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# Data-consistent Approaches to Modeling Population Distributions of Parameters for Biomedical Systems

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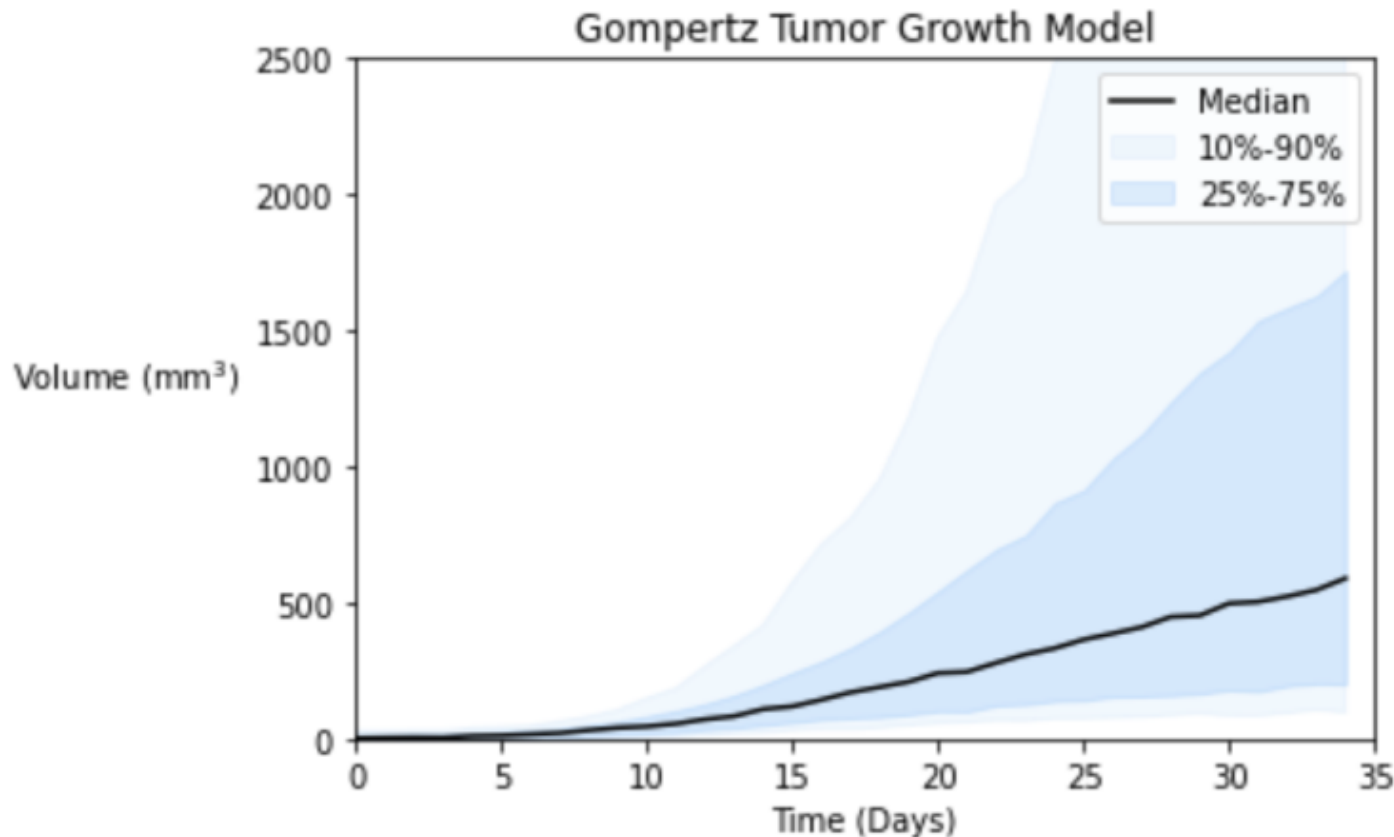
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In many contexts, decision-making and “best practice” is guided by **average** or **typical** behavior...

