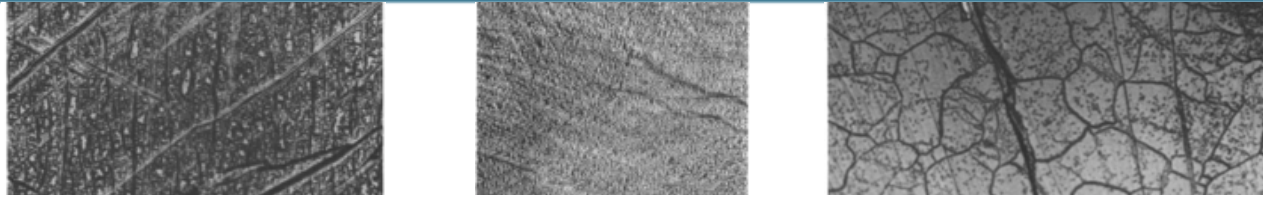
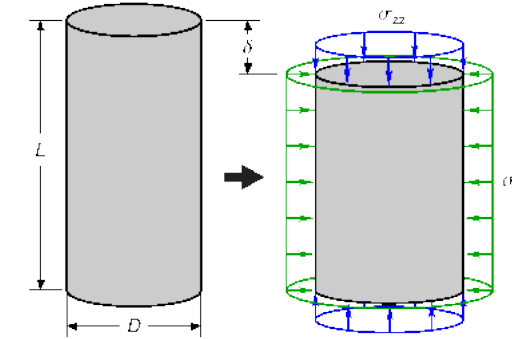




# New Salt Constitutive Model Status



Benjamin Reedlunn

Materials and Failure Modeling Department

WIPP Rock Mechanics Technical Exchange

October 27<sup>th</sup>, 2022



Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525. This research is funded by WIPP programs administered by the Office of Environmental Management (EM) of the U.S. Department of Energy.



1. Motivation
2. Model Overview
3. Calibrations
4. Non-Monotonic Loading
5. Summary

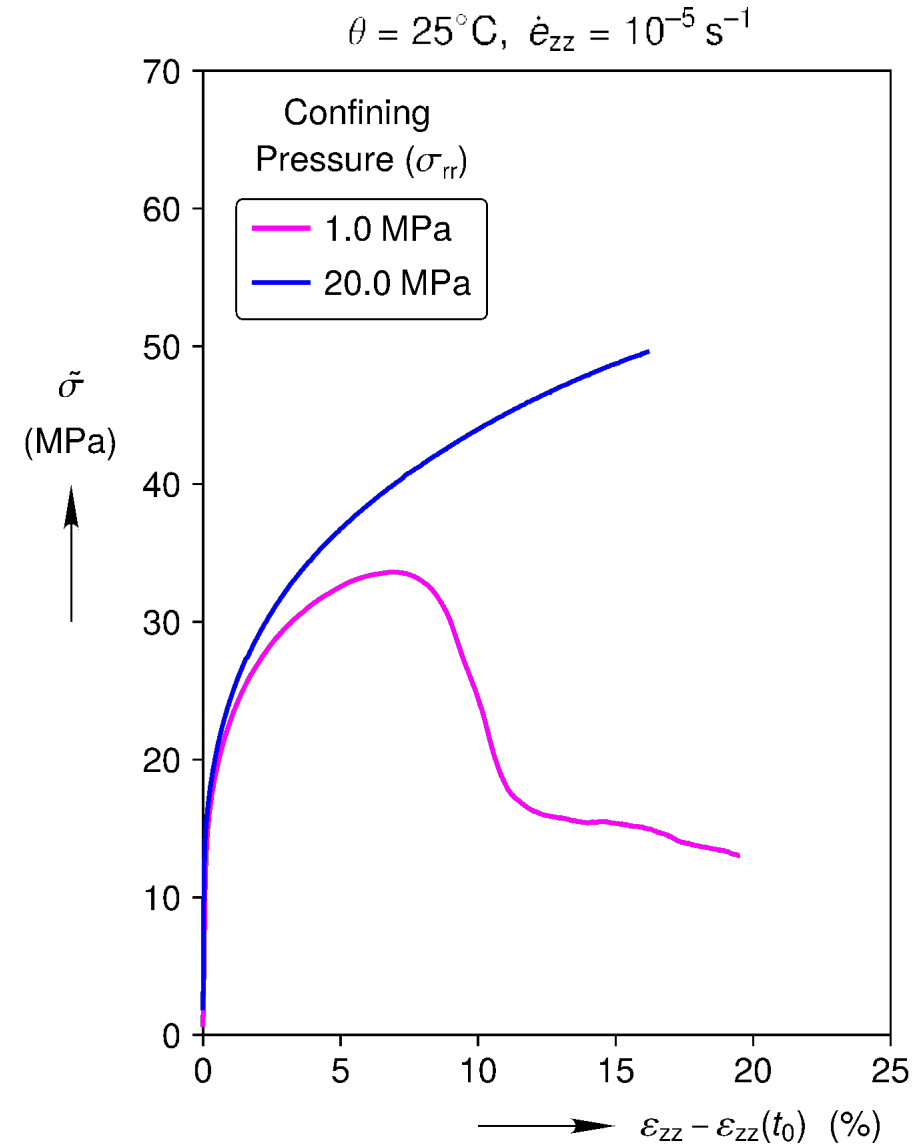
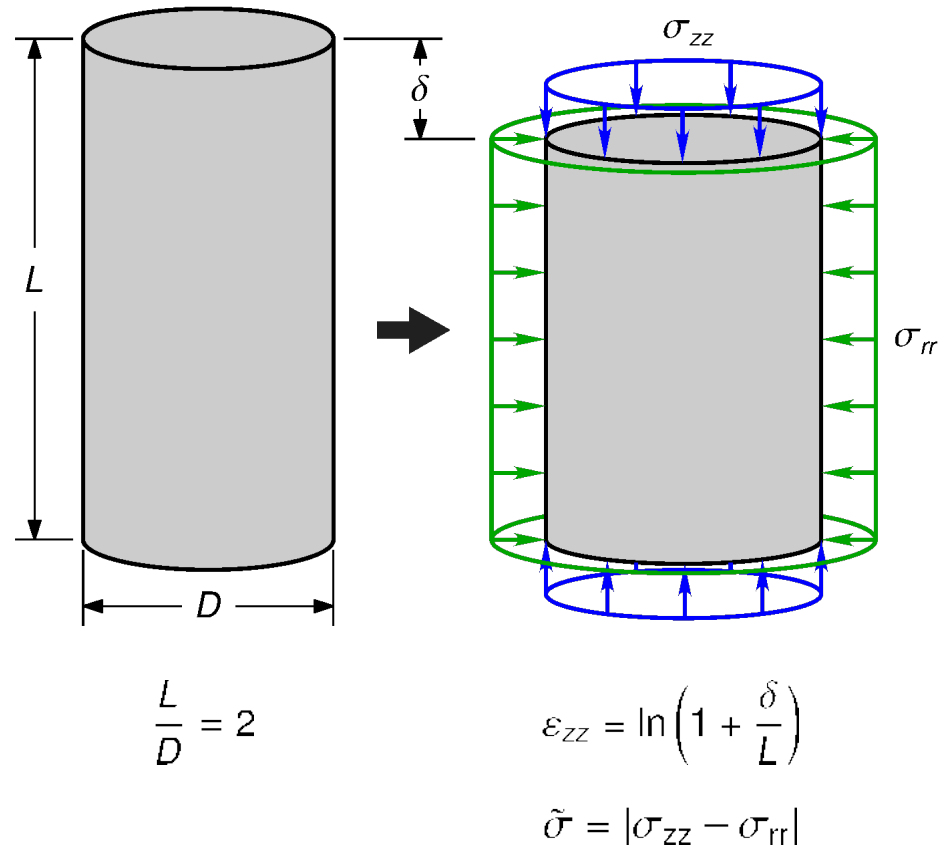
# Motivation



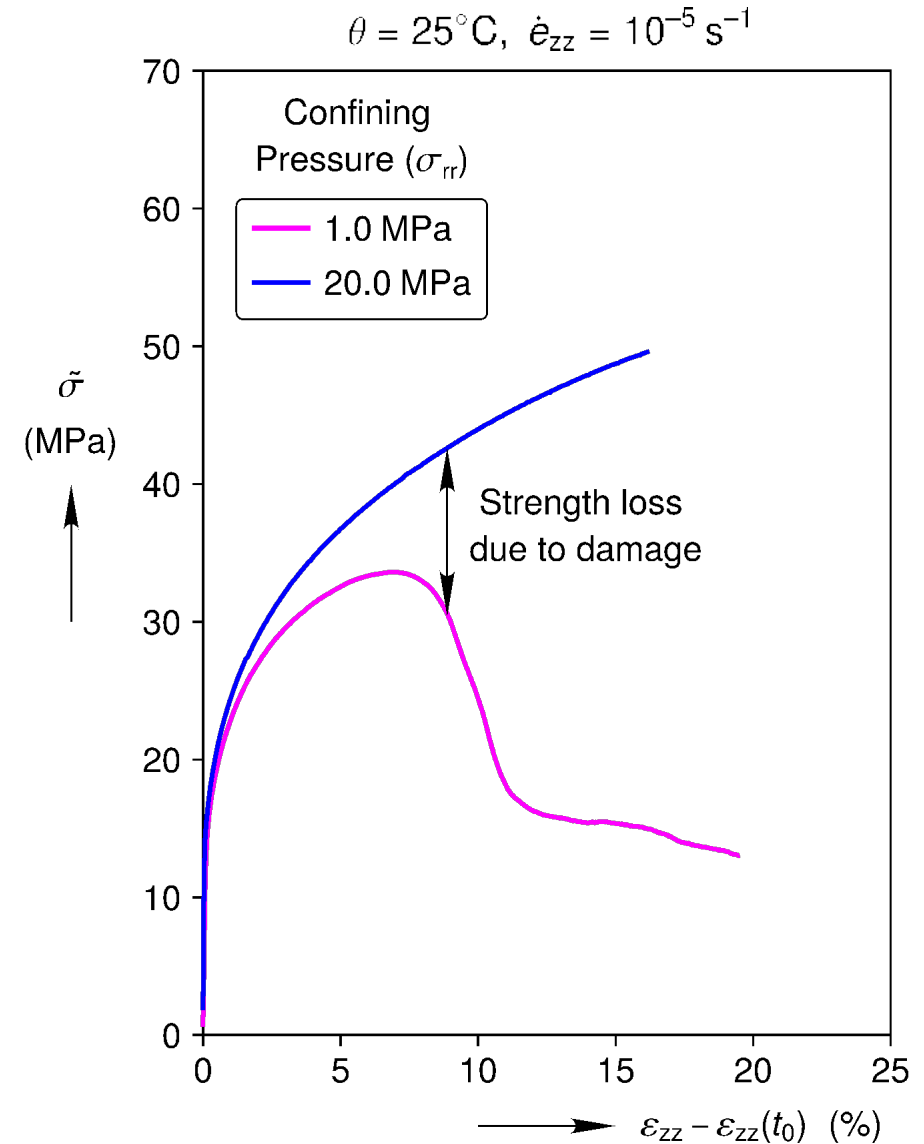
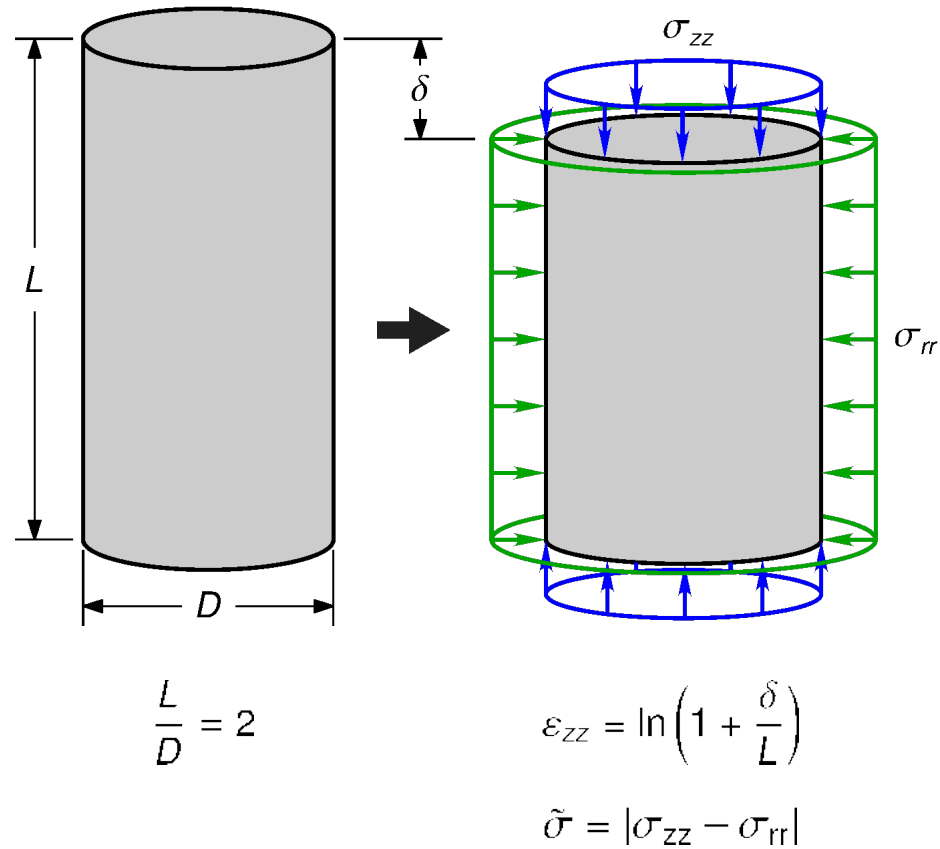
## Munson-Dawson model

1. Advantages
  1. Well-known
  2. Easy to calibrate against constant stress tests
  3. Captures most low and medium stress behavior
2. Small Disadvantages
  1. Highly phenomenological
    1.  $\dot{\epsilon}^{vp} = \dot{\epsilon}^{tr} + \dot{\epsilon}^{ss}$
  2. Cannot be fit into framework of Rational Thermodynamics
3. Larger Disadvantages Relevant to Empty Areas at WIPP
  1. Does not include damage and healing
  2. Cannot capture high strain rate (high stress) behavior
  3. Cannot capture re-hardening during non-monotonic loading

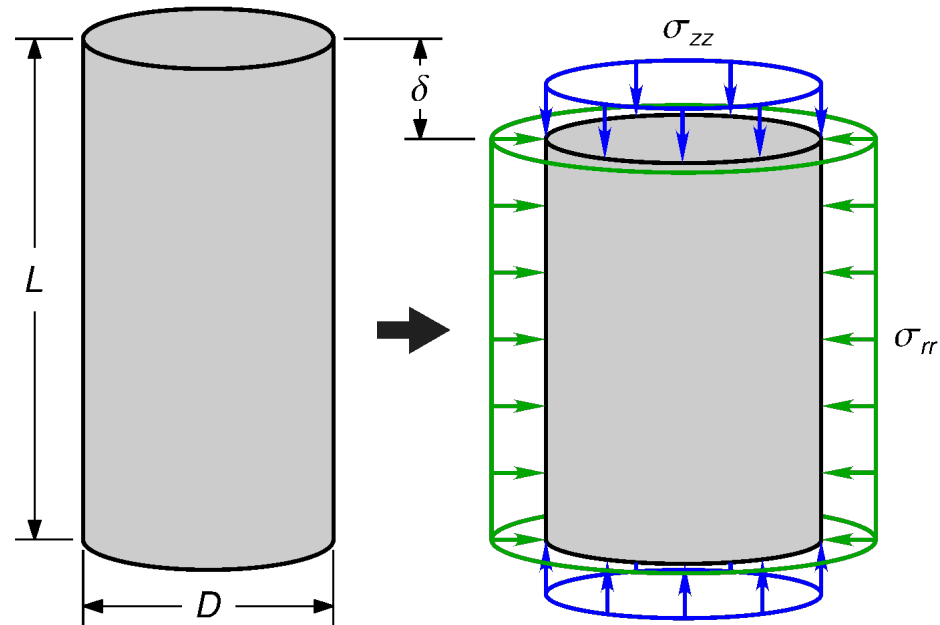
# Damaged, High Strain Rate, Behavior



# Damaged, High Strain Rate, Behavior



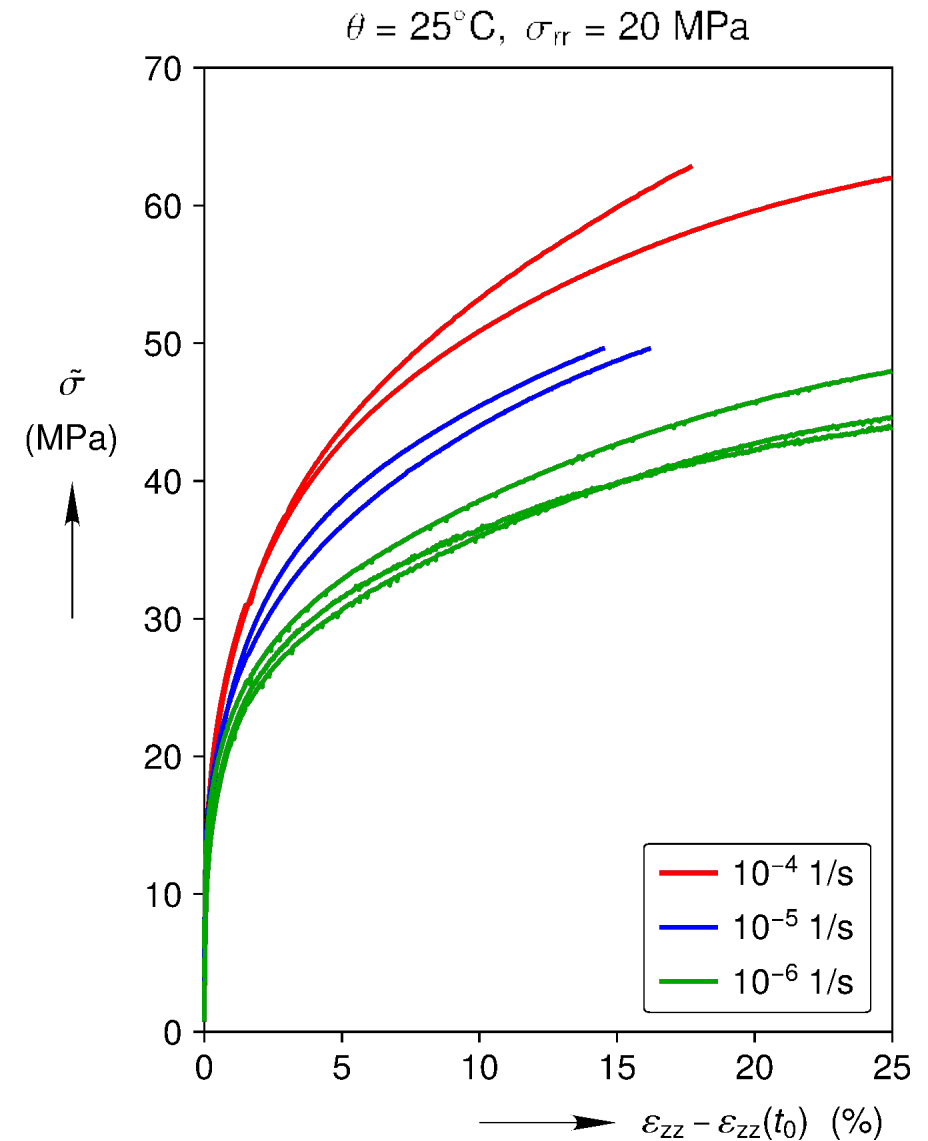
# Damage-Free, High Strain Rate, Behavior



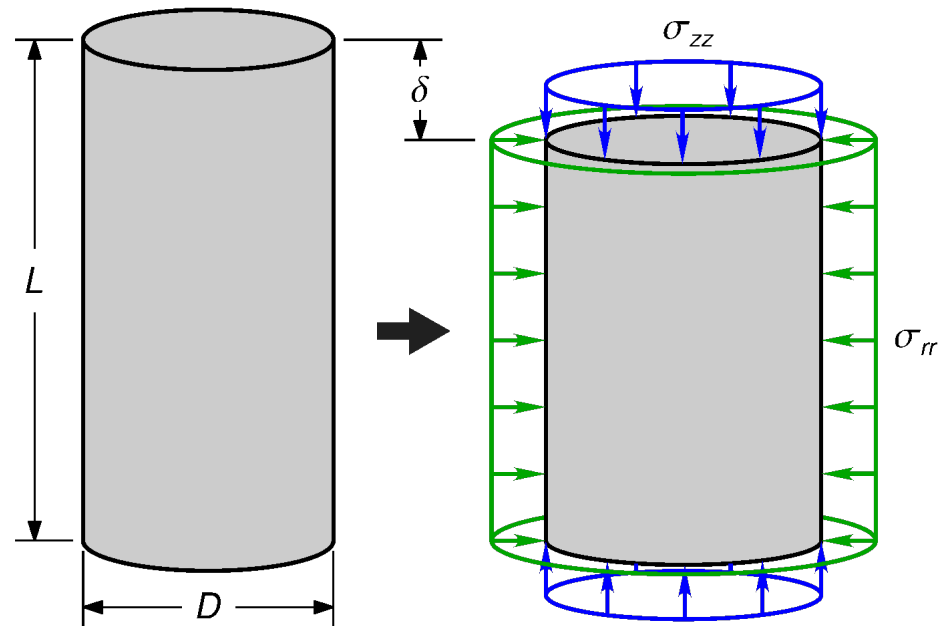
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln\left(1 + \frac{\delta}{L}\right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



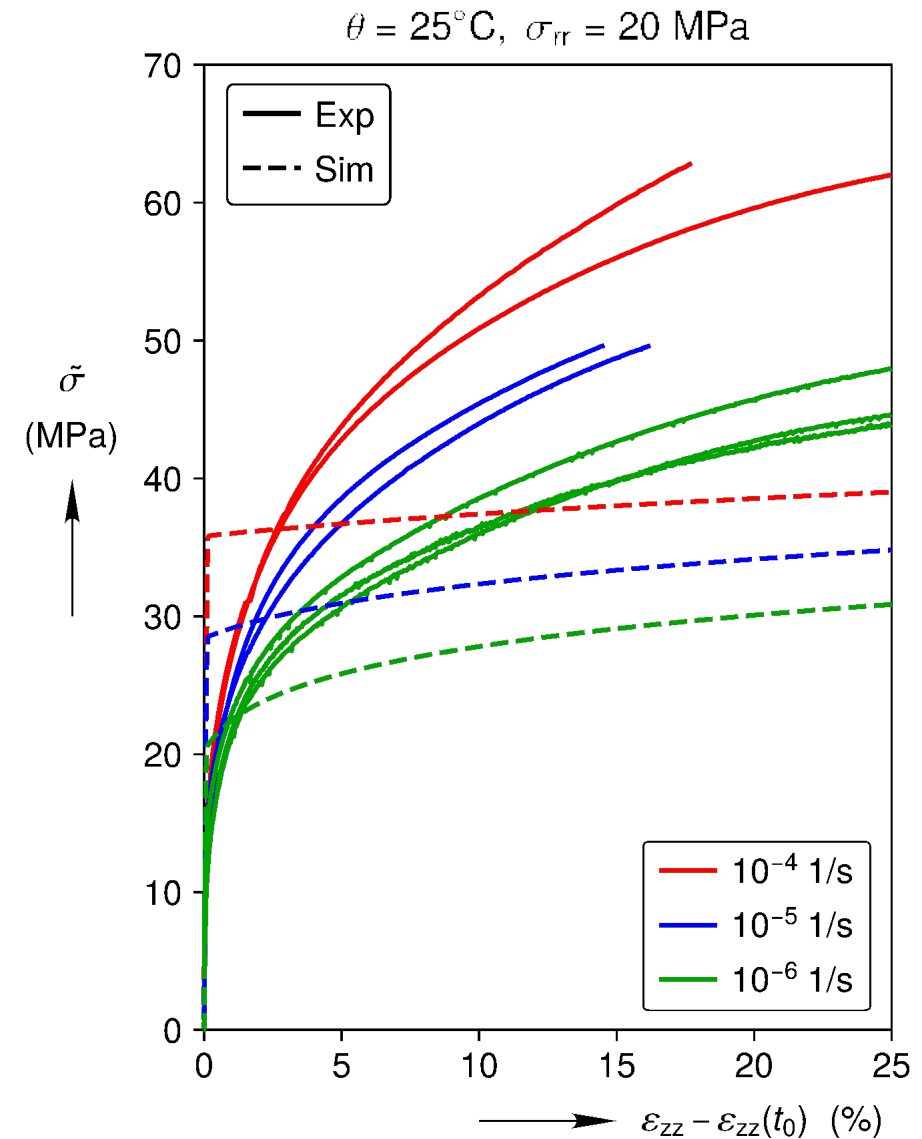
# Damage-Free, High Strain Rate, Munson-Dawson Predictions



$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln\left(1 + \frac{\delta}{L}\right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



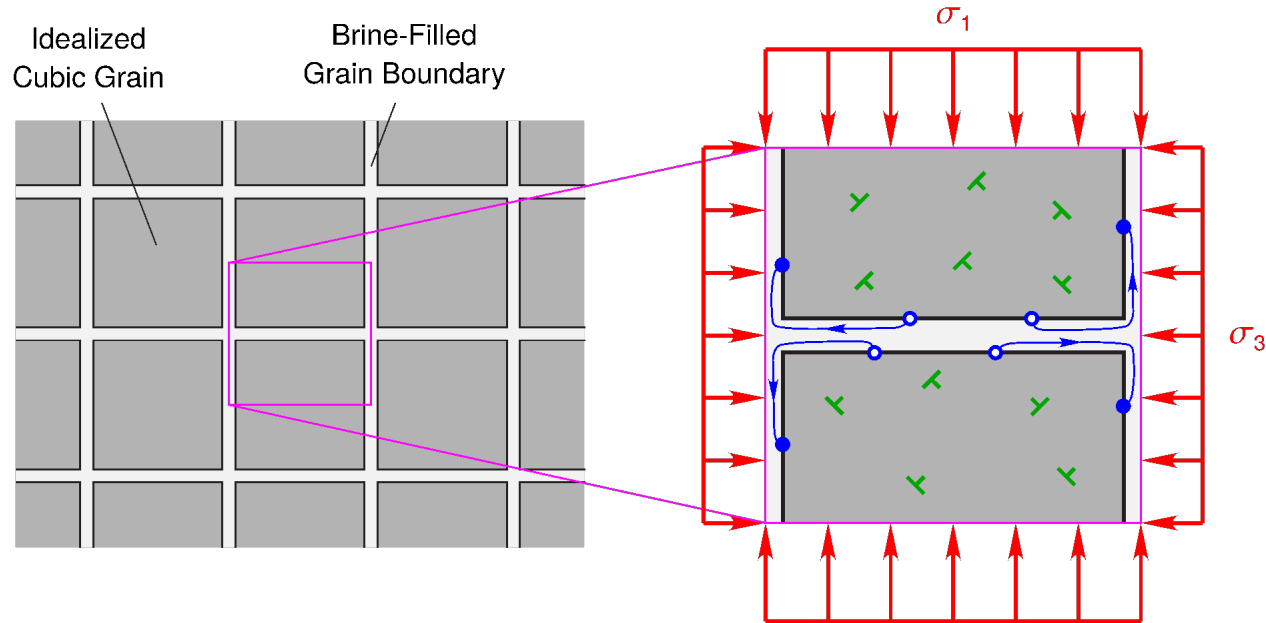


# Model Overview

# Viscoplastic Branches



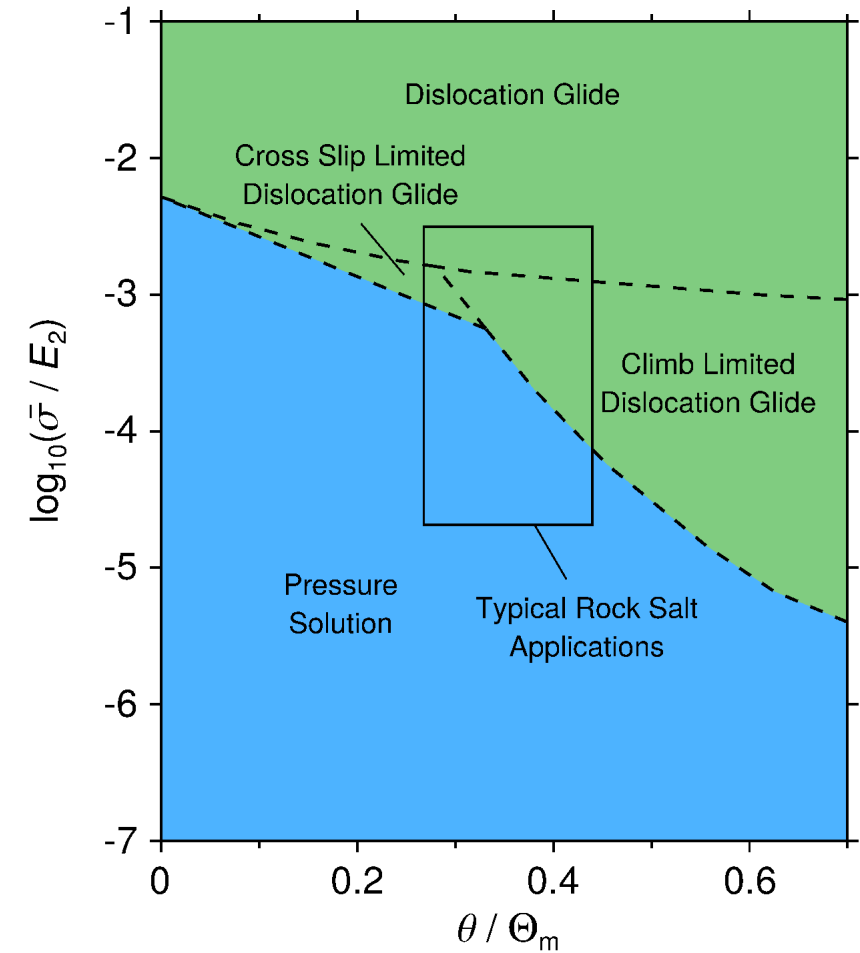
## Pressure Solution and Dislocation Glide



Strain Rate Decomposition  
(proportional, monotonic, loading)

$$\dot{\epsilon}^{vp} = \dot{\epsilon}^{ps} + \dot{\epsilon}^{dg}$$

## A (Rough) Steady-State Deformation Mechanism Map



$\Theta_m = 1077 \text{ K}$

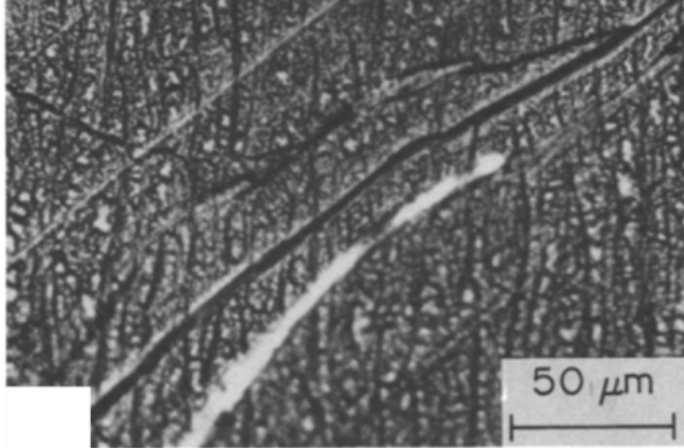
$E_2 = 10 \text{ GPa}$

Average grain size = 10 mm

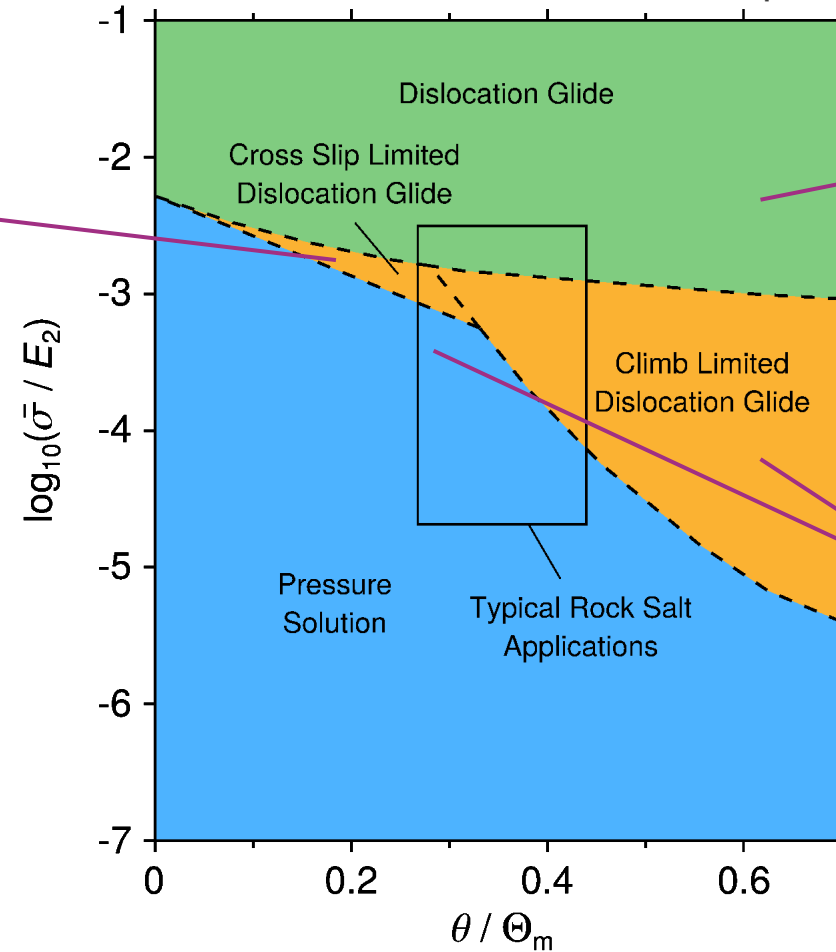
# Microstructural Observations



Wavy Slip Bands



A (Rough) Steady-State Deformation Mechanism Map

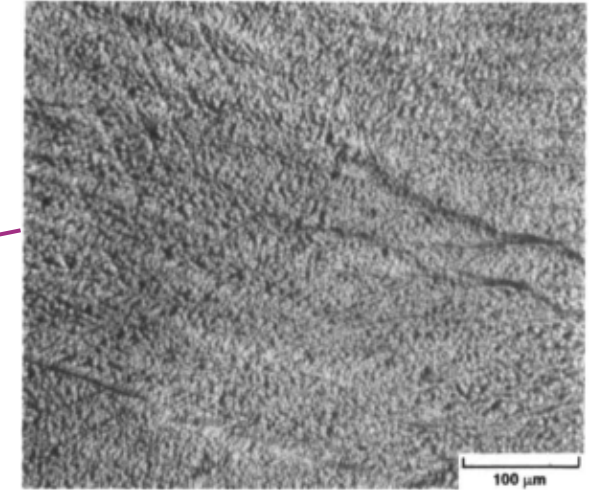


$$\Theta_m = 1077 \text{ K}$$

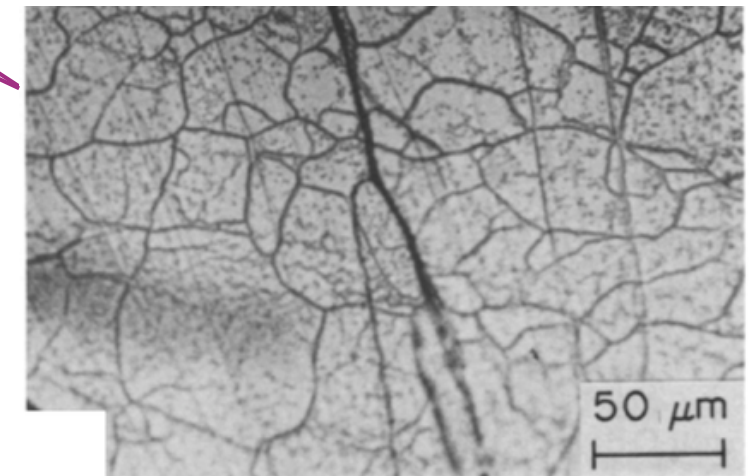
$$E_2 = 10 \text{ GPa}$$

Average grain size = 10 mm

Uniform Dislocation Density



Subgrains



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233-242.

