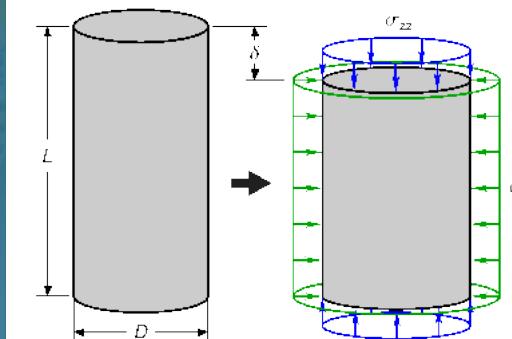
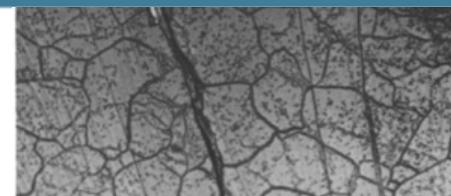




Sandia
National
Laboratories

New Salt Constitutive Model Status



Benjamin Reedlunn

Materials and Failure Modeling Department

WIPP Rock Mechanics Technical Exchange

October 27th, 2022



Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525. This research is funded by WIPP programs administered by the Office of Environmental Management (EM) of the U.S. Department of Energy.



1. Motivation
2. Model Overview
3. Calibrations
4. Non-Monotonic Loading
5. Summary

Motivation



Munson-Dawson model

1. Advantages

1. Well-known
2. Easy to calibrate against constant stress tests
3. Captures most low and medium stress behavior

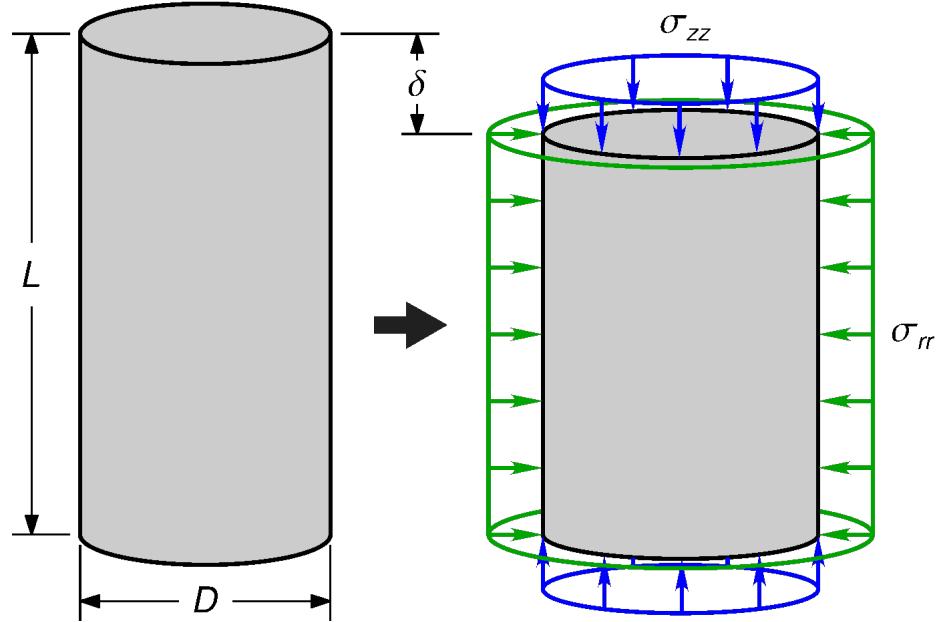
2. Small Disadvantages

1. Highly phenomenological
 1. $\dot{\varepsilon}^{\text{vp}} = \dot{\varepsilon}^{\text{tr}} + \dot{\varepsilon}^{\text{ss}}$
 2. Cannot be fit into framework of Rational Thermodynamics

3. Larger Disadvantages Relevant to Empty Areas at WIPP

1. Does not include damage and healing
2. Cannot capture high strain rate (high stress) behavior
3. Cannot capture re-hardening during non-monotonic loading

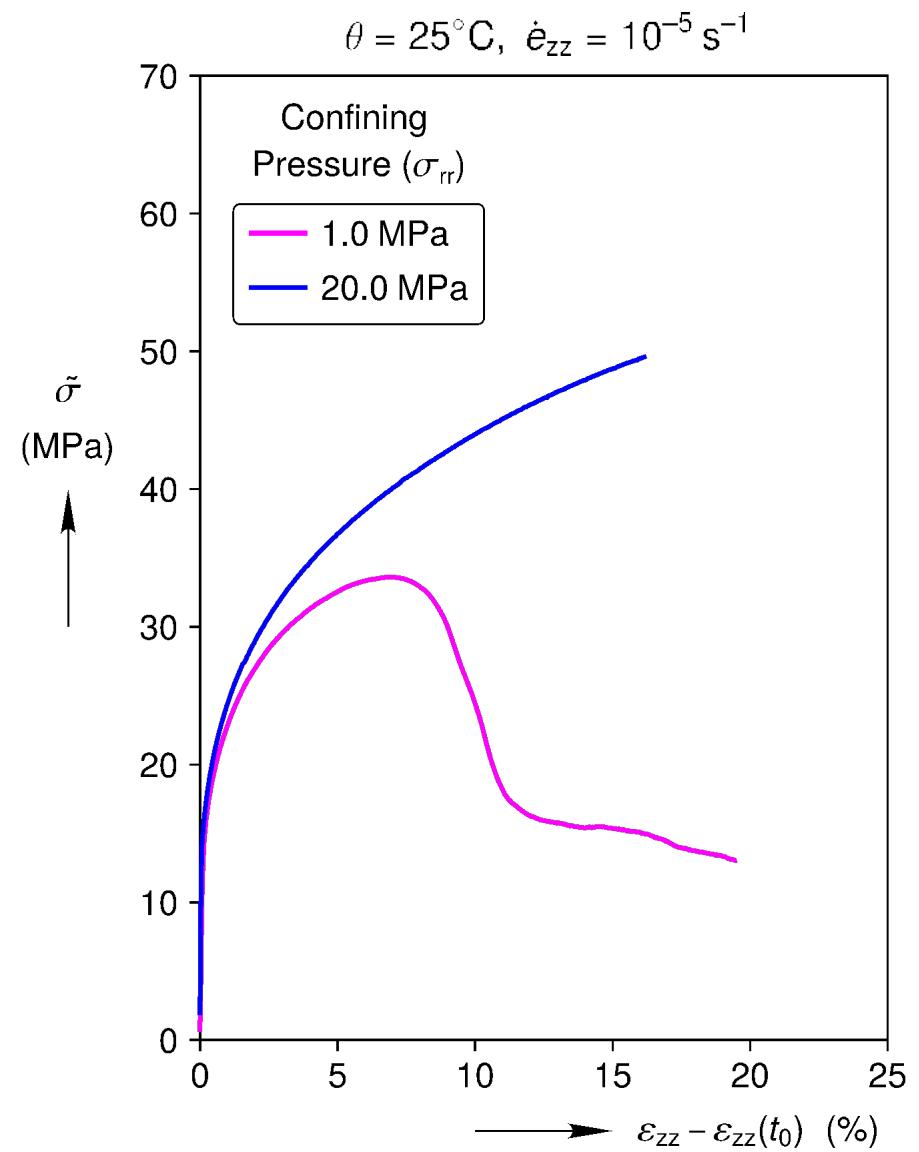
Damaged, High Strain Rate, Behavior



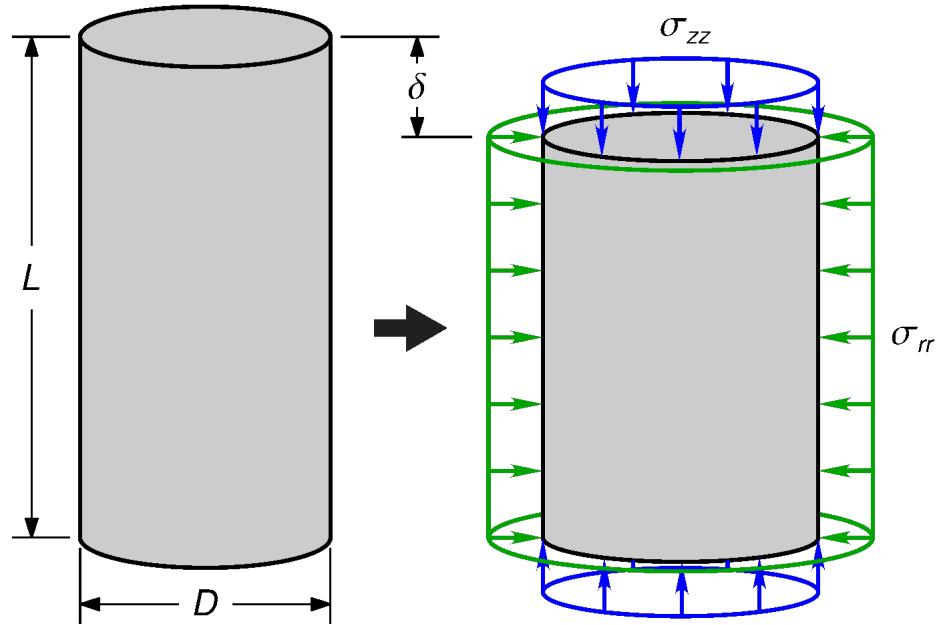
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



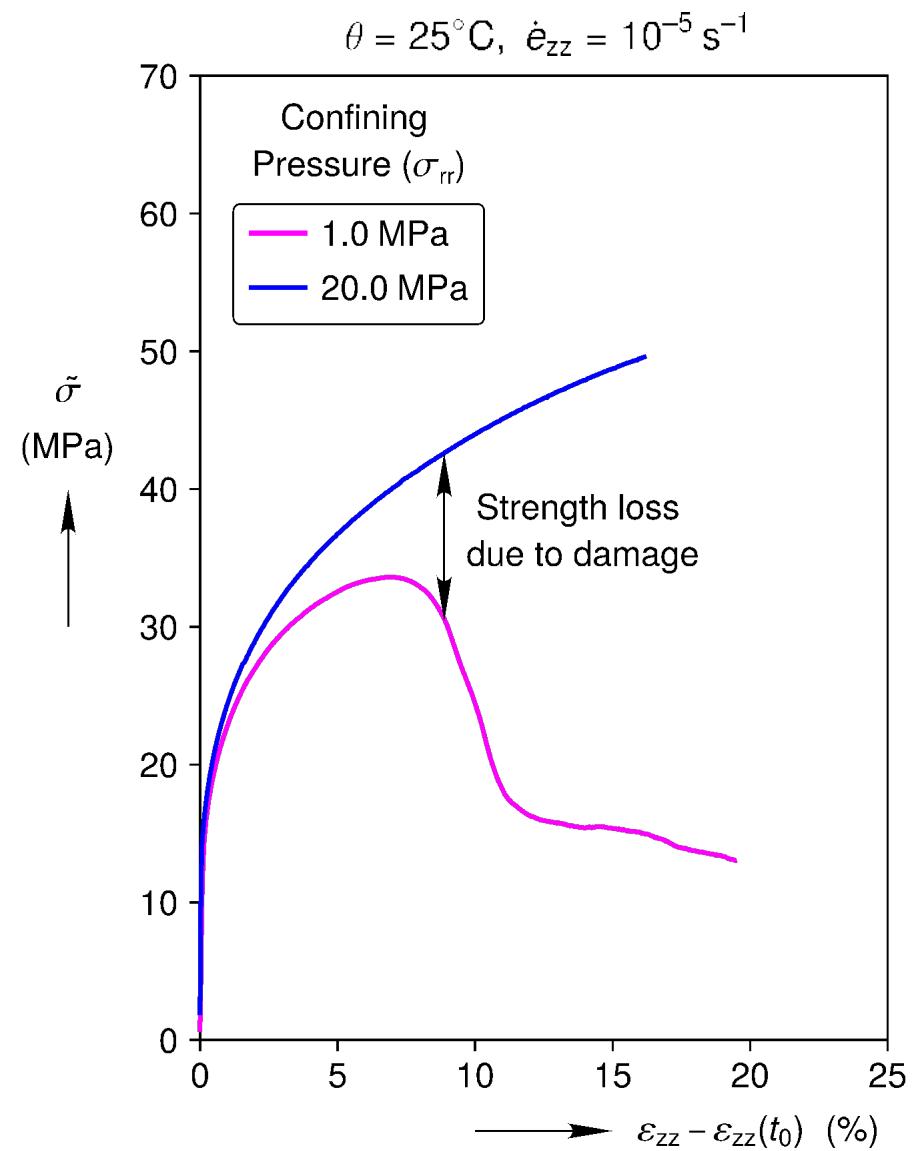
Damaged, High Strain Rate, Behavior



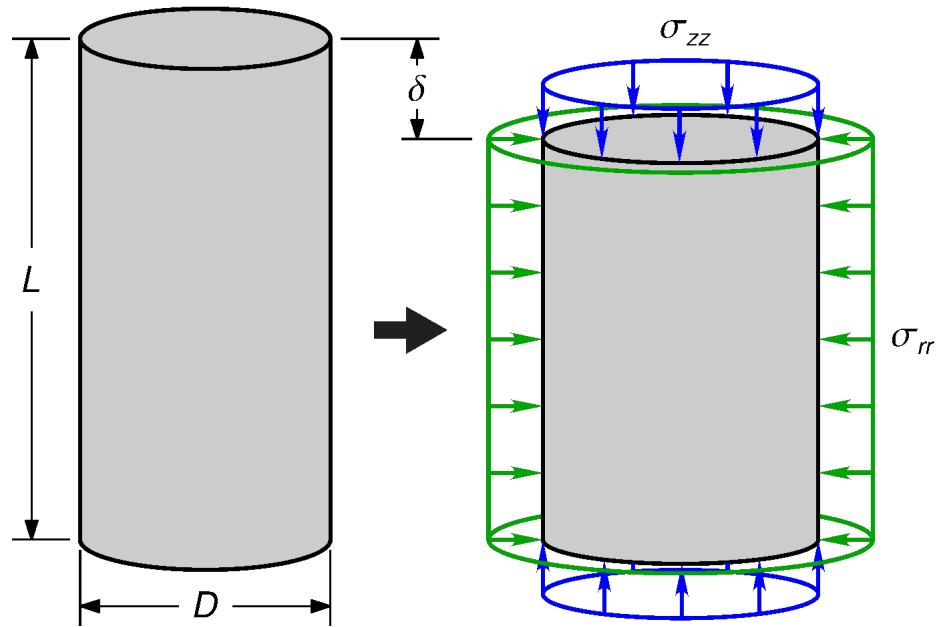
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$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