### Oncology Example:

Tumor growth kinetics used to prescribe radiation treatment dosage and schedules



$$\frac{dV}{dt} = \left( \alpha - \beta \log \left( \frac{V}{V_{inj}} \right) \right) V$$

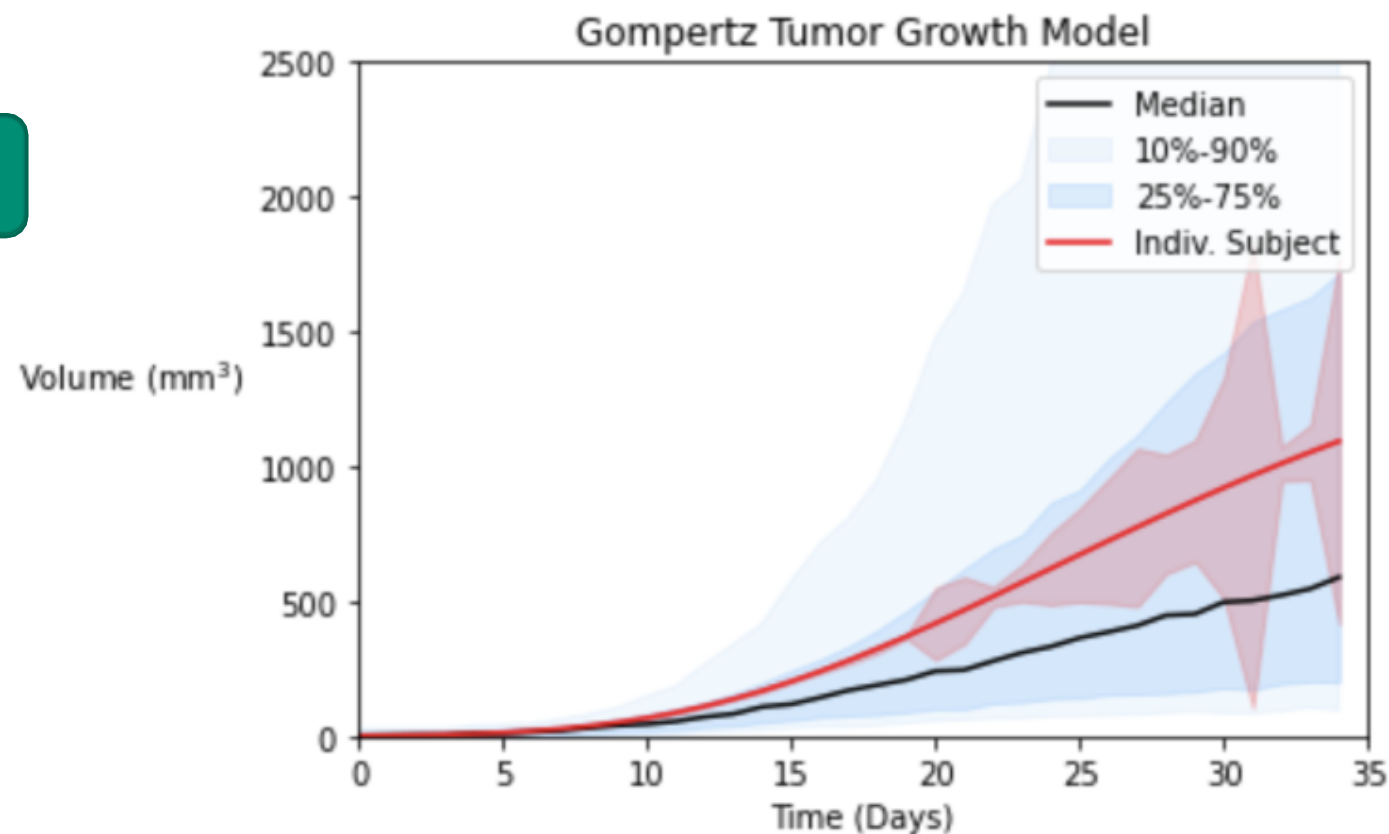


# The promise of “precision medicine” requires effective uncertainty quantification...

Personalized care and effective decision-making needs to account for uncertainty...

