



Sandia
National
Laboratories

Exceptional service in the national interest

Data-consistent Approaches to Modeling Population Distributions of Parameters for Biomedical Systems

Tian Yu Yen

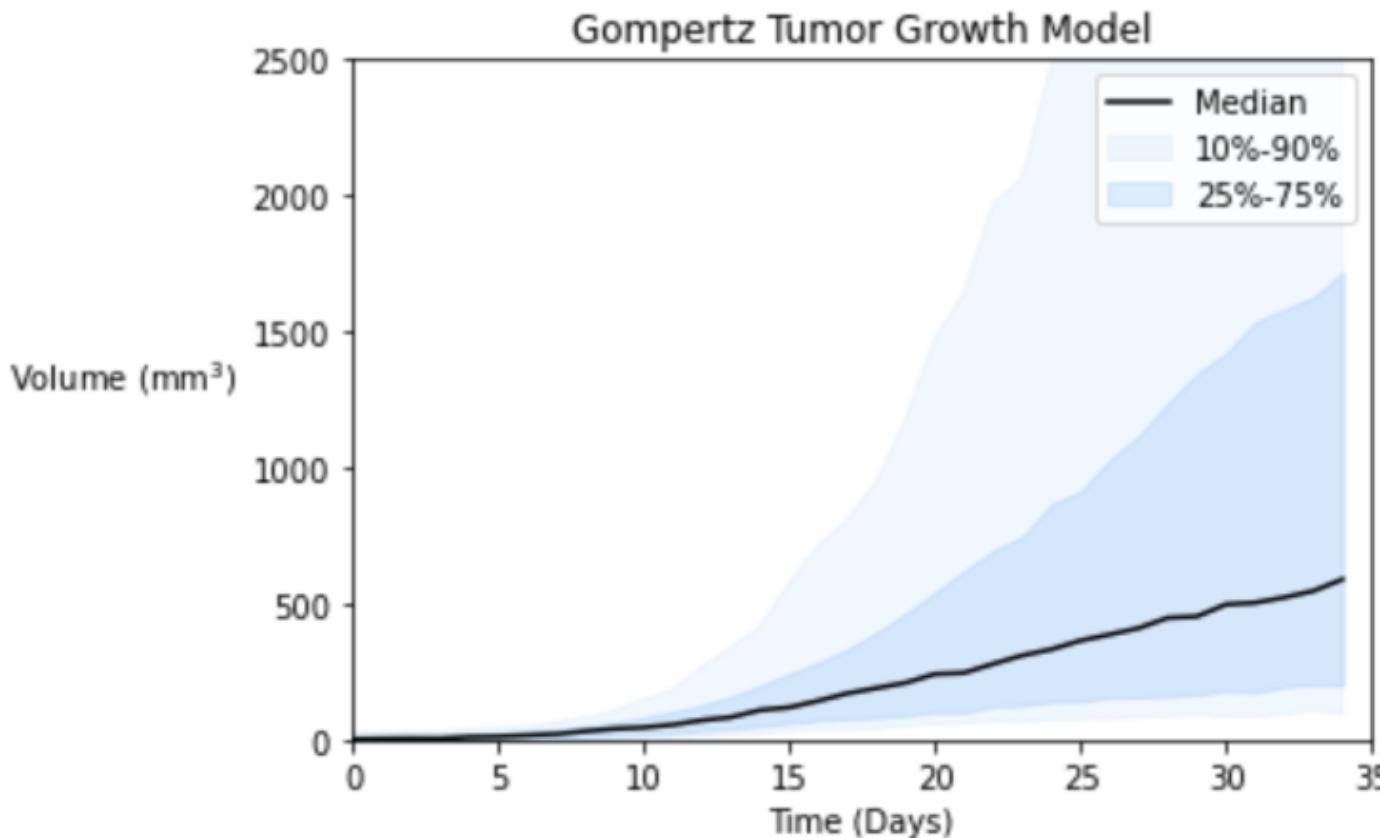
ASA CO-WY Chapter Meeting

11/11/2022

In many contexts, decision-making and “best practice” is guided by **average** or **typical** behavior...