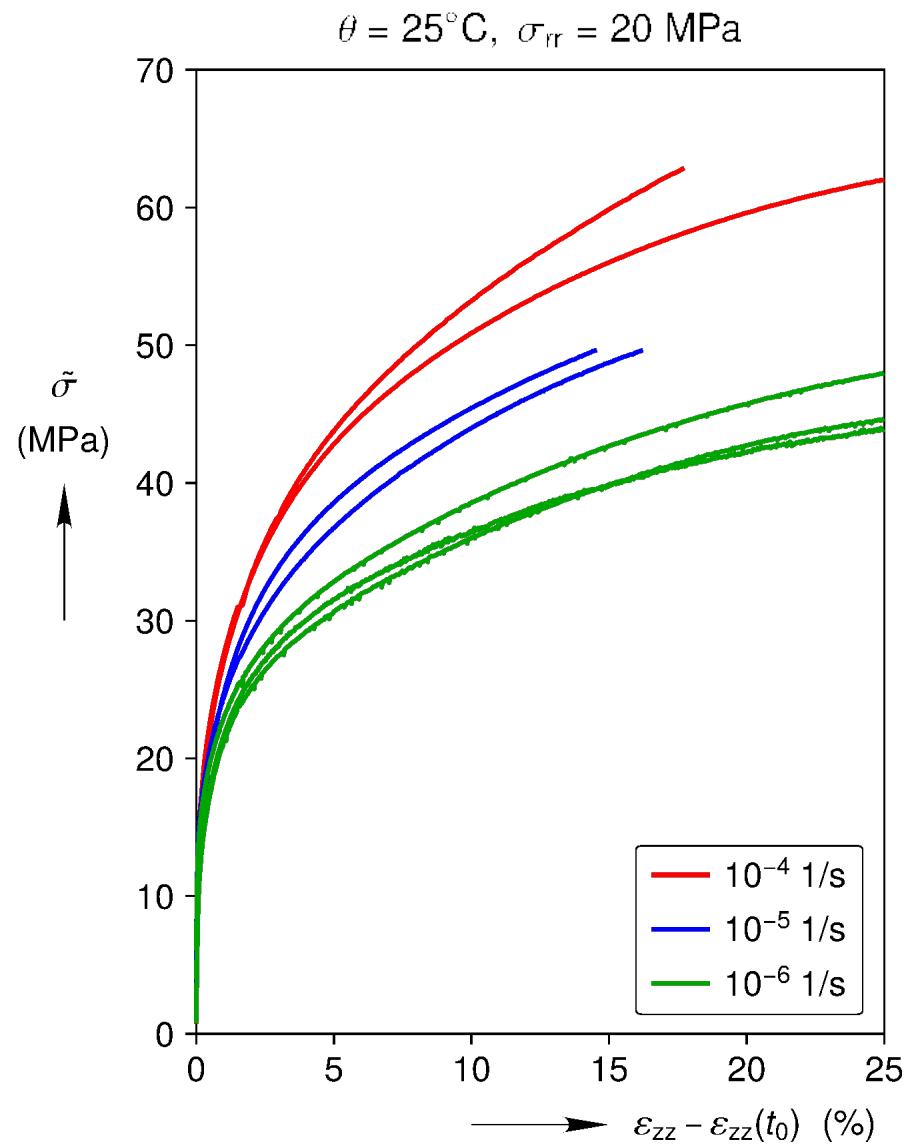
Damage-Free, High Strain Rate, Behavior



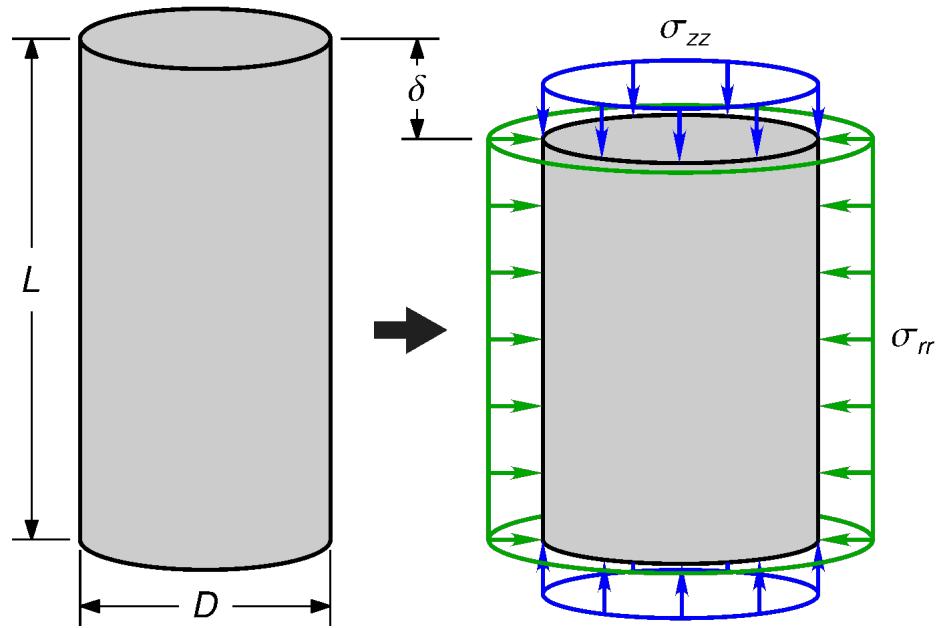
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



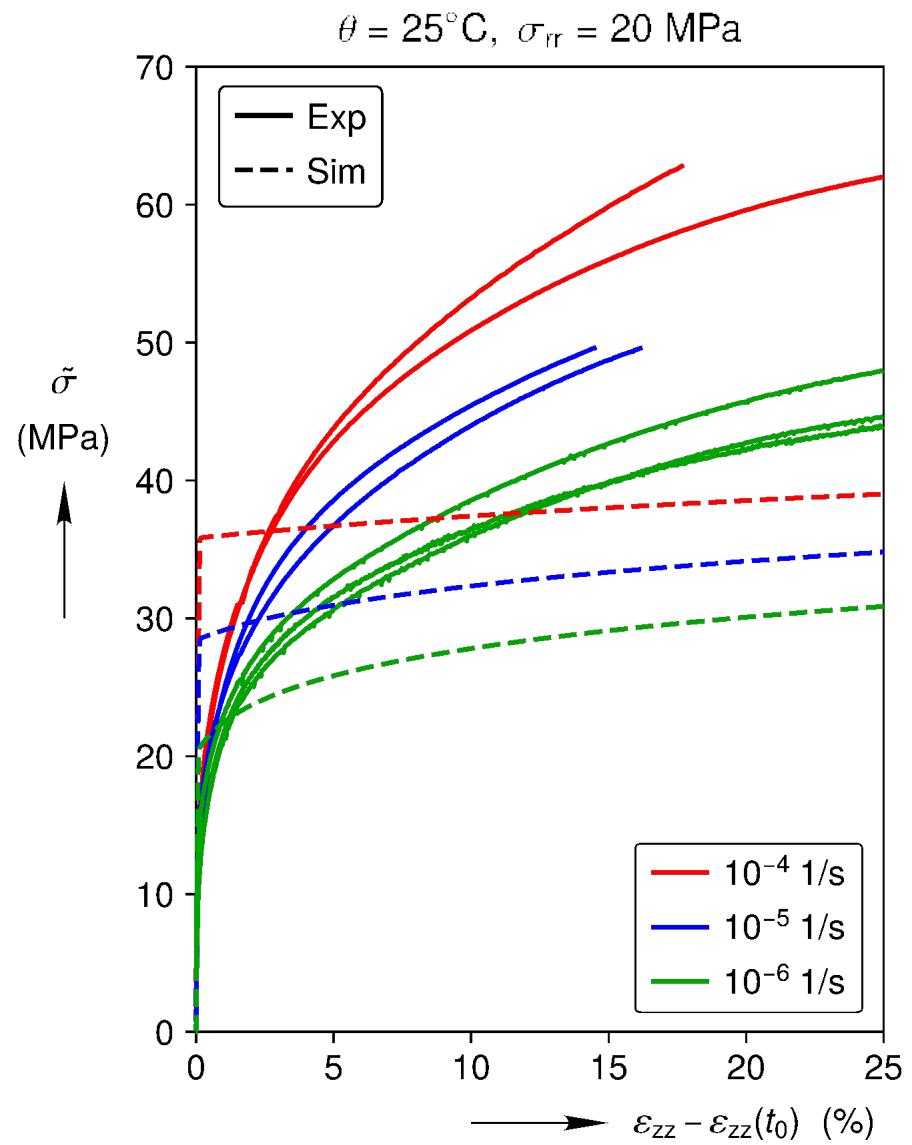
Damage-Free, High Strain Rate, Munson-Dawson Predictions



$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

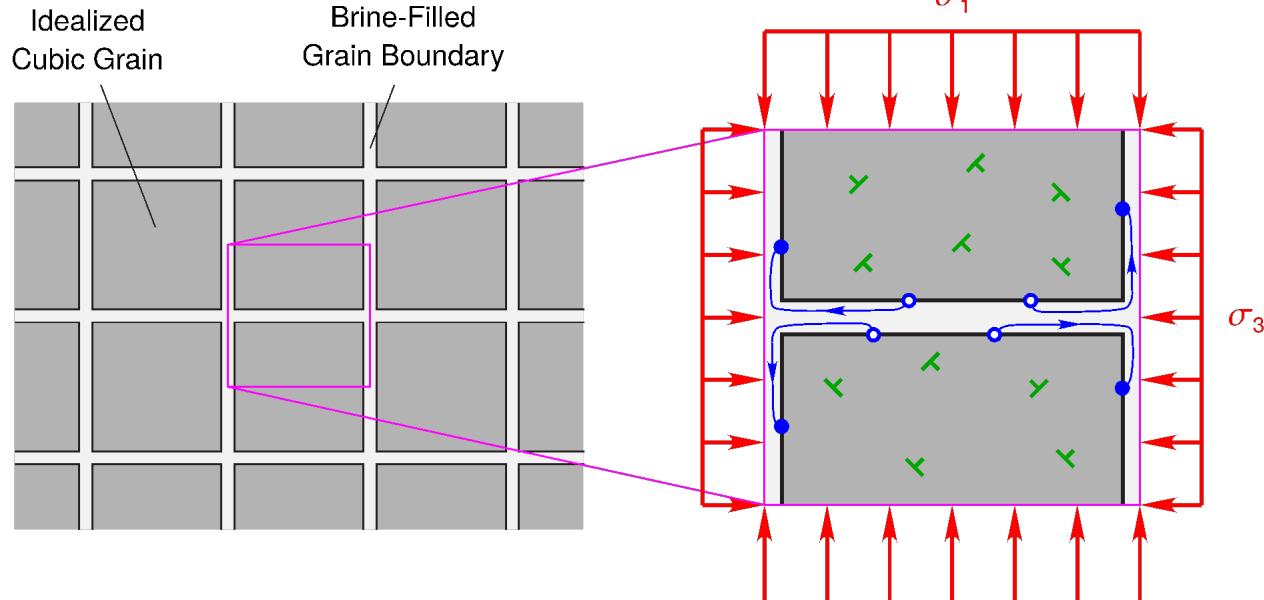
$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



Model Overview

Viscoplastic Branches

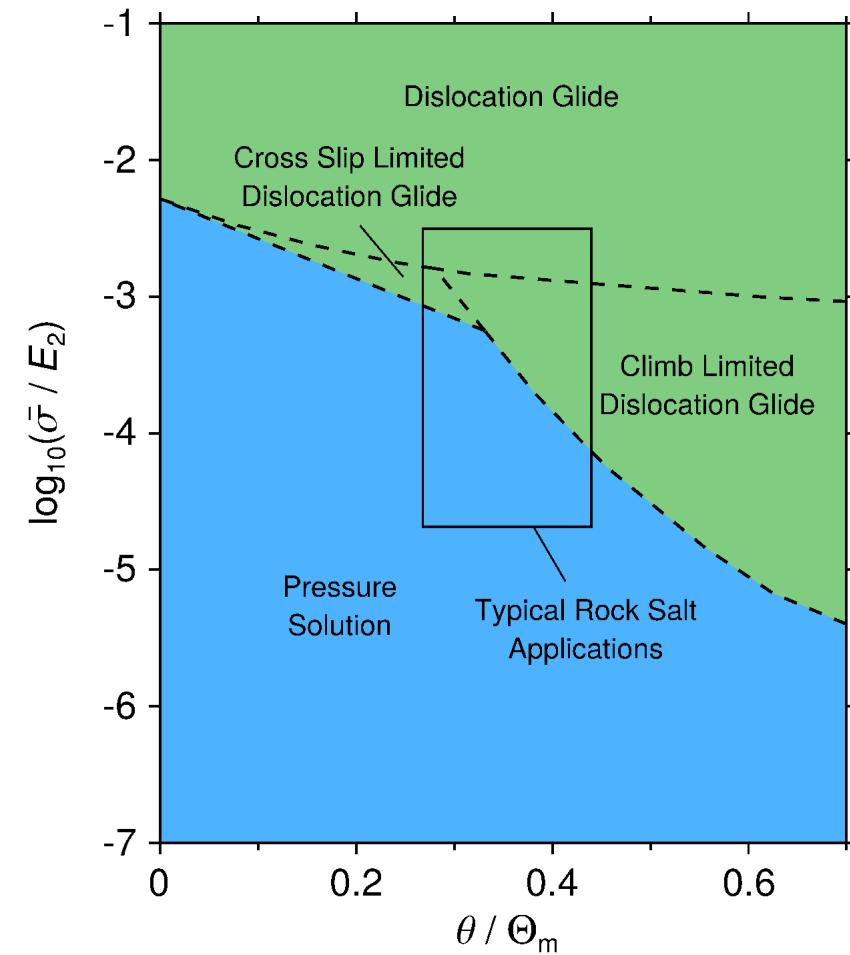
Pressure Solution and Dislocation Glide



Strain Rate Decomposition
(proportional, monotonic, loading)

$$\dot{\varepsilon}^{\text{vp}} = \dot{\varepsilon}^{\text{ps}} + \dot{\varepsilon}^{\text{dg}}$$