## Bayesian Analysis:

$$\pi^{post}(\alpha, \beta) \propto \pi^{prior}(\alpha, \beta) \cdot \pi^{like}(d|\alpha, \beta)$$





# Bayesian approach to personalized uncertainty will be strongly influenced by “population” prior distributions...

Limited data on individuals

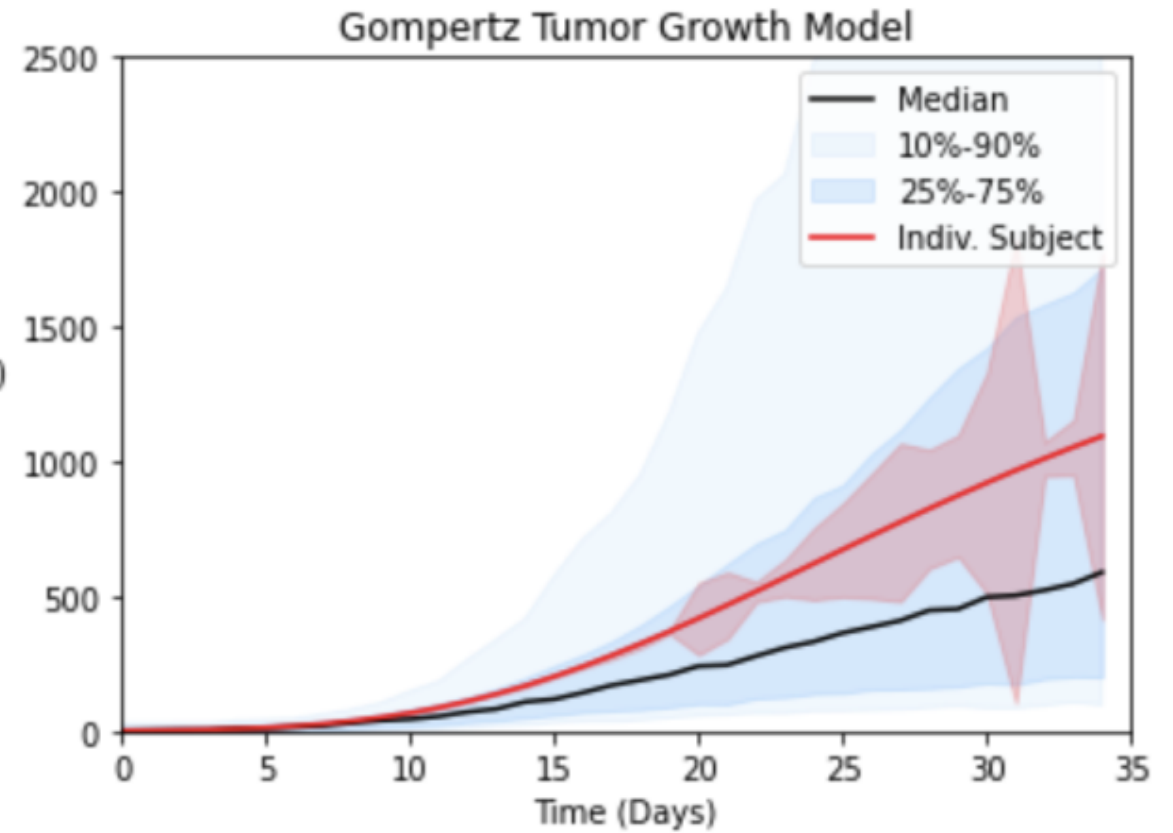
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Population prior will bias decisions...

## Bayesian Analysis:

$$\pi^{post}(\alpha, \beta) \propto \pi^{prior}(\alpha, \beta) \cdot \pi^{like}(d|\alpha, \beta)$$

Volume (mm<sup>3</sup>)





## Data-consistent inversion may be a powerful tool to obtaining suitable prior distributions for Bayesian analyses.

Data-consistent inversion is a measure-theoretic framework for stochastic inversion.

- The updated “population” parameter distribution is **consistent** with observed variation
- The updates from data-consistent inversion occur **only influence data-informed** directions
- The update is a **non-parametric** density estimate
- Generally, **less computationally expensive** than hierarchical Bayesian methods