Oncology Example:

Tumor growth kinetics used to prescribe radiation treatment dosage and schedules



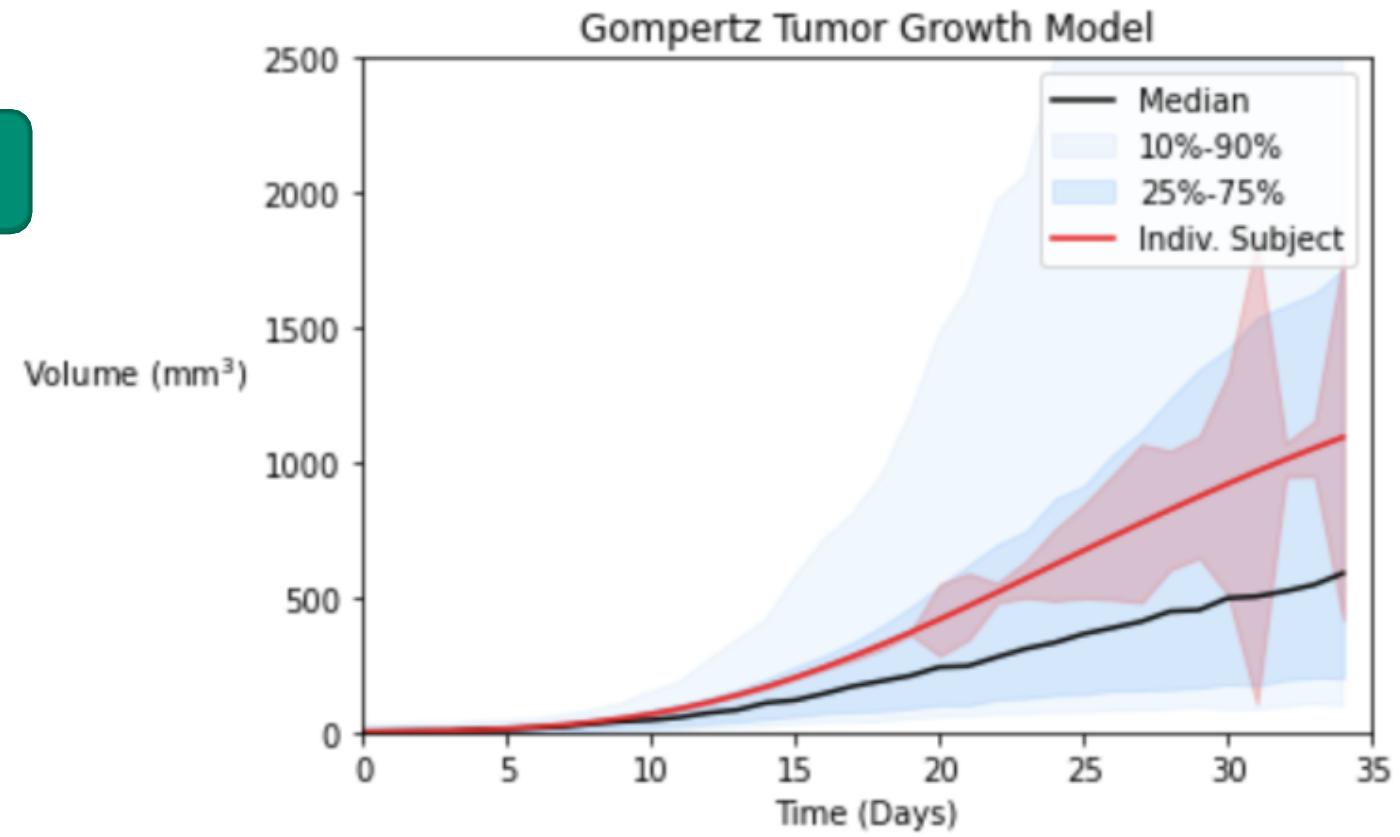
$$\frac{dV}{dt} = \left(\alpha - \beta \log \left(\frac{V}{V_{inj}} \right) \right) V$$

The promise of "precision medicine" requires effective uncertainty quantification...