Carter, N.L., Horne, S.T., Russell, J.E., and Handin, J. (1993). Rheology of rock salt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

# Steady-State Strain Rate Calibration

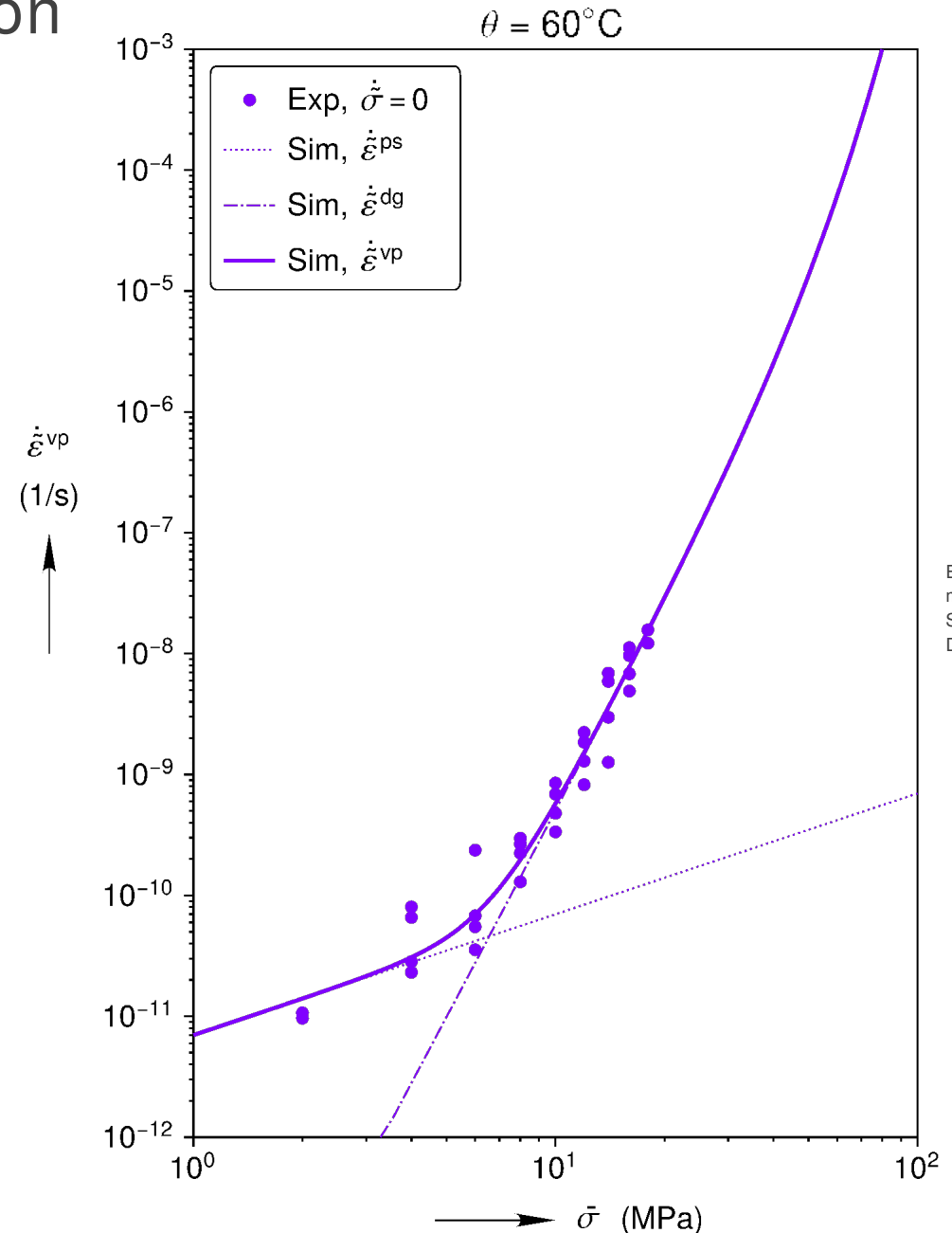
## Steady-State Strain Rates

$$\dot{\epsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\epsilon}^{\text{dg}} = Y_3 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{Y_4}\right)\right]^{Y_5}$$

Spiers, C., Schutjens, P., Brzesowsky, R., Peach, C., Liezenberg, J., and Zwart, H. (1990). Experimental determination of constitutive parameters governing creep of rocksalt by pressure solution. In: Geological Society, London, Special Publications 54.1, pp. 215–227.

Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.



# Steady-State Strain Rate Calibration

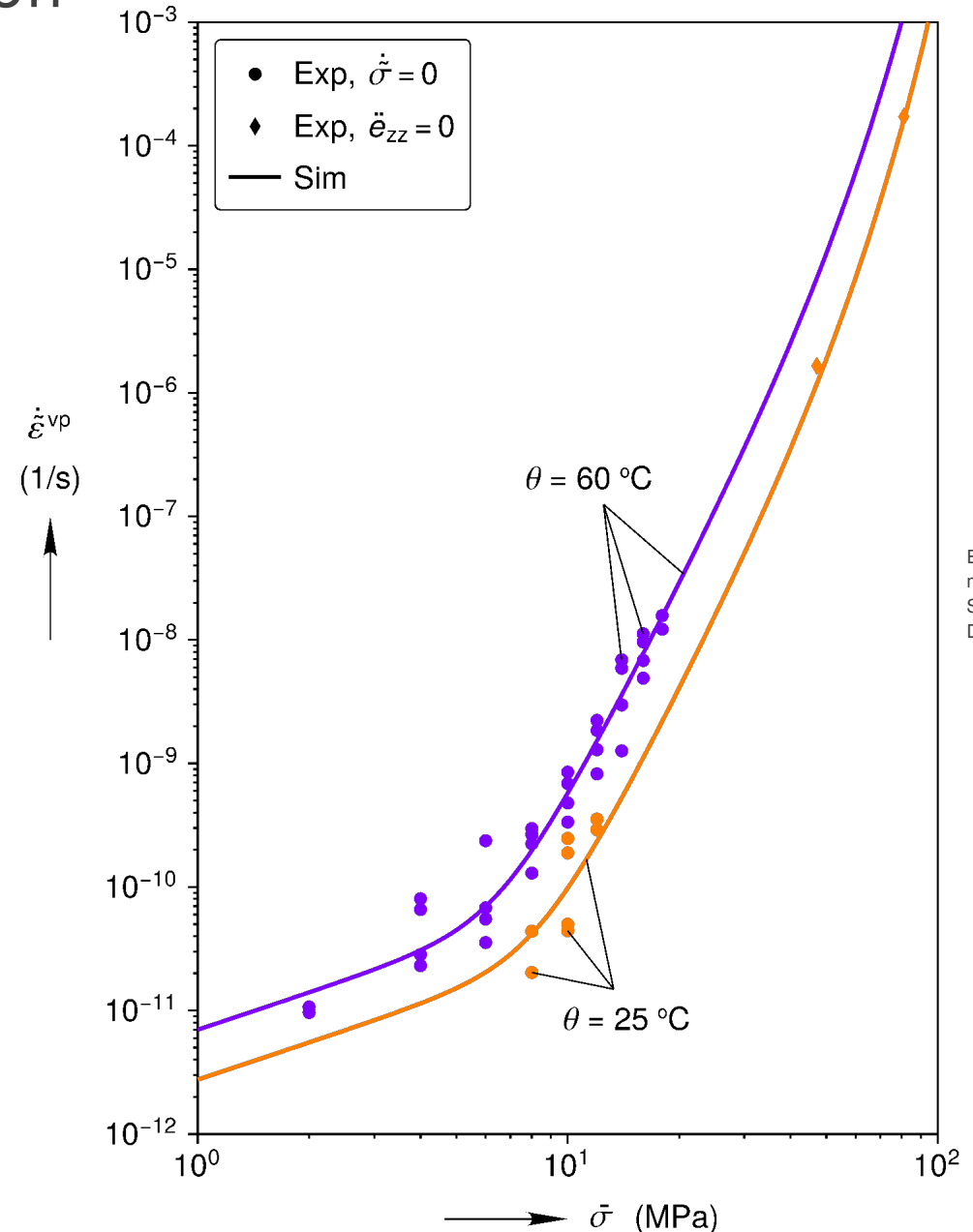
## Steady-State Strain Rates

$$\dot{\epsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

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Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.



# Strain Rates While Hardening



Strain Rates

(proportional, monotonic, loading)

$$\dot{\tilde{\epsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\epsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

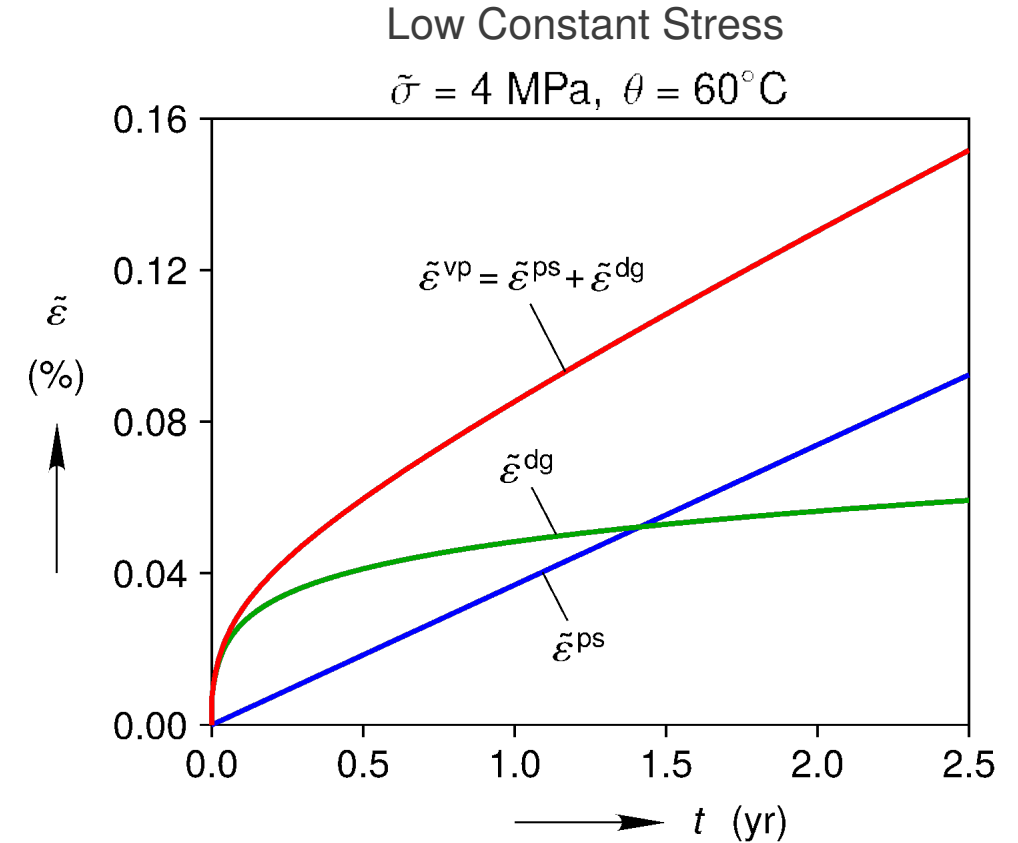
# Strain Rates While Hardening



Strain Rates  
(proportional, monotonic, loading)

$$\dot{\tilde{\epsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\epsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$



# Strain Rates While Hardening



Strain Rates

(proportional, monotonic, loading)

$$\dot{\tilde{\epsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\epsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$



# Dislocation Glide Hardening



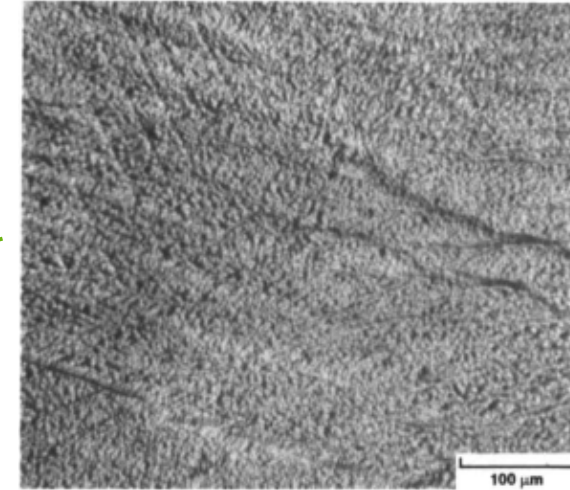
Equivalent Stress Decomposition  
(proportional, monotonic, loading)

$$\tilde{\sigma} = \tilde{b} + \tilde{\sigma}^{\text{dg}}$$

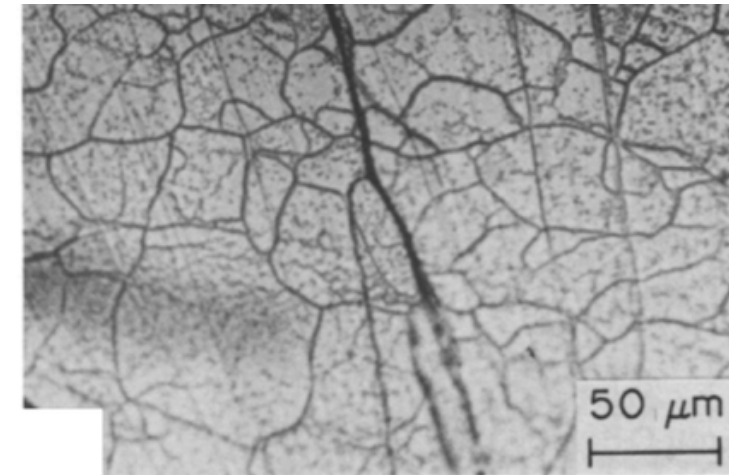
Drag Stress Contribution

$$\tilde{\sigma}^{\text{dg}} = y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$

Uniform Dislocation Density



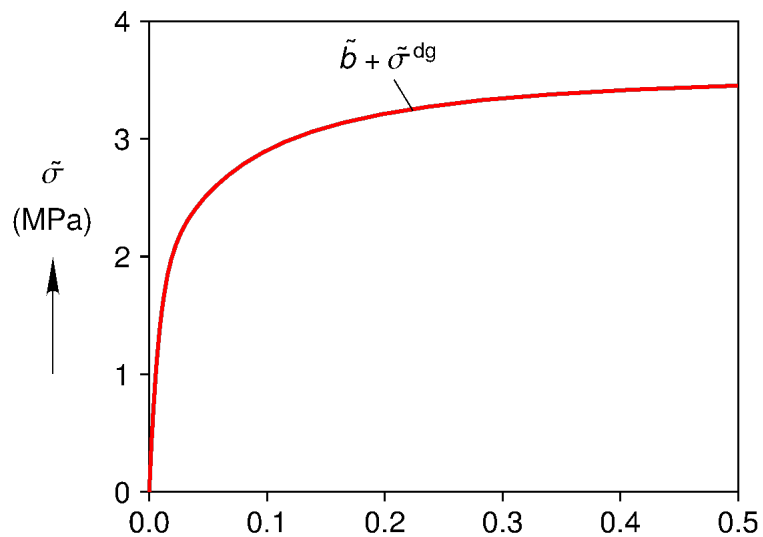
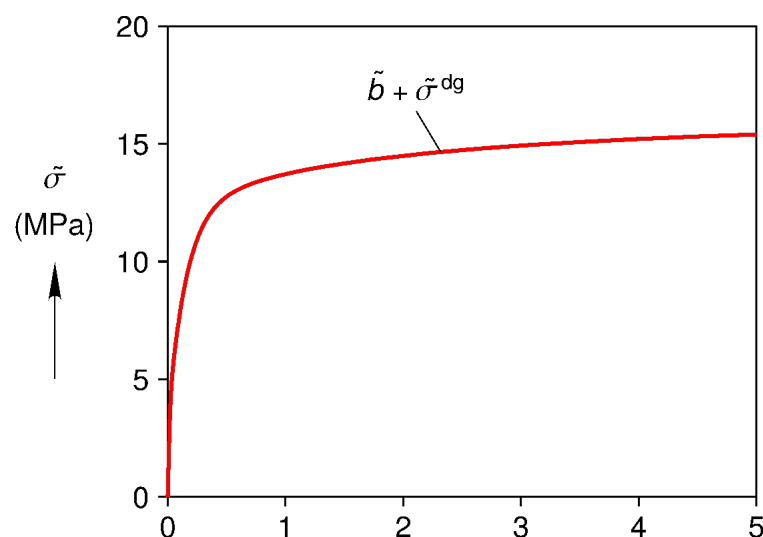
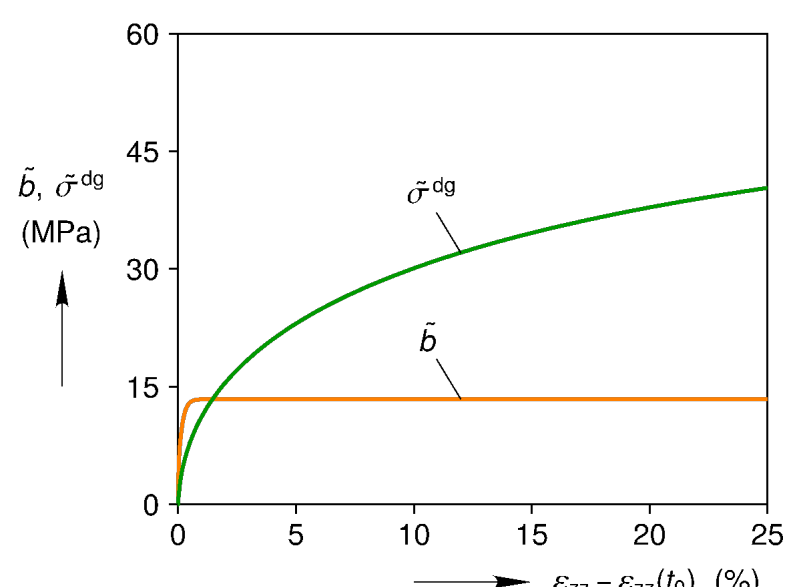
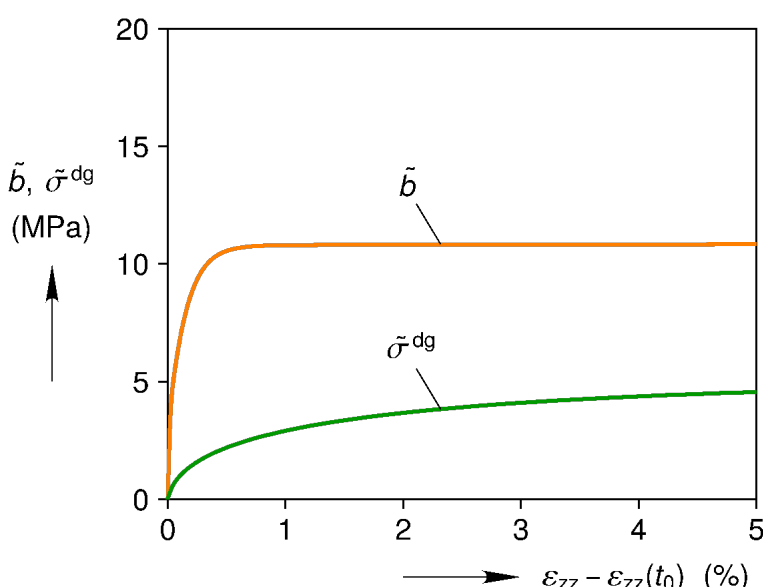
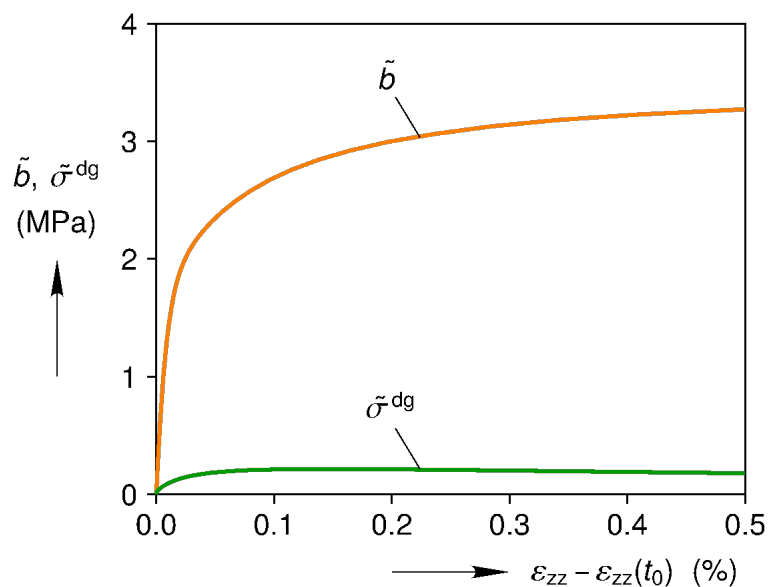
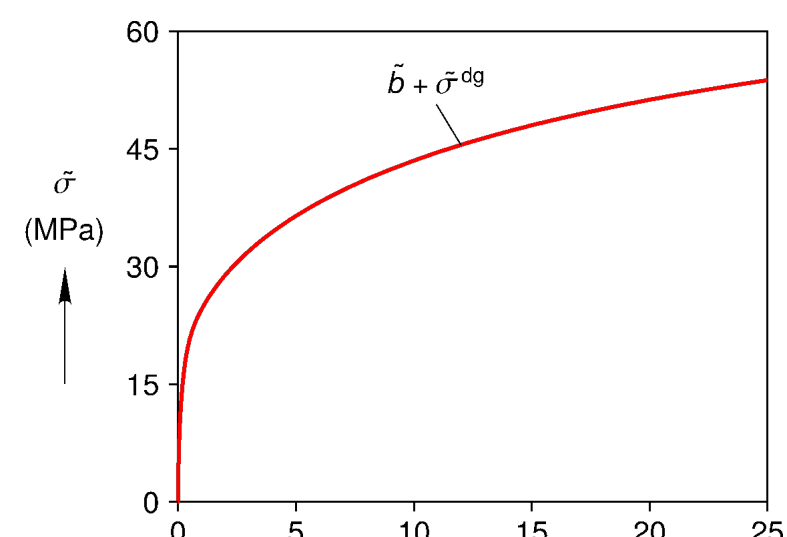
Non-Uniform Dislocation Density



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233–242.

Carter, NL, Horseman, ST, Russell, JE, and Handin, J (1993). Rheology of rocksalt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

# Constant Strain Rate Behavior

Low Strain Rate ( $10^{-11}$  1/s)Medium Strain Rate ( $10^{-9}$  1/s)High Strain Rate ( $10^{-5}$  1/s)

