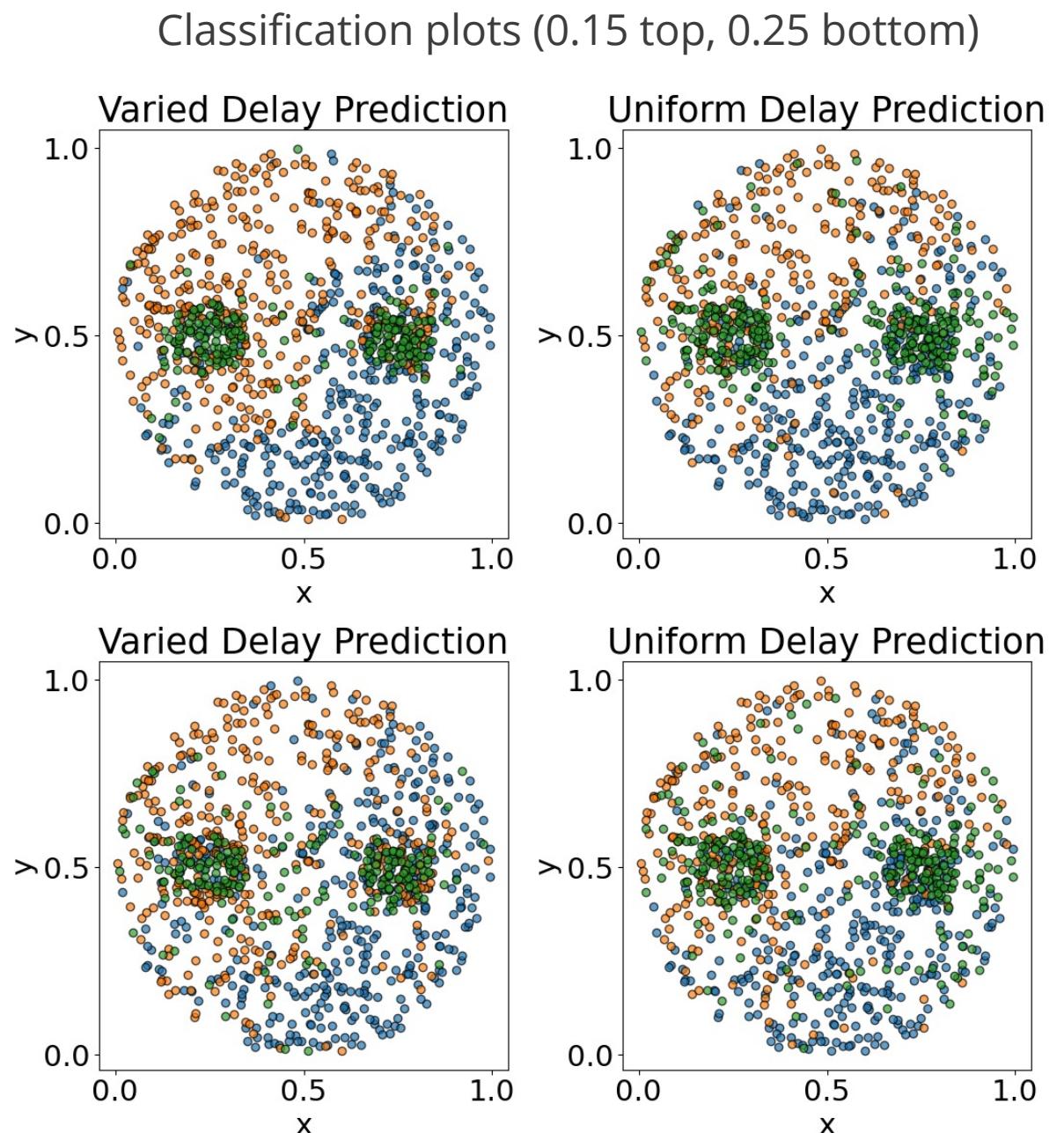
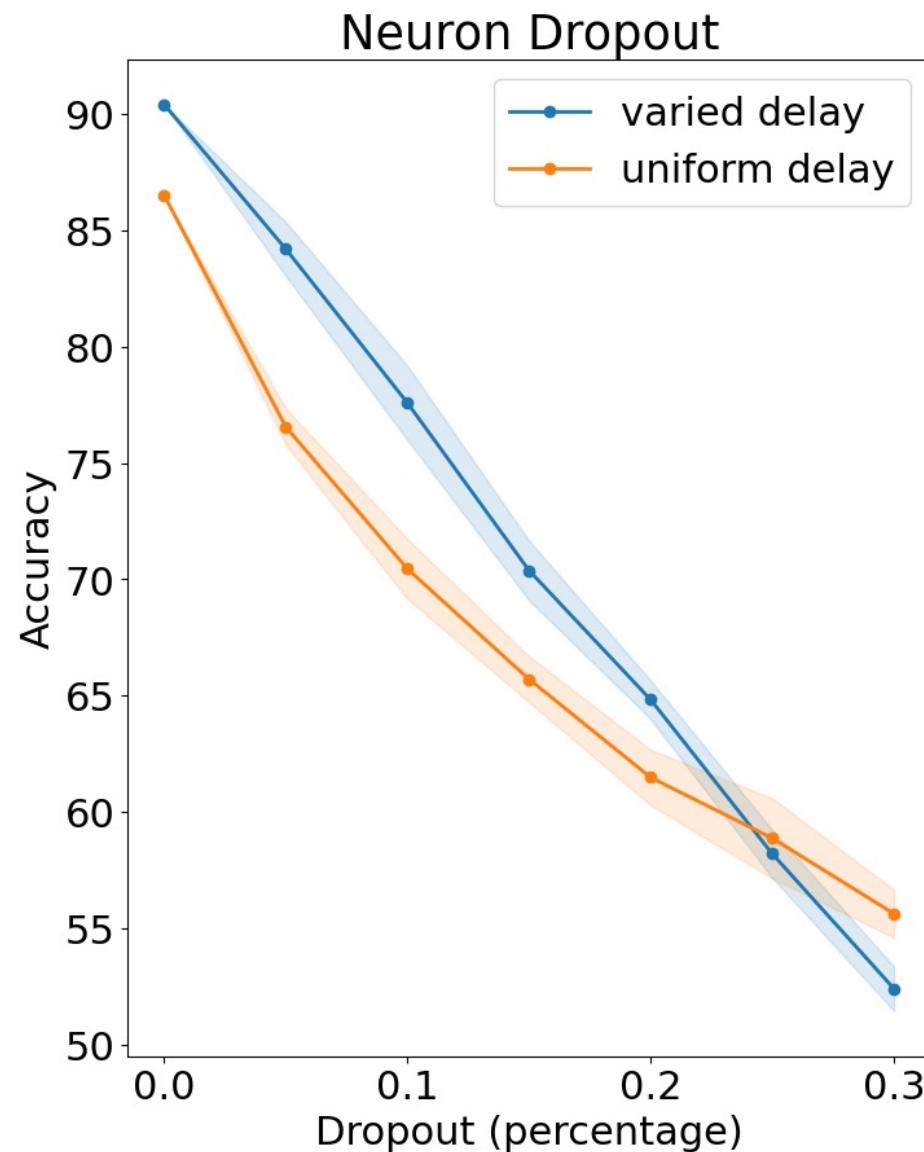
Uniform Delay Prediction