Personalized care and effective decision-making needs to account for uncertainty...

Bayesian Analysis:

$$\pi^{post}(\alpha, \beta) \propto \pi^{prior}(\alpha, \beta) \cdot \pi^{like}(d|\alpha, \beta)$$

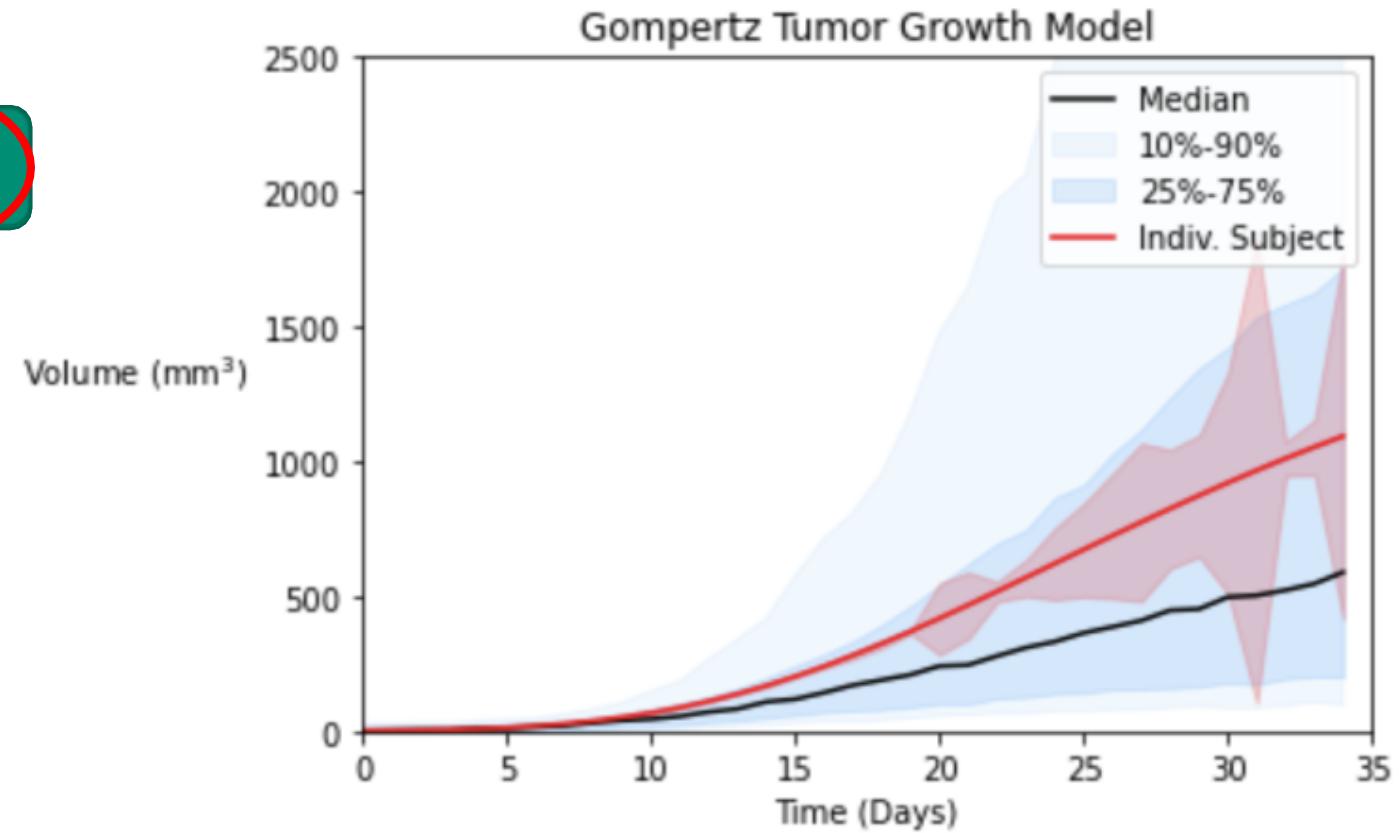


Bayesian approach to personalized uncertainty will be strongly influenced by “population” prior distributions...