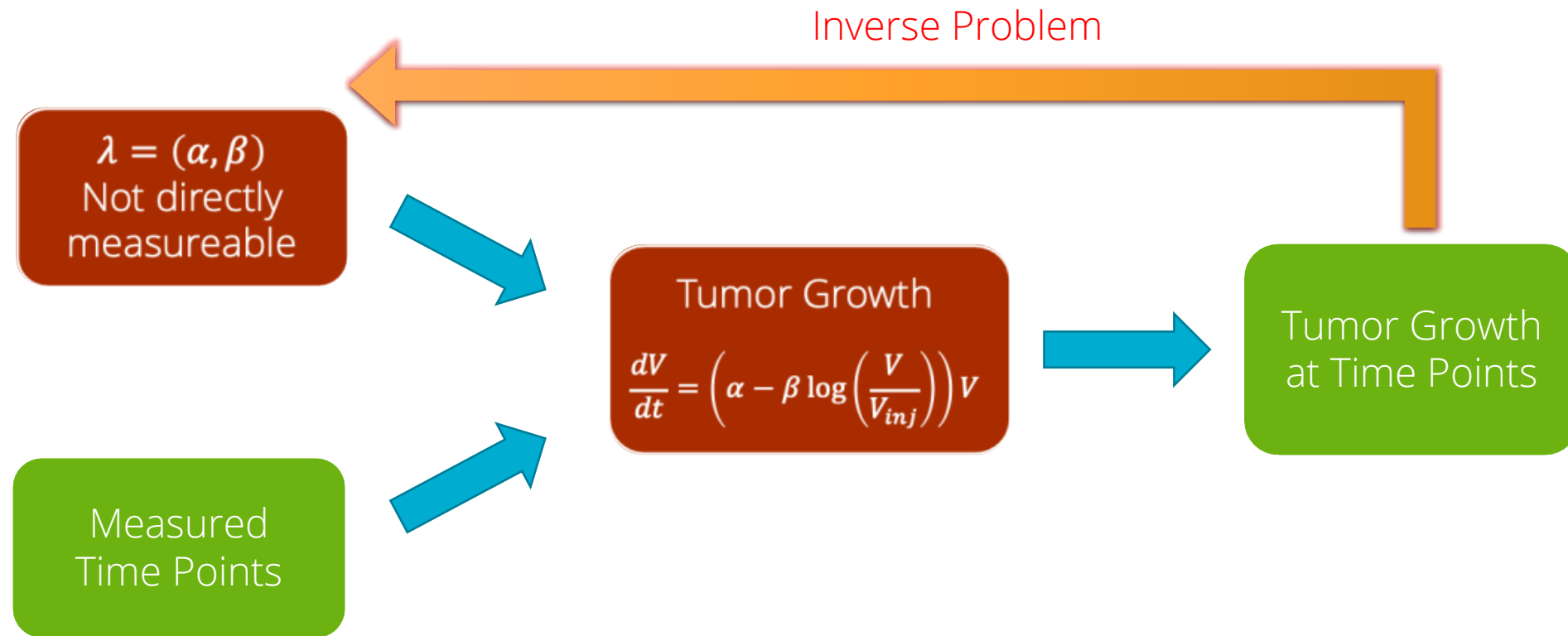
$$\pi_{update}(\lambda) = \pi_{init}(\lambda) \frac{\pi_{obs}(Q(\lambda))}{\pi_{predict}(Q(\lambda))}$$

# Brief Overview of Data-consistent Inversion





The goal of an inverse problem with **a known dynamics model** is to obtain a better characterization of **input parameters**.