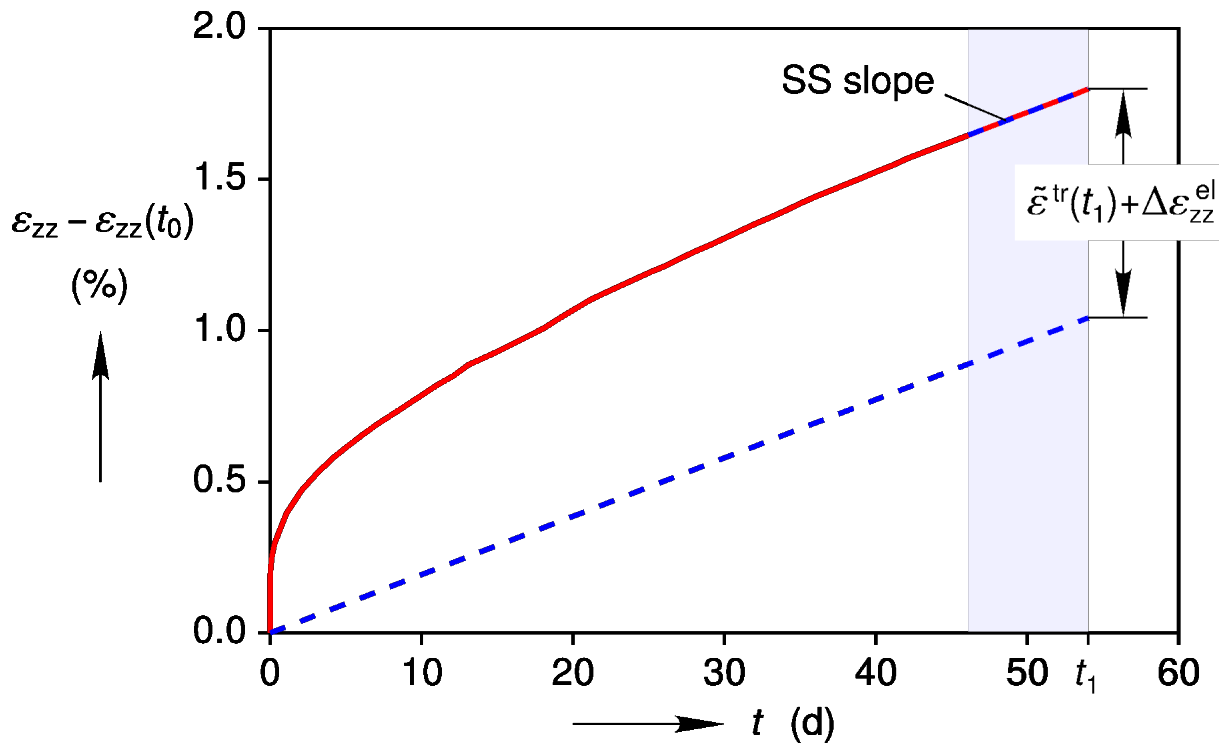
# Calibrations

# Hardening Transition from Low to Medium Stresses (Strain Rates)

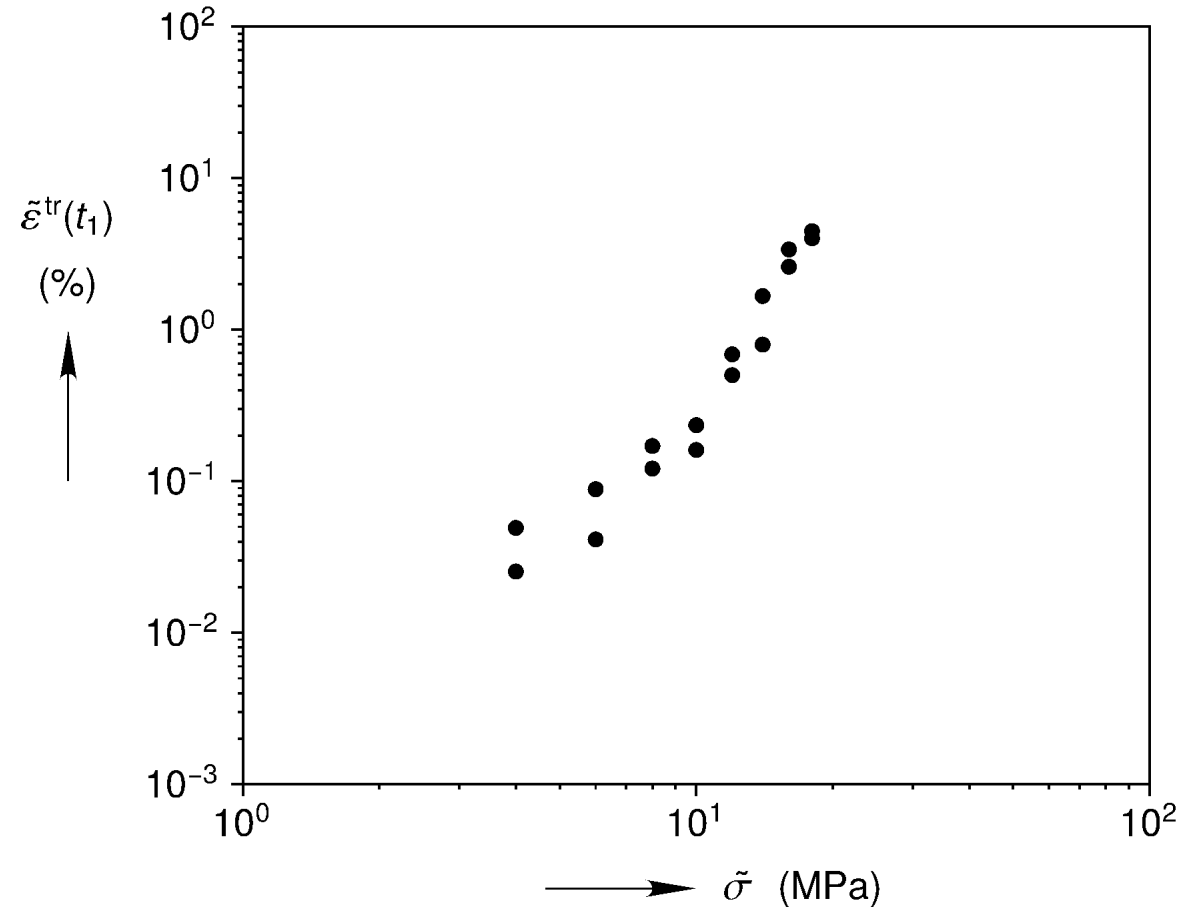


WIPP Salt, Constant Stress, Strain History

$\tilde{\sigma} = 12 \text{ MPa}$ ,  $\theta = 60 \text{ }^{\circ}\text{C}$



Transient Strain After 50 days at  $\theta = 60 \text{ }^{\circ}\text{C}$

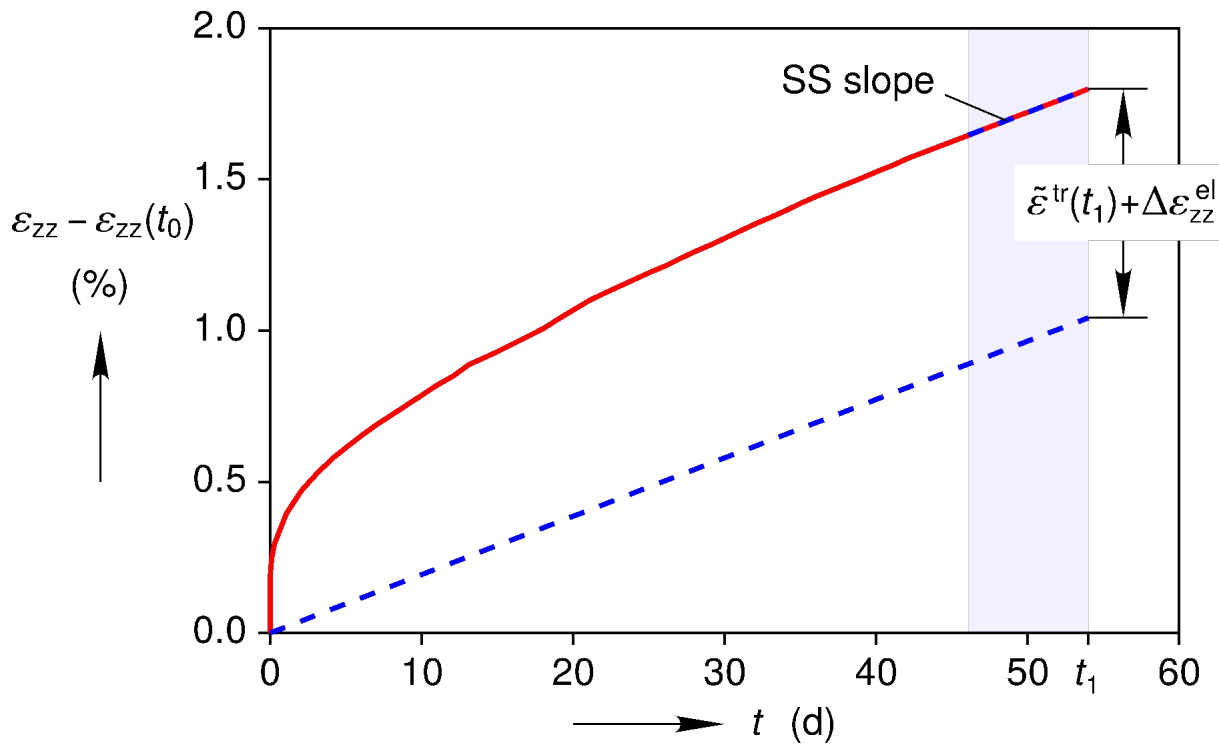


# Hardening Transition from Low to Medium Stresses (Strain Rates)

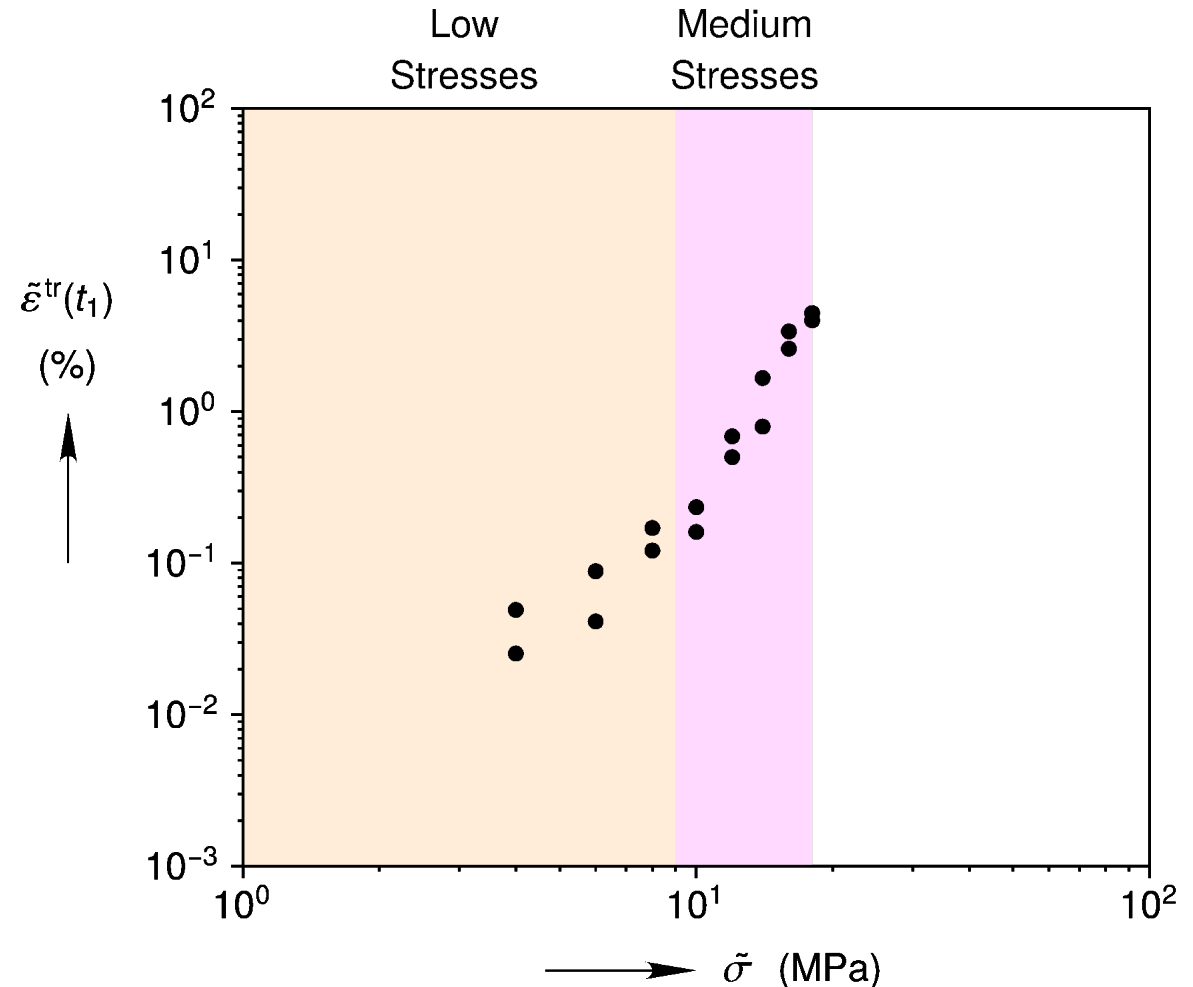


WIPP Salt, Constant Stress, Strain History

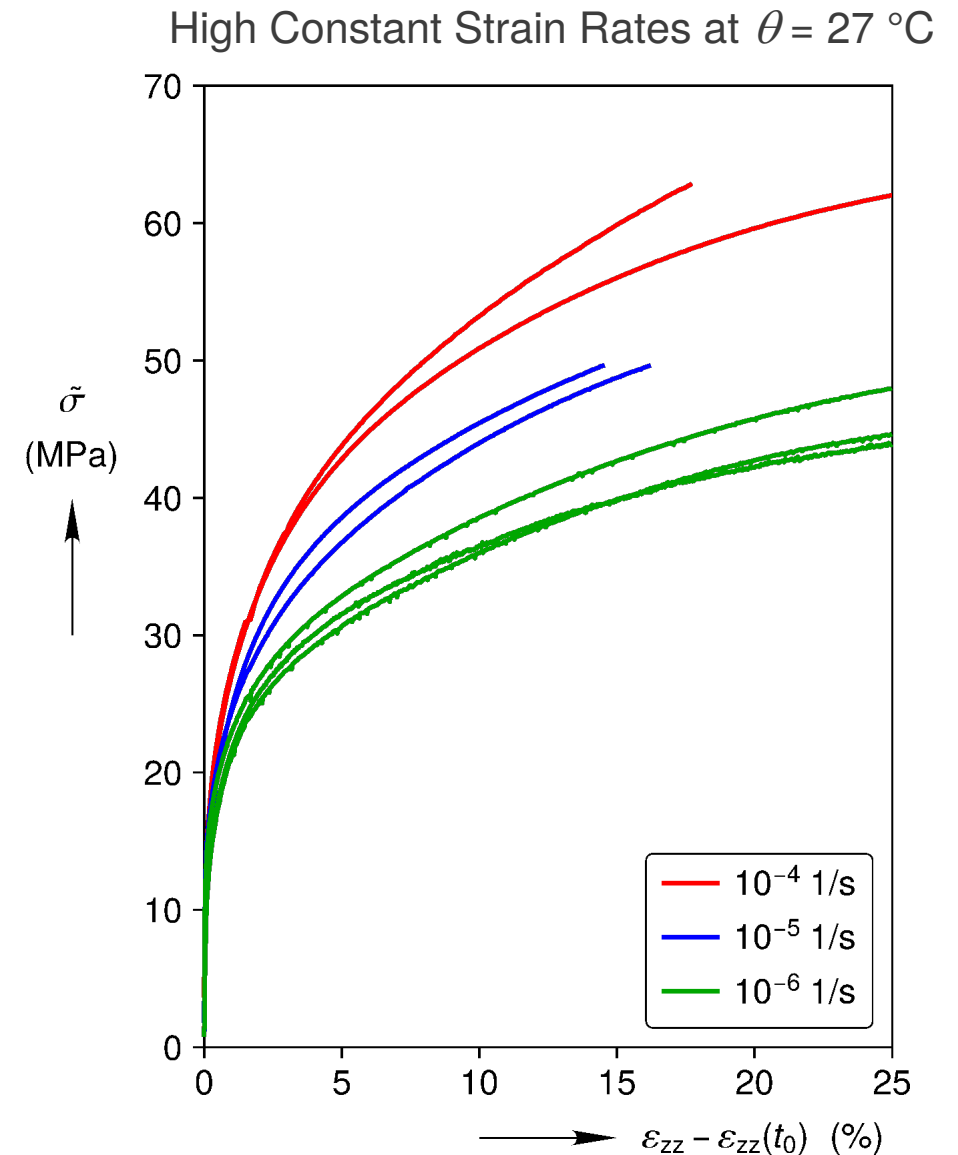
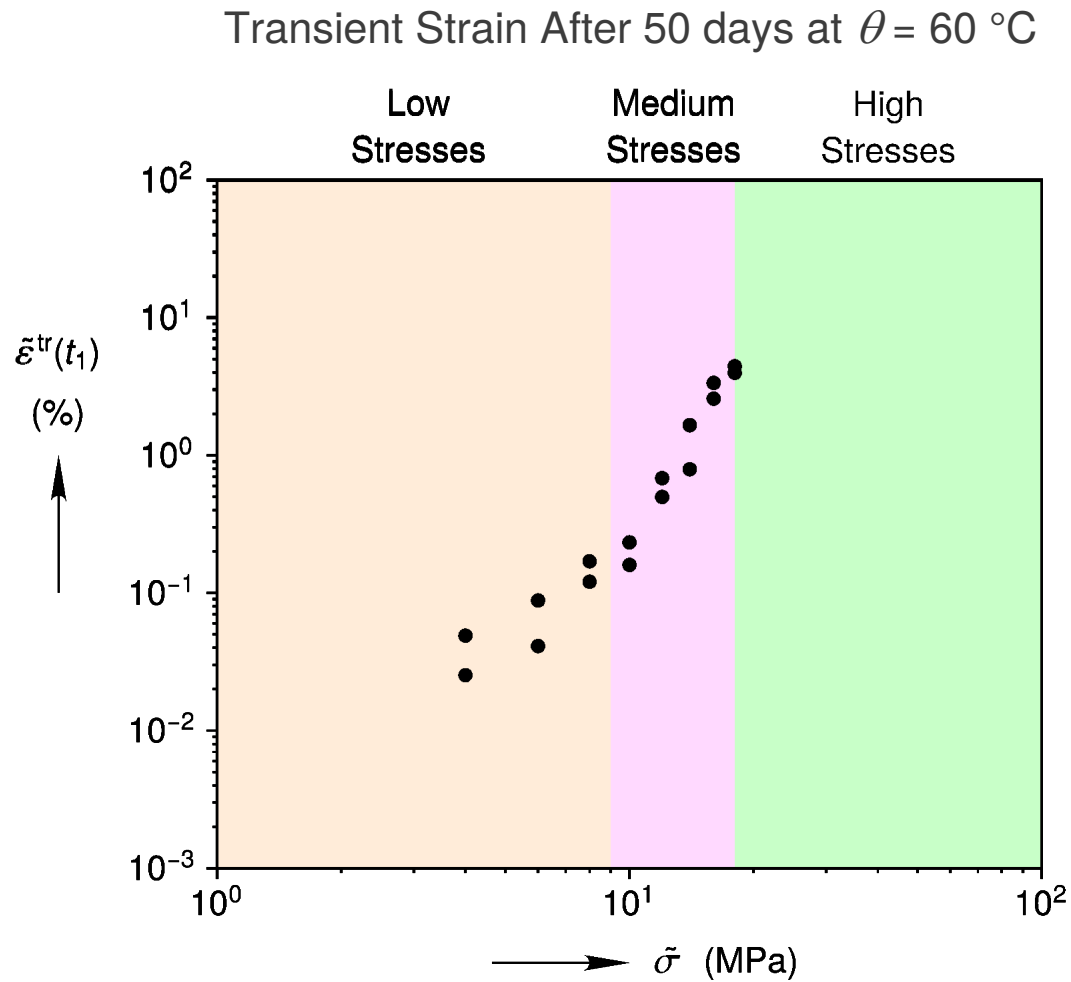
$\tilde{\sigma} = 12 \text{ MPa}$ ,  $\theta = 60^\circ \text{C}$



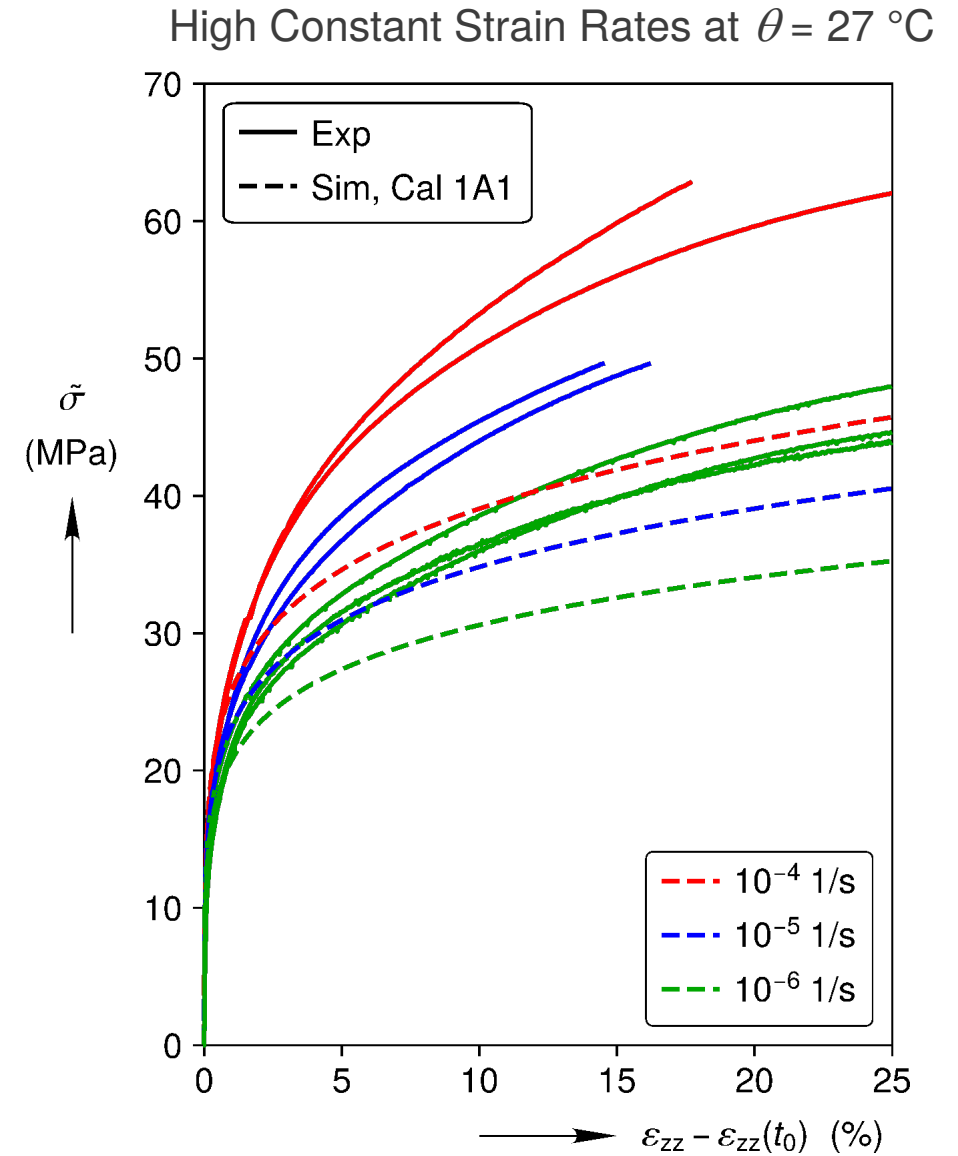
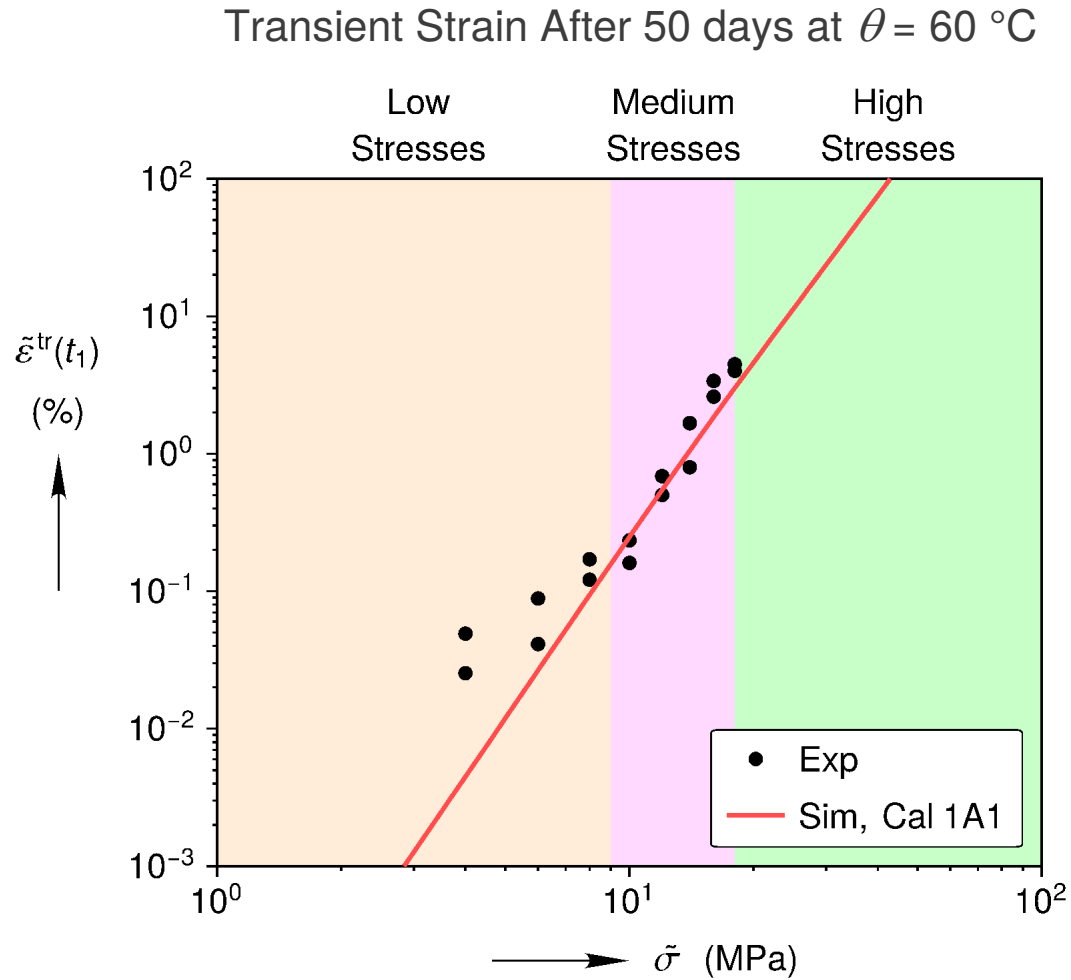
Transient Strain After 50 days at  $\theta = 60^\circ \text{C}$



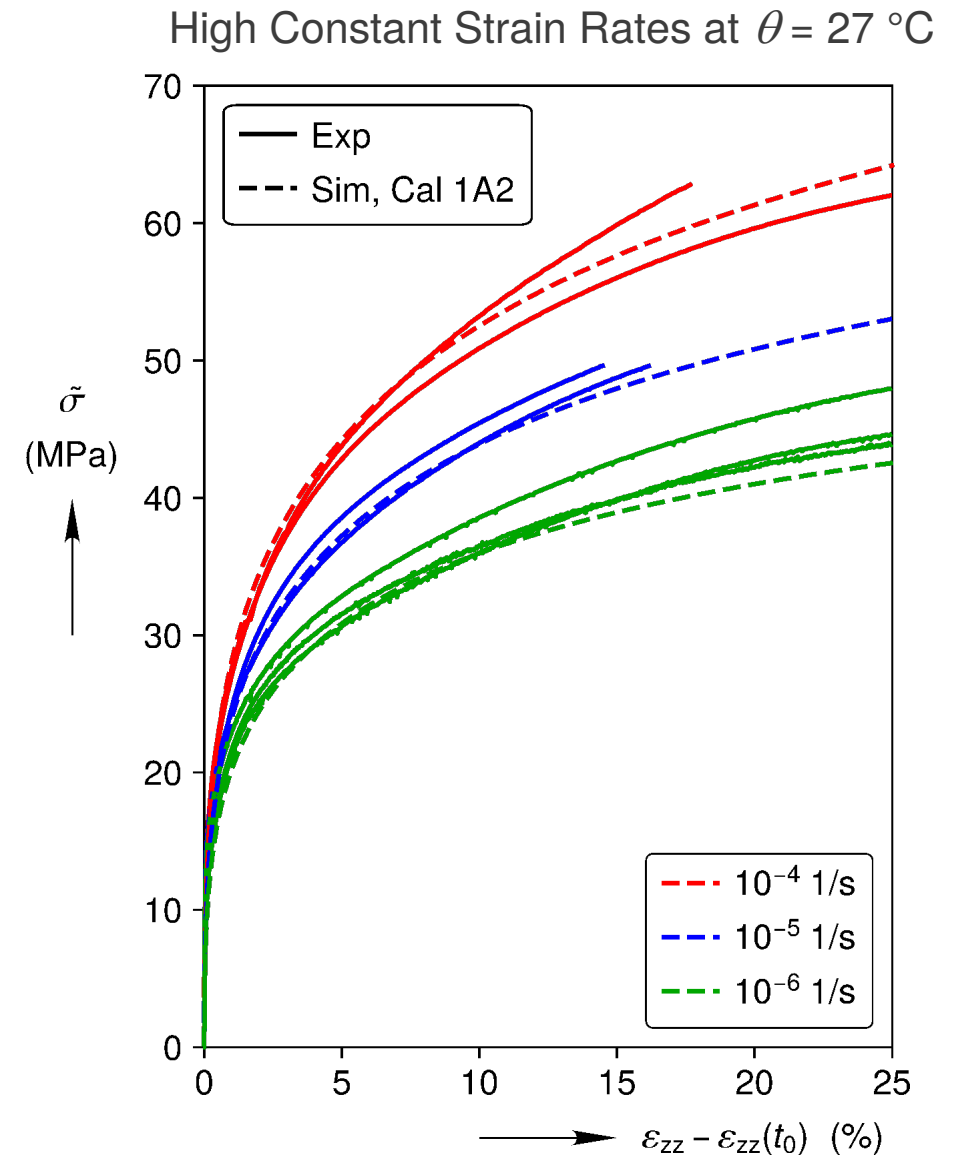
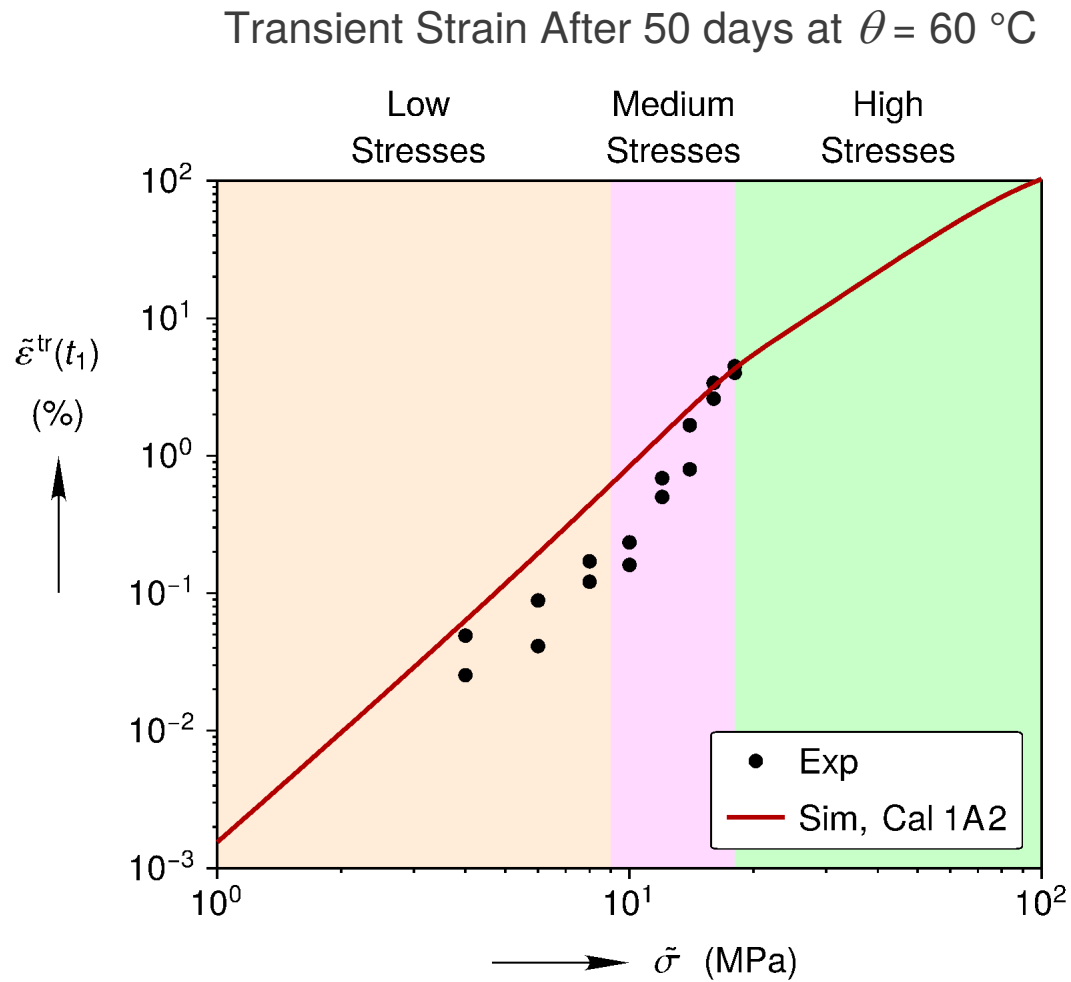
# Selected Hardening Measurements on WIPP Salt



# Calibration 1A1: Drag Stress Only

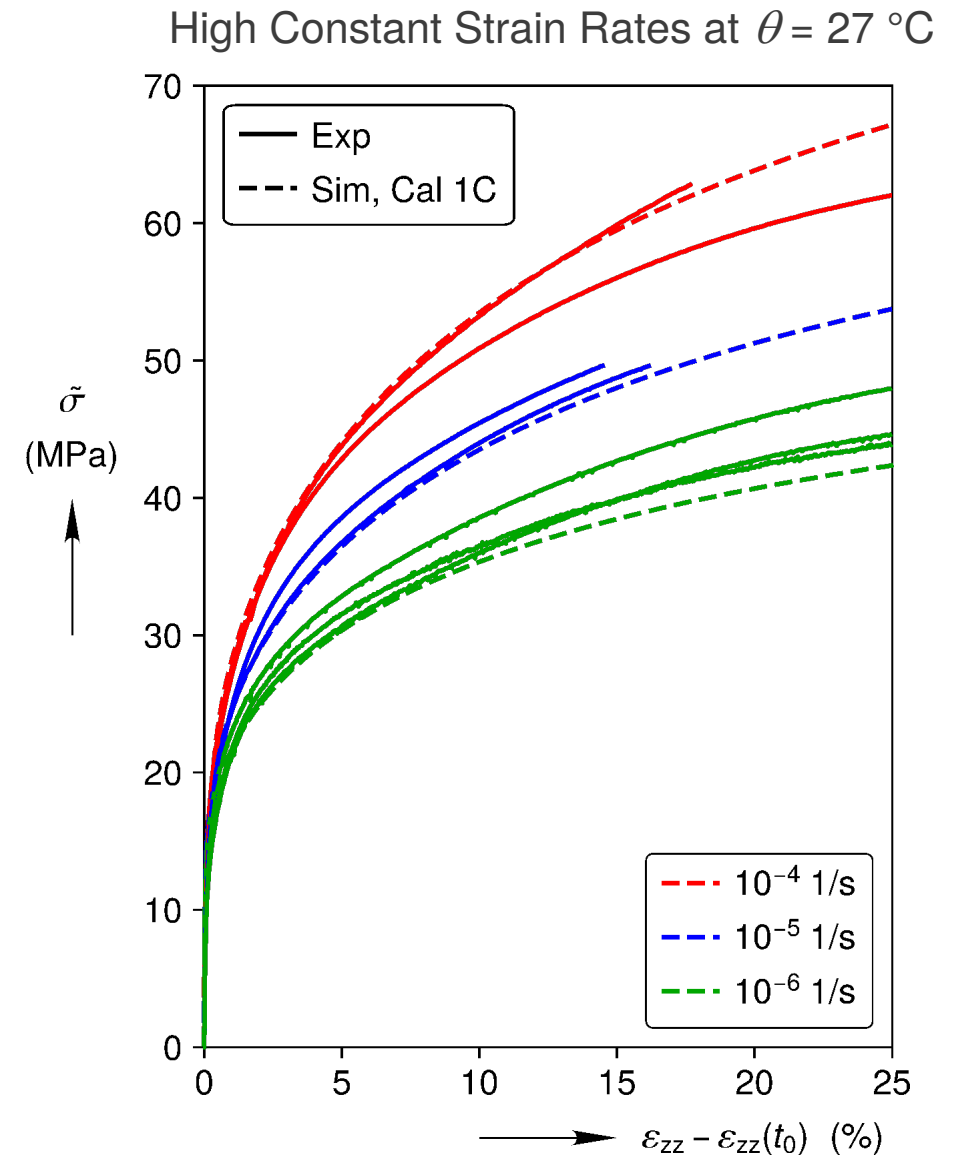
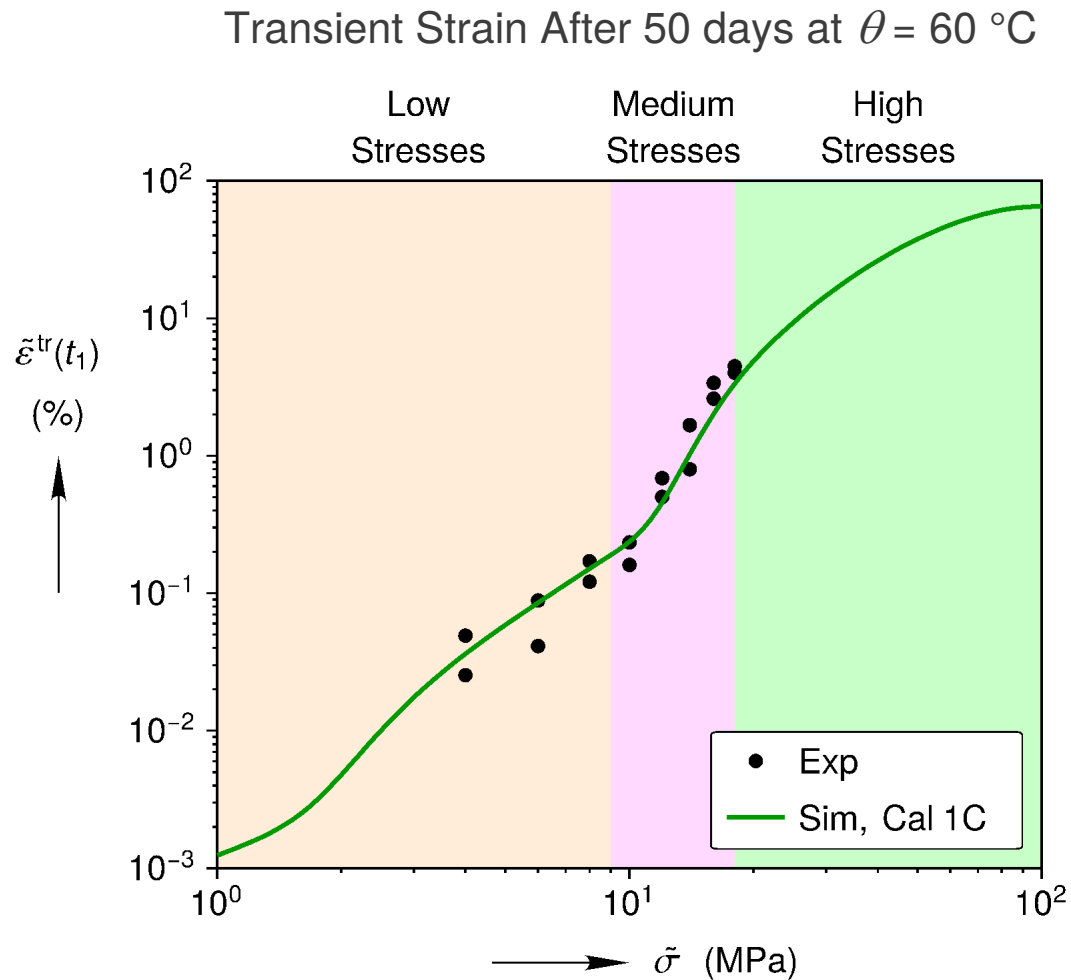


# Calibration 1A2: Drag Stress Only





# Calibration 1C: Drag Stress and Back Stress

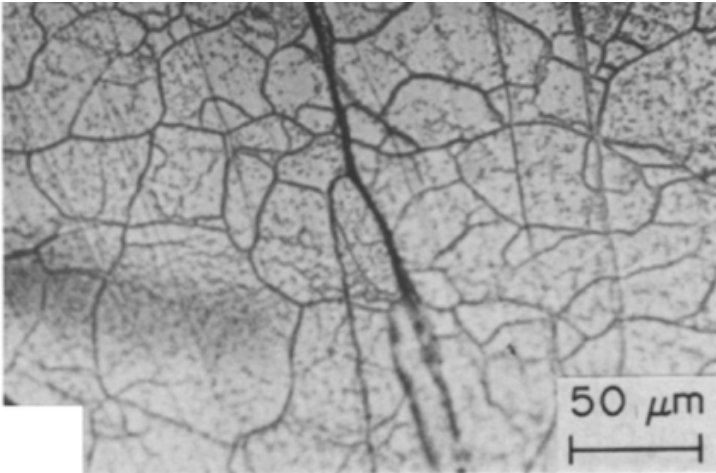


# Non-Monotonic Loading

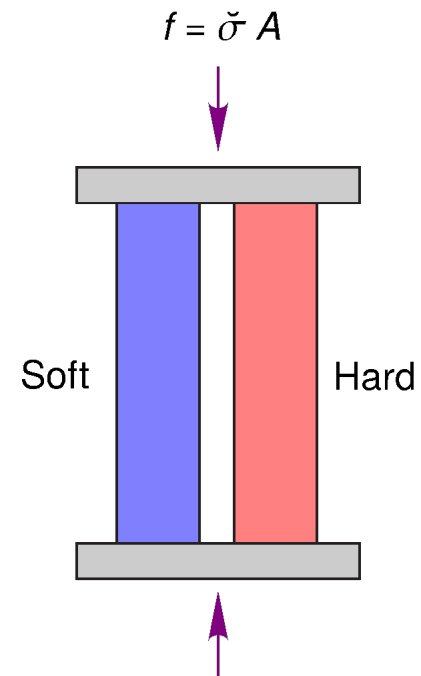
# A “Gedankenexperiment”