Limited data on individuals \Rightarrow Population prior will bias decisions...

Bayesian Analysis:

$$\pi^{post}(\alpha, \beta) \propto \pi^{prior}(\alpha, \beta) \cdot \pi^{like}(d|\alpha, \beta)$$





Data-consistent inversion may be a powerful tool to obtaining suitable prior distributions for Bayesian analyses.

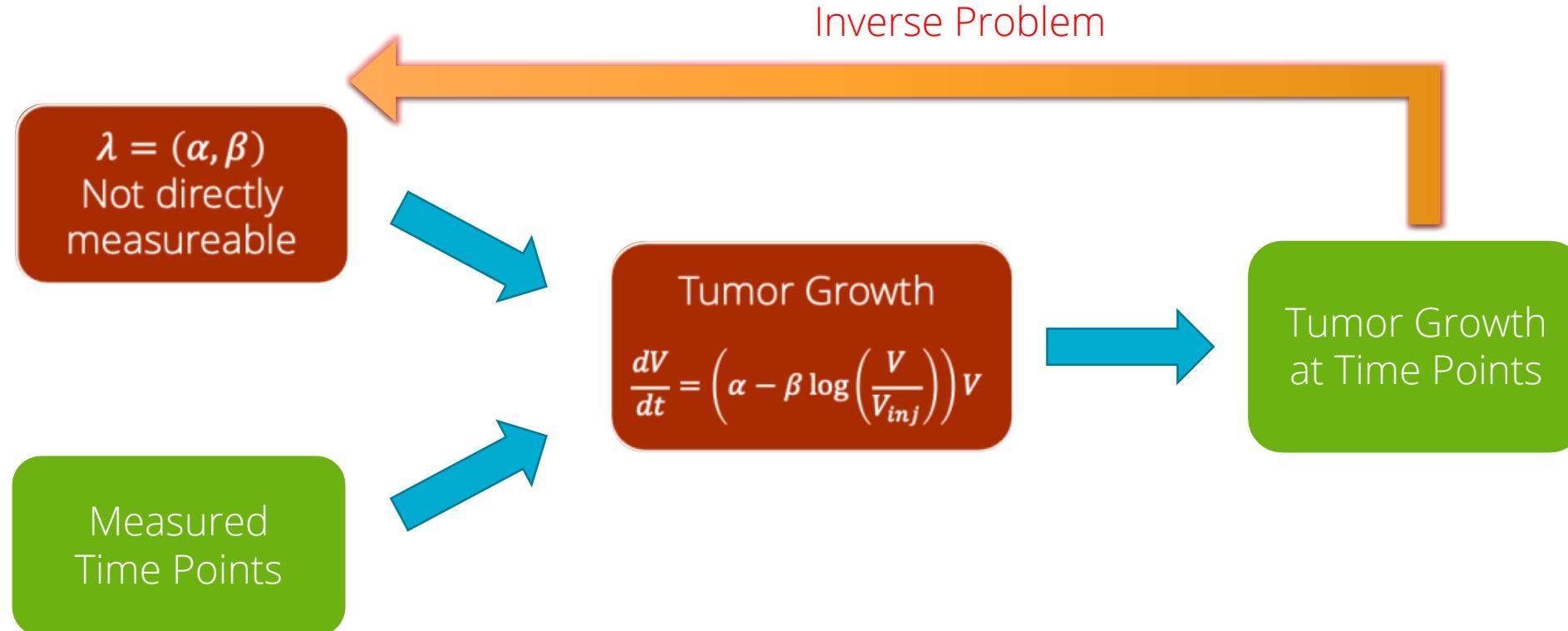
Data-consistent inversion is a measure-theoretic framework for stochastic inversion.