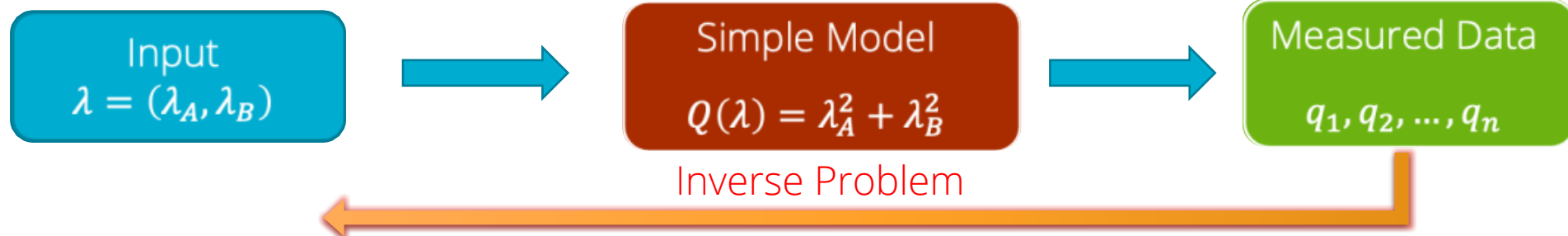
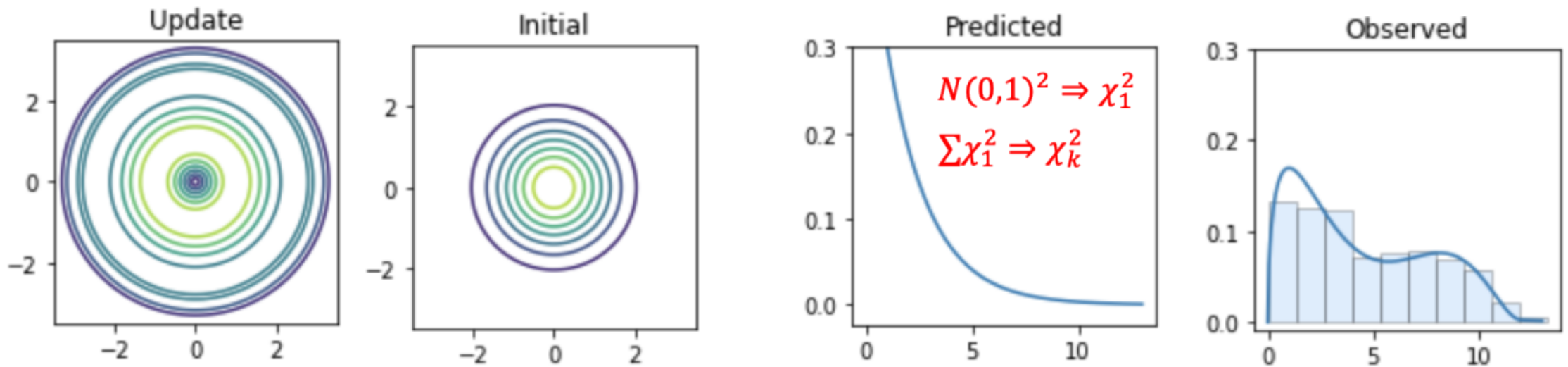
Key Point: In our stochastic inverse problem, we want to find the population distribution of  $\lambda$  !



The data-consistent approach finds a **consistent** update to the initial using the discrepancy between observations and predictions.

1. Propose initial distribution
2. Determine the predicted distribution
3. Compare predictions to observations
4. Re-weight initial distribution by ratio of pdfs

$$\pi_{update}(\lambda) = \pi_{init}(\lambda) \frac{\pi_{obs}(Q(\lambda))}{\pi_{predict}(Q(\lambda))}$$







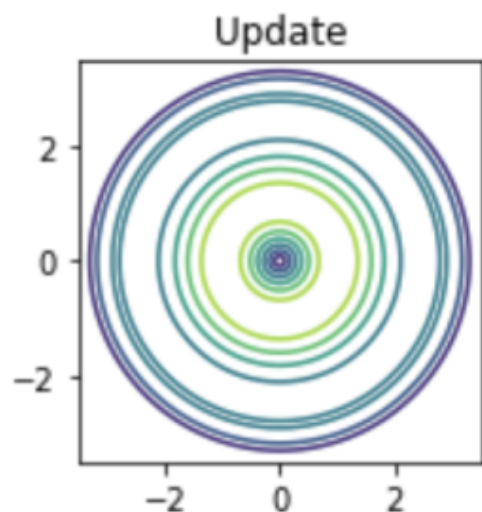
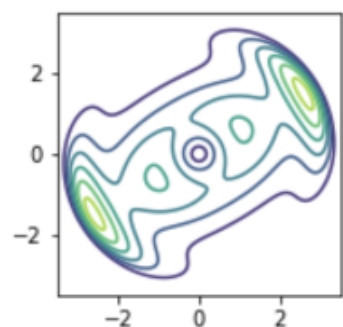
## Features of Data-consistent Approach

The update is **consistent** with the observed distribution.

The data-consistent approach is **non-parametric**.

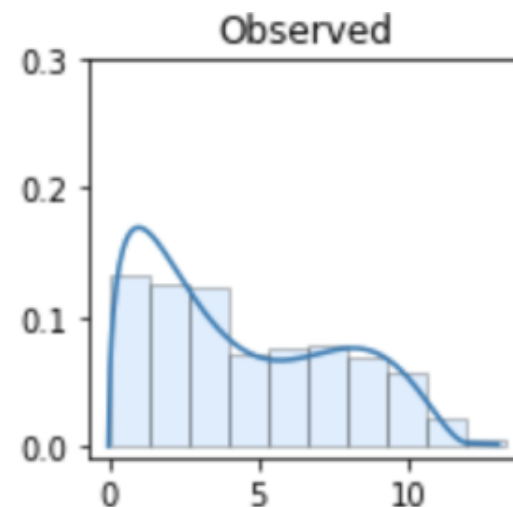
The update ***only influences data-informed*** directions and it is ***unbiased*** in these directions.