A (Rough) Steady-State Deformation Mechanism Map



$$\Theta_m = 1077 \text{ K}$$

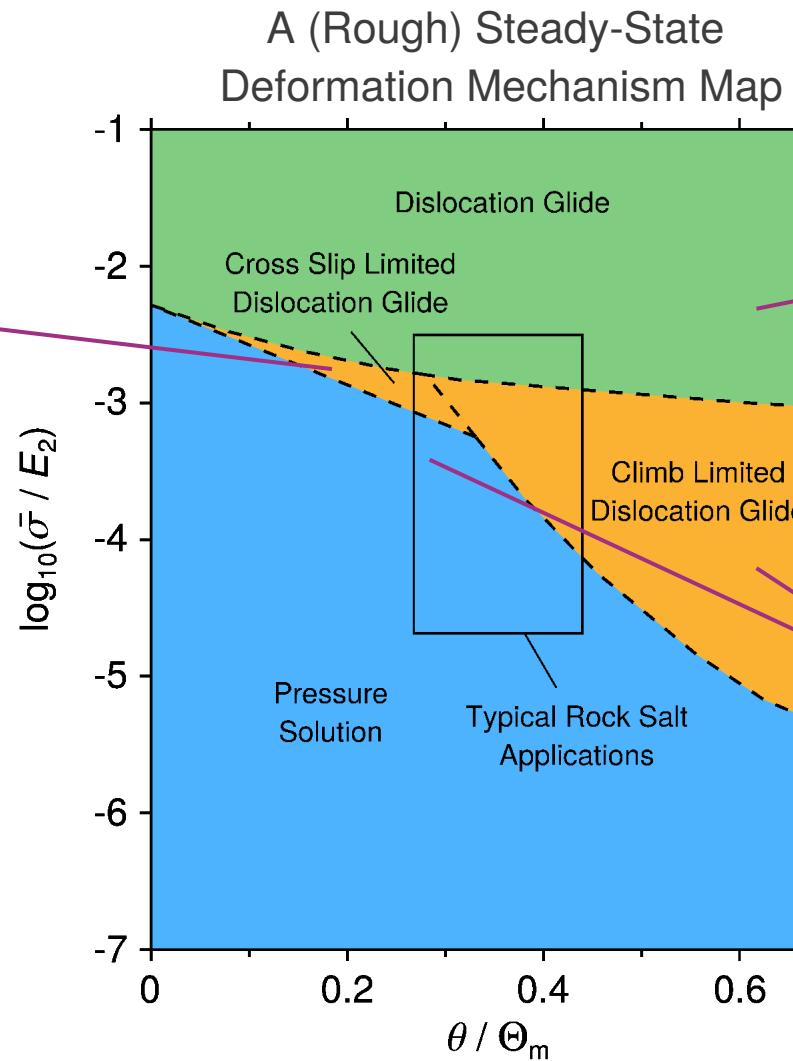
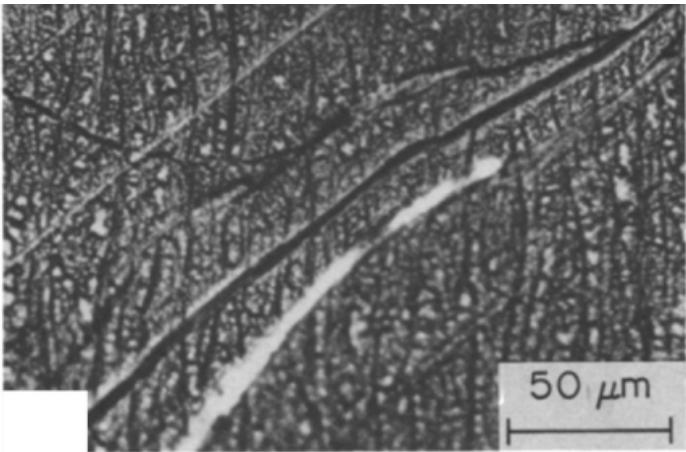
$$E_2 = 10 \text{ GPa}$$

Average grain size = 10 mm

Microstructural Observations

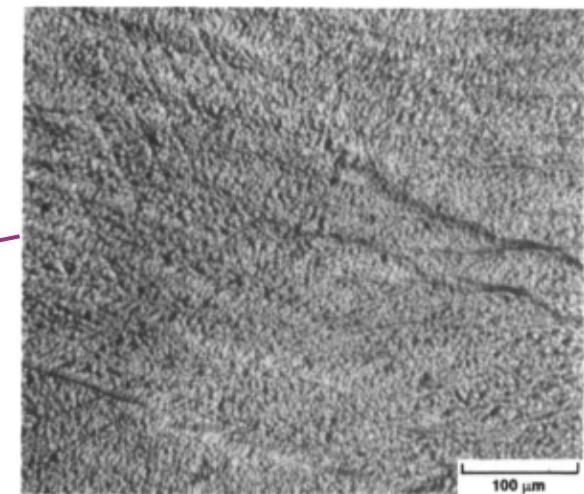


Wavy Slip Bands

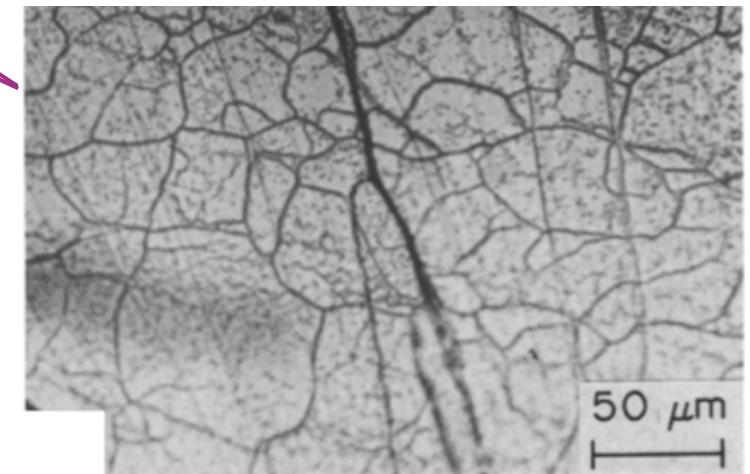


$\Theta_m = 1077$ K
 $E_2 = 10$ GPa
Average grain size = 10 mm

Uniform Dislocation Density



Subgrains



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233–242.

Carter, NL, Horsemann, ST, Russell, JE, and Handin, J (1993). Rheology of rocksalt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

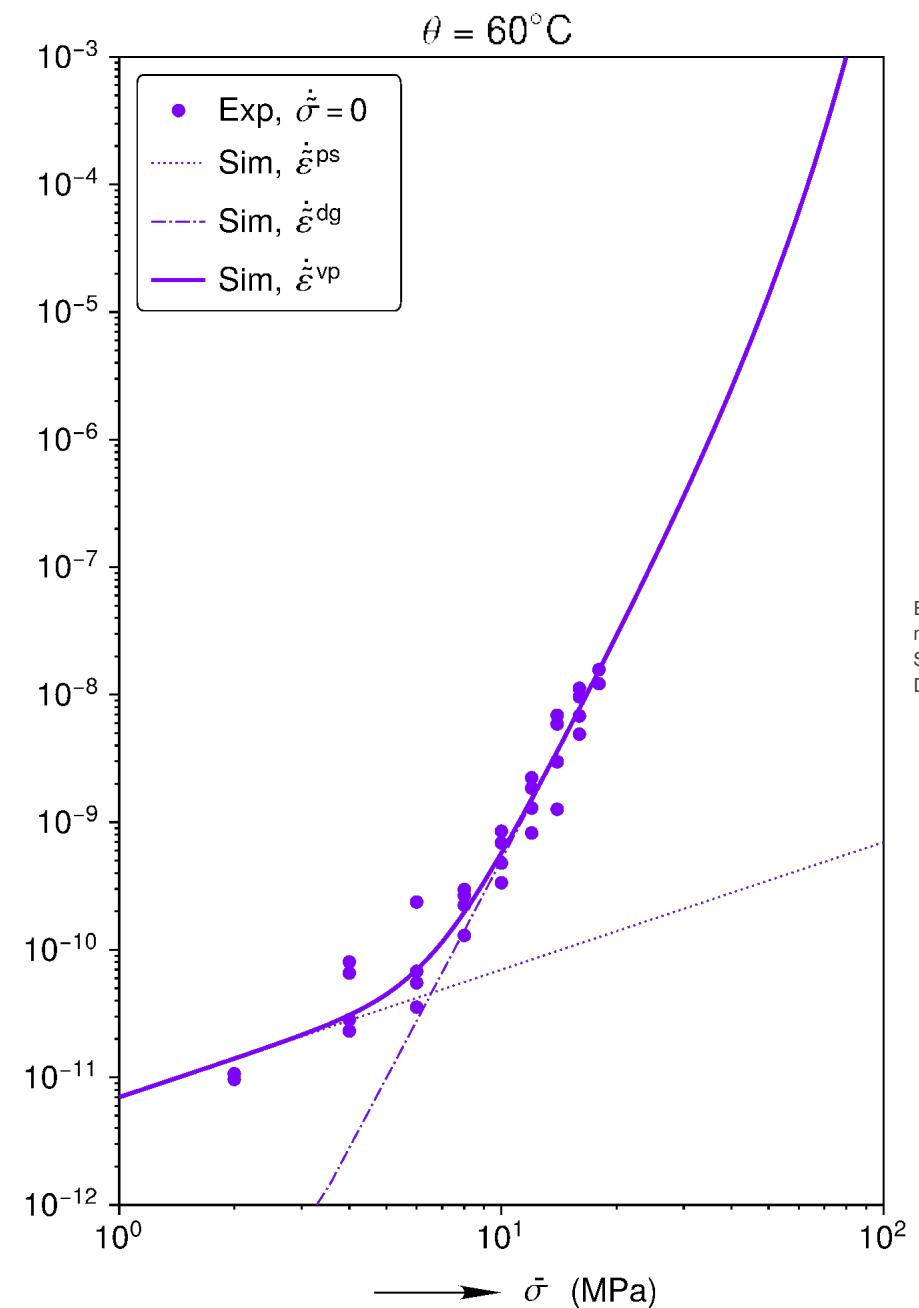
Steady-State Strain Rates

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\varepsilon}^{\text{dg}} = Y_3 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{Y_4}\right) \right]^{Y_5}$$

Spies, C., Schutjens, P., Brzesowsky, R., Peach, C., Liezenberg, J., and Zwart, H. (1990). Experimental determination of constitutive parameters governing creep of rocksalt by pressure solution. In: Geological Society, London, Special Publications 54.1, pp. 215–227.

Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.



Experimental measurements from: Salzer et al. (2015) and Düsterloh et al. (2015).

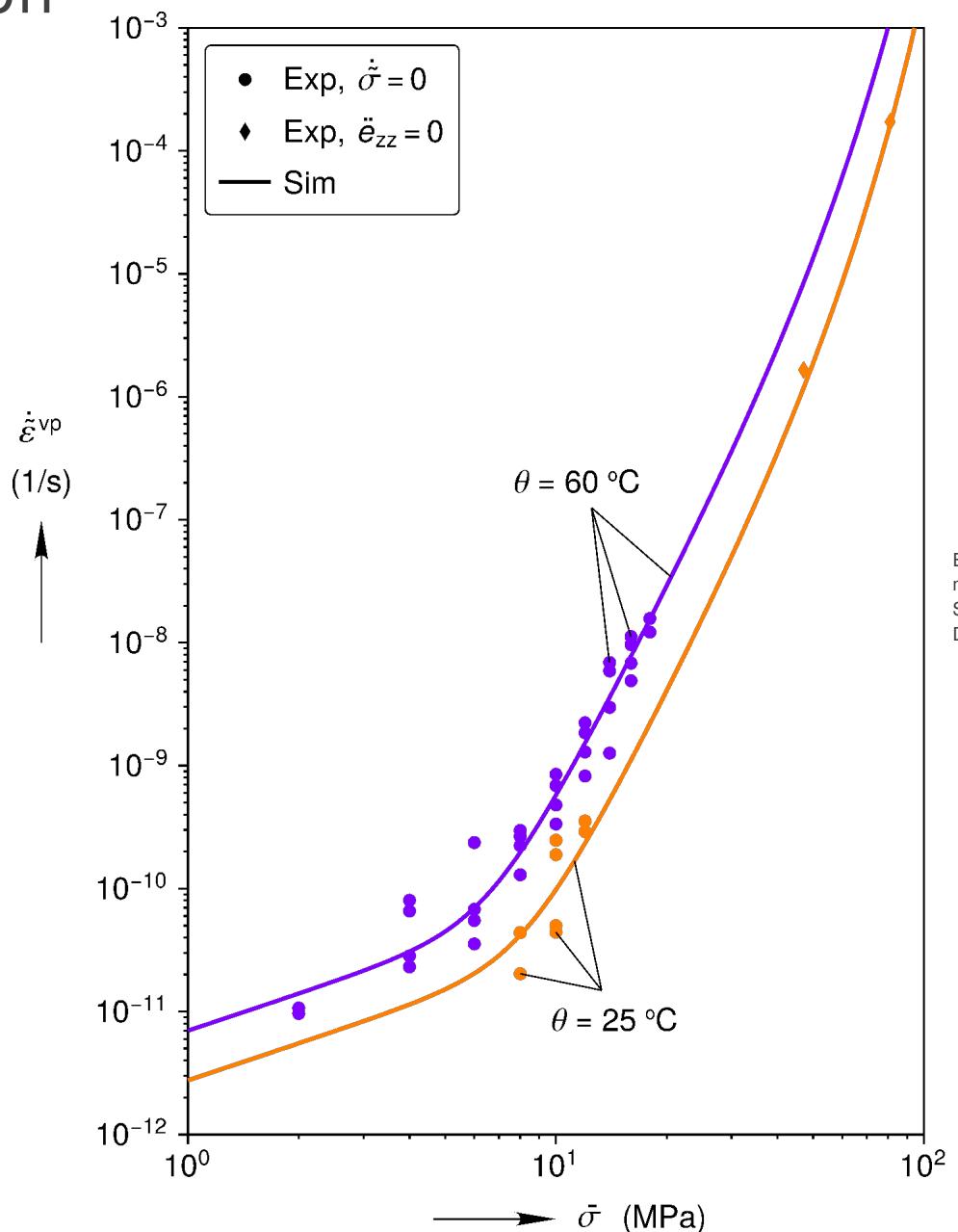
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Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.