- The updated “population” parameter distribution is ***consistent*** with observed variation
- The updates from data-consistent inversion occur ***only influence data-informed*** directions
- The update is a ***non-parametric*** density estimate
- Generally, ***less computationally expensive*** than hierarchical Bayesian methods

$$\pi_{update}(\lambda) = \pi_{init}(\lambda) \frac{\pi_{obs}(Q(\lambda))}{\pi_{predict}(Q(\lambda))}$$

Brief Overview of Data-consistent Inversion

The goal of an inverse problem with a known dynamics model is to obtain a better characterization of input parameters.

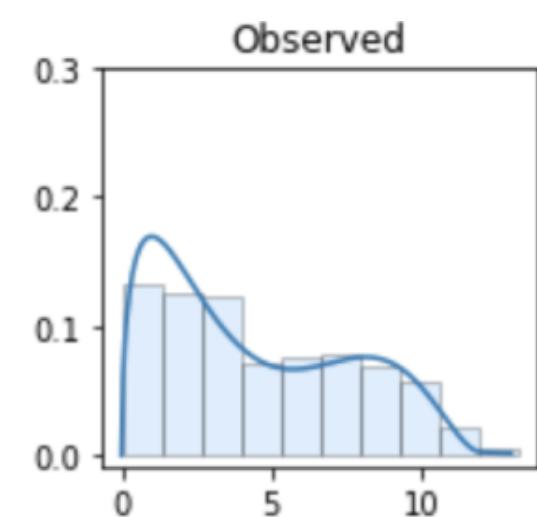
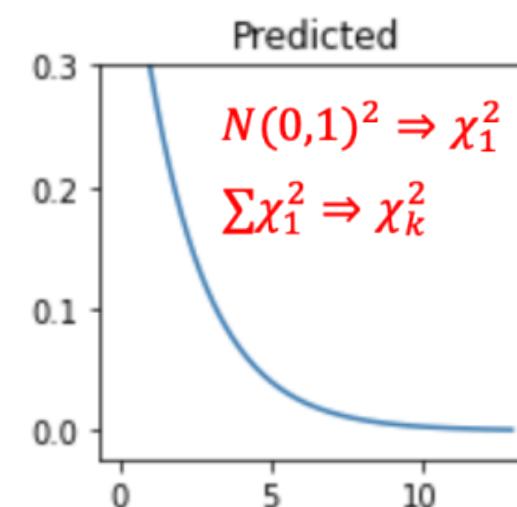
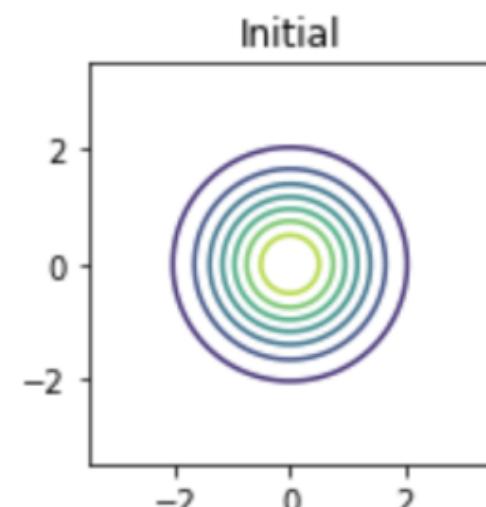
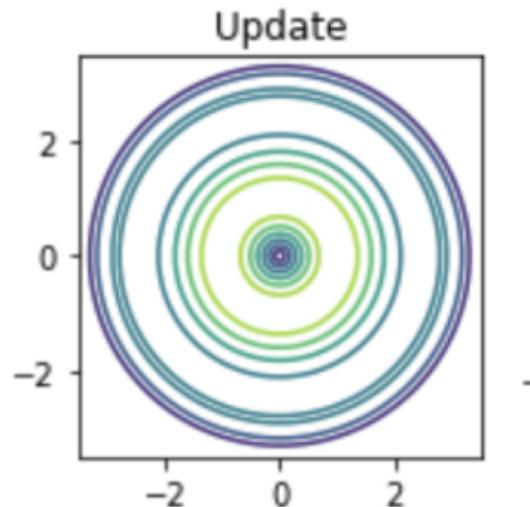


Key Point: In our *stochastic* inverse problem, we want to find the population *distribution* of λ !