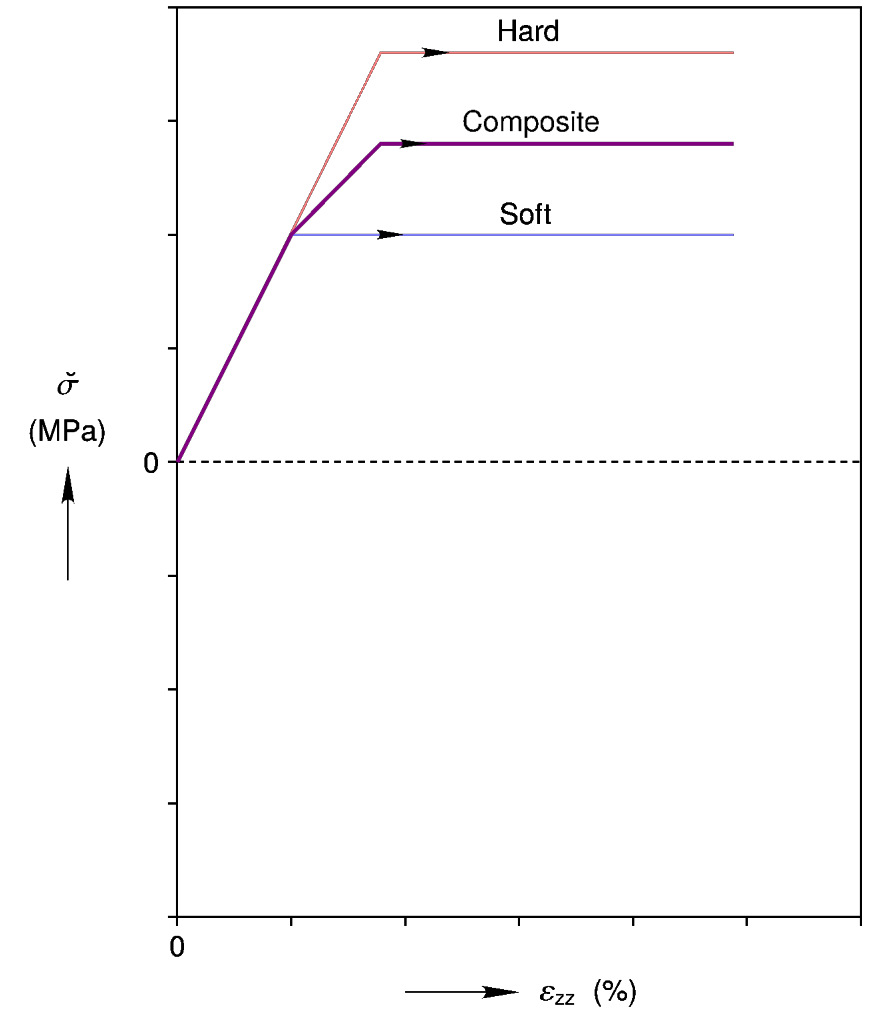
Subgrains



Schematic



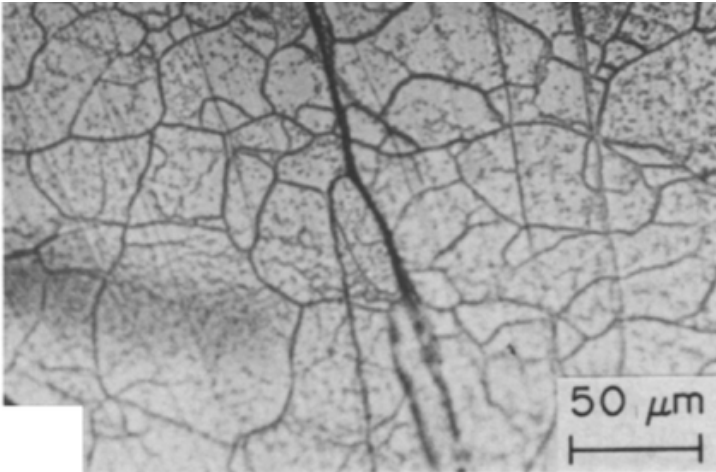
Mechanical Responses



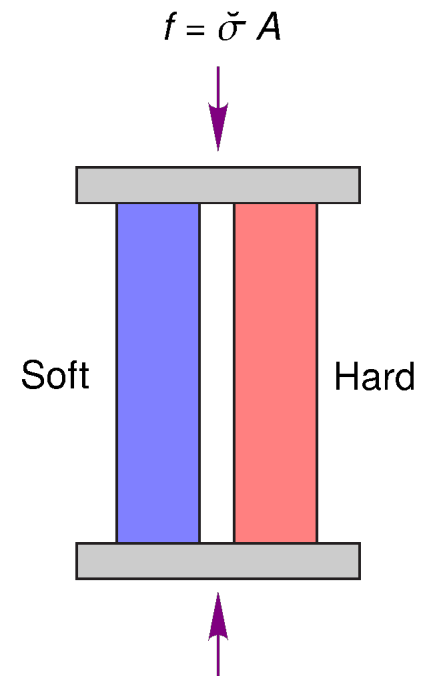
# A “Gedankenexperiment”



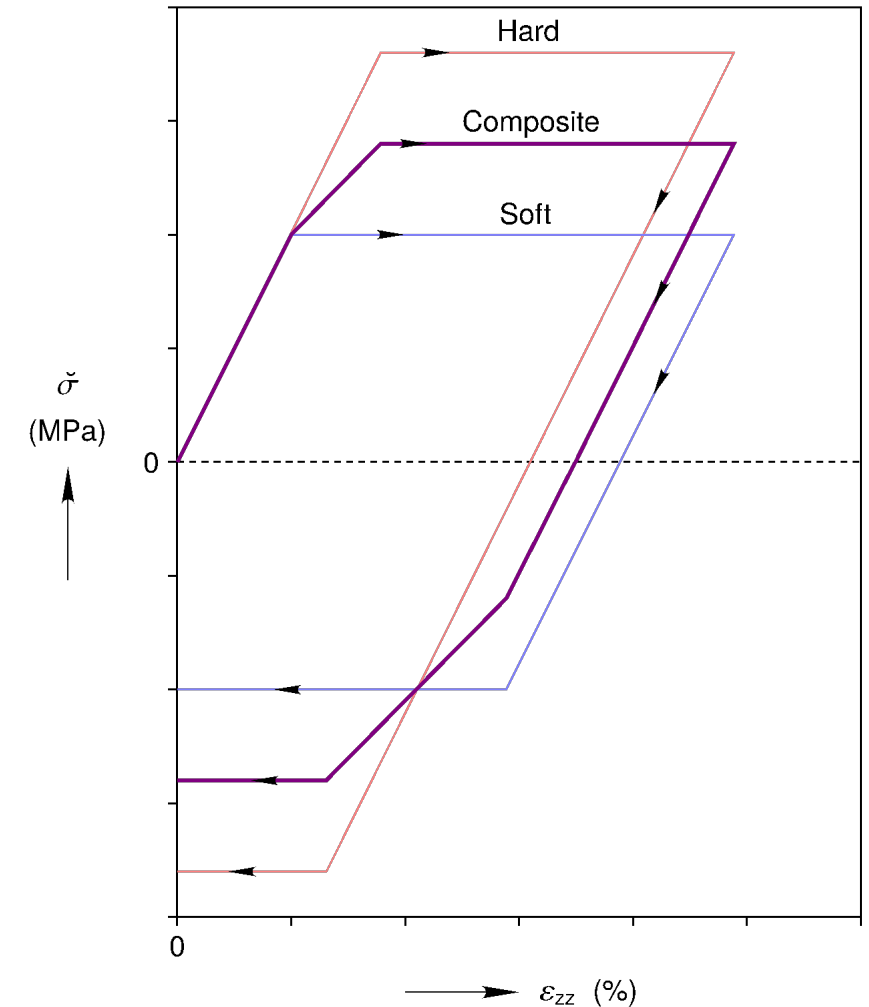
Subgrains



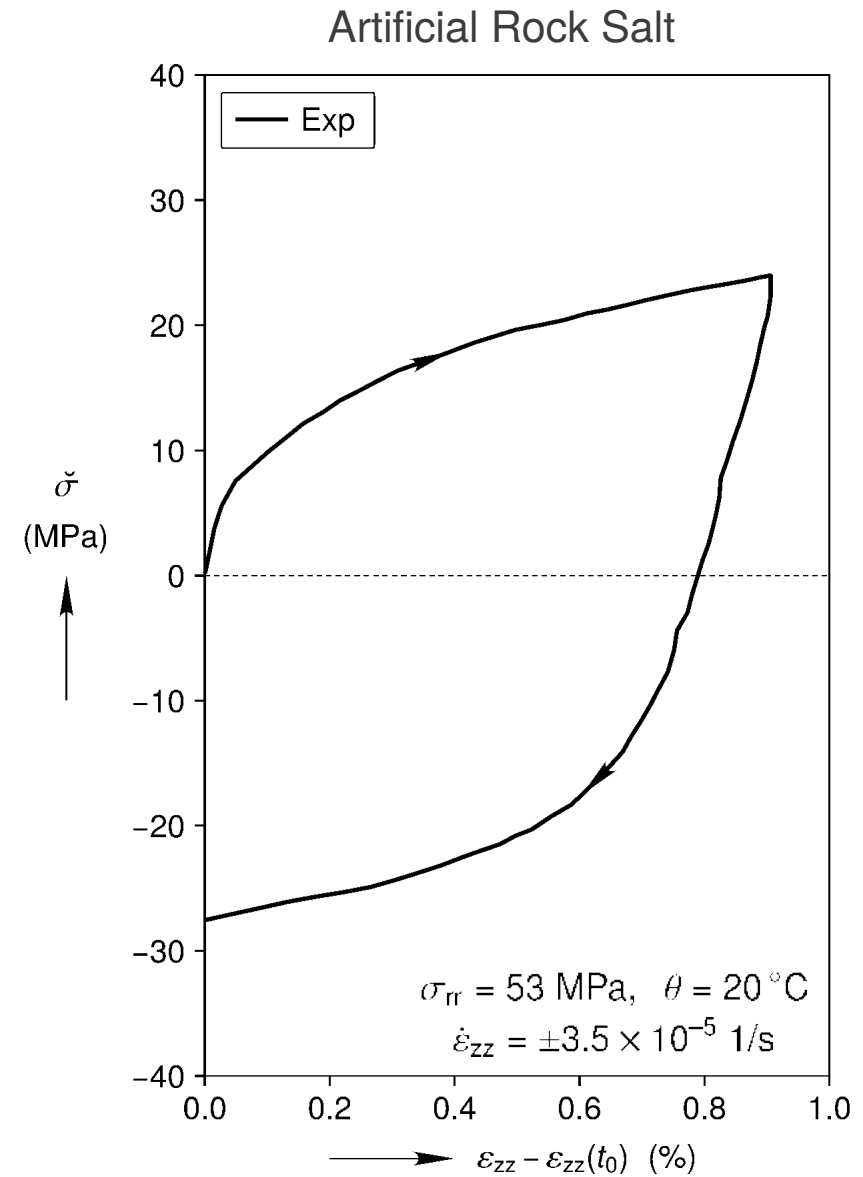
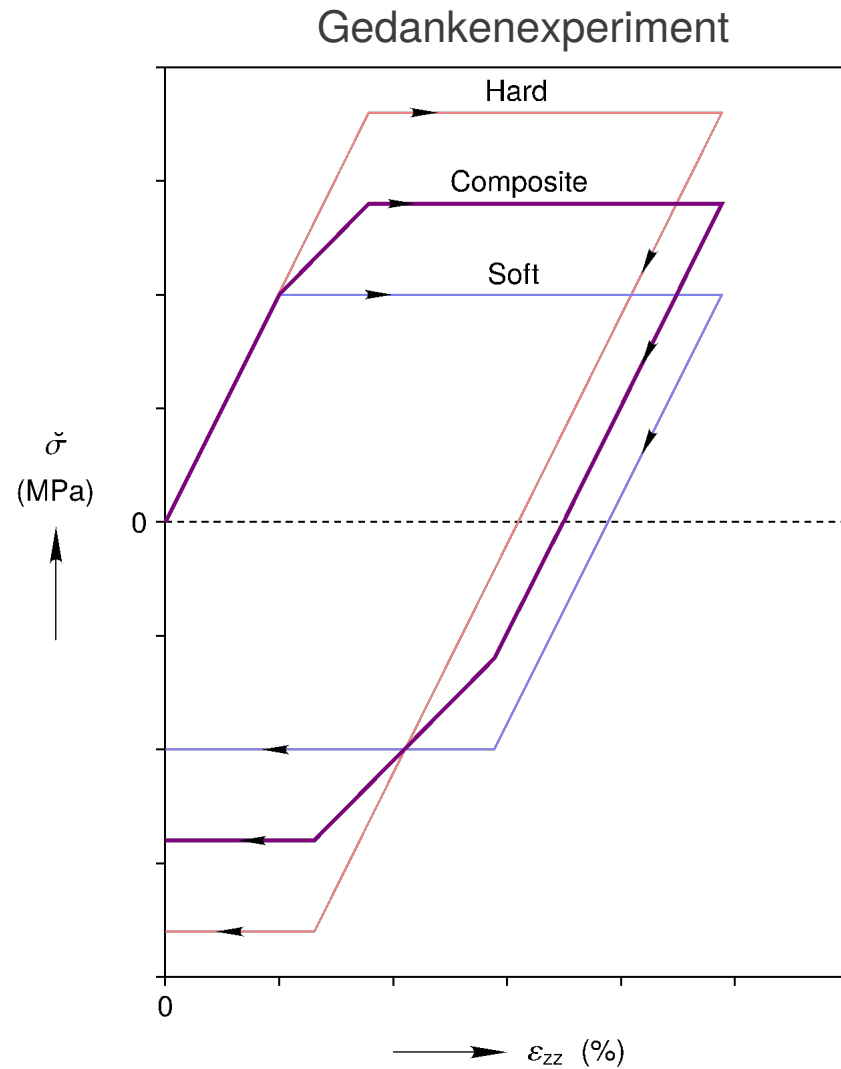
Schematic



Mechanical Responses



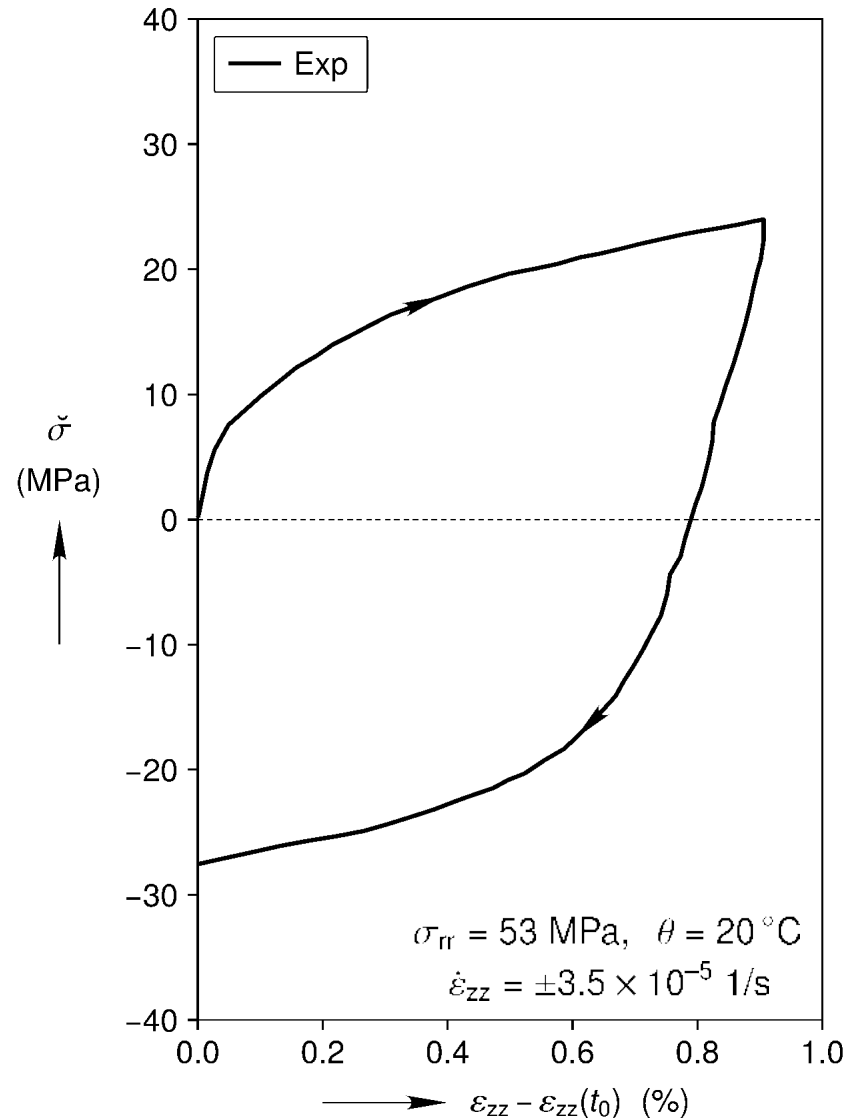
# Bauschinger Effect



# Re-hardening during Non-Monotonic Loading

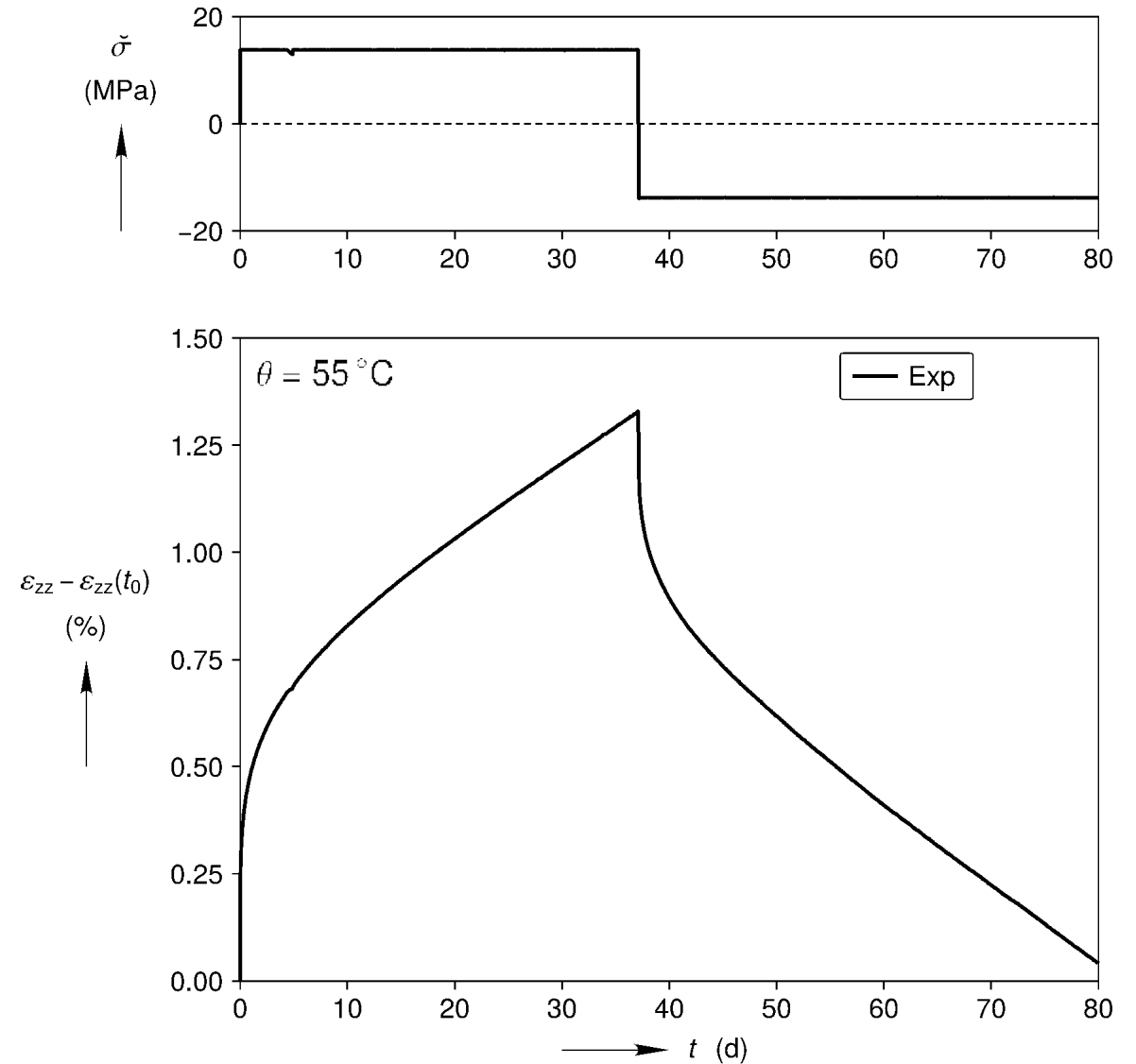


Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

Multi-Stage Constant Stress Test on Cayuta Salt

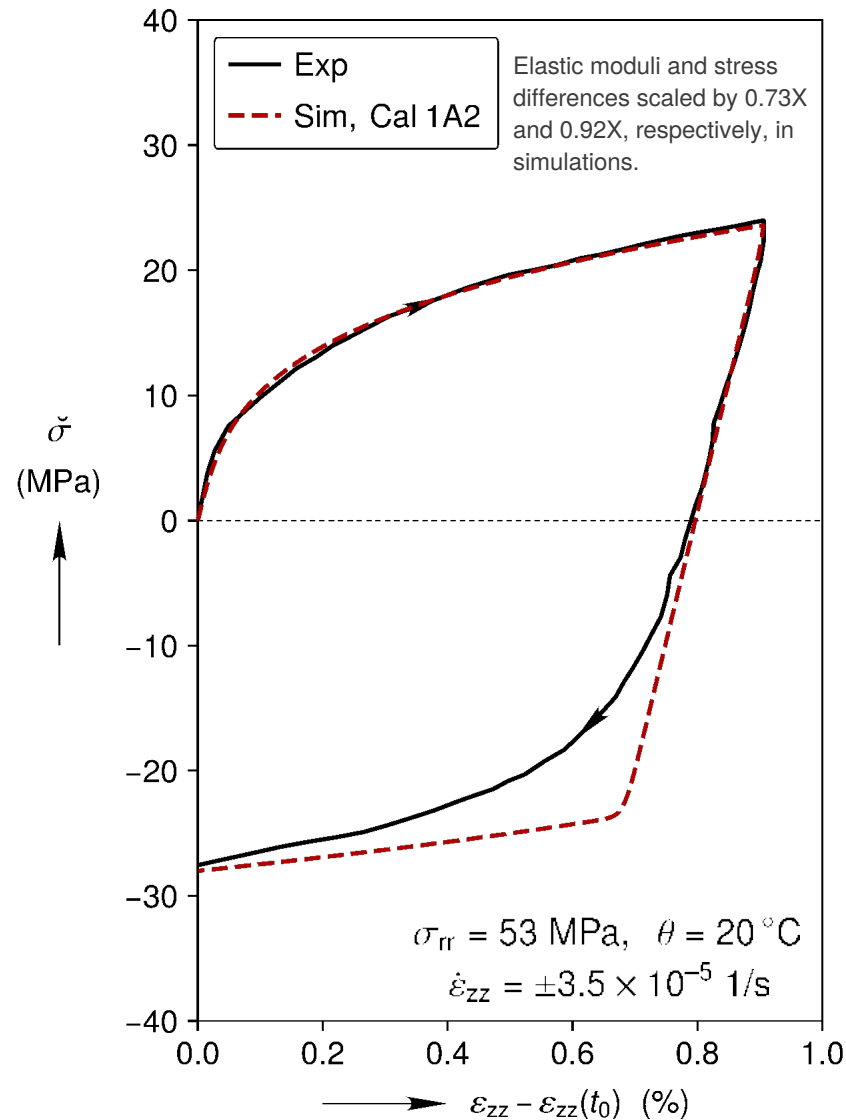


Experimental measurements from: Mellegard et al. (2007)

# Re-hardening during Non-Monotonic Loading

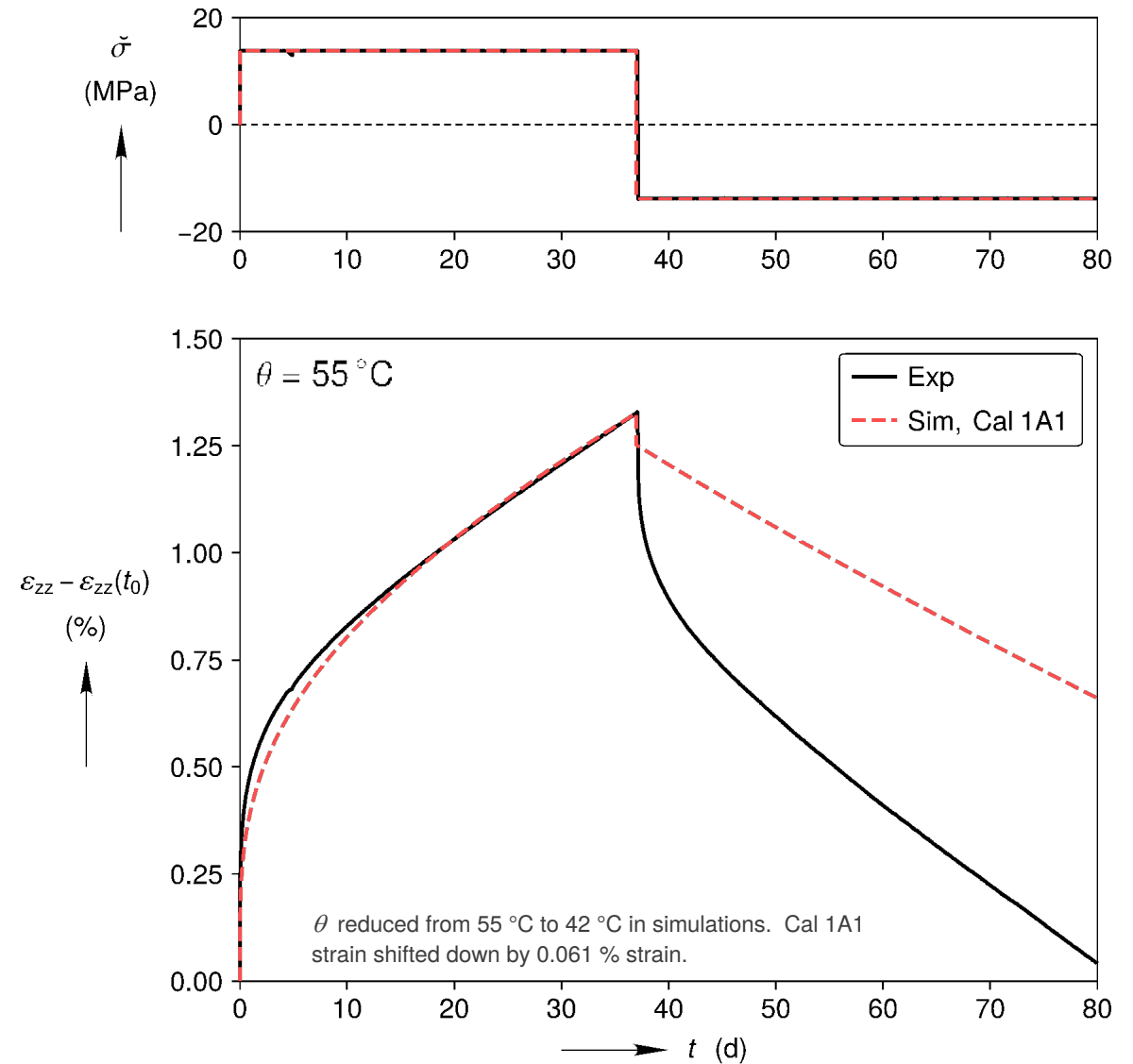


## Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

## Multi-Stage Constant Stress Test on Cayuta Salt

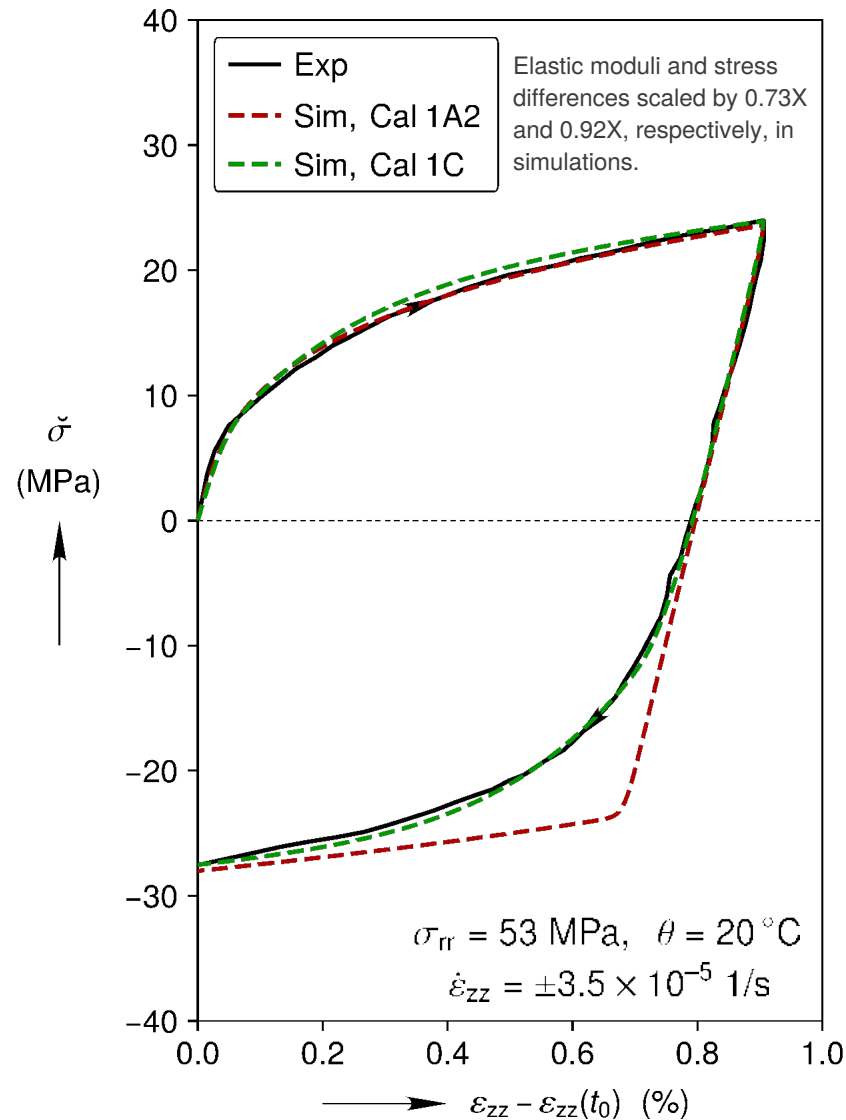


Experimental measurements from: Mellegard et al. (2007)

# Re-hardening during Non-Monotonic Loading

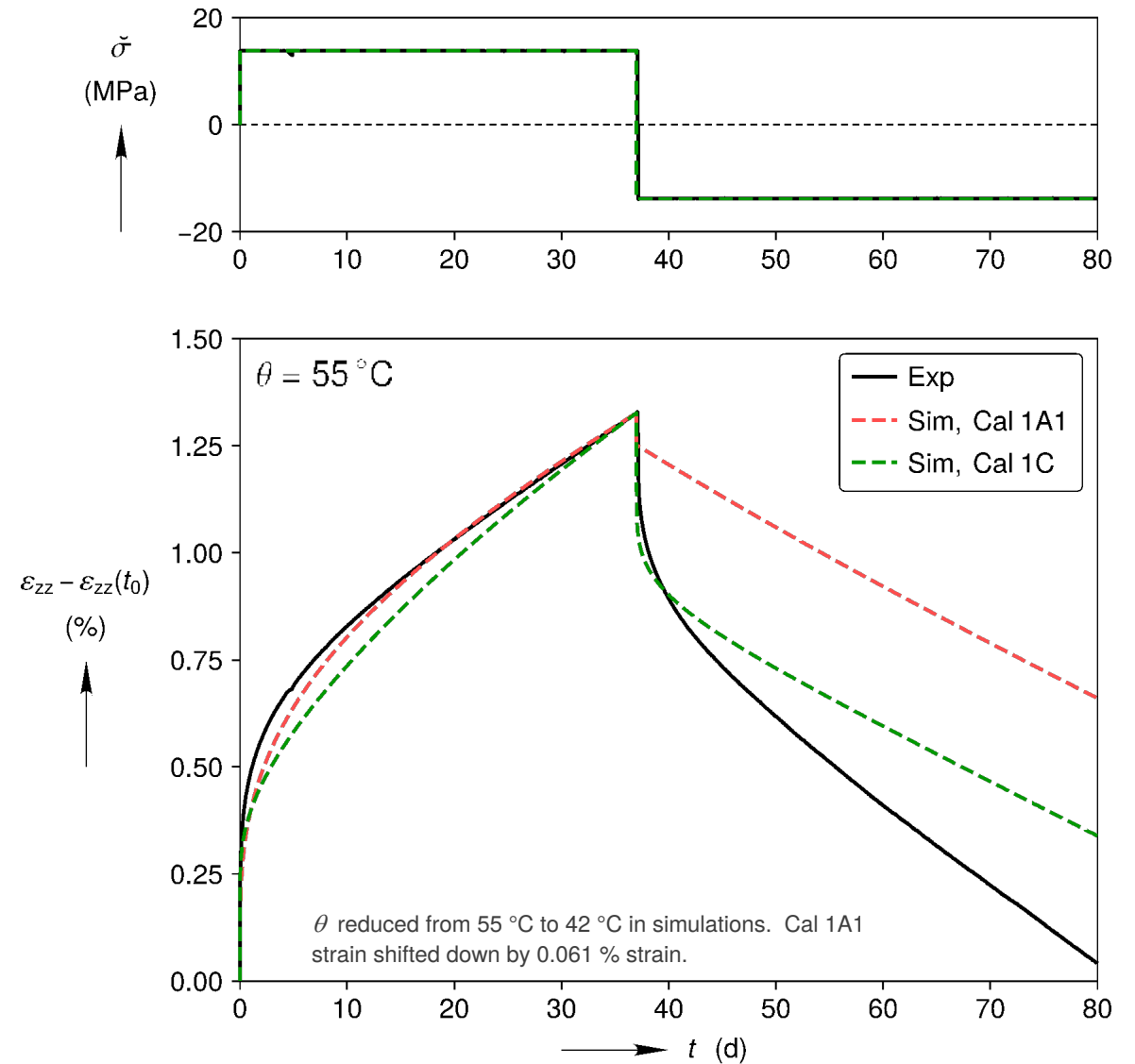


## Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

## Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)



# Summary & Future Work



## 1. Summary

1. Largely phenomenological model, but key decisions were motivated by micro-physical observations.
2. Pressure solution and dislocation glide branches
3. Combined drag and back stress hardening enables one to capture hardening at low, medium, and high strain rates, as well as re-hardening behavior after non-monotonic loading.