Strain Rates While Hardening



Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

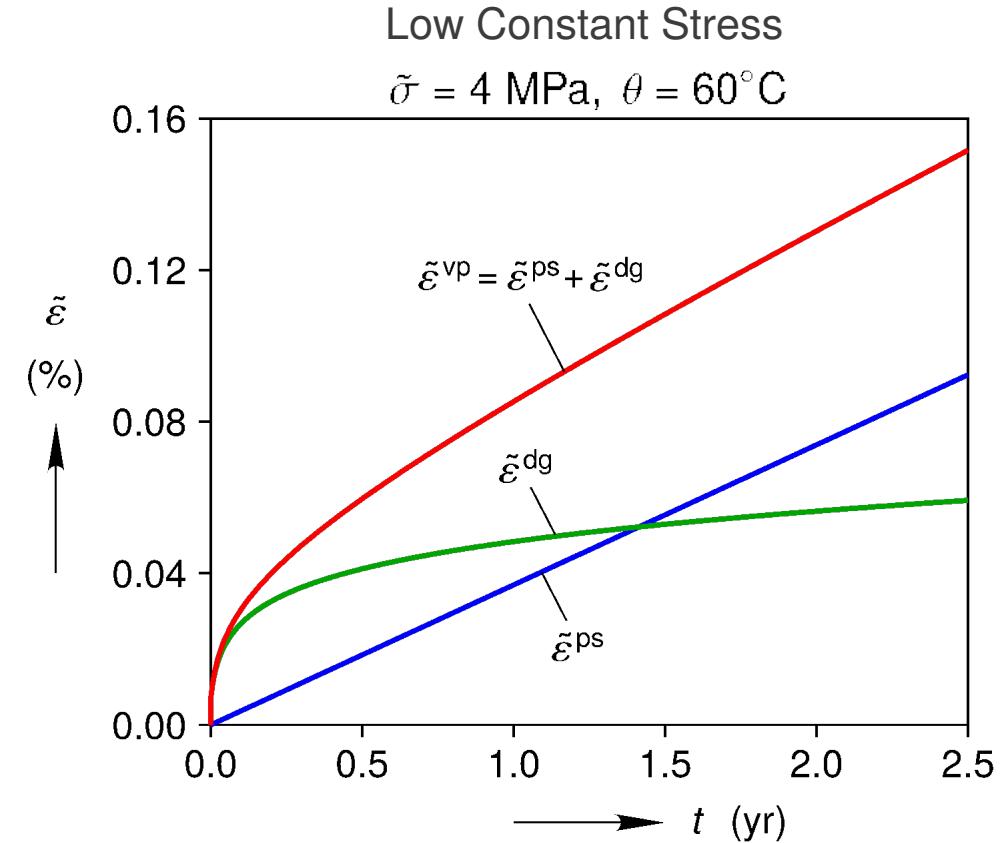
Strain Rates While Hardening



Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$





Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

Dislocation Glide Hardening



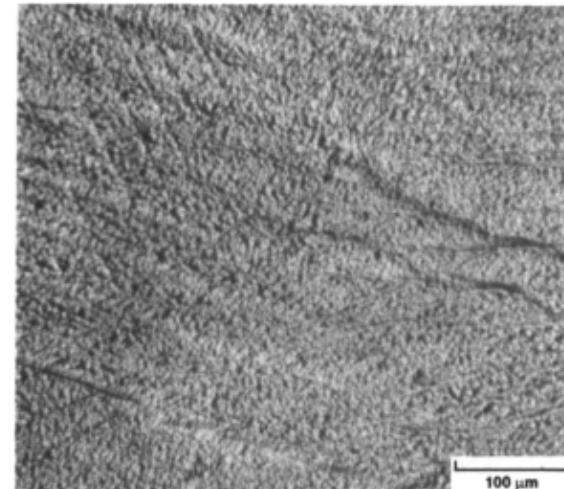
Equivalent Stress Decomposition
(proportional, monotonic, loading)

$$\tilde{\sigma} = \tilde{b} + \tilde{\sigma}^{dg}$$

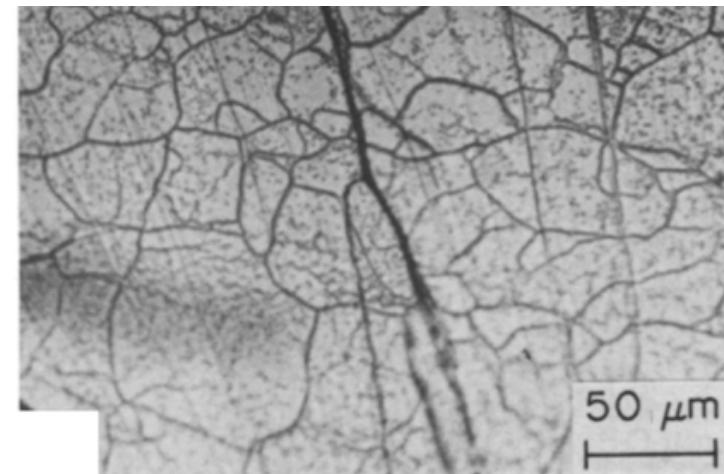
Drag Stress Contribution

$$\tilde{\sigma}^{dg} = y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$

Uniform Dislocation Density

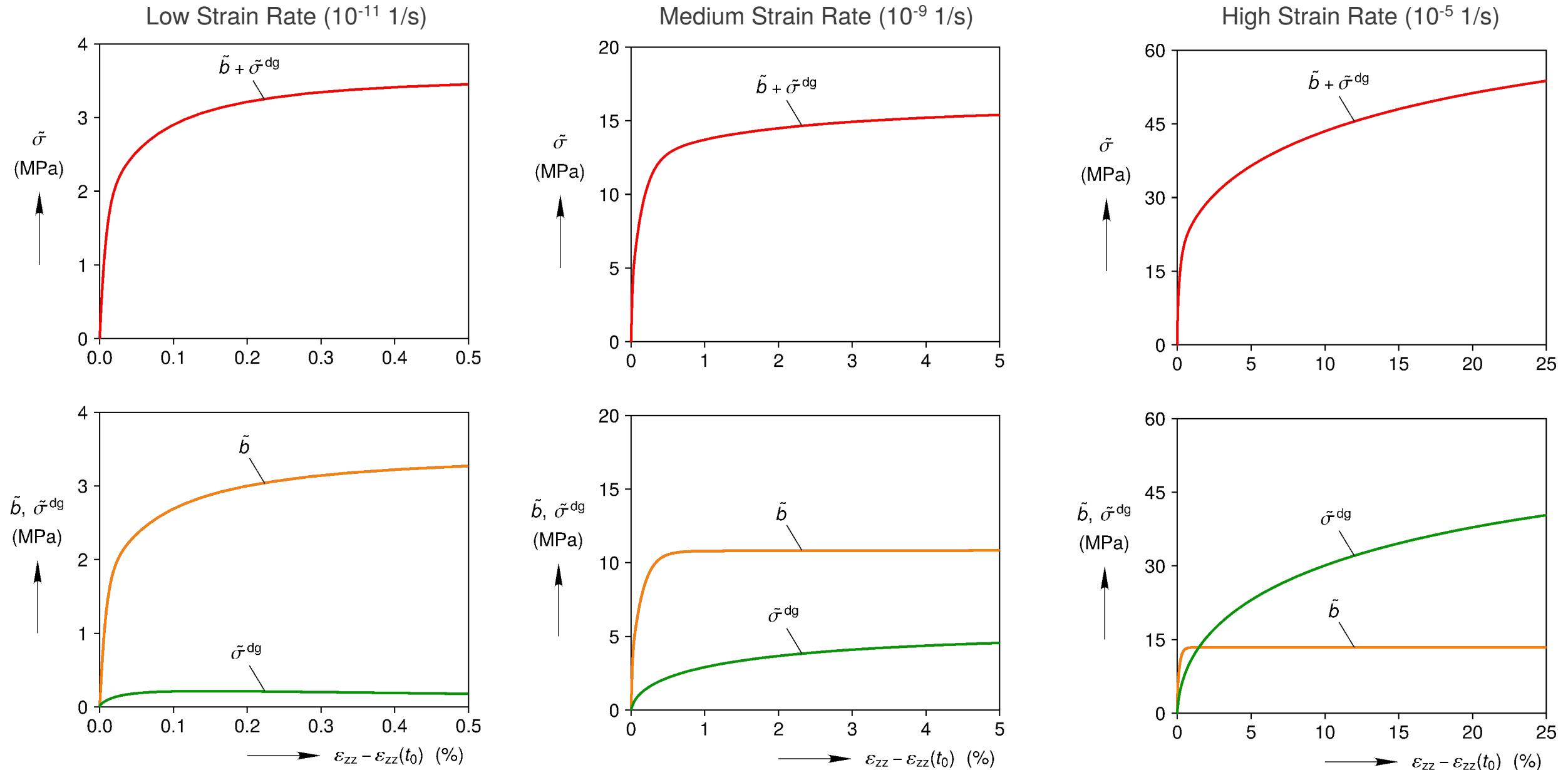


Non-Uniform Dislocation Density



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233–242.
Carter, NL, Horsemann, ST, Russell, JE, and Handin, J (1993). Rheology of rocksalt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

Constant Strain Rate Behavior



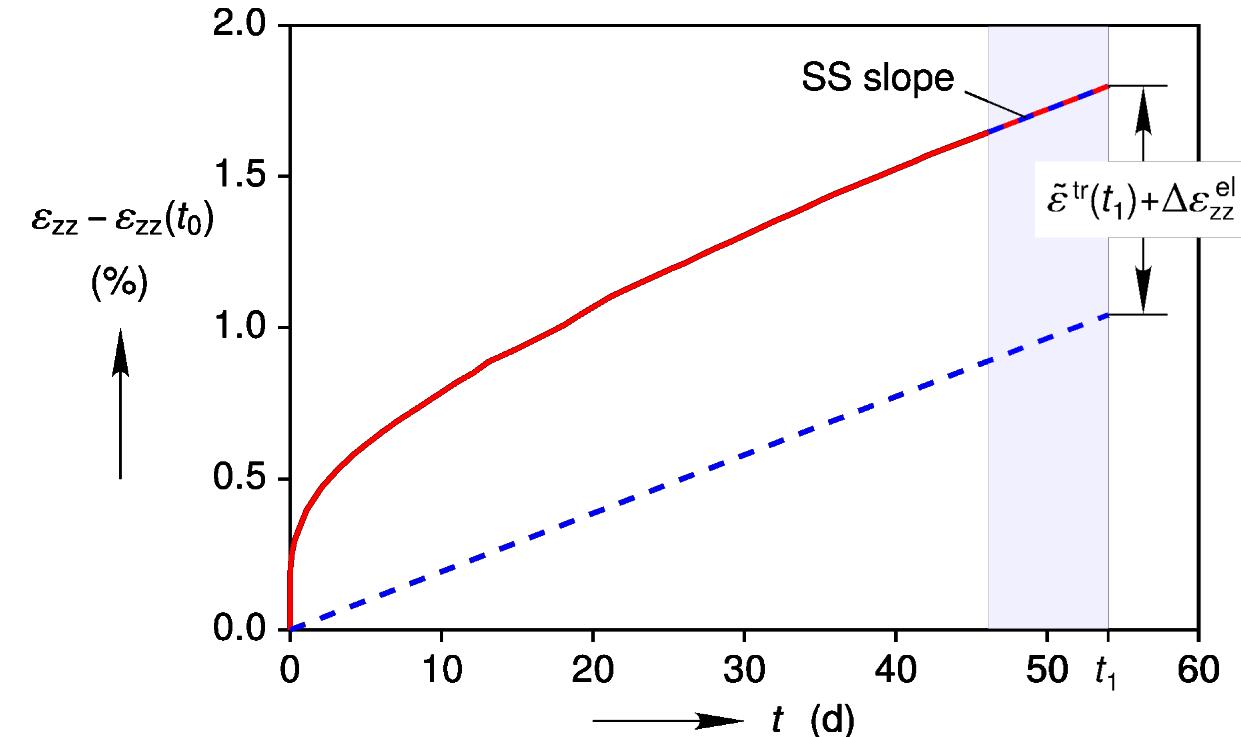
Calibrations

Hardening Transition from Low to Medium Stresses (Strain Rates)