The data-consistent approach finds a **consistent** update to the initial using the discrepancy between observations and predictions.

1. Propose initial distribution
2. Determine the predicted distribution
3. Compare predictions to observations
4. Re-weight initial distribution by ratio of pdfs

$$\pi_{update}(\lambda) = \pi_{init}(\lambda) \frac{\pi_{obs}(Q(\lambda))}{\pi_{predict}(Q(\lambda))}$$



Input
 $\lambda = (\lambda_A, \lambda_B)$



Simple Model
 $Q(\lambda) = \lambda_A^2 + \lambda_B^2$



Measured Data
 q_1, q_2, \dots, q_n

Inverse Problem

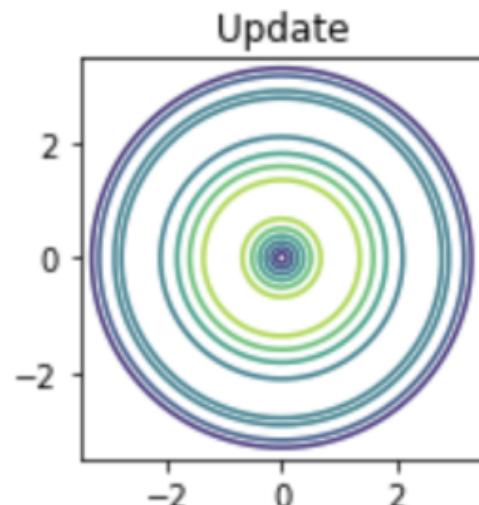
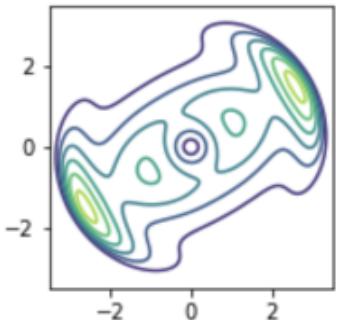
Features of Data-consistent Approach

The update is consistent with the observed distribution.

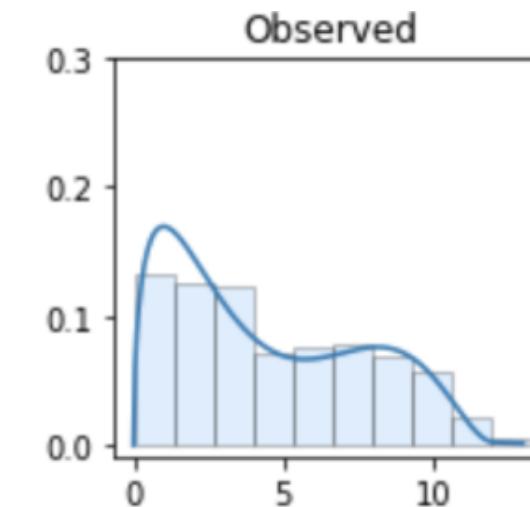
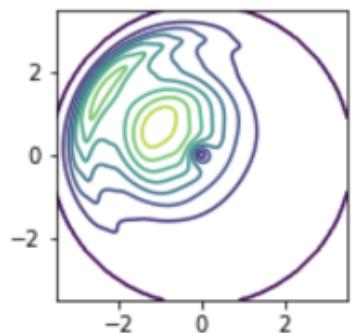
The data-consistent approach is non-parametric.

The update **only influences data-informed** directions and it is **unbiased** in these directions.

Generally, **less computationally expensive** than hierarchical Bayesian methods.