## 2. Future work

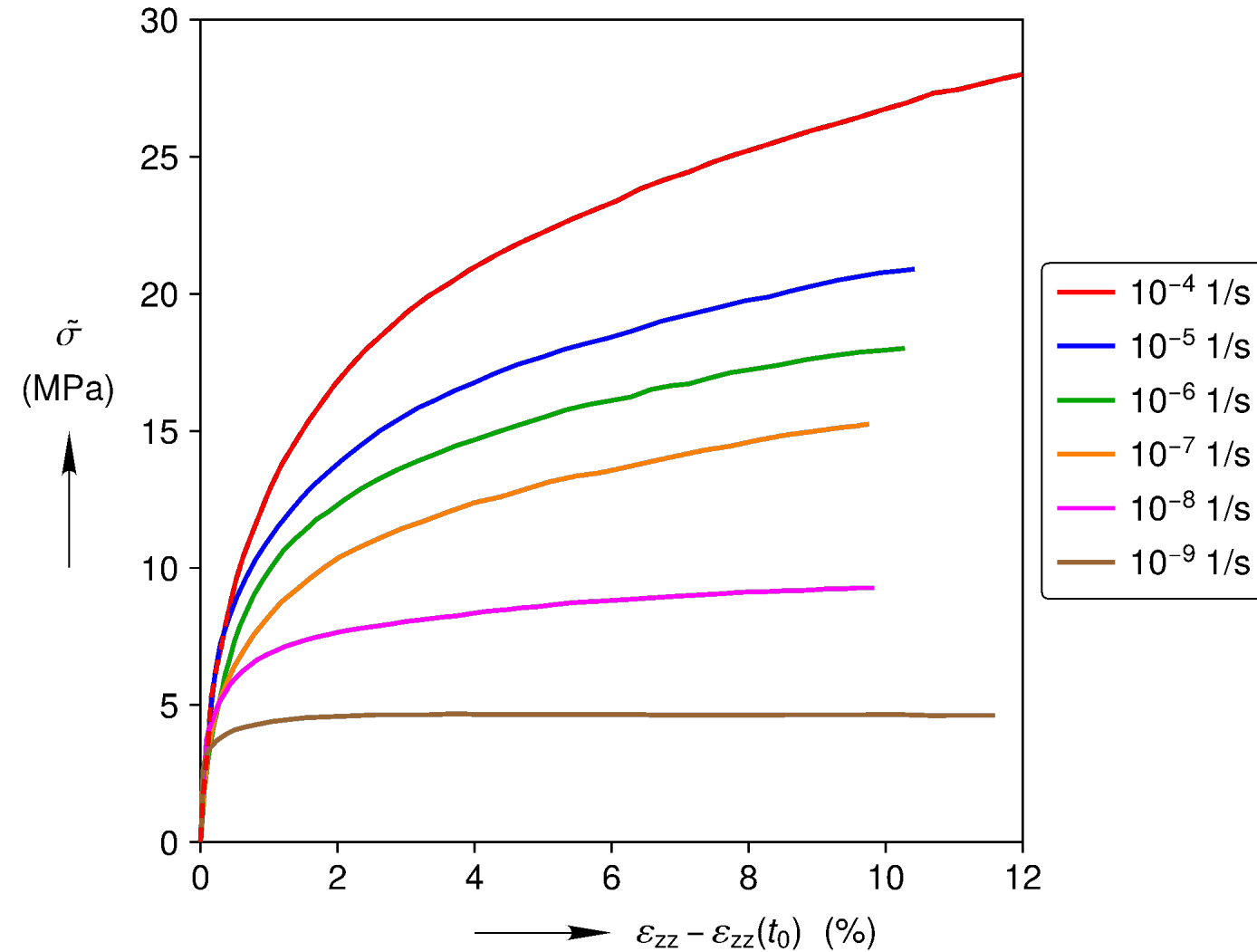
1. Polish numerical implementation
2. Simulate more underground structures
3. Add damage and healing

# Extra Slides

# Hardening Transition from Medium to High Strain Rates (Stresses)



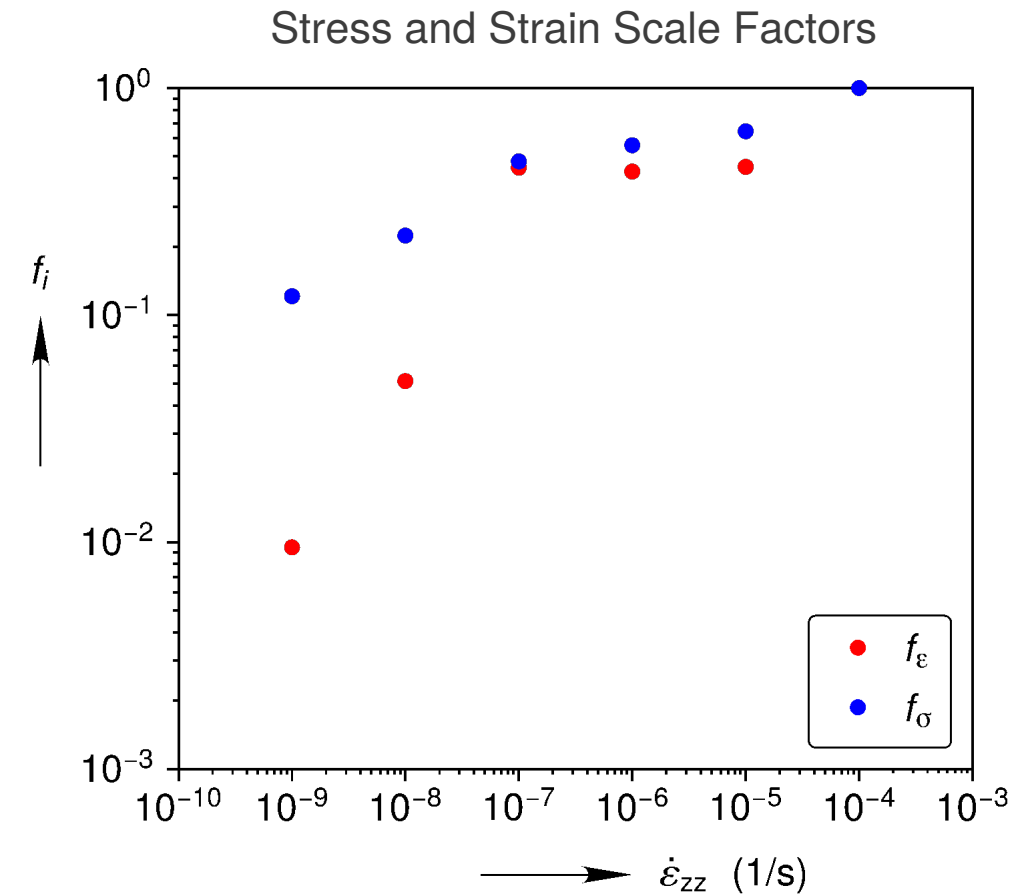
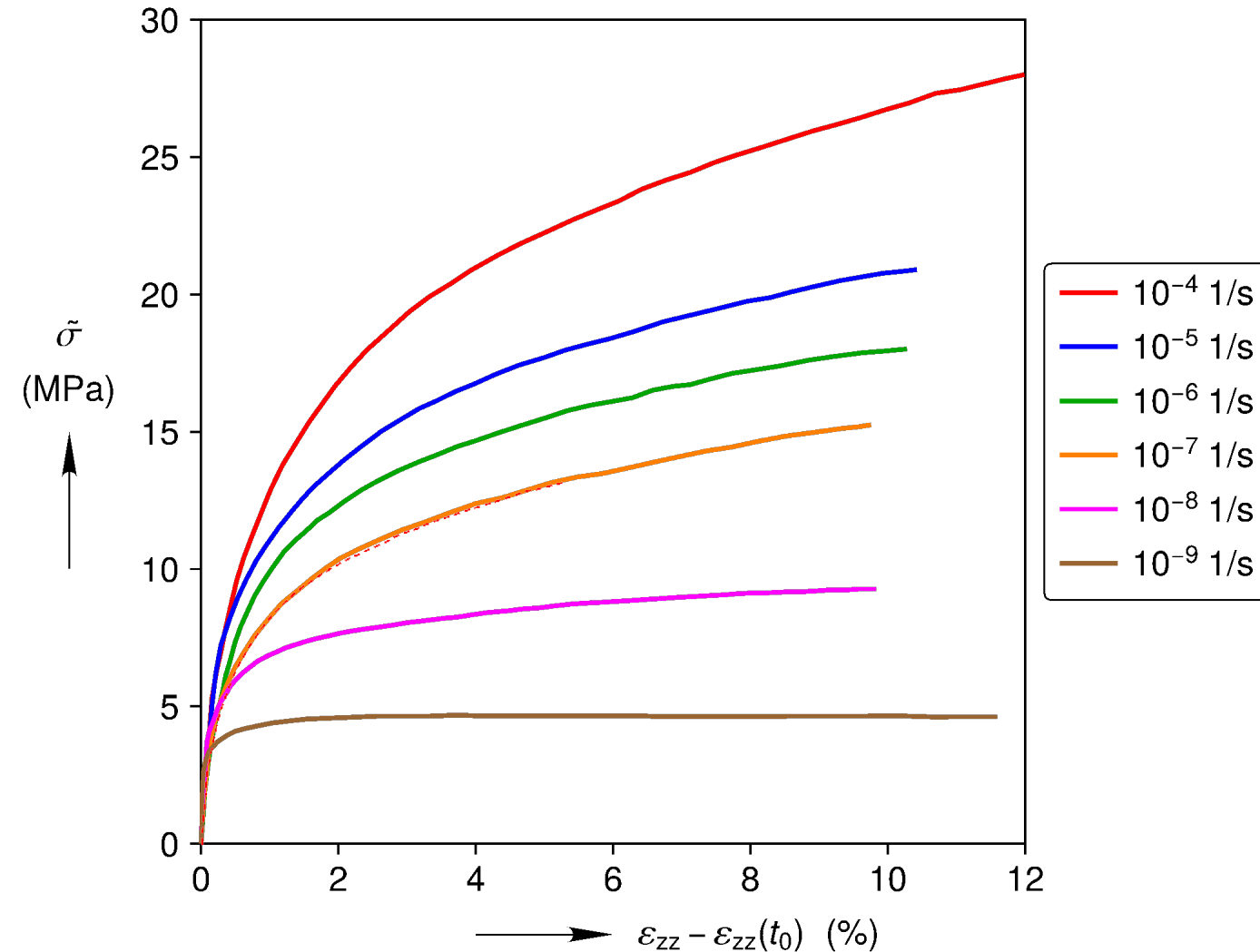
Avery Island Salt, Stress vs. Strain Curves at  $\theta = 100^\circ\text{C}$



# Hardening Transition from Medium to High Strain Rates (Stresses)



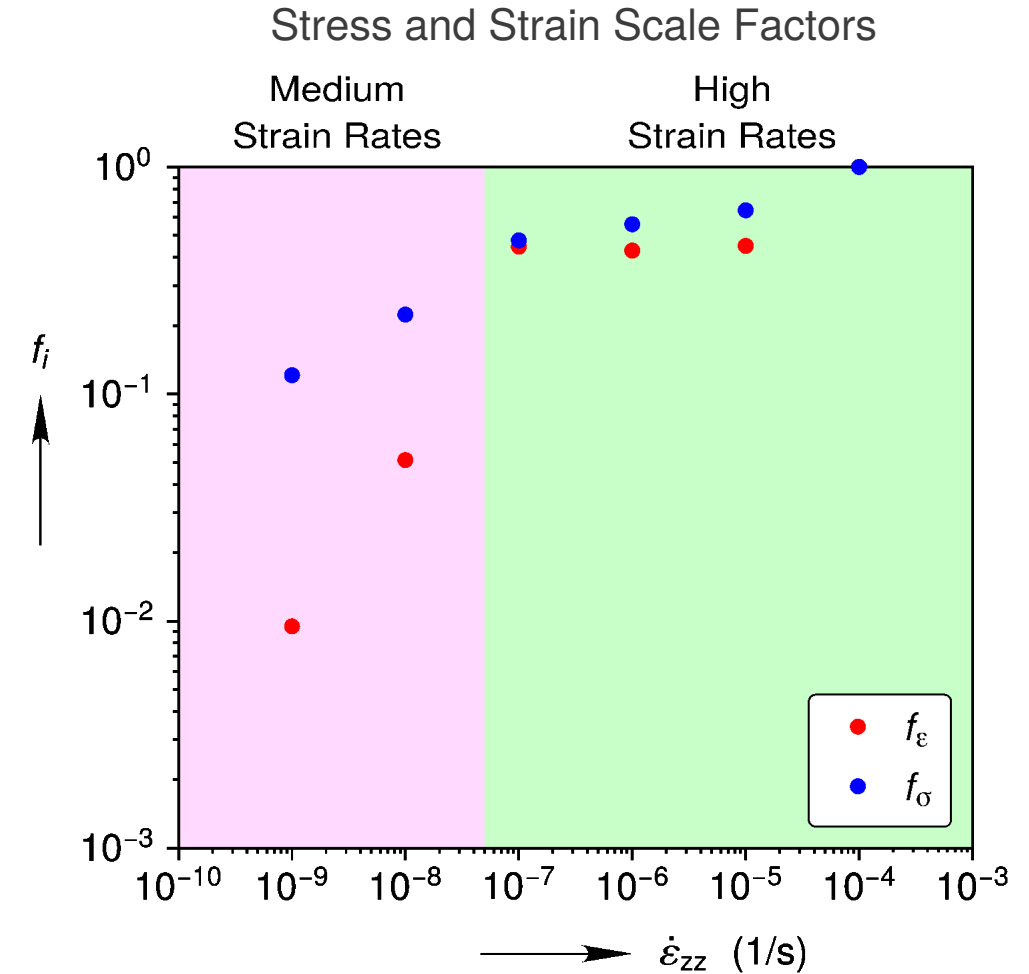
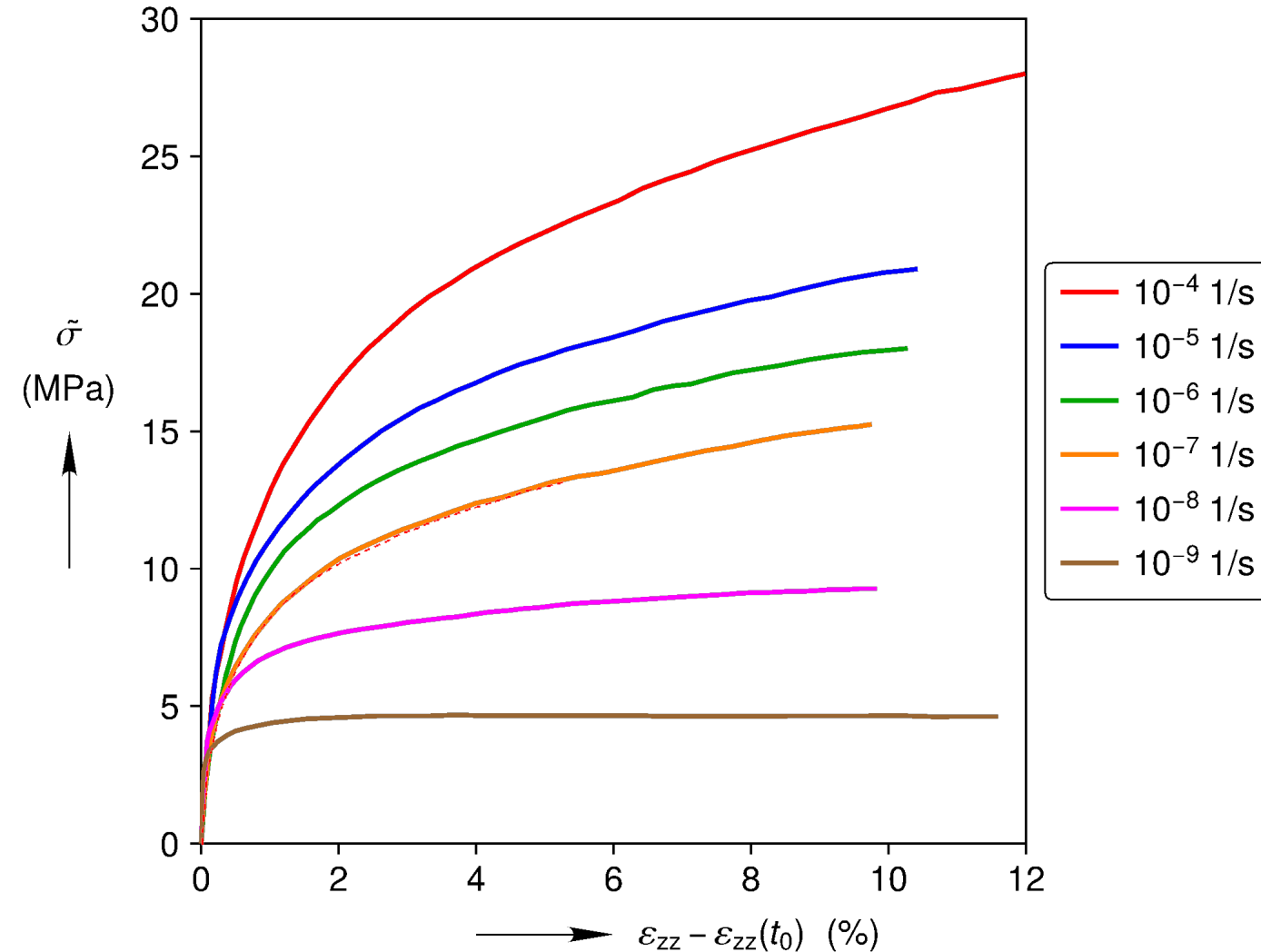
Avery Island Salt, Stress vs. Strain Curves at  $\theta = 100^\circ\text{C}$



# Hardening Transition from Medium to High Strain Rates (Stresses)



Avery Island Salt, Stress vs. Strain Curves at  $\theta = 100^\circ\text{C}$

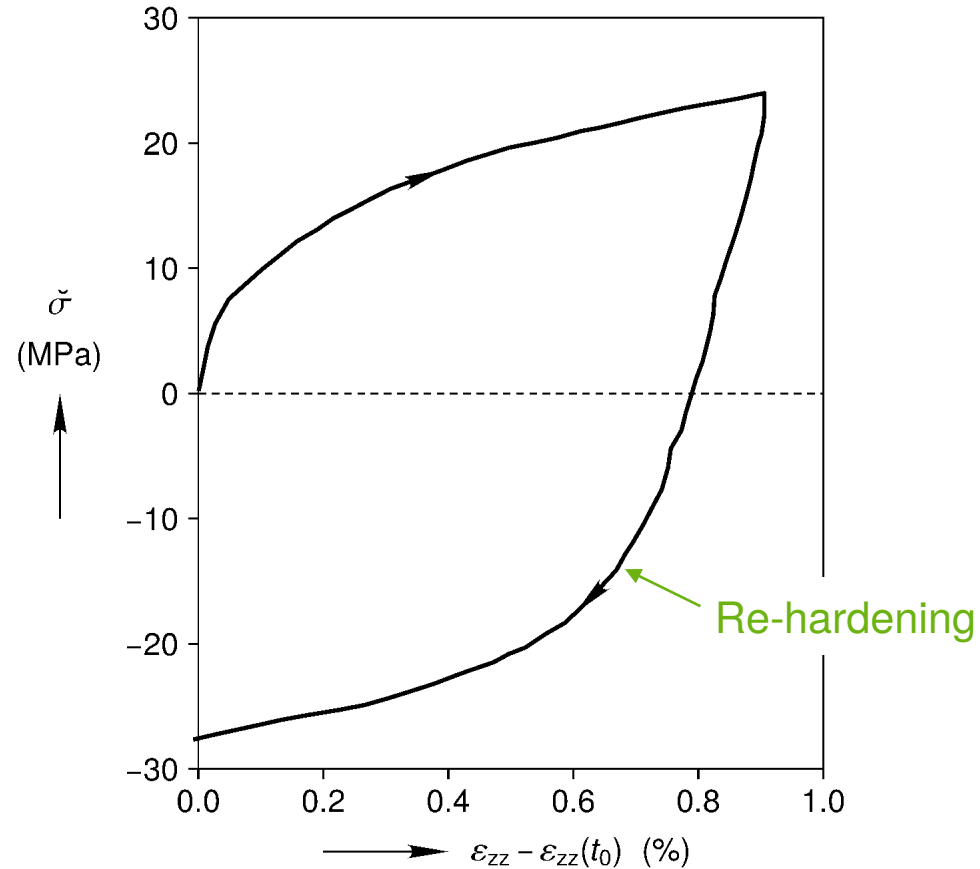


# Re-hardening During Non-Monotonic Loading



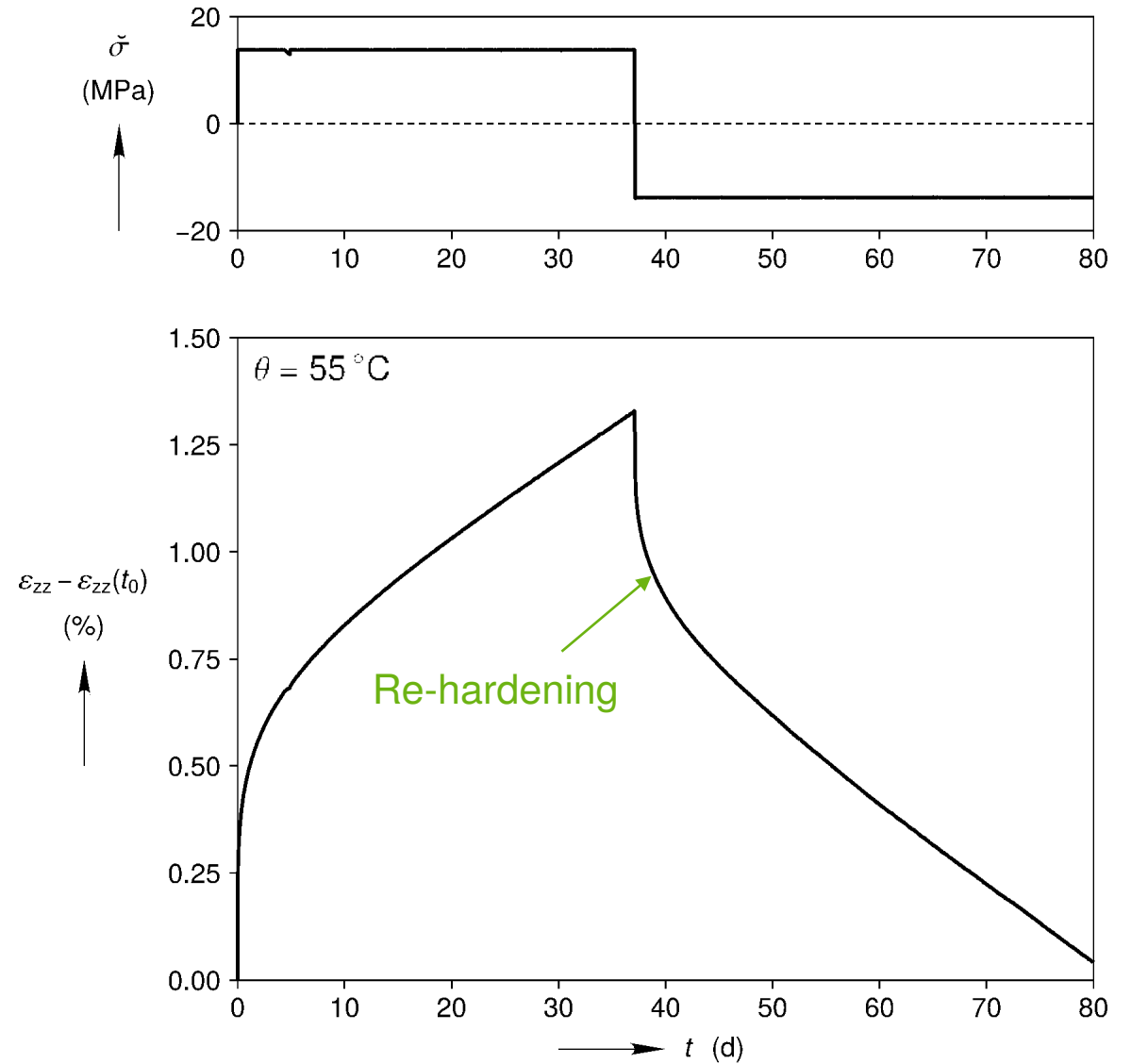
Artificial Salt, Stress vs. Strain Curve

$$\sigma_{rr} = 53 \text{ MPa}, \theta = 293 \text{ K}, \dot{\varepsilon}_{zz} = \pm 3.5 \times 10^{-5} \text{ 1/s}$$



Experimental measurements from Aubertin et al. (1999)

Cayuta Salt, Stress and Strain Histories



Experimental measurements from: Mellegard et al. (2007)

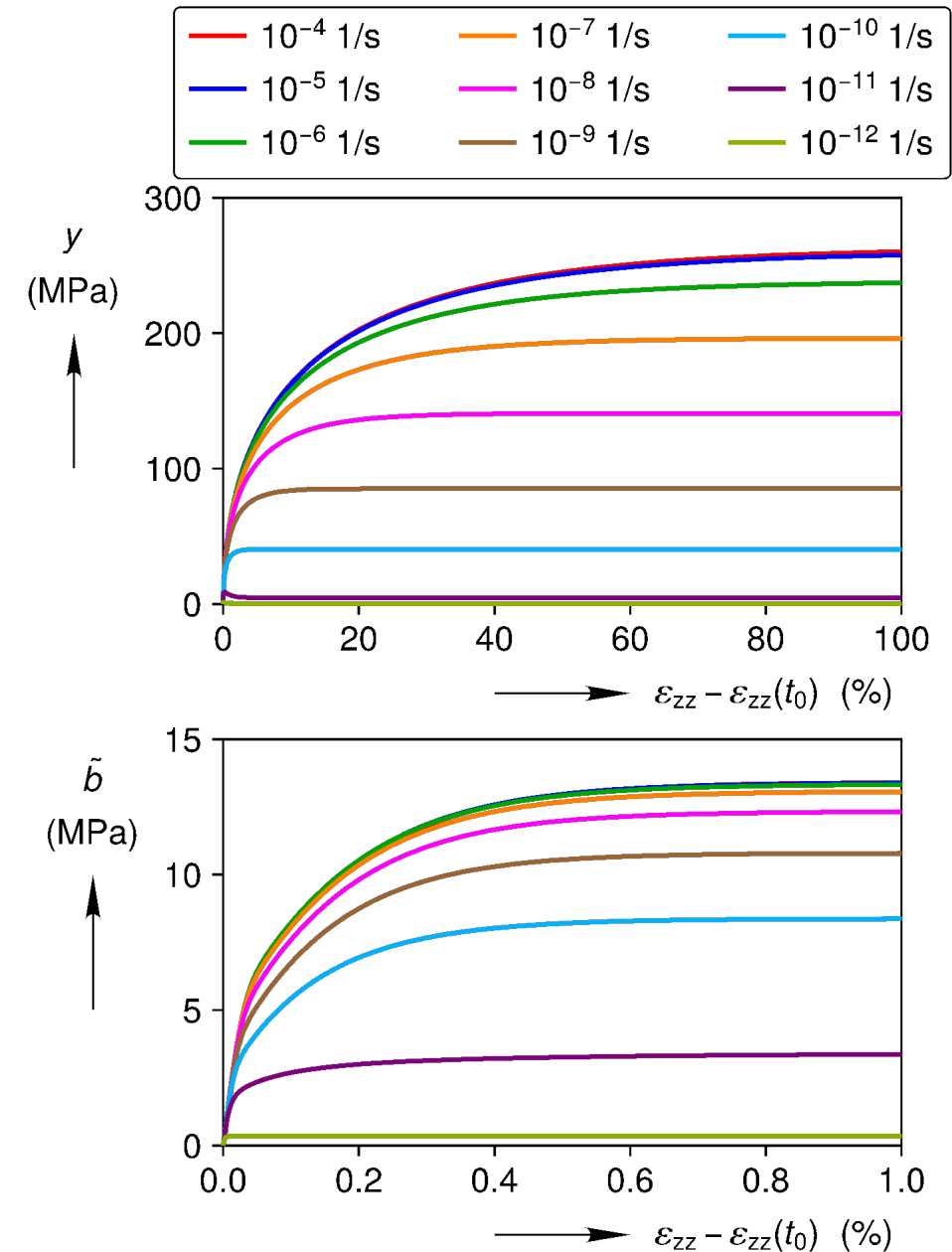
# Dislocation Glide Hardening

Drag Stress Evolution Equation

$$\dot{y} = Y_1 \left( \frac{Y_1}{y} \right)^{Y_2} \left( 1 - \frac{y}{\bar{y}} \right) \dot{\varepsilon}^{\text{dg}}$$

Back Stress Evolution Equation  
(proportional, monotonic, loading)

$$\dot{\tilde{b}} = B_1 \left( 1 - \frac{\tilde{b}}{\bar{b}} \right) \dot{\varepsilon}^{\text{dg}}$$





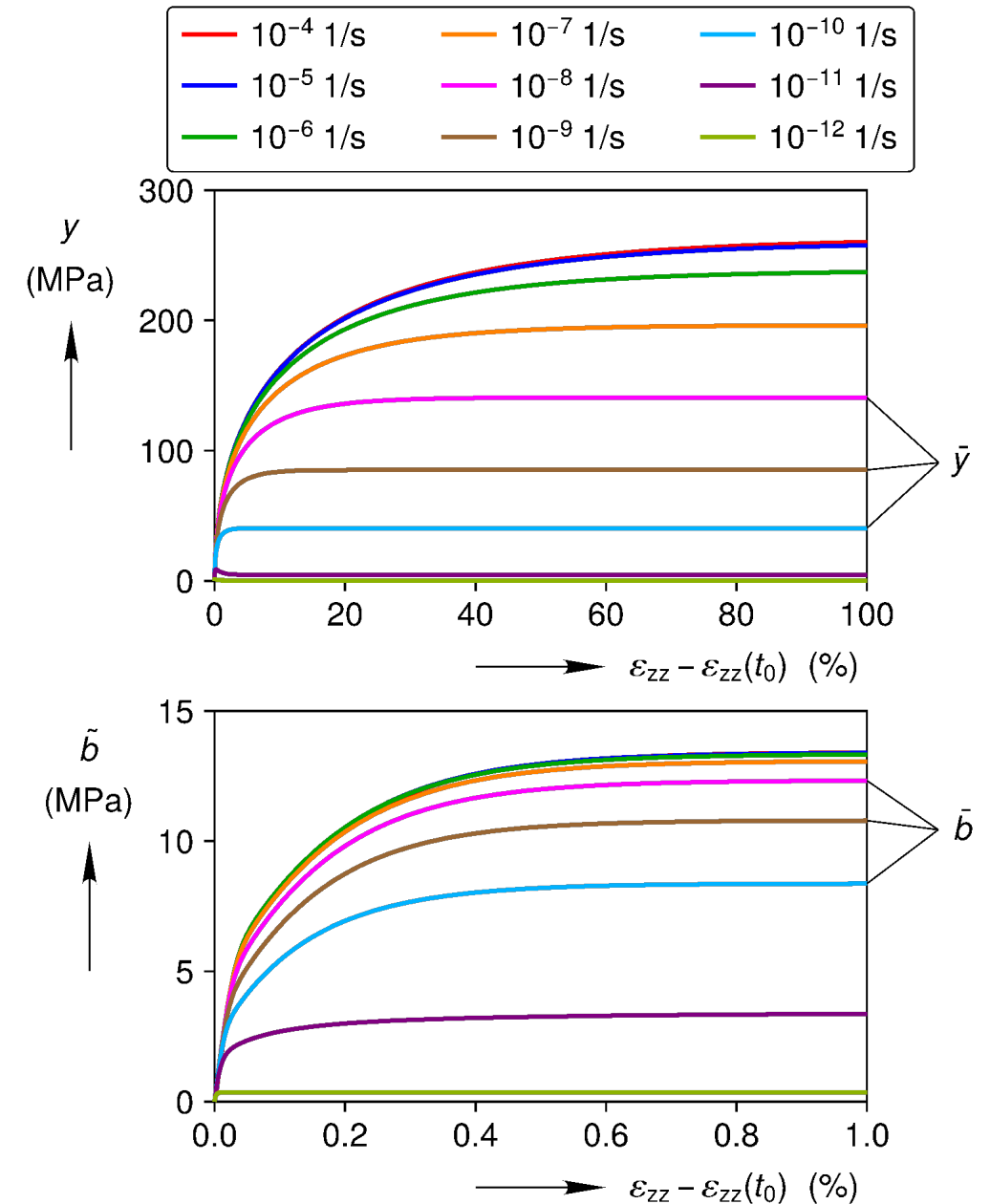
# Dislocation Glide Hardening

Drag Stress Evolution Equation

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Back Stress Evolution Equation  
(proportional, monotonic, loading)

$$\dot{\tilde{b}} = B_1 \left( 1 - \frac{\tilde{b}}{\bar{b}} \right) \dot{\epsilon}^{\text{dg}}$$