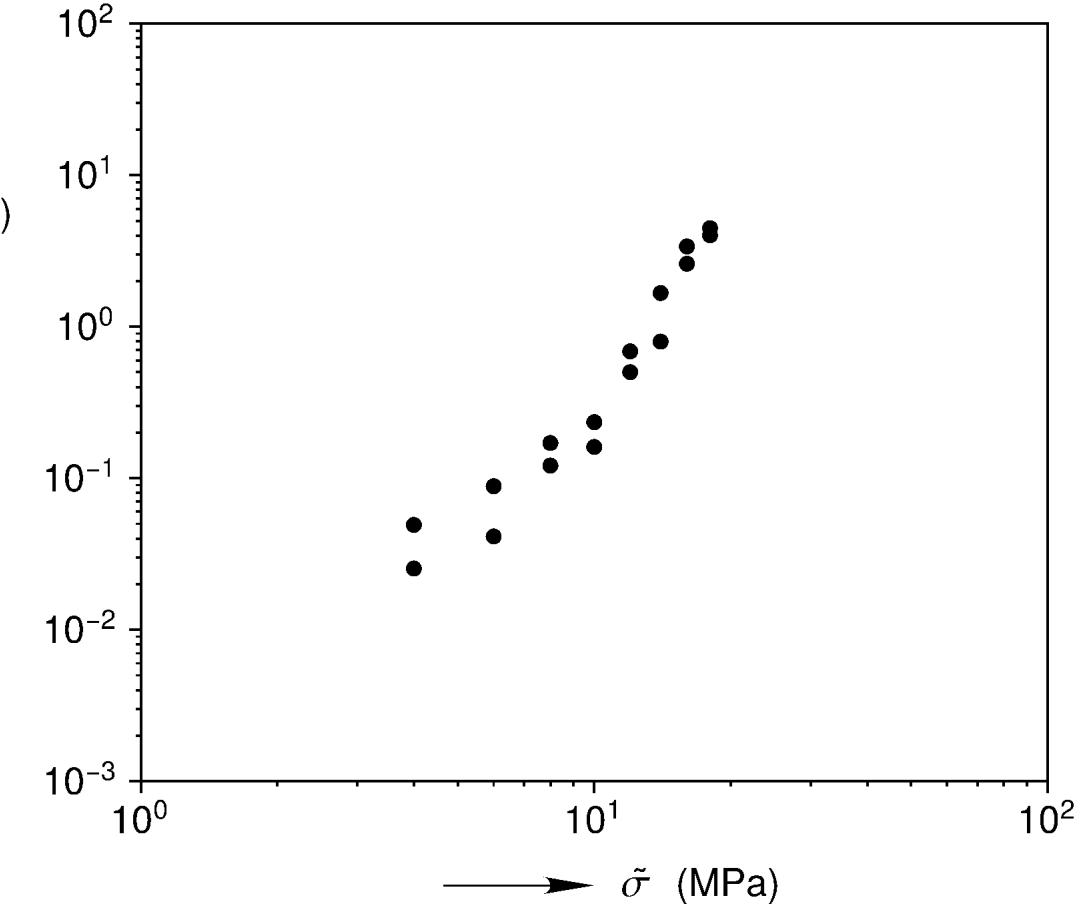


WIPP Salt, Constant Stress, Strain History

$$\tilde{\sigma} = 12 \text{ MPa}, \theta = 60^\circ\text{C}$$



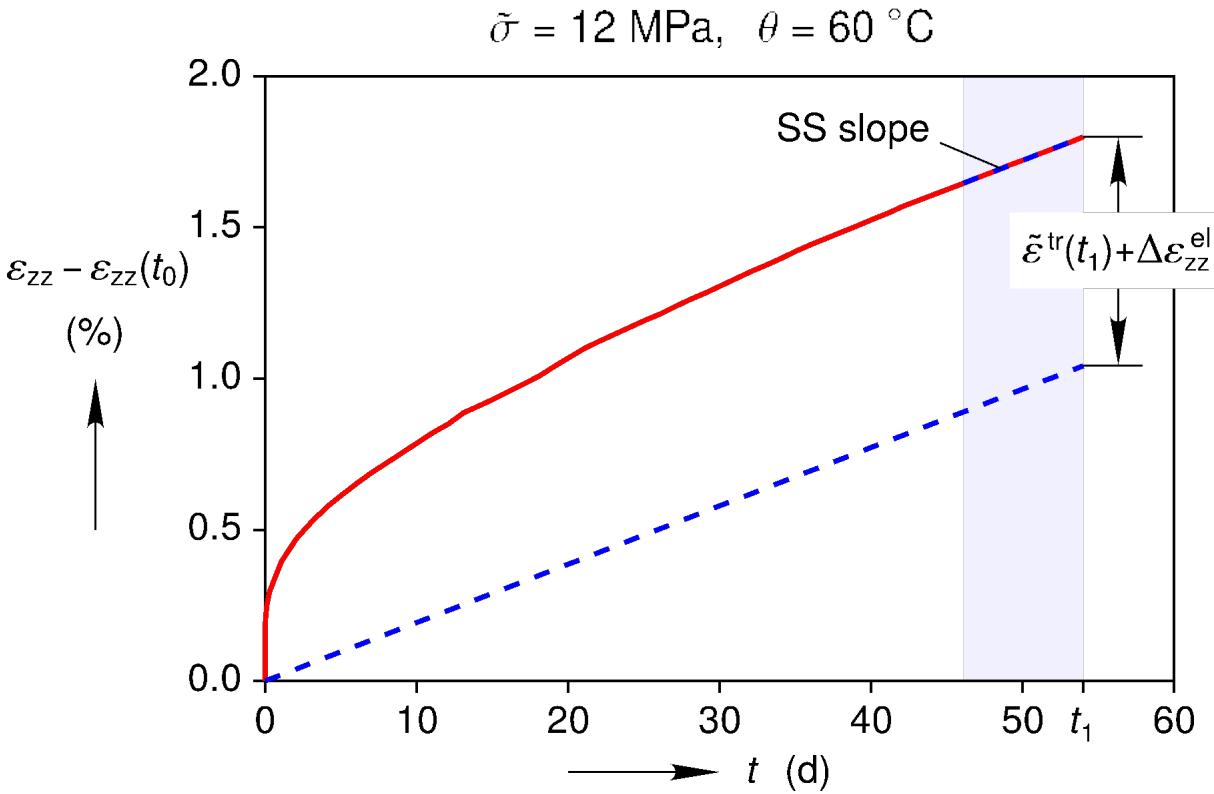
Transient Strain After 50 days at $\theta = 60^\circ\text{C}$



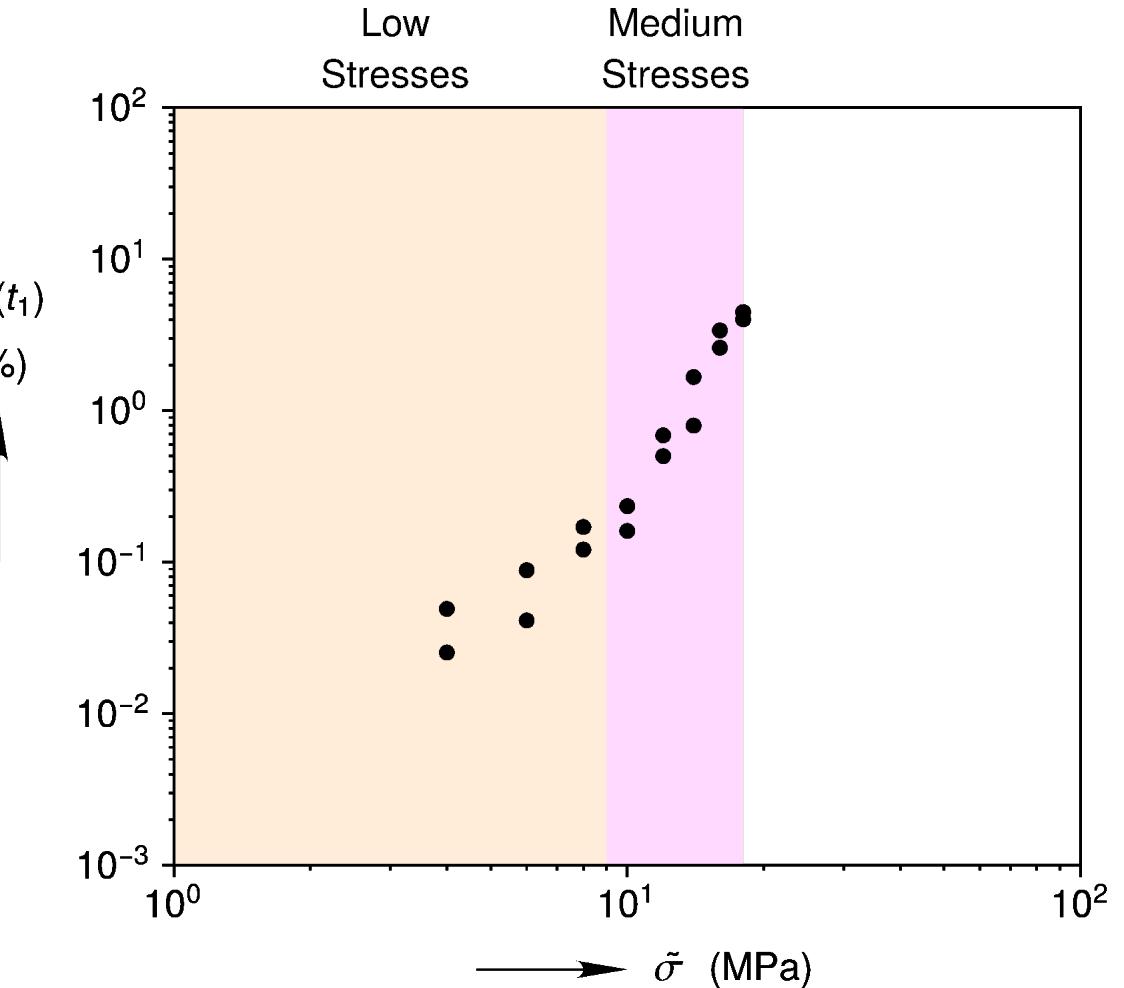
Hardening Transition from Low to Medium Stresses (Strain Rates)



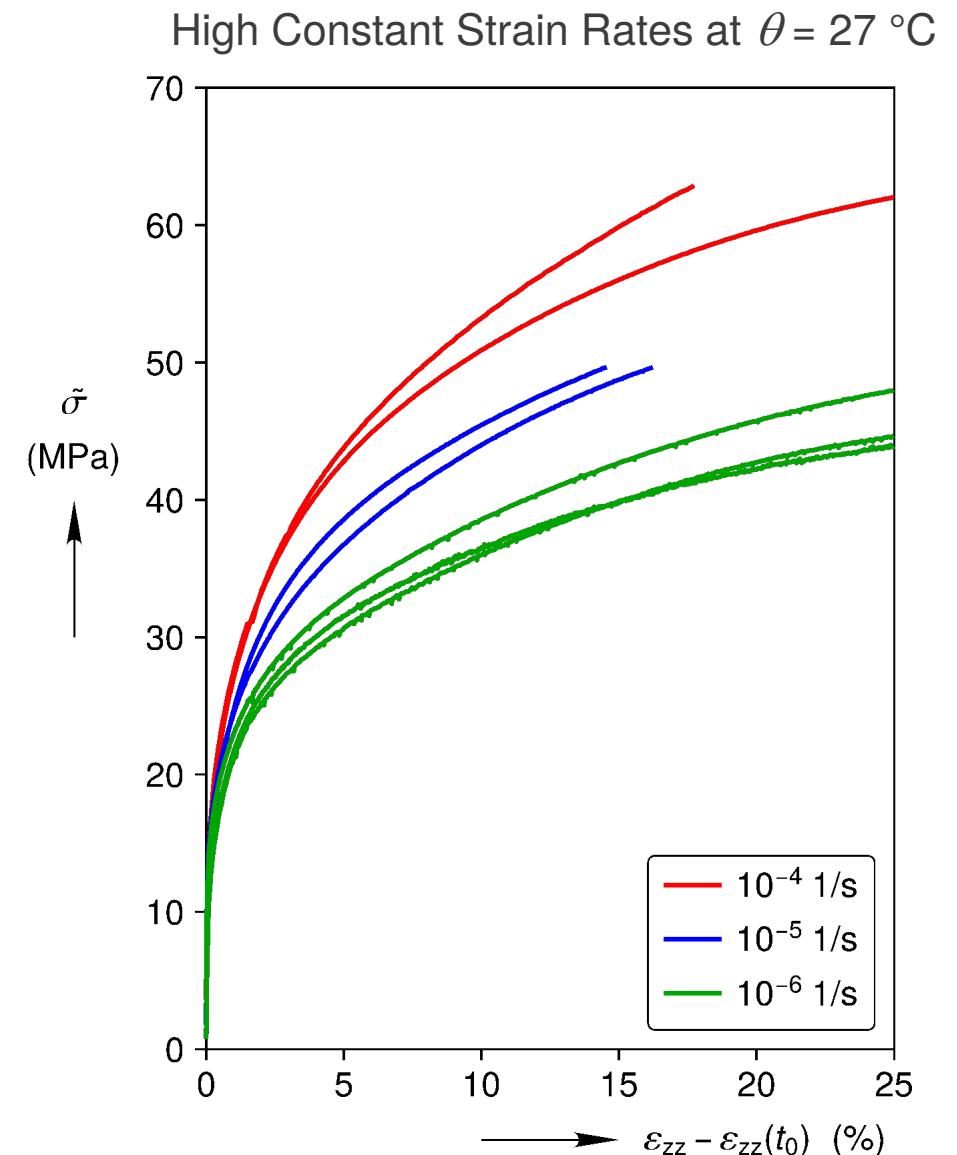
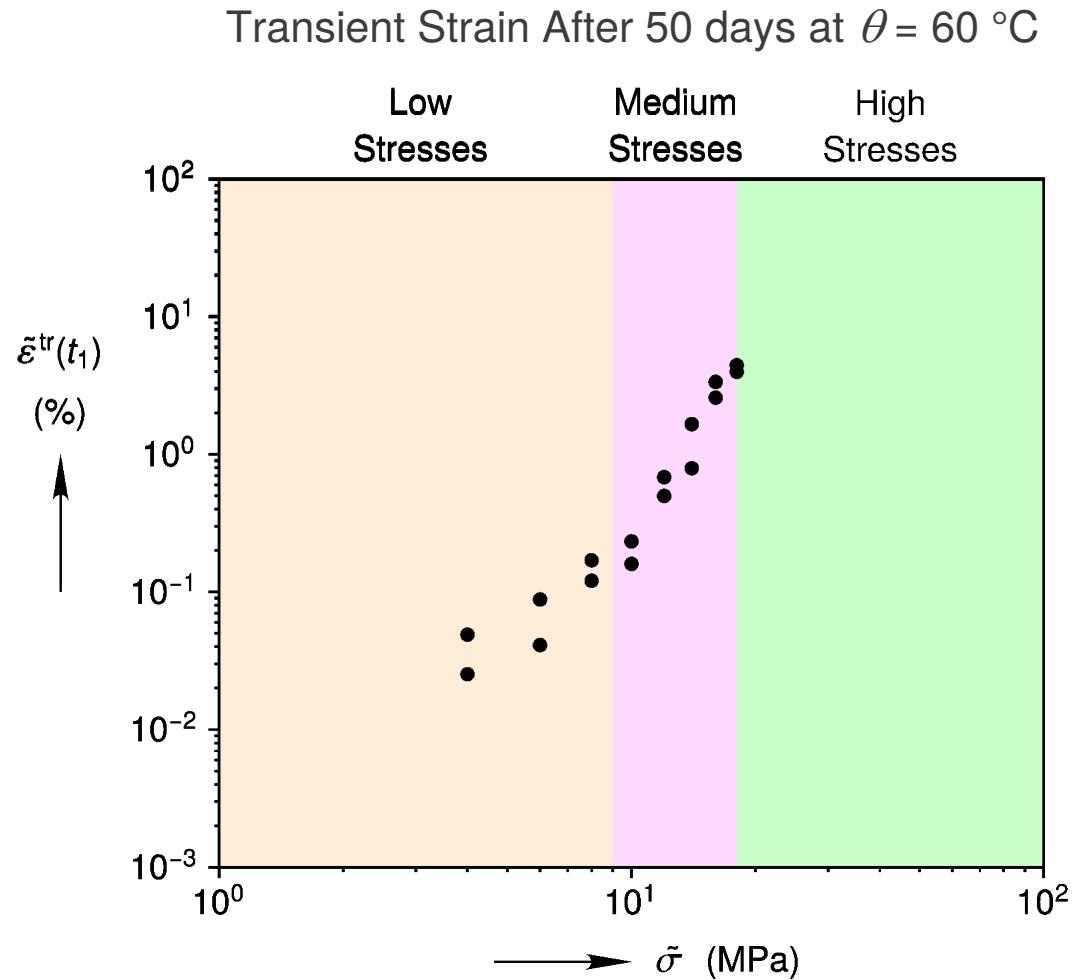
WIPP Salt, Constant Stress, Strain History