Generally, ***less computationally expensive*** than hierarchical Bayesian methods.



Simple Model

$$Q(\lambda) = \lambda_A^2 + \lambda_B^2$$



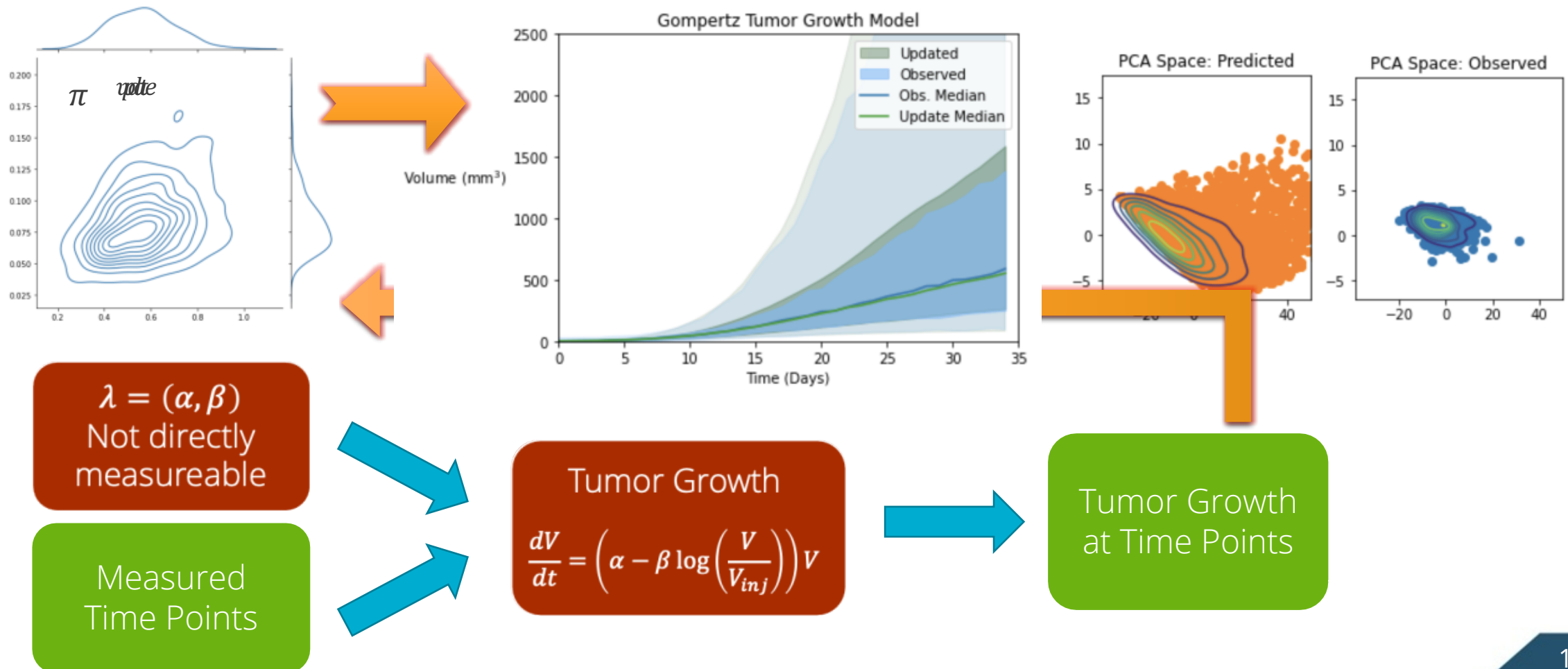
# Data-consistent Inversion for Tumor Growth





# A Data-consistent Approach to an Oncology Example

**Goal:** Obtain a population distribution of tumor growth parameters that is consistent with observed tumor growth curves.





## Conclusion: I am looking for collaborators...

Data-consistent inversion is a **powerful** new approach to stochastic inversion...

- Implementing data-consistent inversion is **straightforward**...
- Applying the data-consistent approach requires the **right perspective**...

The data-consistent approach may be particularly suited for estimating **population** distributions for use in precision medicine!

Interested? Reach out!

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