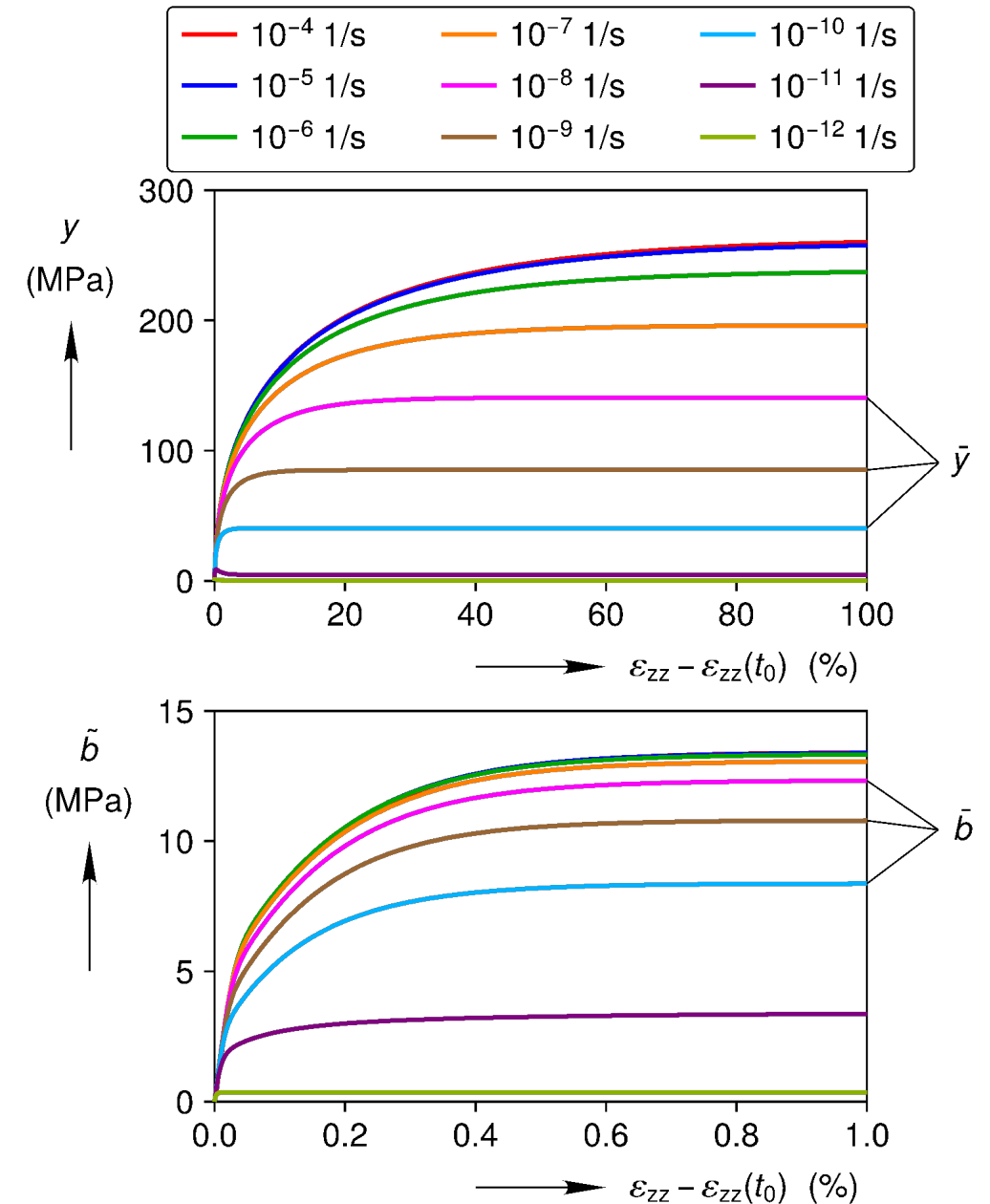


# Dislocation Glide Hardening

Strain Rates  
(proportional, monotonic, loading)

$$\dot{\varepsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

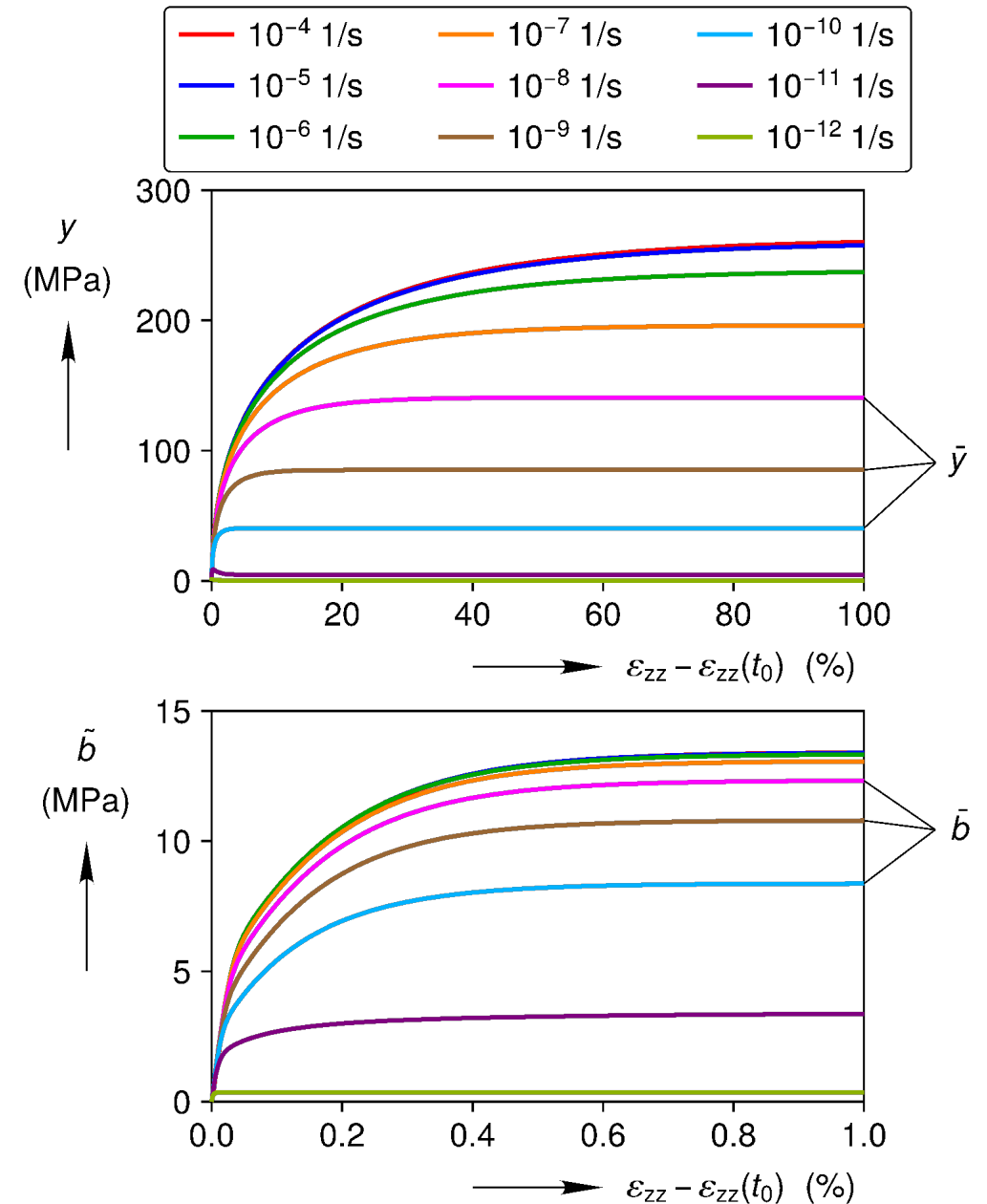


# Dislocation Glide Steady-State

Steady-State Strain Rates

$$\dot{\varepsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma} - \bar{b}}{\bar{y}}\right) \right]^{G_3}$$

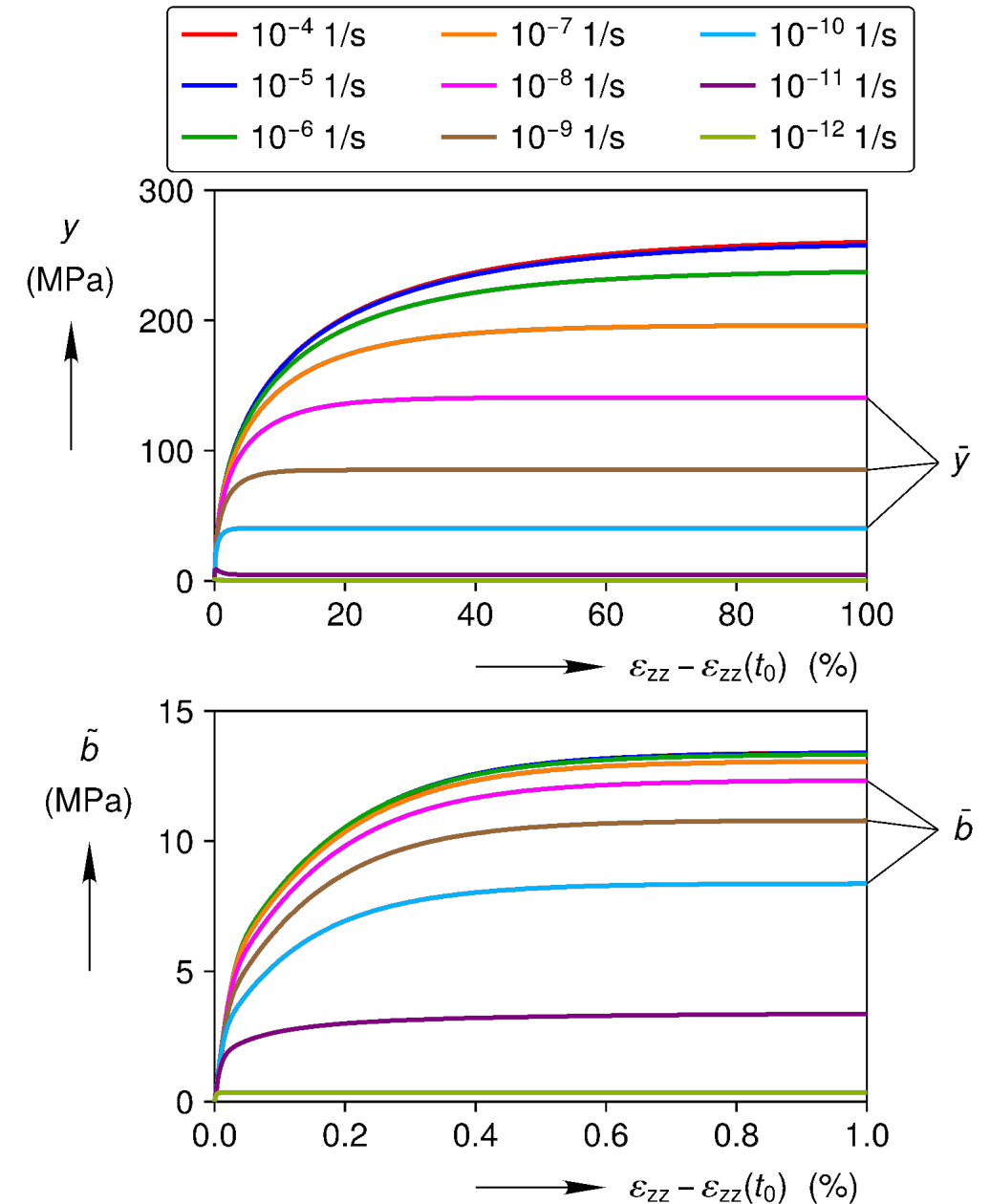


# Dislocation Glide Steady-State

Steady-State Strain Rates

$$\dot{\epsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\epsilon}^{\text{dg}} = Y_3 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{Y_4}\right)\right]^{Y_5}$$



# Dislocation Glide Hardening



Equivalent Dislocation Glide Strain Rate

$$\dot{\epsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma}^{\text{dg}}}{y}\right) \right]^{G_3}$$

# Isotropic Dislocation Glide Hardening



Equivalent Dislocation Glide Strain Rate

$$\dot{\epsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{\tilde{\sigma}}{y}\right) \right]^{G_3}$$

Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$

Flow Rule

$$\dot{\epsilon}^{\text{dg}} = \dot{\epsilon}^{\text{dg}} \frac{\partial f_i}{\partial \sigma}$$

# Isotropic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_i = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

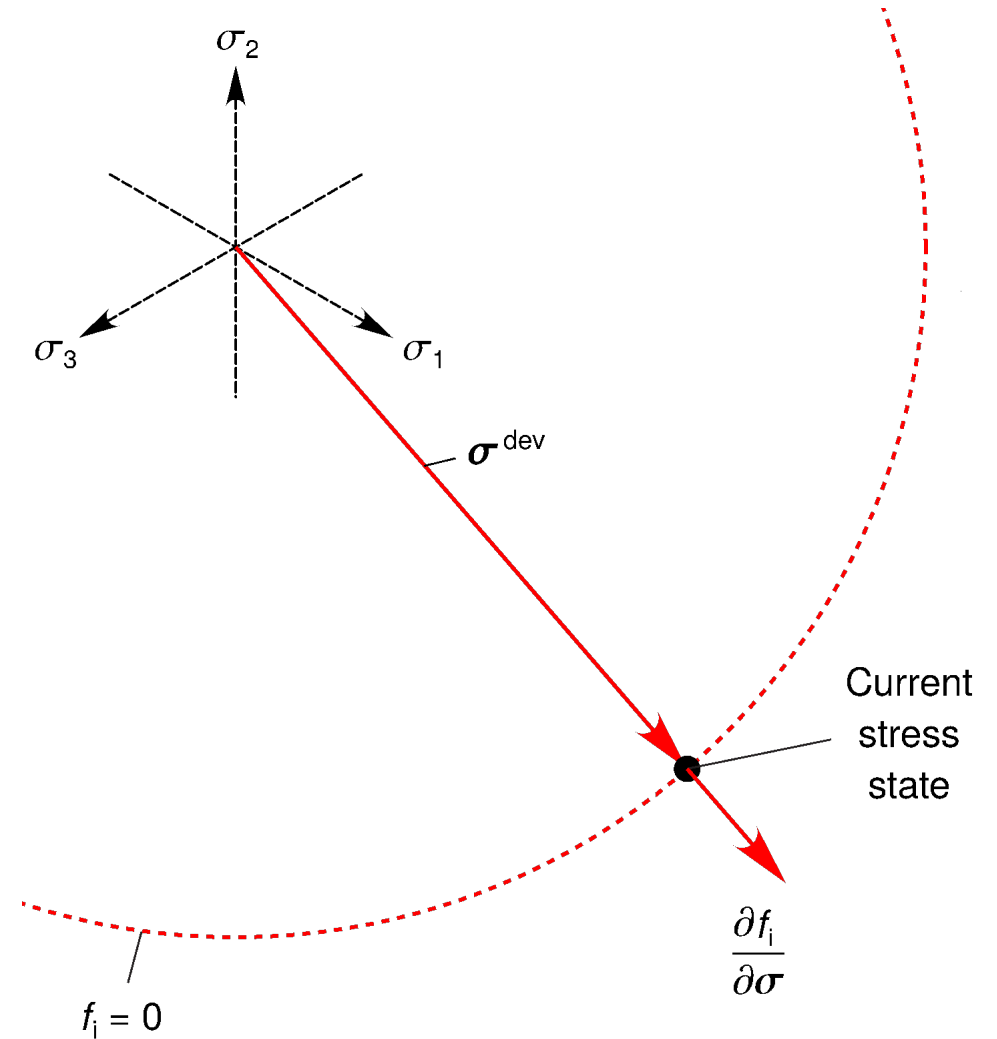
Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$

Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\epsilon}^{dg} \frac{\partial f_i}{\partial \sigma}$$

Deviatoric Plane



# Isotropic Dislocation Glide Hardening



Dynamic Yield Surface

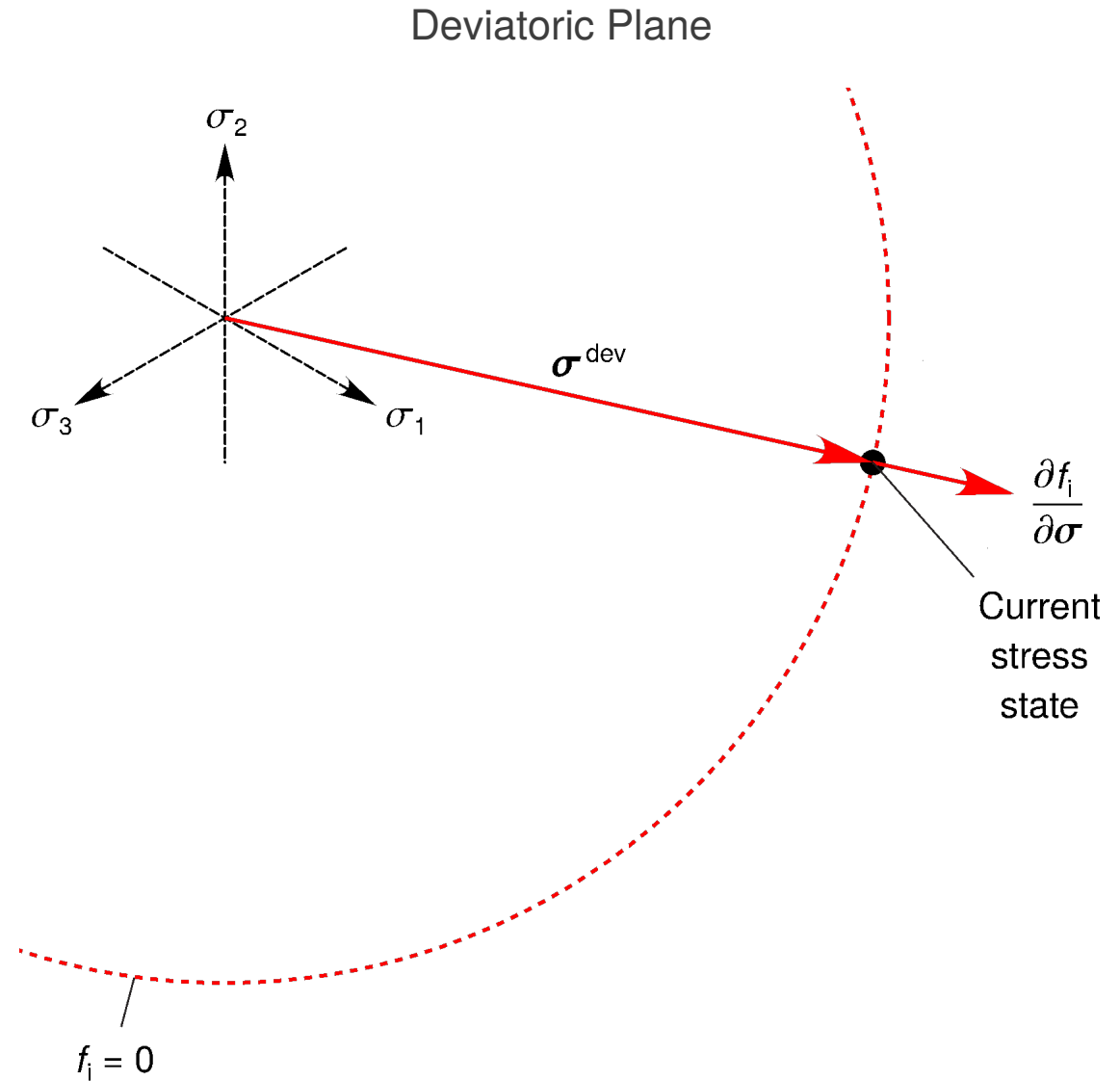
$$f_i = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$

Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\epsilon} \frac{\partial f_i}{\partial \sigma}$$





# Isotropic and Kinematic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Dislocation Glide Stress

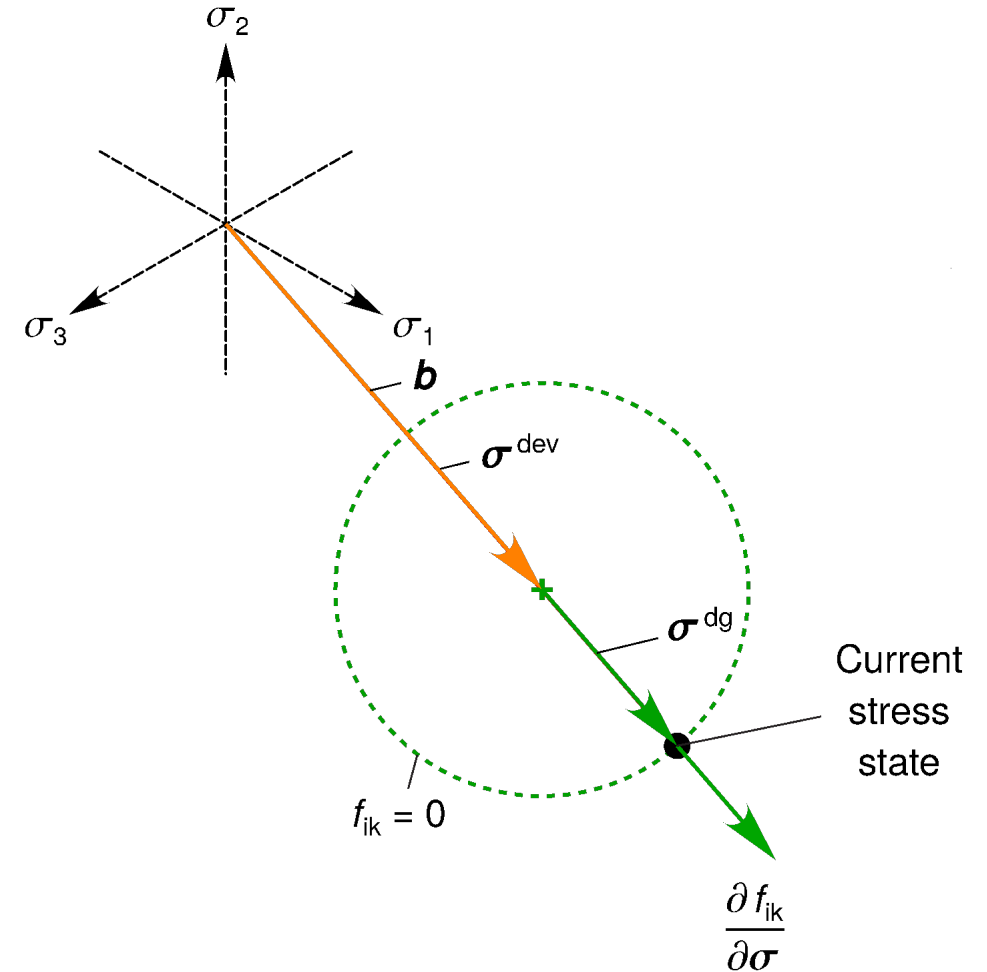
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\epsilon}^{dg} \frac{\partial f_{ik}}{\partial \sigma}$$

Deviatoric Plane



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

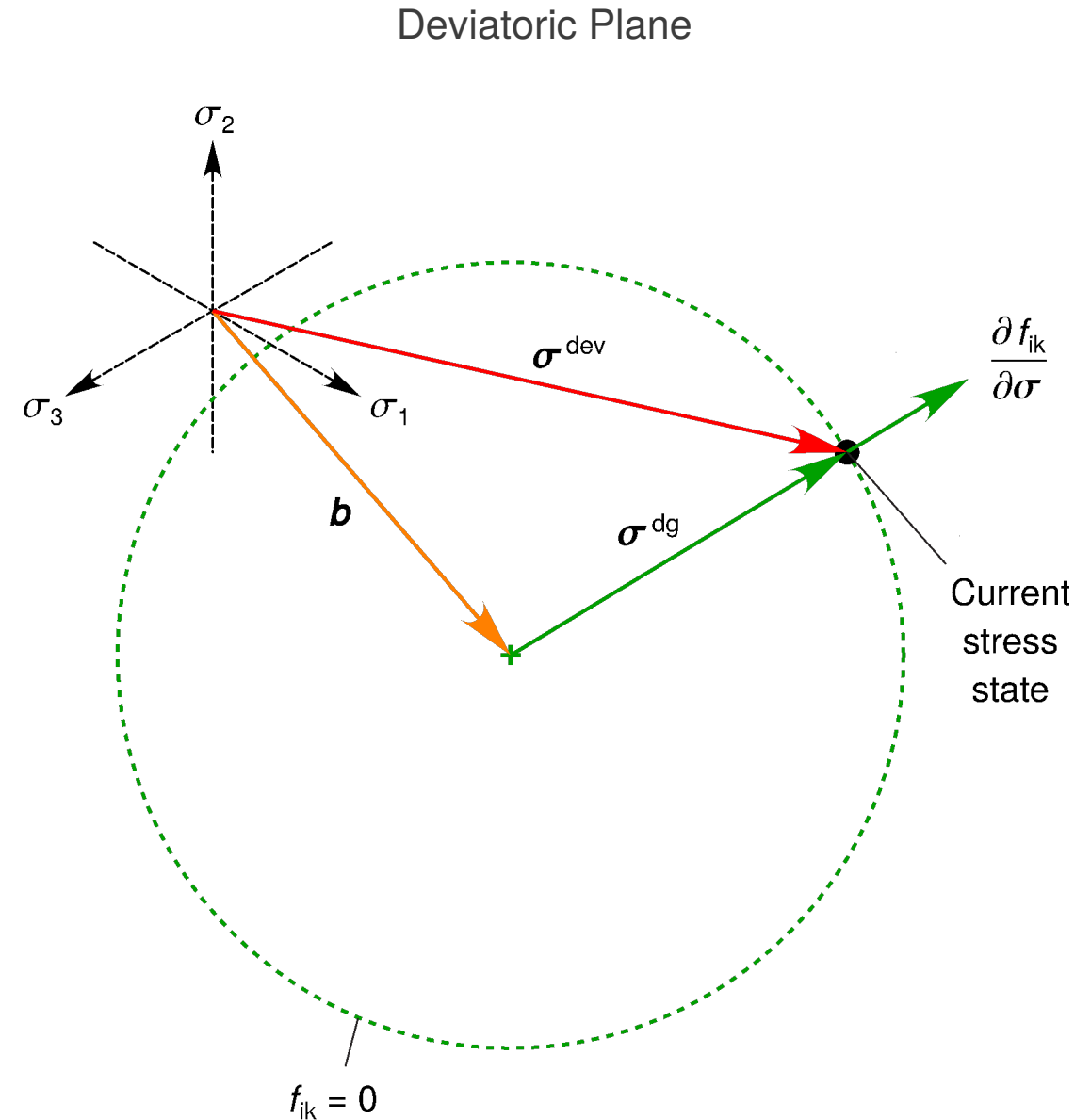
Dislocation Glide Stress

$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\epsilon} \frac{\partial f_{ik}}{\partial \sigma}$$



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Dislocation Glide Stress

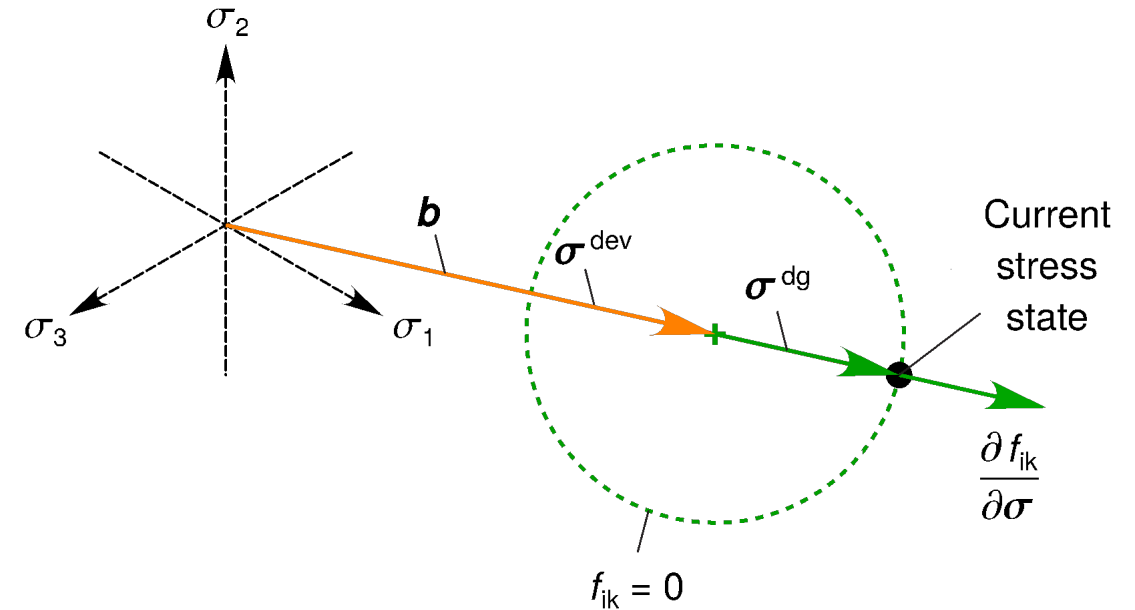
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\epsilon} \frac{\partial f_{ik}}{\partial \sigma}$$

Deviatoric Plane



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{\text{dg}} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

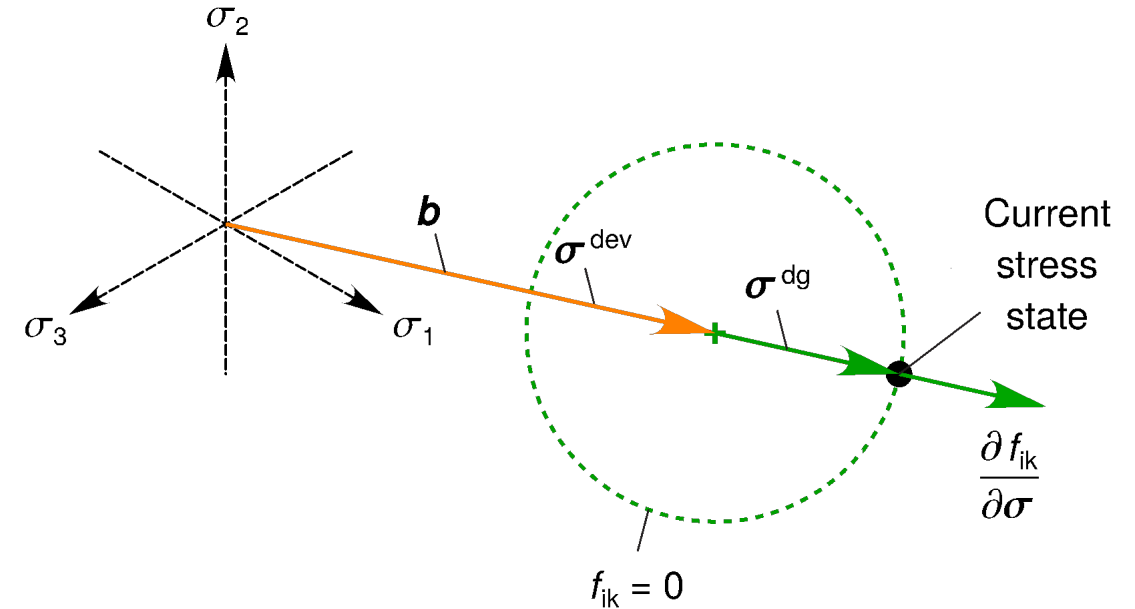
Equivalent Dislocation Glide Stress  
(proportional, monotonic, loading, or steady-state)

$$\tilde{\sigma}^{\text{dg}} = \tilde{\sigma} - \tilde{b}$$

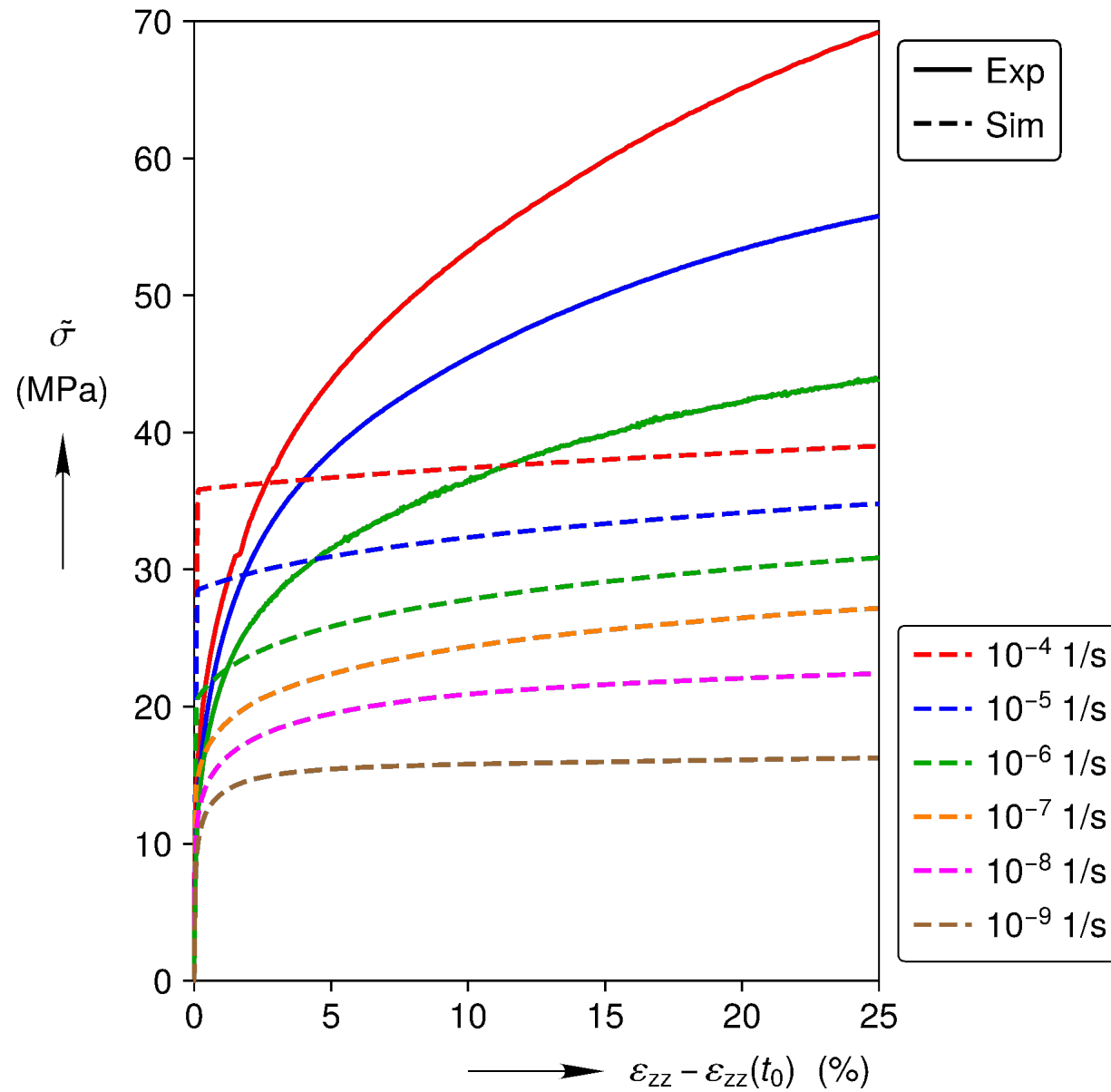
Flow Rule

$$\dot{\epsilon}^{\text{dg}} = \dot{\epsilon} \frac{\partial f_{ik}}{\partial \sigma}$$

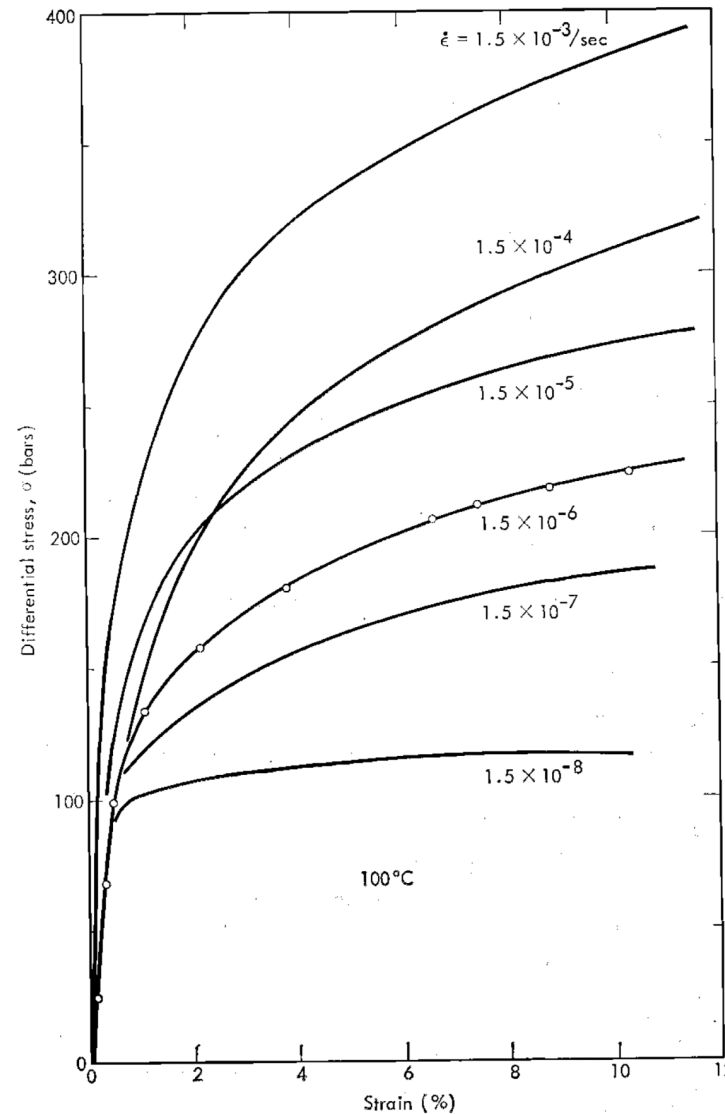
Deviatoric Plane



# Munson-Dawson Model Constant Strain Rate Predictions



# Constant Strain Rate Tests on Artificial Salt

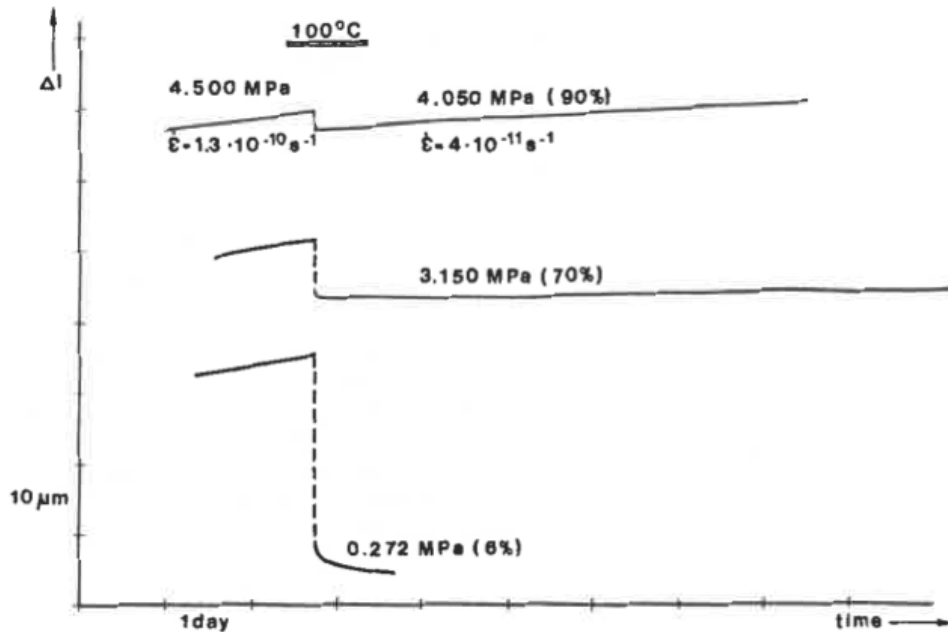


Heard, H. (1972): Steady-state flow in polycrystalline halite at pressure of 2 kilobars. *Flow and fracture of rocks*. Vol 16. pp 191-209.