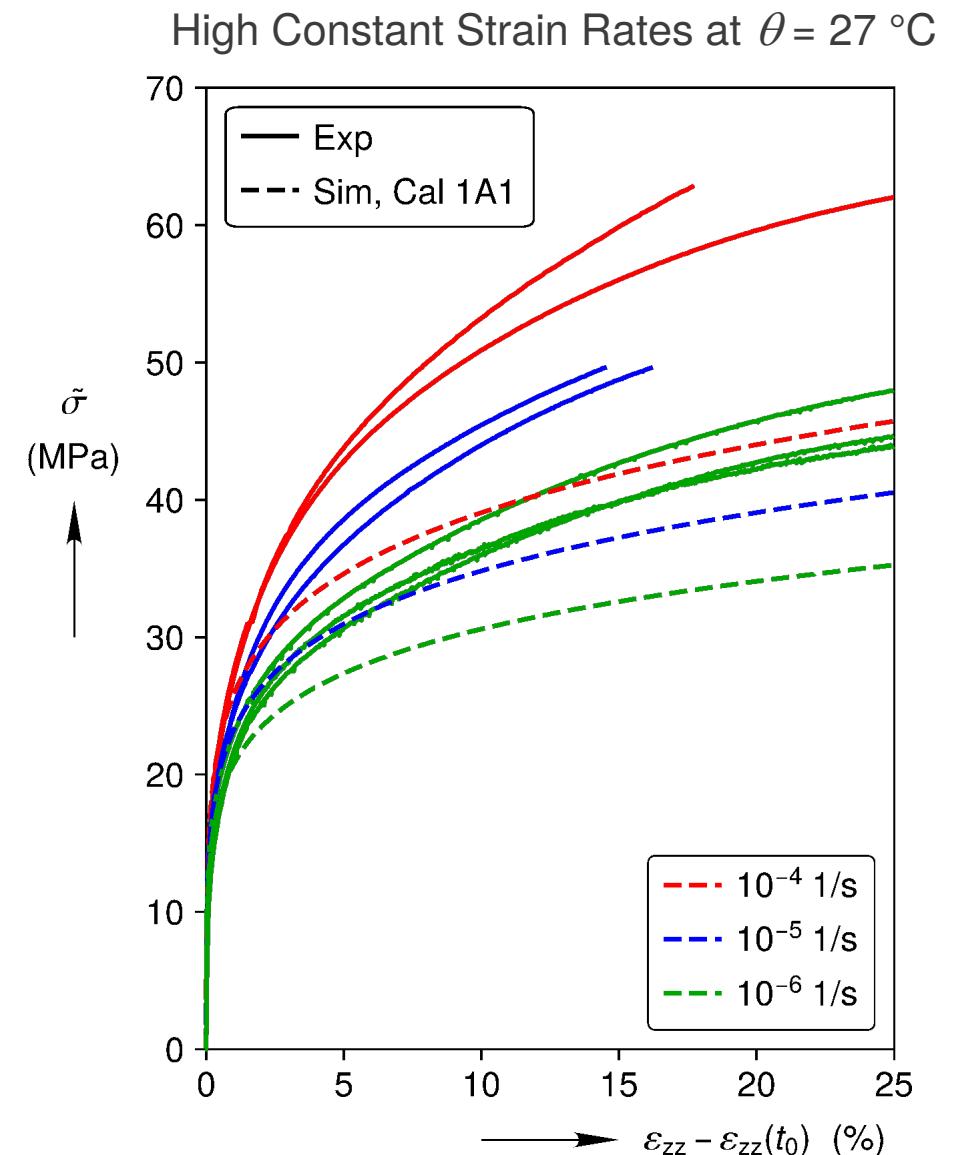
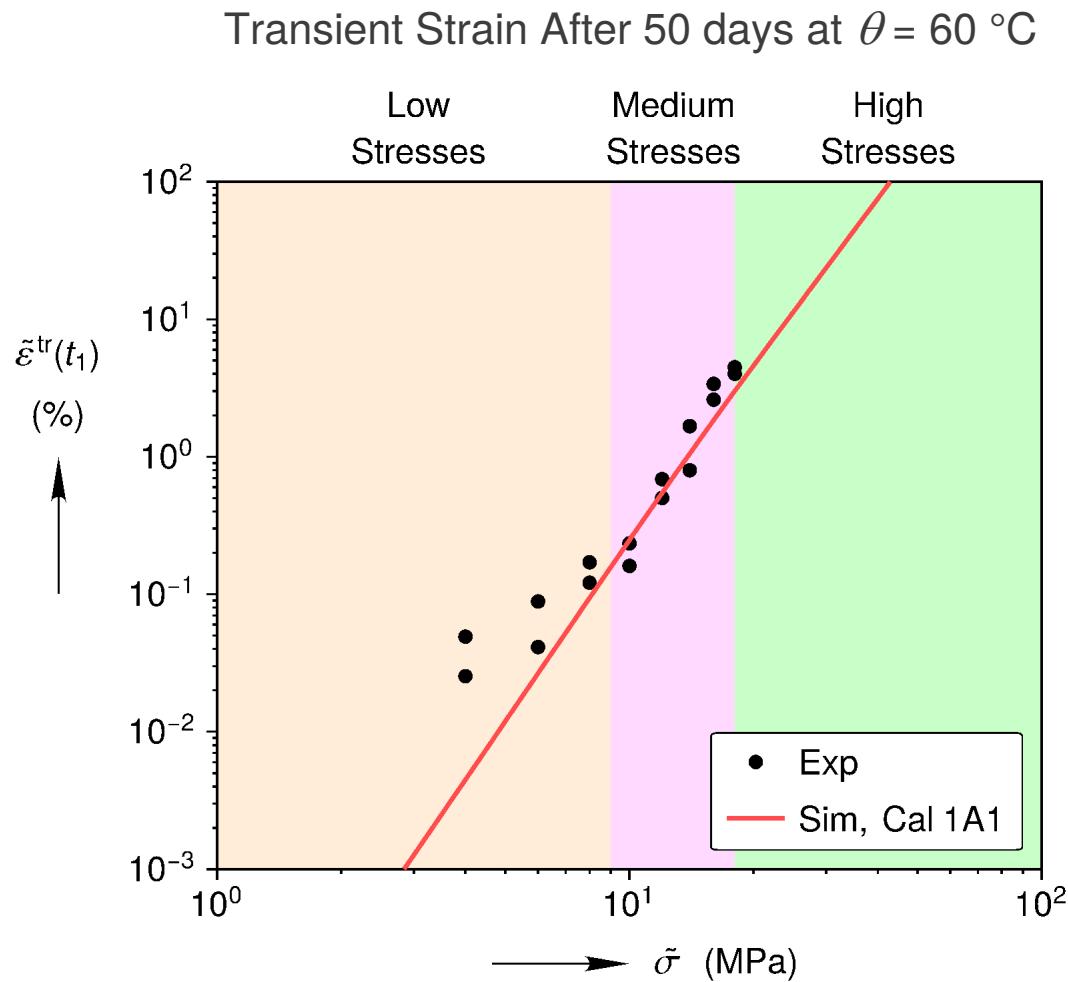
Transient Strain After 50 days at $\theta = 60^\circ\text{C}$



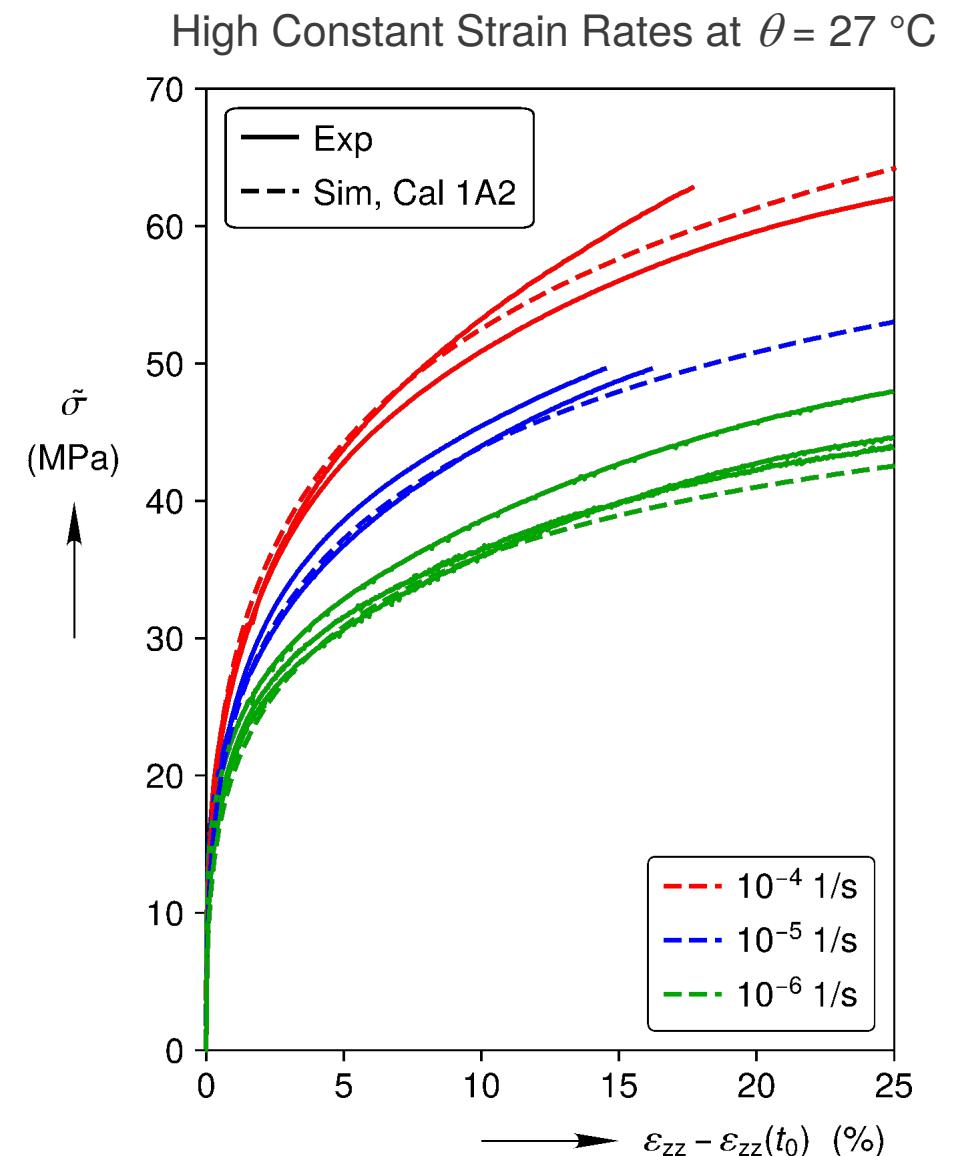
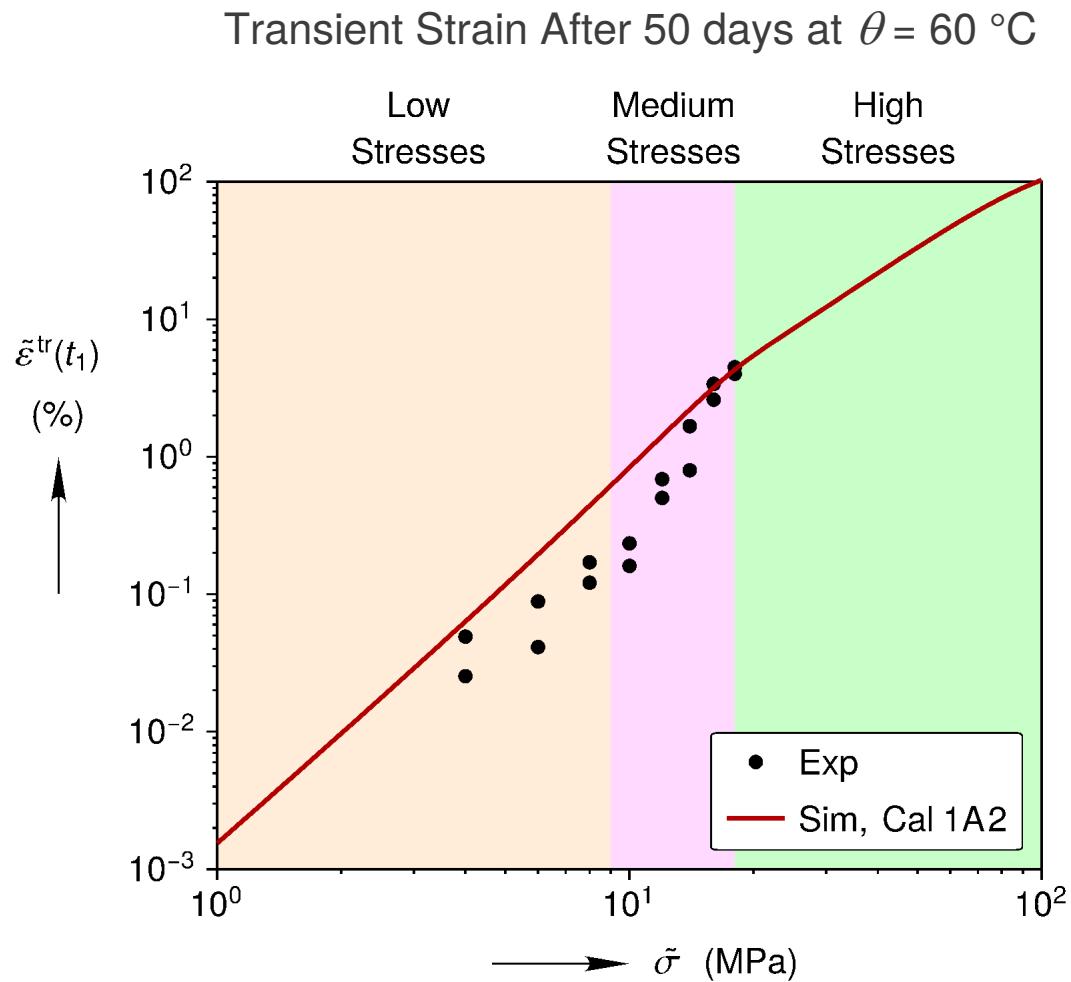
Selected Hardening Measurements on WIPP Salt