Simple Model
 $Q(\lambda) = \lambda_A^2 + \lambda_B^2$

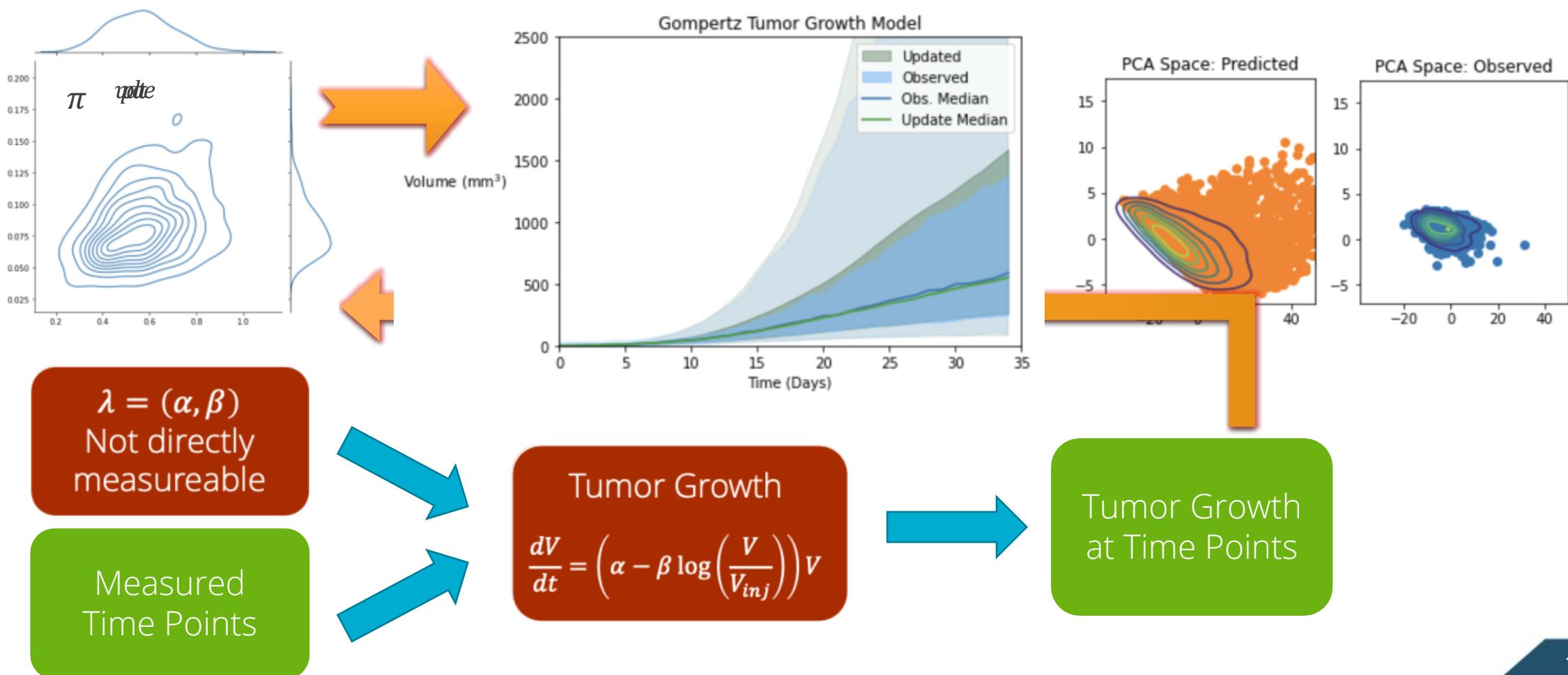


Data-consistent Inversion for Tumor Growth



A Data-consistent Approach to an Oncology Example

Goal: Obtain a population distribution of tumor growth parameters that is consistent with observed tumor growth curves.





Conclusion: I am looking for collaborators...

Data-consistent inversion is a **powerful** new approach to stochastic inversion...

- Implementing data-consistent inversion is **straightforward**...
- Applying the data-consistent approach requires the **right perspective**...

The data-consistent approach may be particularly suited for estimating **population** distributions for use in precision medicine!

Interested? Reach out!

Tian Yu Yen

tyen@sandia.gov