Fig. 1. Differential stress-strain curves for polycrystalline halite extended at 2 kb,  $\dot{\epsilon} = 1.5 \times 10^{-3}$  to  $1.5 \times 10^{-8} \text{ sec}^{-1}$ , and  $100^\circ\text{C}$ .

# Back Stress Measurements

Creep Responses due to Small Stress Changes  
Asse Rock Salt

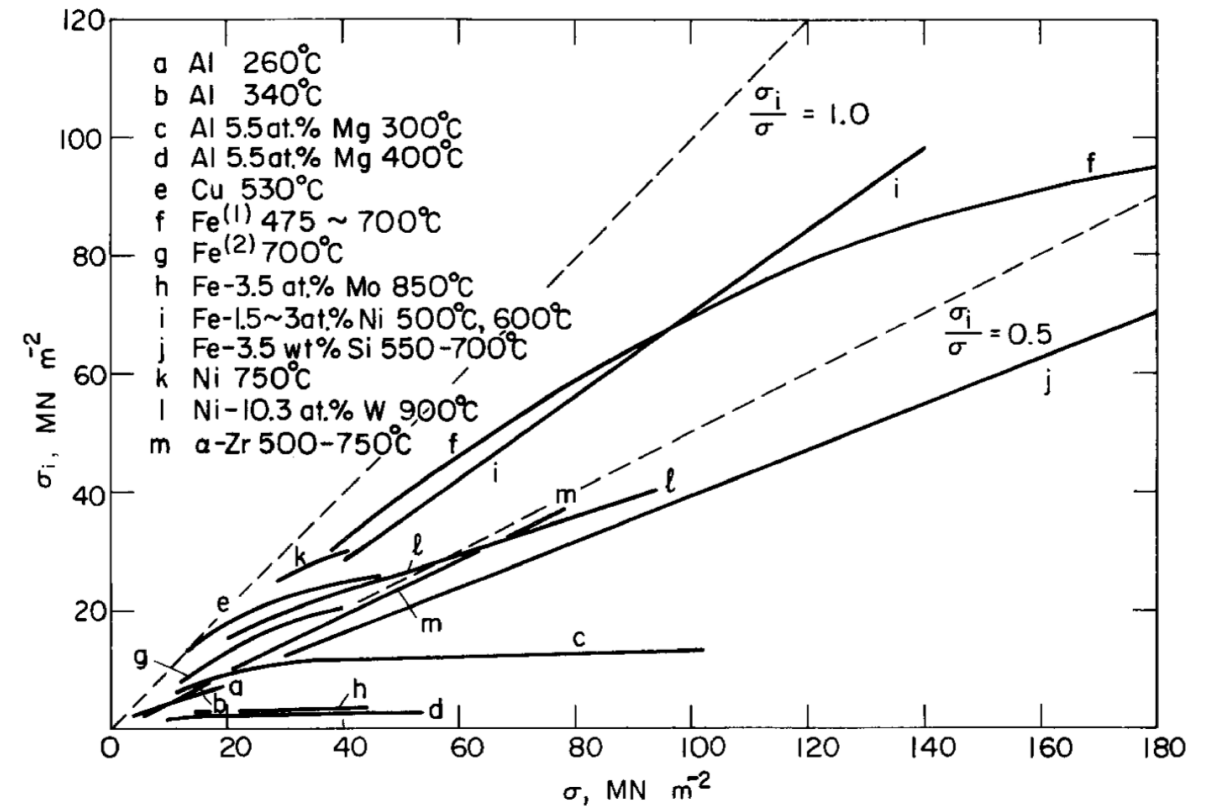


Hunsche, U. 1988. Measurement of creep in rock salt at small strain rates. Proceedings of the 2<sup>nd</sup> Conference on the Mechanical Behavior of Salt. Pg. 187-196

Dislocation Glide Strain Rate  
(proportional loading)

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[ \sinh\left(\frac{|\check{\sigma} - \check{b}|}{y}\right) \right]^{G_3} \text{sign}(\check{\sigma} - \check{b})$$

## Back Stress Measurements on Single Phase Metals

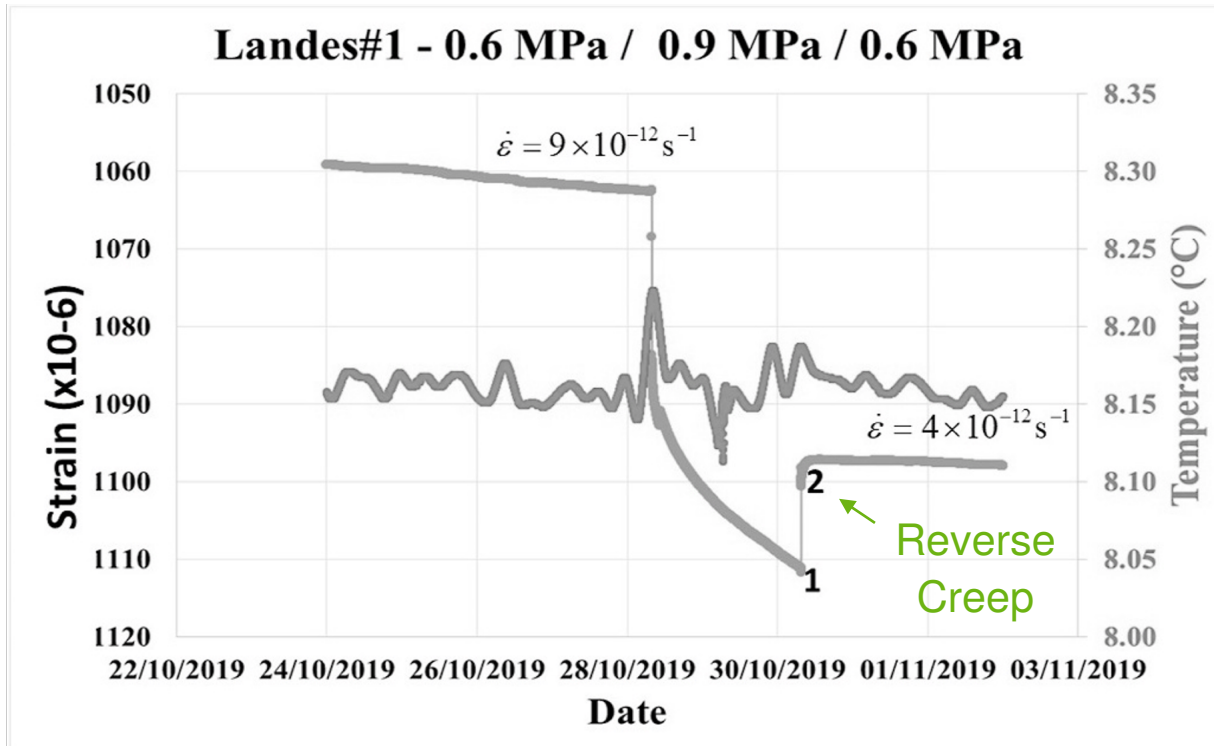


Takeuchi, S. and Argon, A. (1976). "Steady-state creep of single-phase crystalline matter at high temperature". In: Journal of Materials Science 11.8, pp. 1542-1566.

# Reverse Creep at Low Stress and Room Temperature

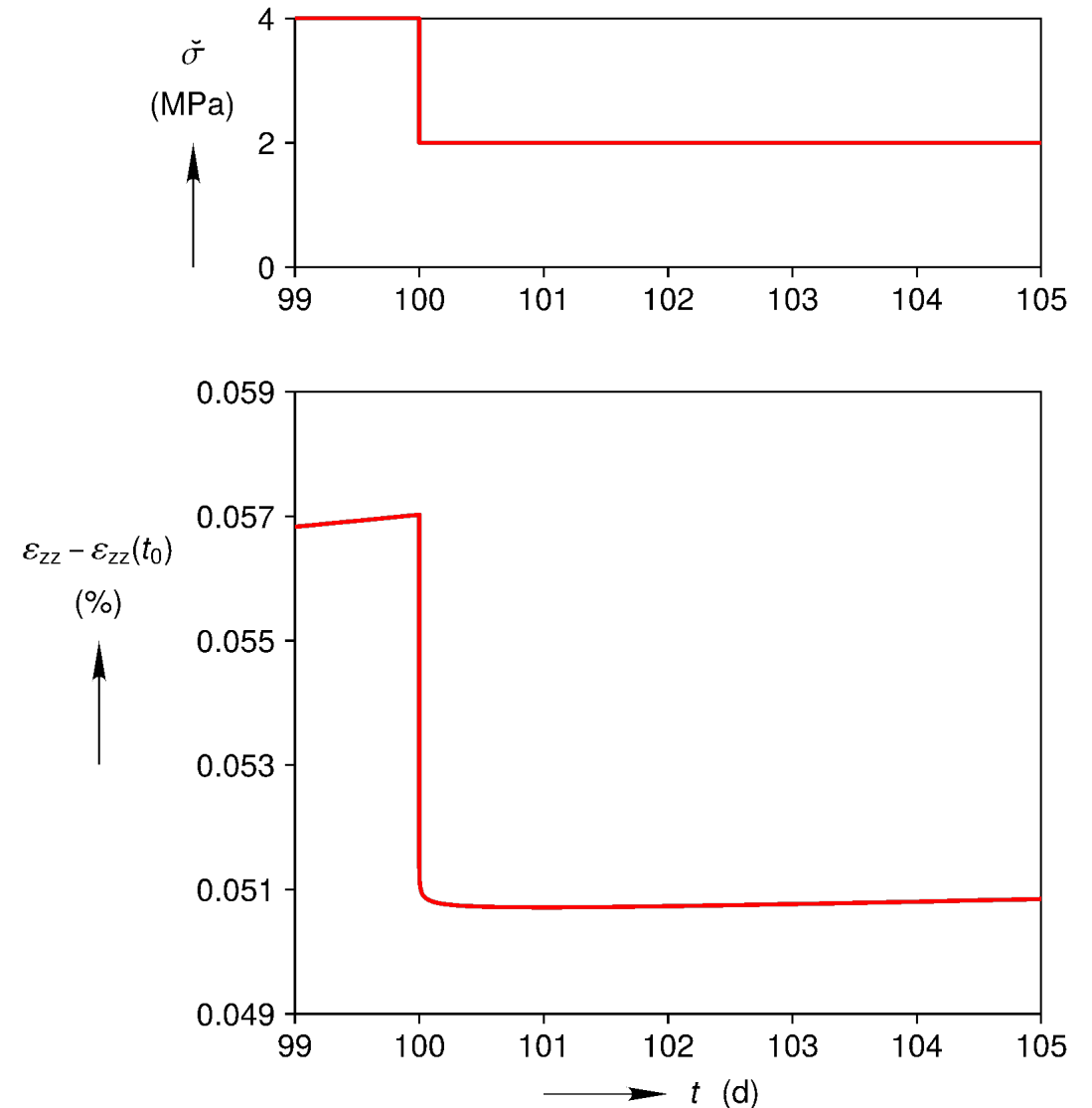


Multi-stage Constant Stress Test on Landes Salt



Gharbi, H., Bérest, P., Blanco-Martín, L., and Brouard, B. (Oct. 2020). "Determining upper and lower bounds for steady state strain rate during a creep test on a salt sample". In: International Journal of Rock Mechanics and Mining Sciences 134, p. 104452

Multi-stage Constant Stress Predictions (Cal 1C)  
(different stresses than on Landes salt)





# Dislocation Glide Hardening Saturation (Steady-State)

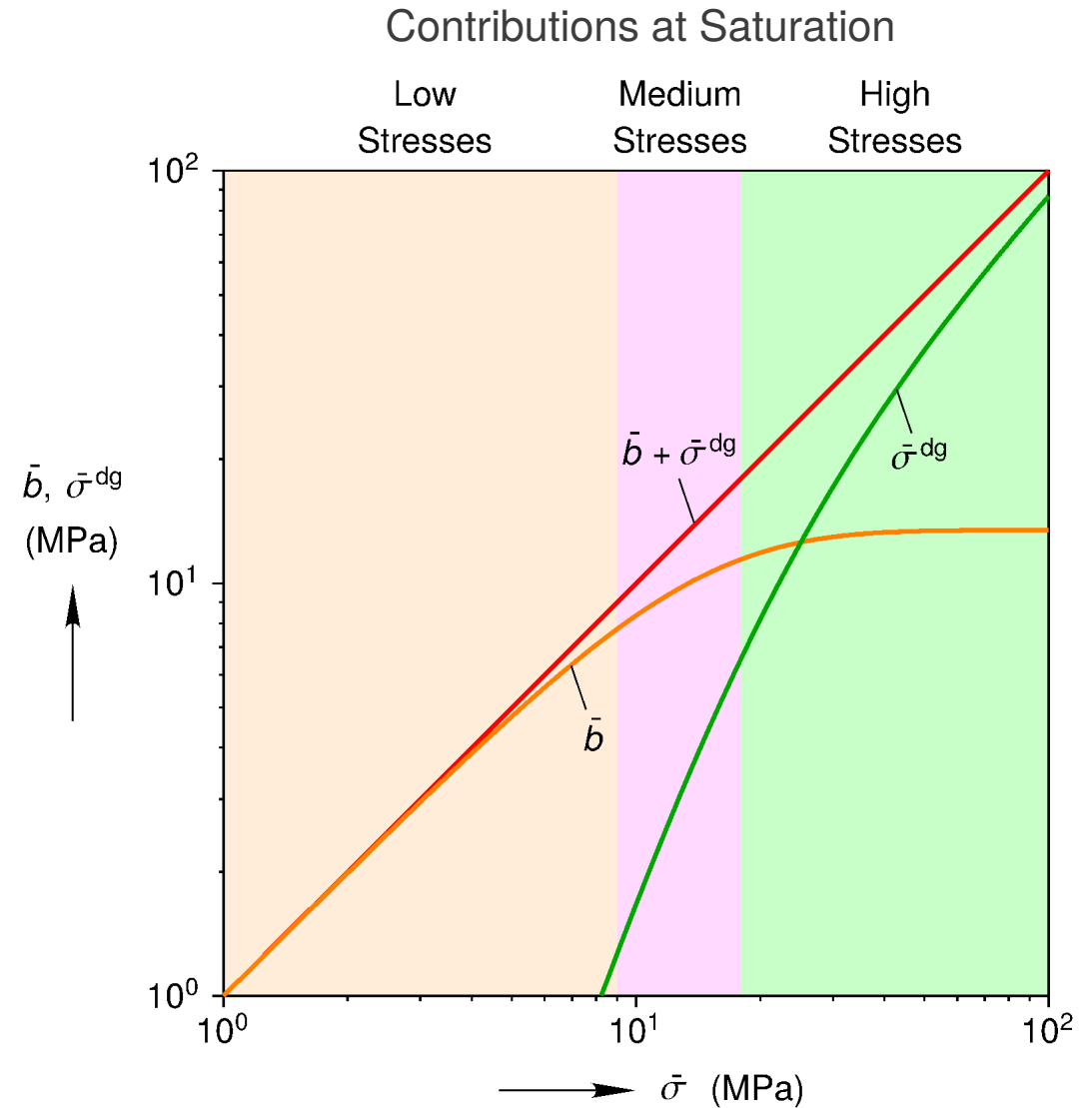


Equivalent Stress Decomposition  
(proportional, monotonic, loading)

$$\bar{\sigma} = \bar{b} + \bar{\sigma}^{\text{dg}}$$

Drag Stress Contribution

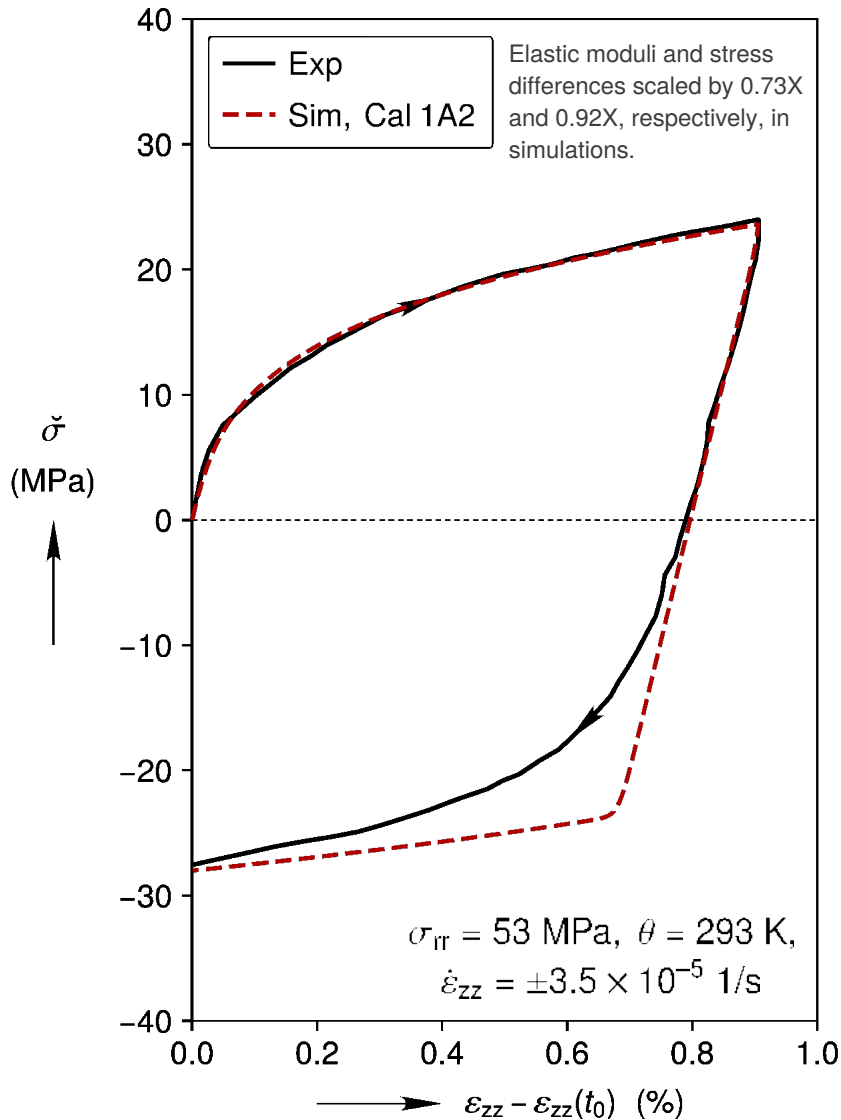
$$\bar{\sigma}^{\text{dg}} = \bar{y} \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$



# Bauschinger Effect: Isotropic Hardening Only



## Artificial Rock Salt



Experimental measurements from Aubertin et al. (1999)

## Strain Rate

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{y}\right)\right]^{G_3}$$

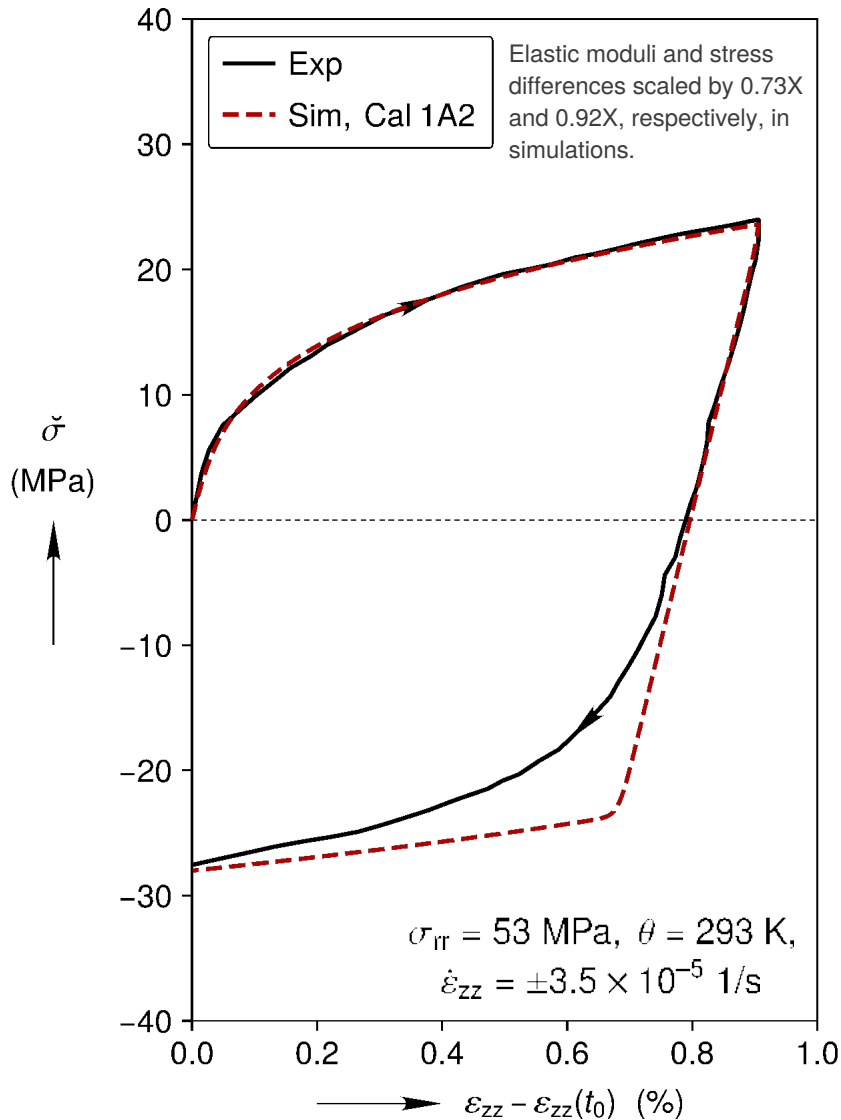
## Equivalent Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \sigma^{\text{dev}} : \sigma^{\text{dev}}}$$

# Bauschinger Effect: Isotropic Hardening Only



## Artificial Rock Salt



## Dynamic Yield Surface

$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

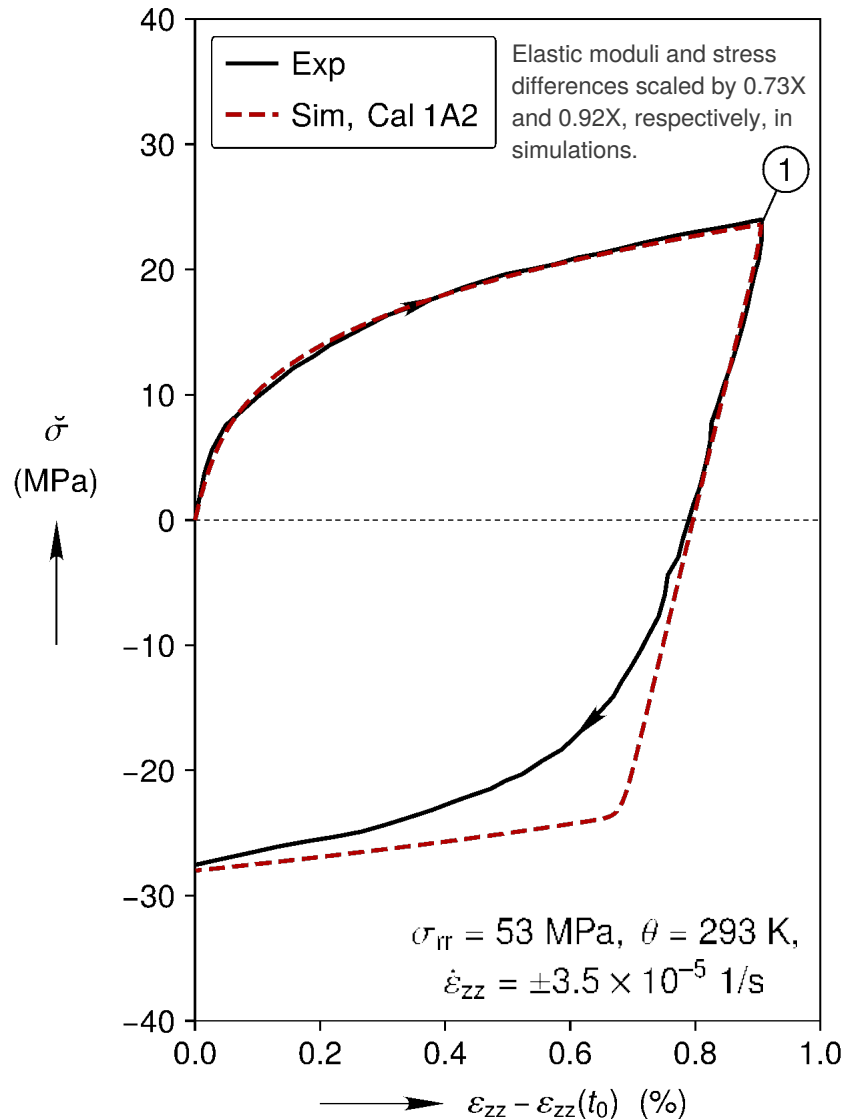
## Equivalent Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \sigma^{\text{dev}} : \sigma^{\text{dev}}}$$

# Bauschinger Effect: Isotropic Hardening Only



Artificial Rock Salt



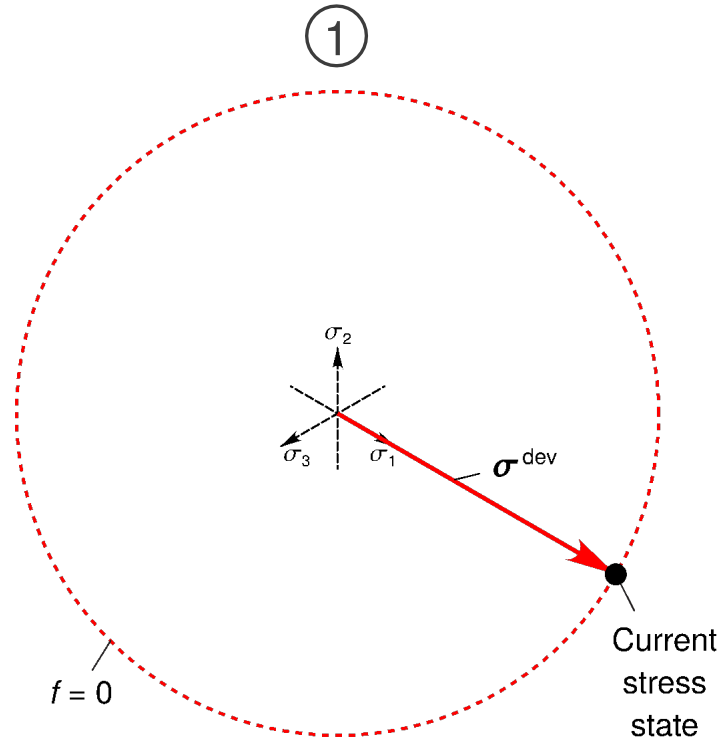
Experimental measurements from Aubertin et al. (1999)

Dynamic Yield Surface

$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Equivalent Stress

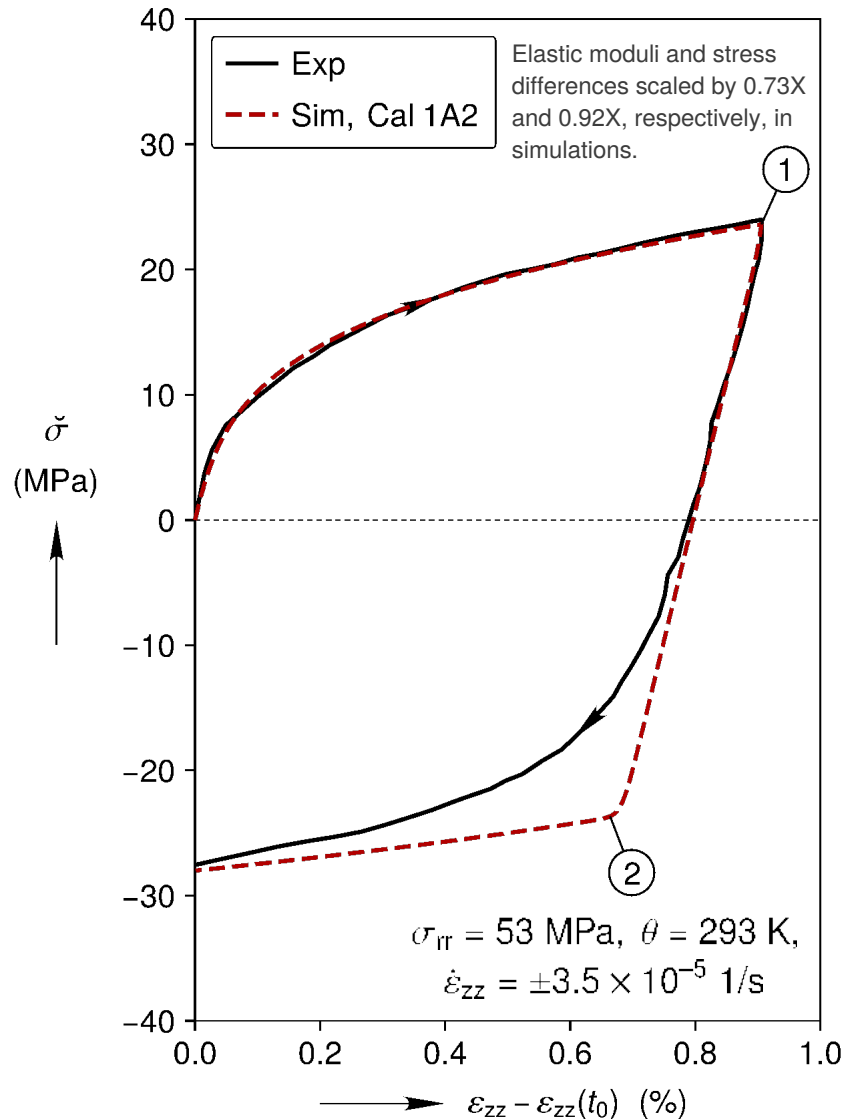
$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$



# Bauschinger Effect: Isotropic Hardening Only



Artificial Rock Salt



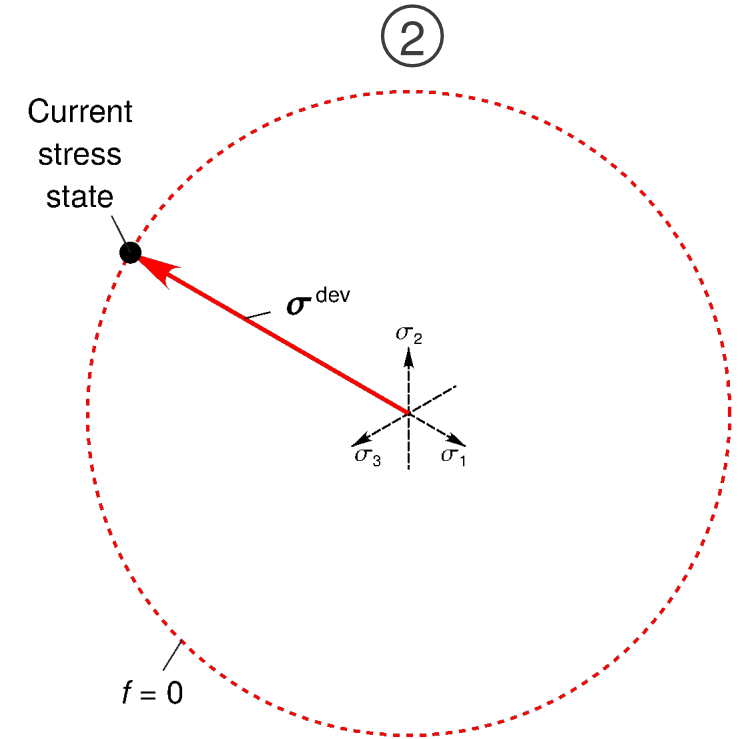
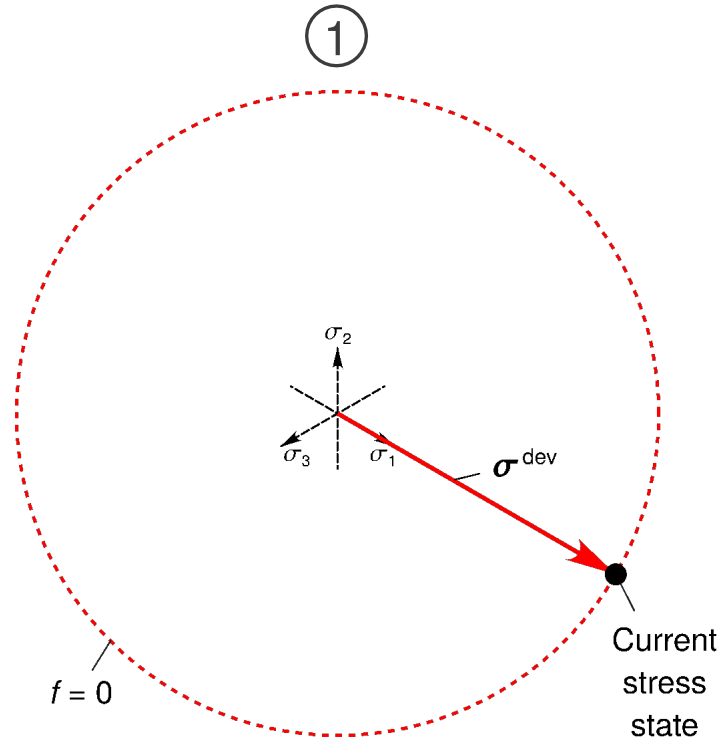
Experimental measurements from Aubertin et al. (1999)