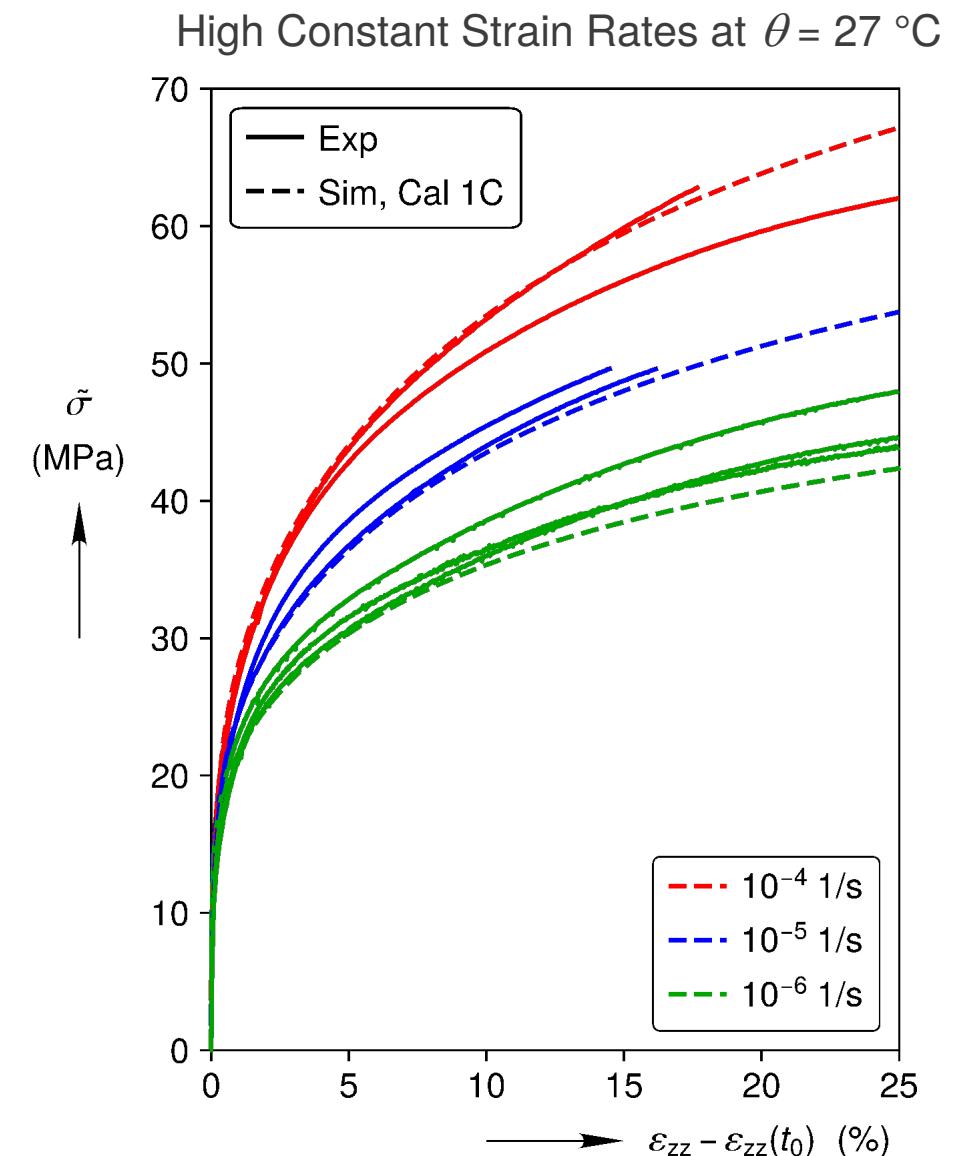
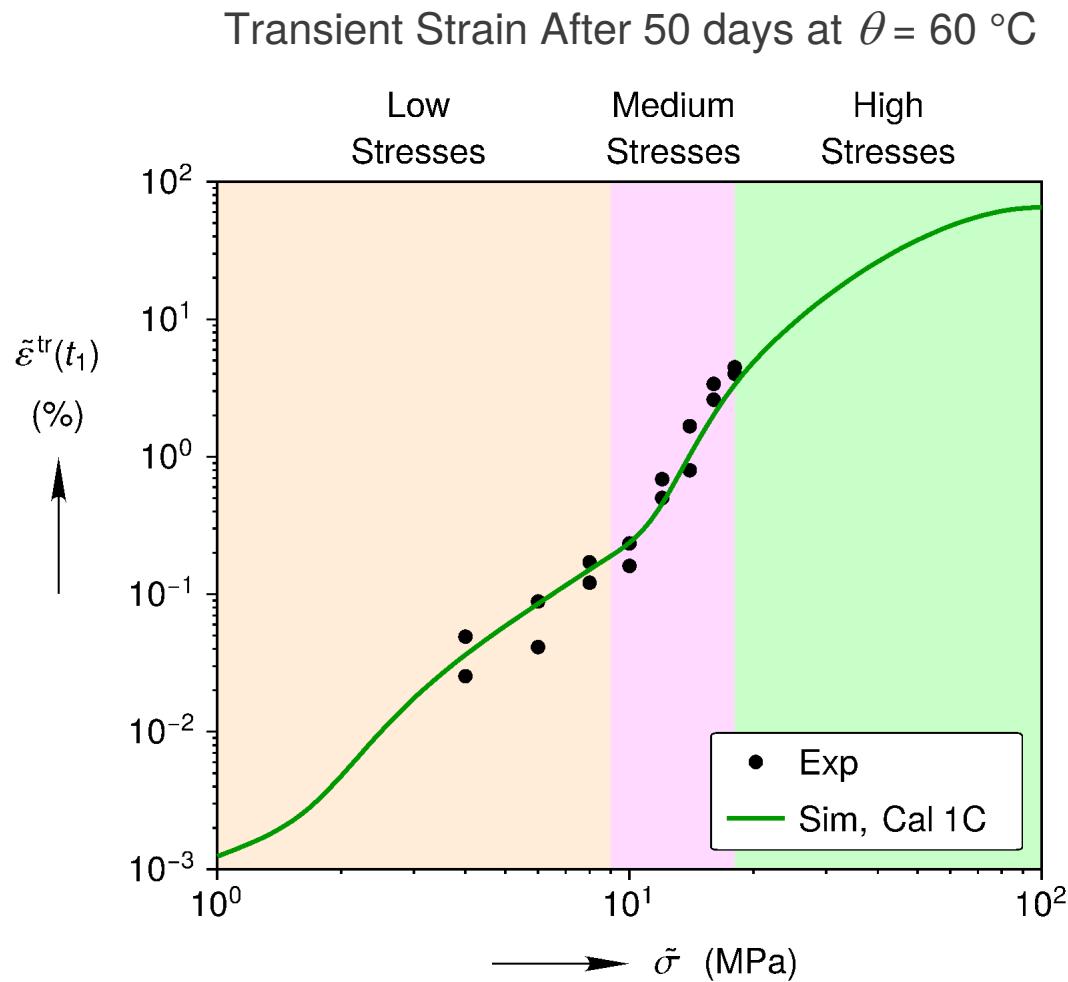
Calibration 1A1: Drag Stress Only



Calibration 1A2: Drag Stress Only

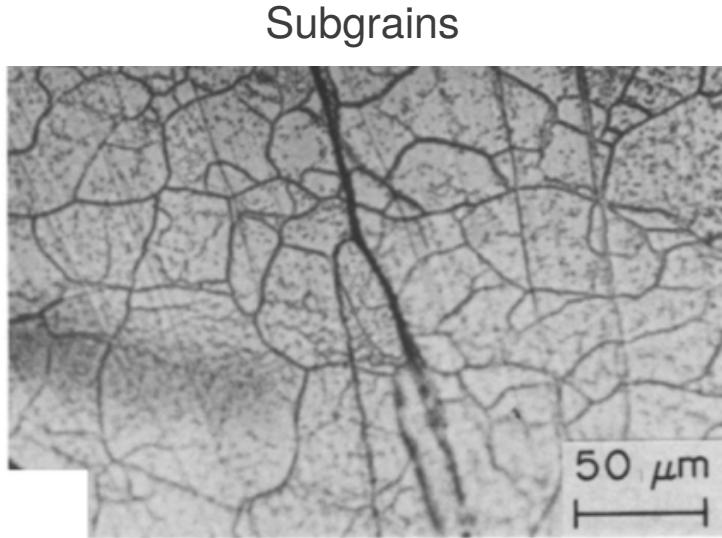


Calibration 1C: Drag Stress and Back Stress



Non-Monotonic Loading

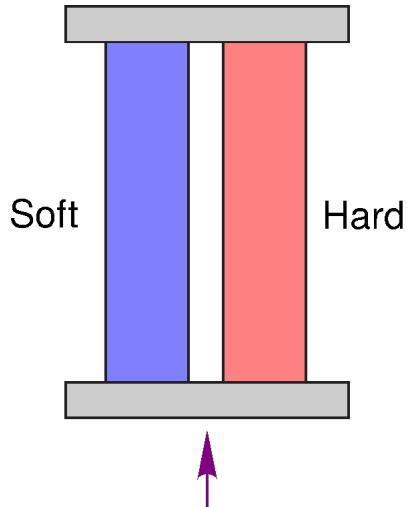
A “Gedankenexperiment”



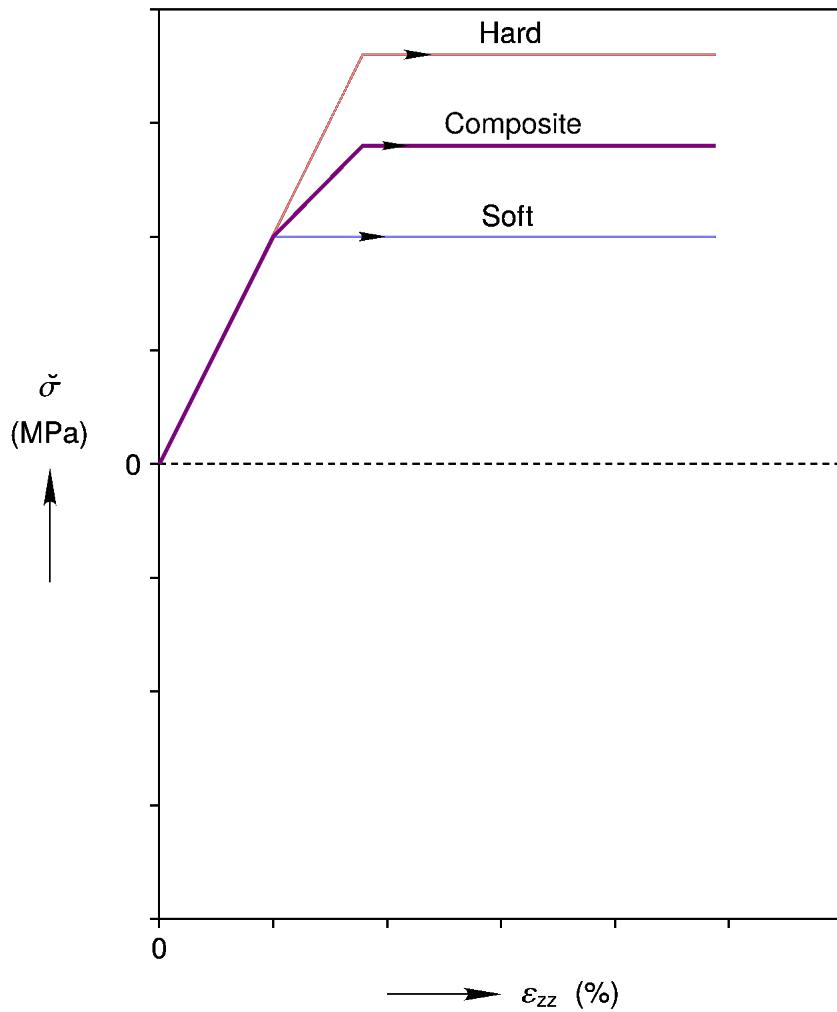
Subgrains

Schematic

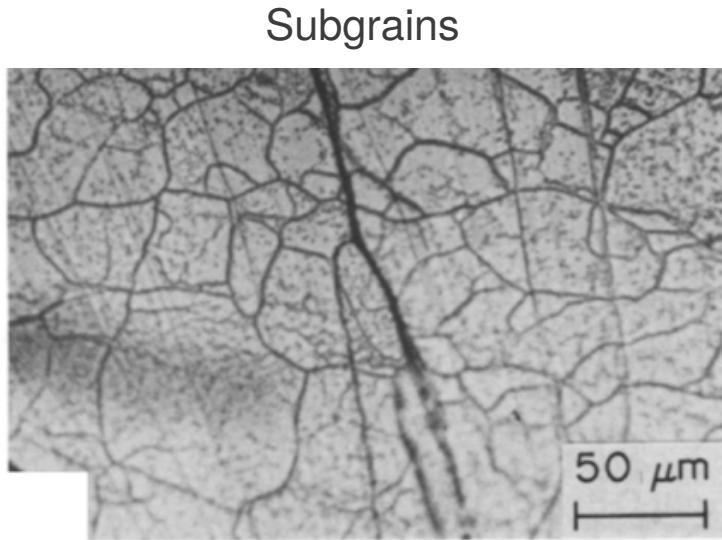
$$f = \check{\sigma} A$$



Mechanical Responses



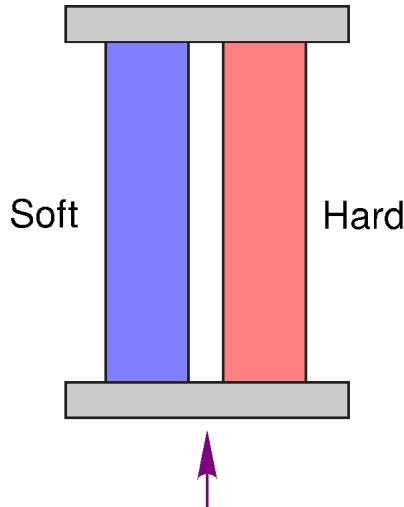
A “Gedankenexperiment”