Varied Delay Prediction



# Spike Dropout in Inference



# Summary and Takeaways



- Varied connection delays are a notable architectural difference between more biological SNNs (e.g. reservoirs) compared to their artificial counterparts (e.g. layers)
  - Learning rules such as STDP may be taking advantage of varied delays at a network level
  - Computationally, varied delays result in differentially evaluated upstream spiking activity
- We extend ANN/DL-based methods (EventProp) to compute gradients through varied delays by introducing a delay matrix and forward/backward temporal indices
  - We perform experiments on a simple temporal classification task (Yin-Yang dataset)
  - We found that delays result in improvements in training (e.g. faster learning, robustness)
- Future work will be required in exploring the hyperparameter and architecture space
  - What determines the relationship between connection delays and the input space?
  - Can we start to merge learning rules? (e.g. STDP + STP + BackProp)
  - Are there additional mechanisms to support training? (e.g. background spiking activity)
- Current tools are typically for either biological SNNs or DL/ANNs, but not both
  - Implementation of connection delays is suboptimal with tools like PyTorch

# References



1. Wunderlich, T. C. and Pehle, C. "Event-based backpropagation can compute exact gradients for spiking neural networks", *Scientific Reports*, vol. 11, pp. 12829, (2021)
2. Kriener, L. Göltz, J. and Petrovici, M. A. "The Yin-Yang dataset", *arXiv CoRR*, abs/2102.08211, (2021)
3. Izhikevich, E. "Polychronization: computation with spikes", *Neural Computation*, vol. 18, pp. 245-282, (2006)