Dynamic Yield Surface

$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Equivalent Stress

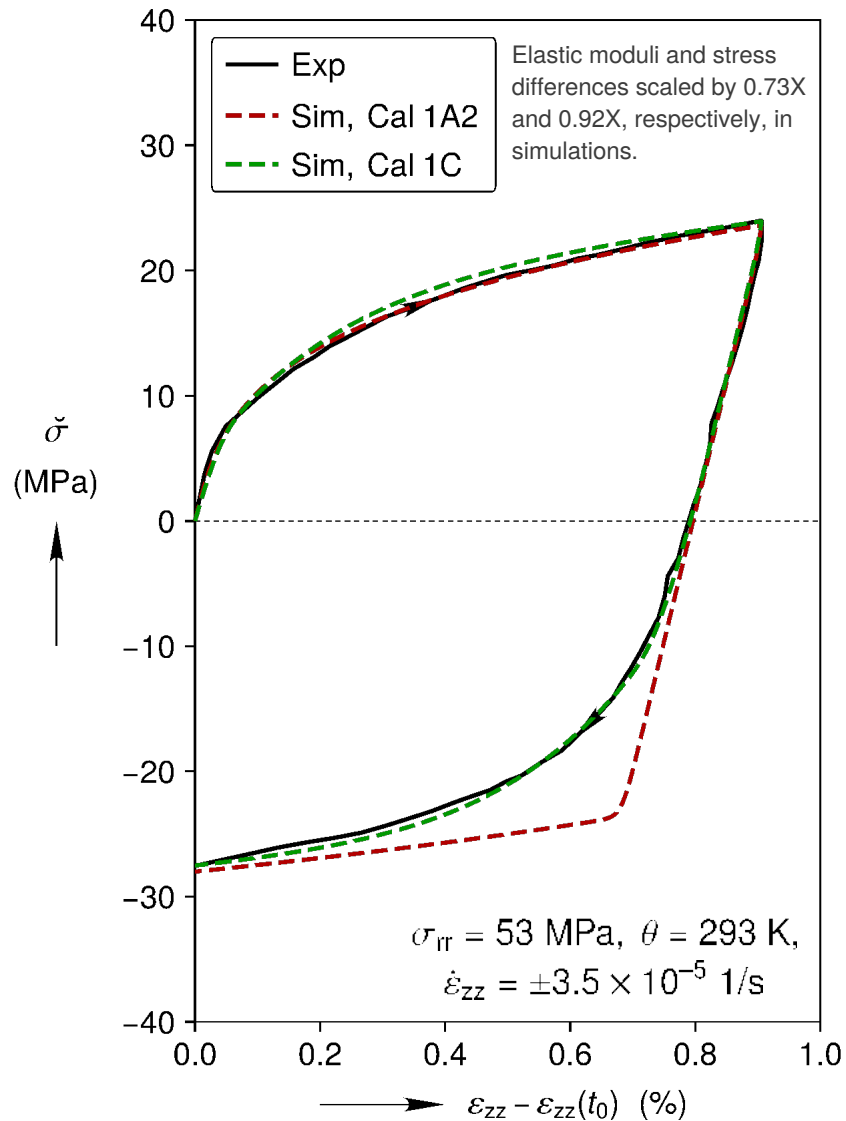
$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$



# Bauschinger Effect: Isotropic + Kinematic Hardening



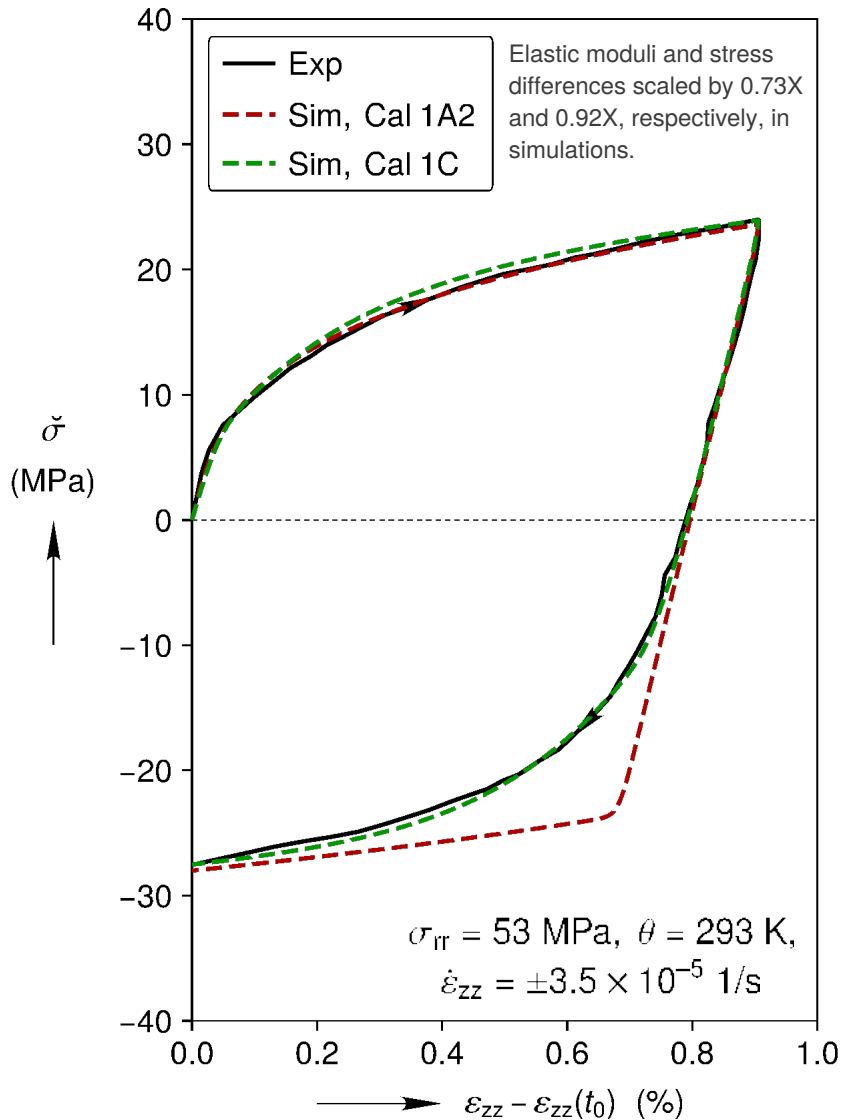
## Artificial Rock Salt



# Bauschinger Effect: Isotropic + Kinematic Hardening



## Artificial Rock Salt



Experimental measurements from Aubertin et al. (1999)

## Dynamic Yield Surface

$$f = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

## Equivalent Stress

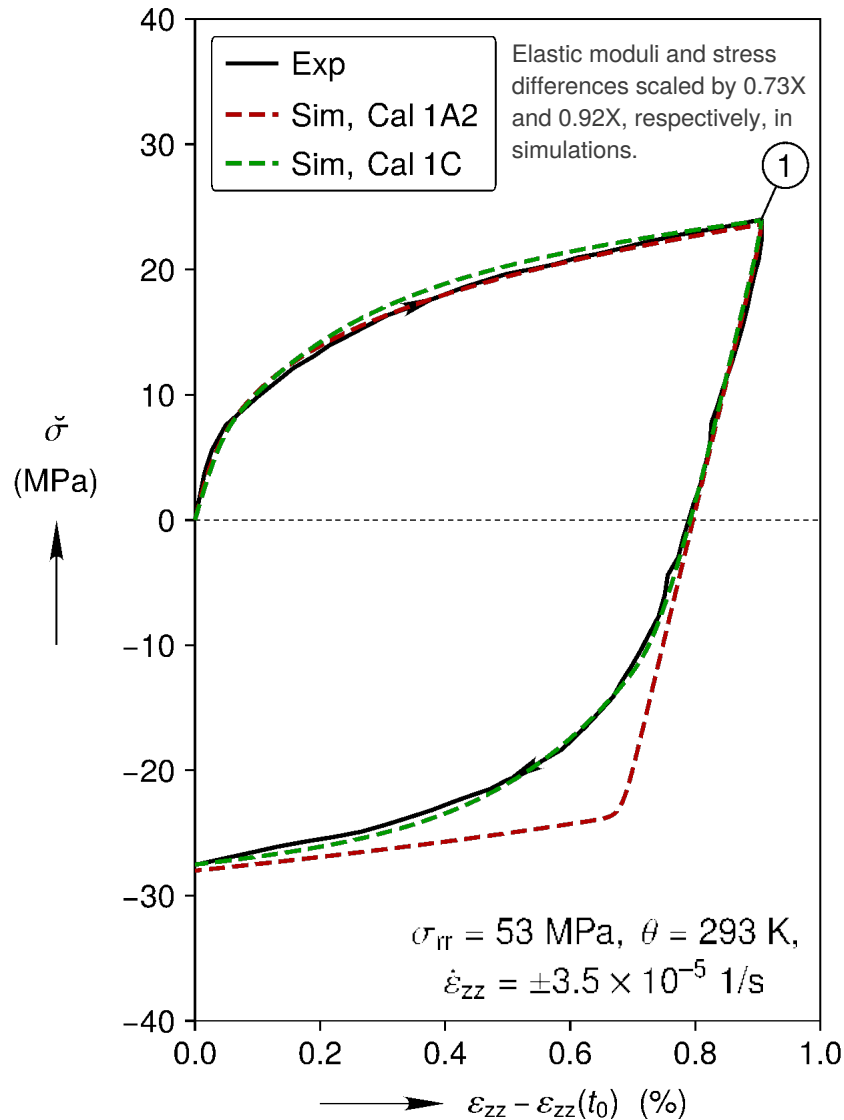
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

# Bauschinger Effect: Isotropic + Kinematic Hardening



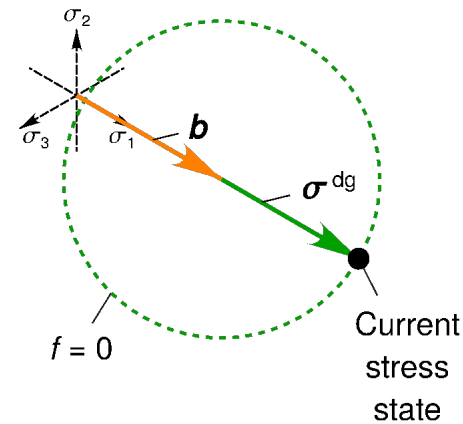
Artificial Rock Salt



Dynamic Yield Surface

$$f = \bar{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

①



Equivalent Stress

$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

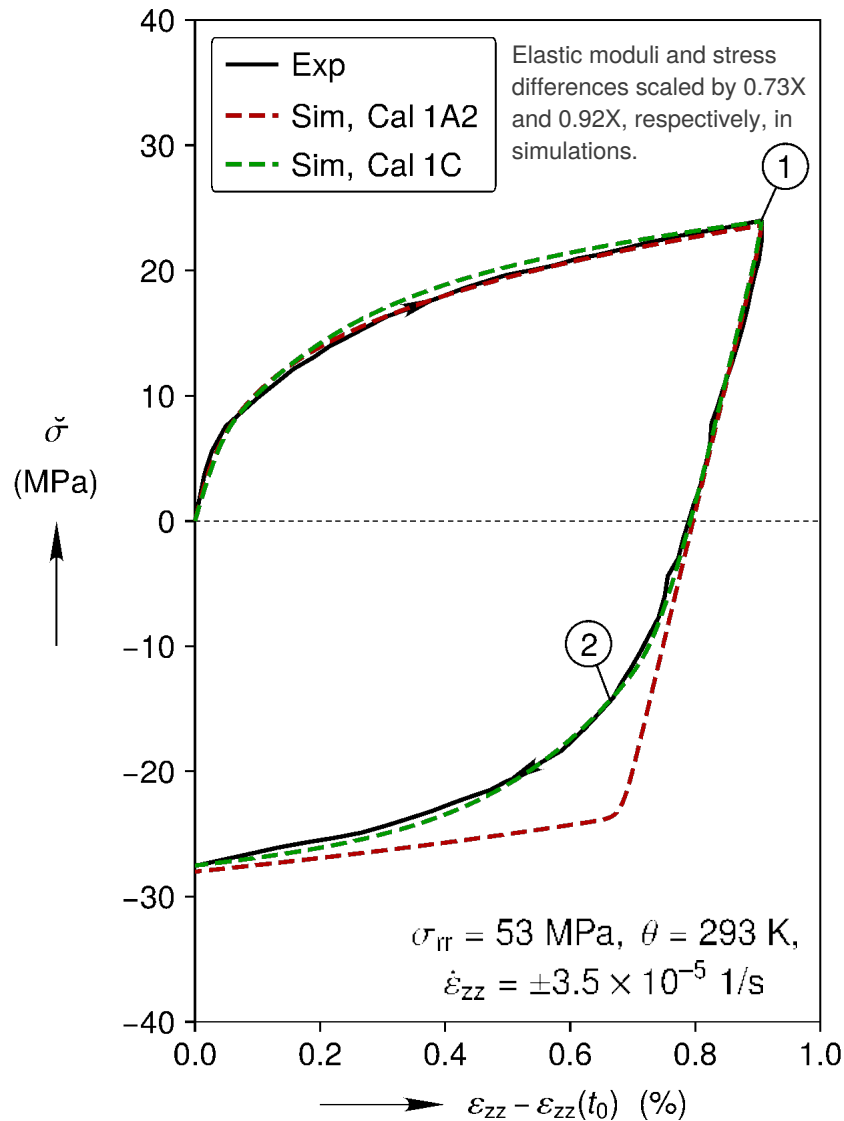
$$\bar{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$



# Bauschinger Effect: Isotropic + Kinematic Hardening



Artificial Rock Salt

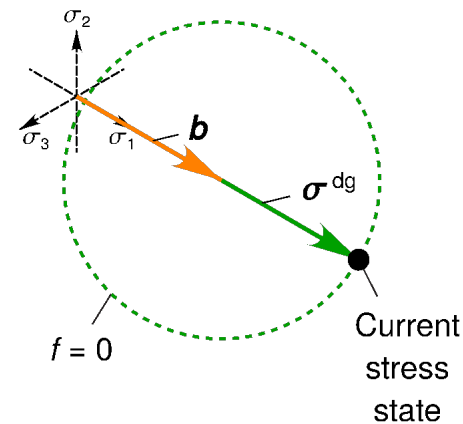


Experimental measurements from Aubertin et al. (1999)

Dynamic Yield Surface

$$f = \bar{\sigma}^{\text{dg}} - y \sinh^{-1} \left\{ \left[ \frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

①

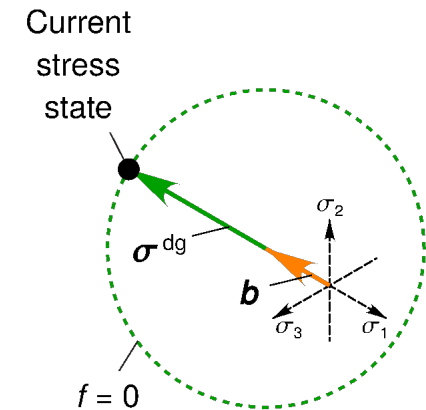


Equivalent Stress

$$\sigma^{\text{dg}} = \sigma^{\text{dev}} - b$$

$$\bar{\sigma}^{\text{dg}} = \sqrt{\frac{3}{2} \sigma^{\text{dg}} : \sigma^{\text{dg}}}$$

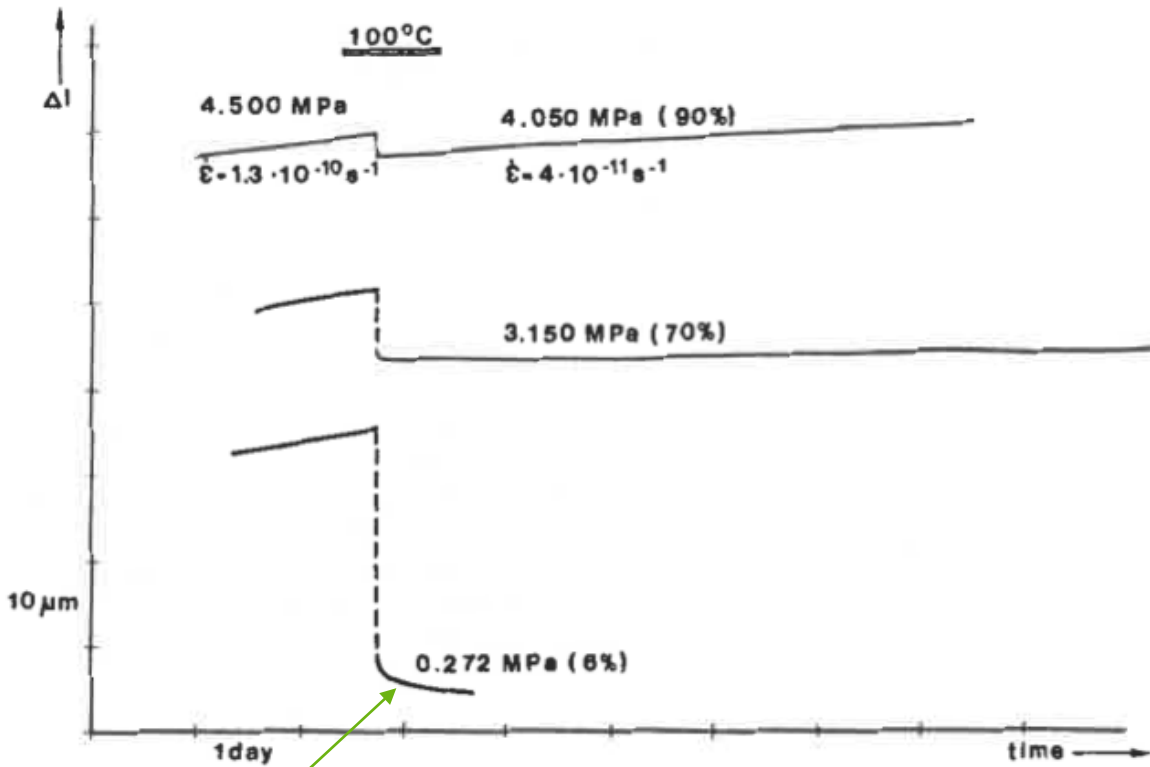
②



# Re-hardening During Non-Monotonic Loading

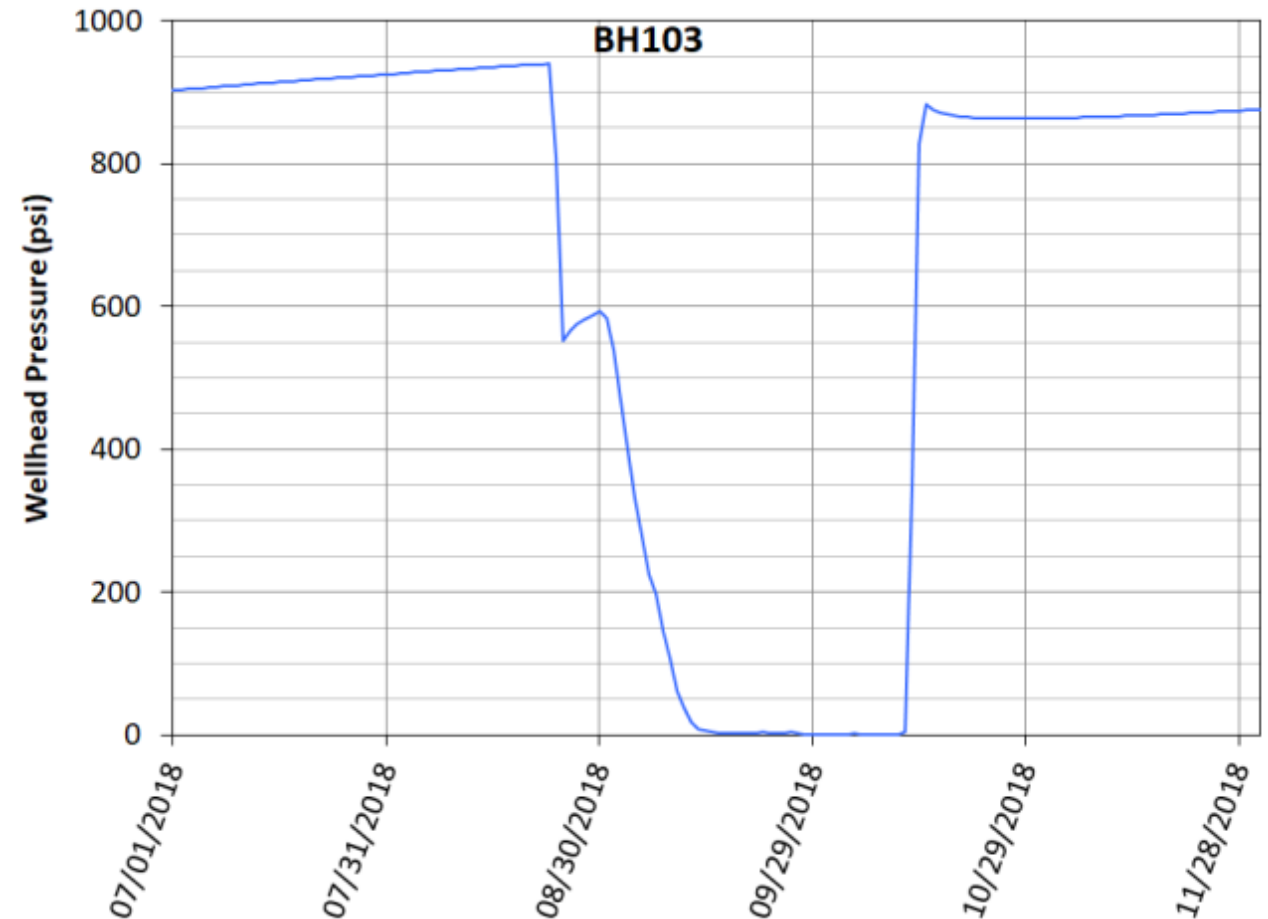


Asse Salt, Strain Histories

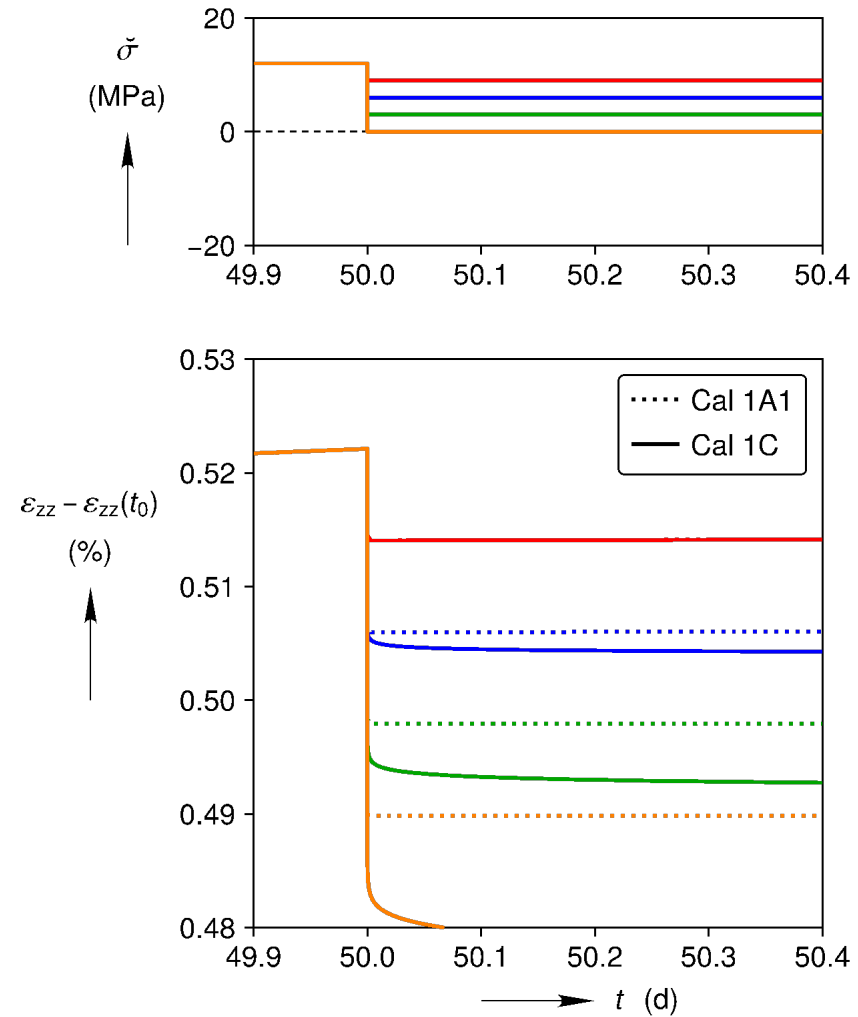
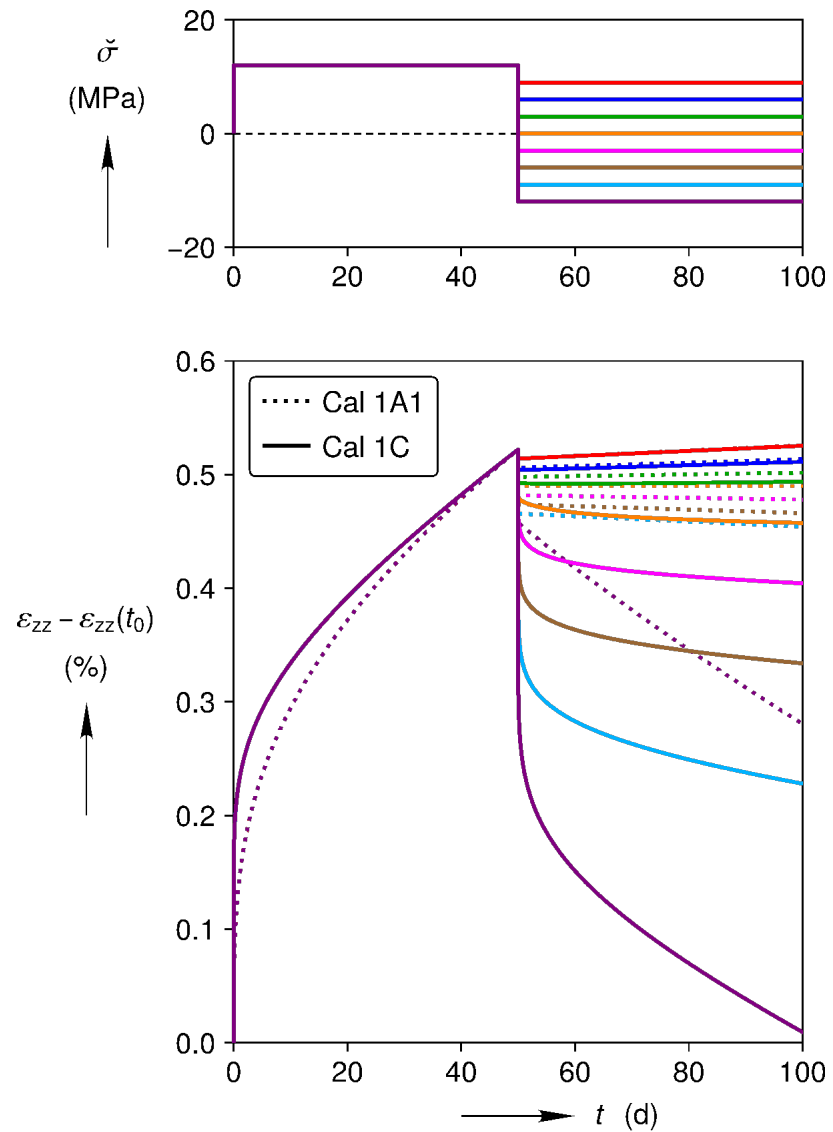


Re-hardening

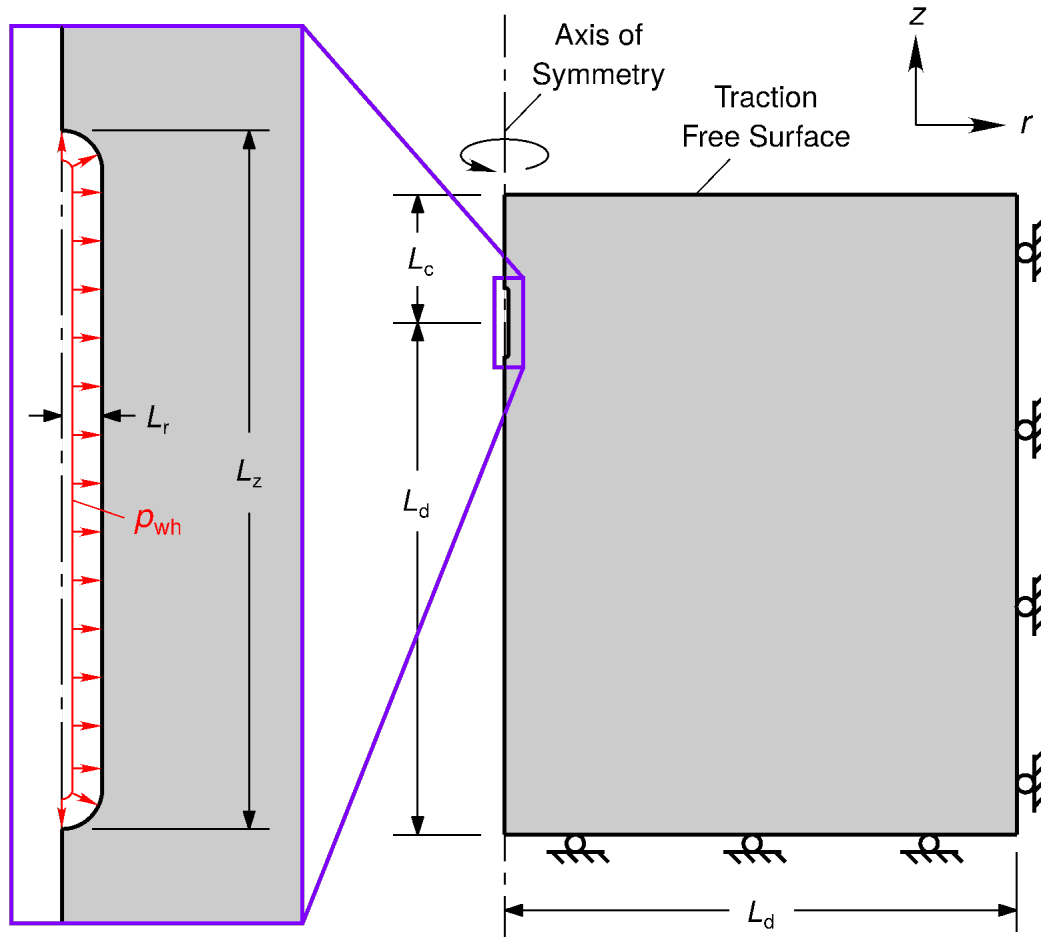
Wellhead Pressure Before, During, and After a Workover



# Stress Drop Behavior

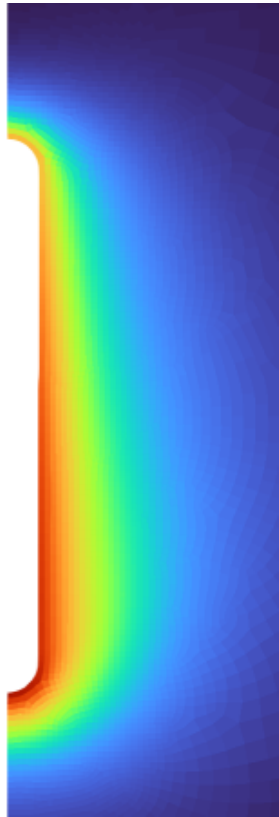
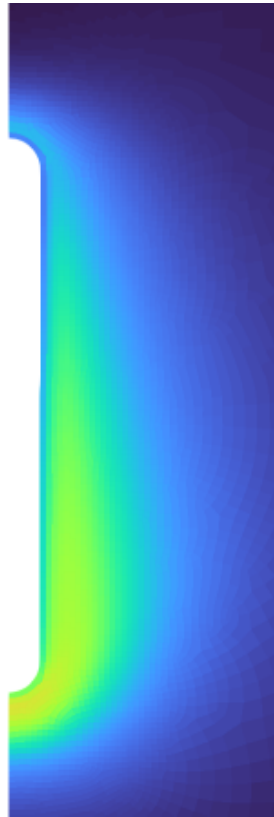
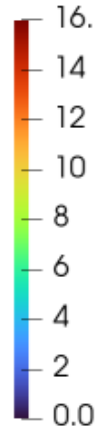


Cal 1A1 strain shifted down by 0.067 % strain.



1. Isolated cigar shaped cavern
  1. Axisymmetric analysis
  2. Center of cavern was 1100 m deep
  3. Wellhead pressure held high for 5 years, held low for 3 days, and then cycled 3X.
2. Constitutive model robustness
  1. Model evaluated  $2.6 \times 10^9$  times
  2. Model failed to converge 8 times
3. Constitutive model adaptive time stepping
  1. Initial time step =  $10^{-3}$  s
  2. Time step after 5 years = 10 d

## von Mises Stress Distributions

 $p_{wh} = 6.4 \text{ MPa}$  $p_{wh} = 12.8 \text{ MPa}$  $\bar{\sigma}$  (MPa)

## Wellhead Pressure and Cavern Volume Loss Histories