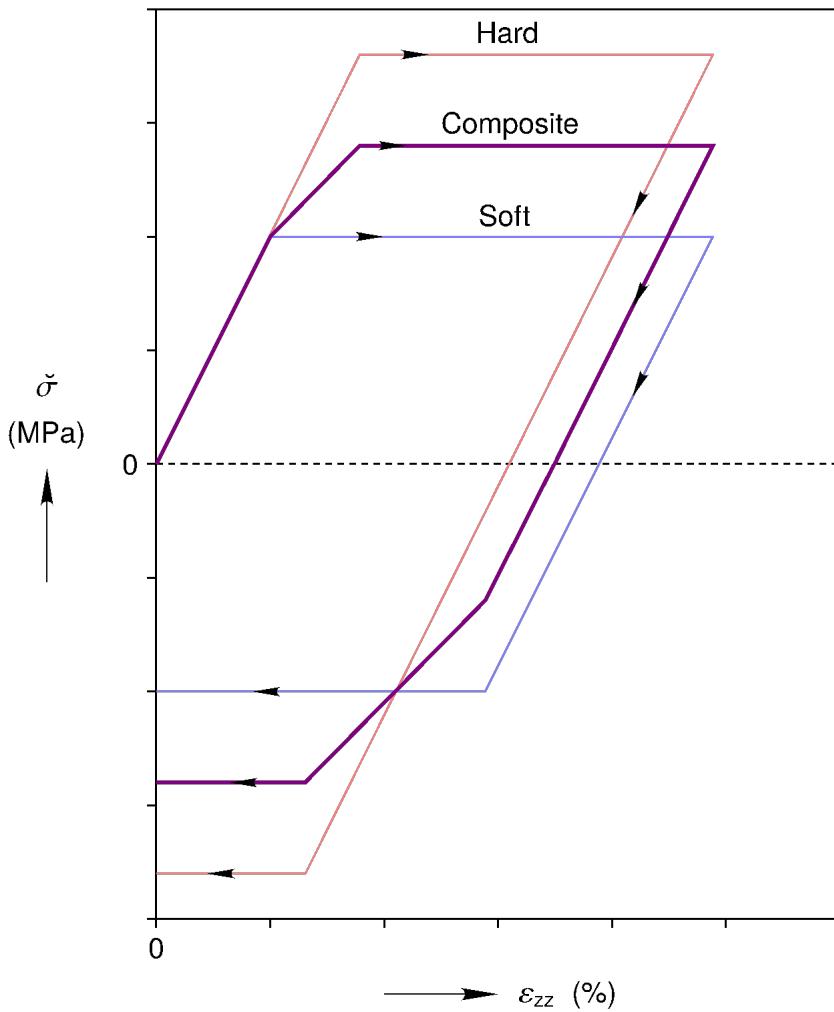
Subgrains

Schematic

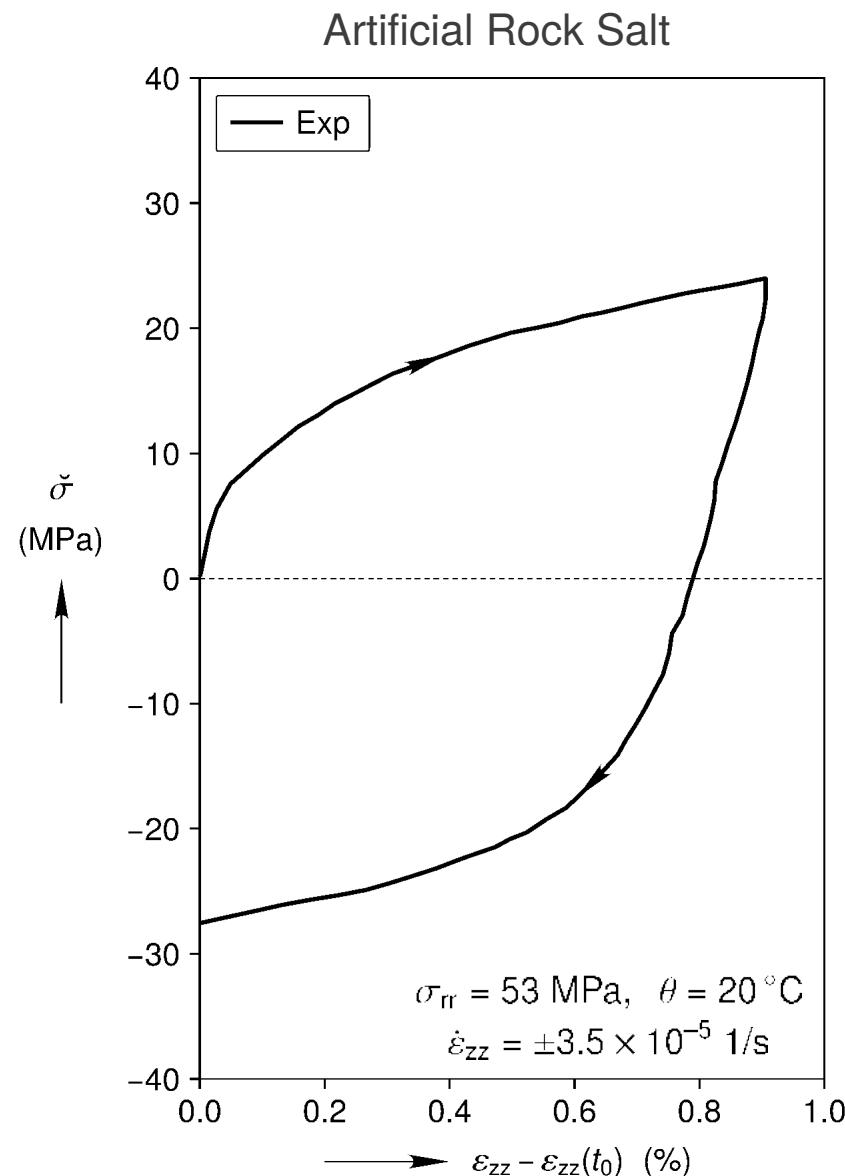
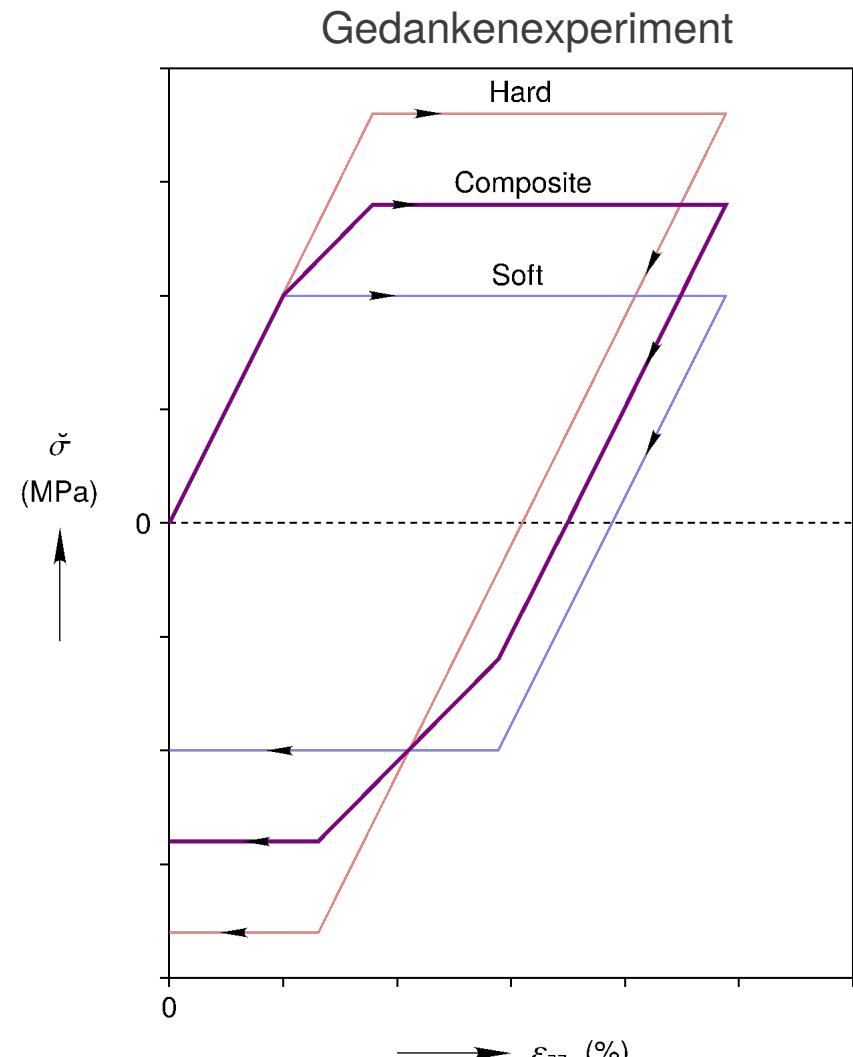
$$f = \check{\sigma} \circ A$$



Mechanical Responses



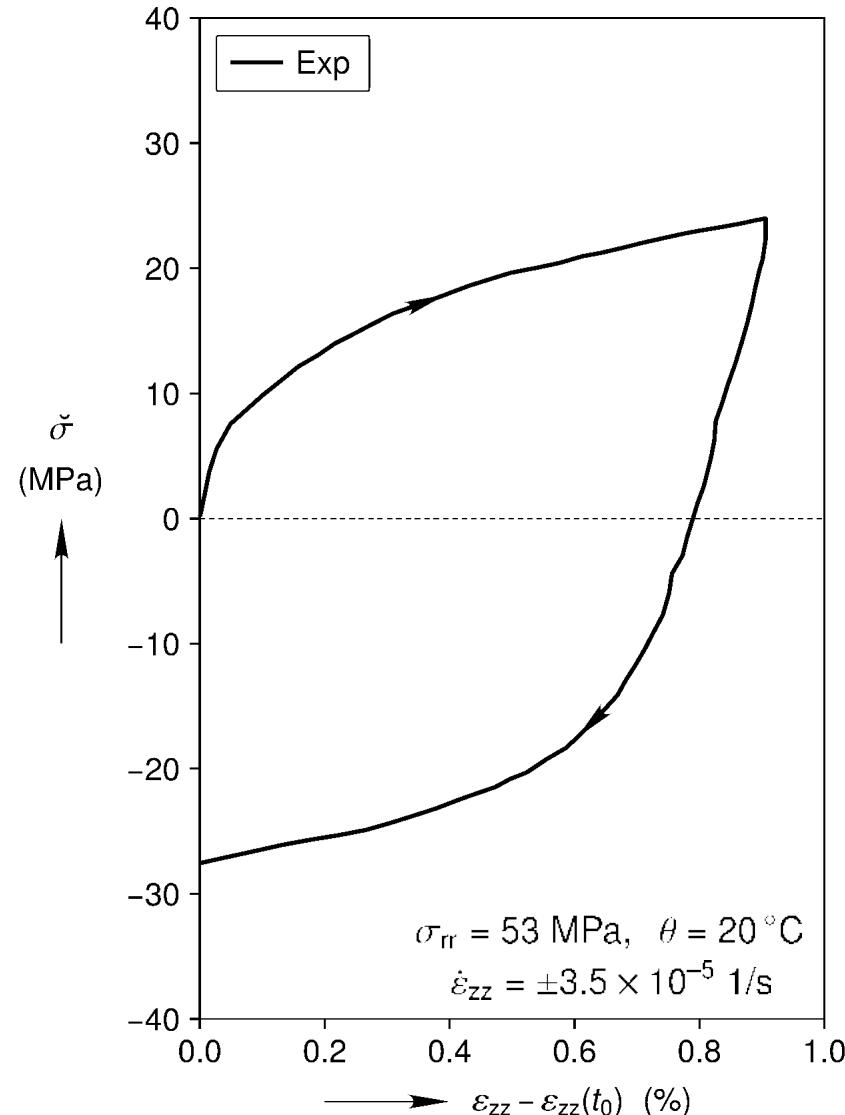
Bauschinger Effect



Re-hardening during Non-Monotonic Loading

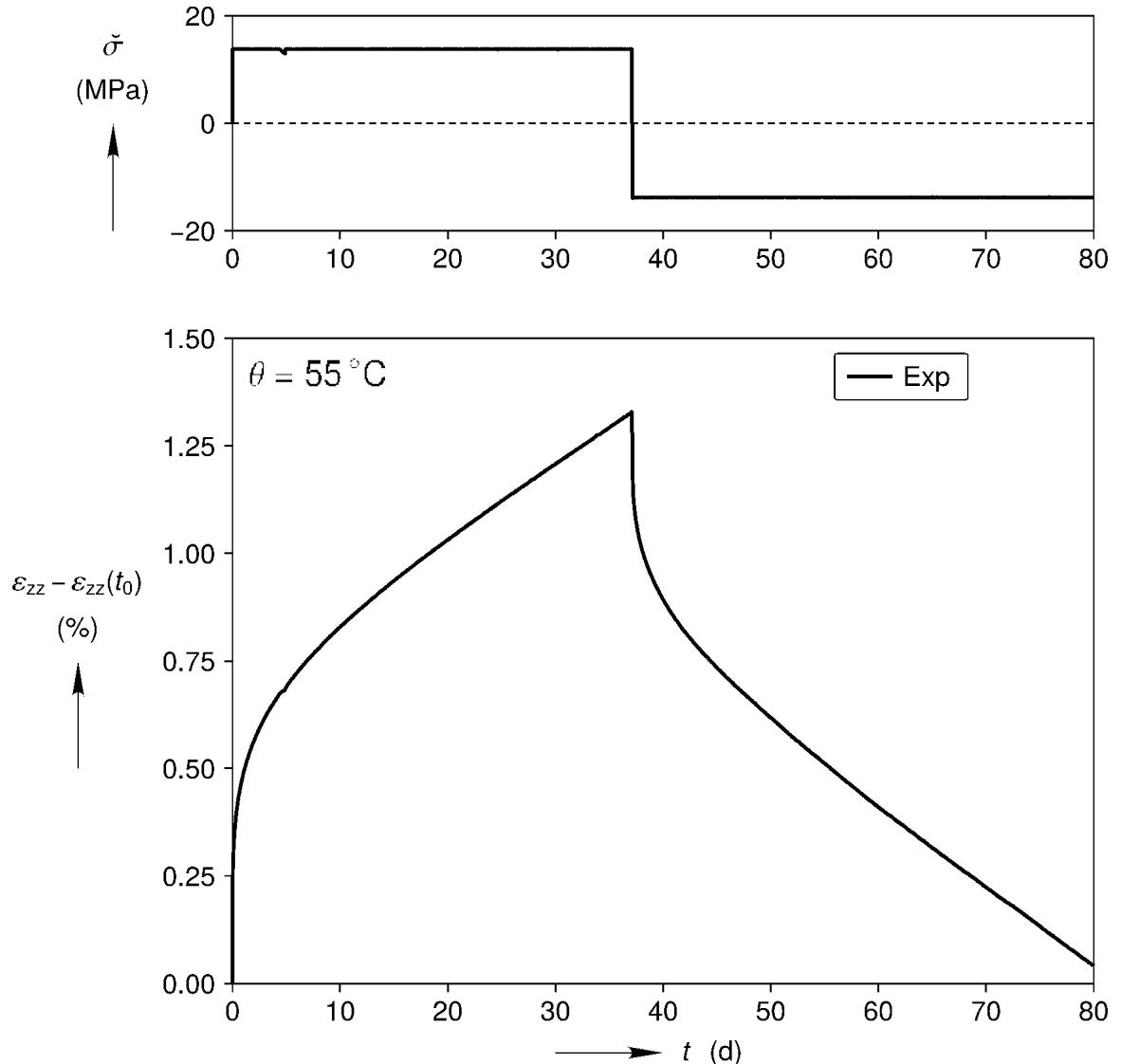


Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

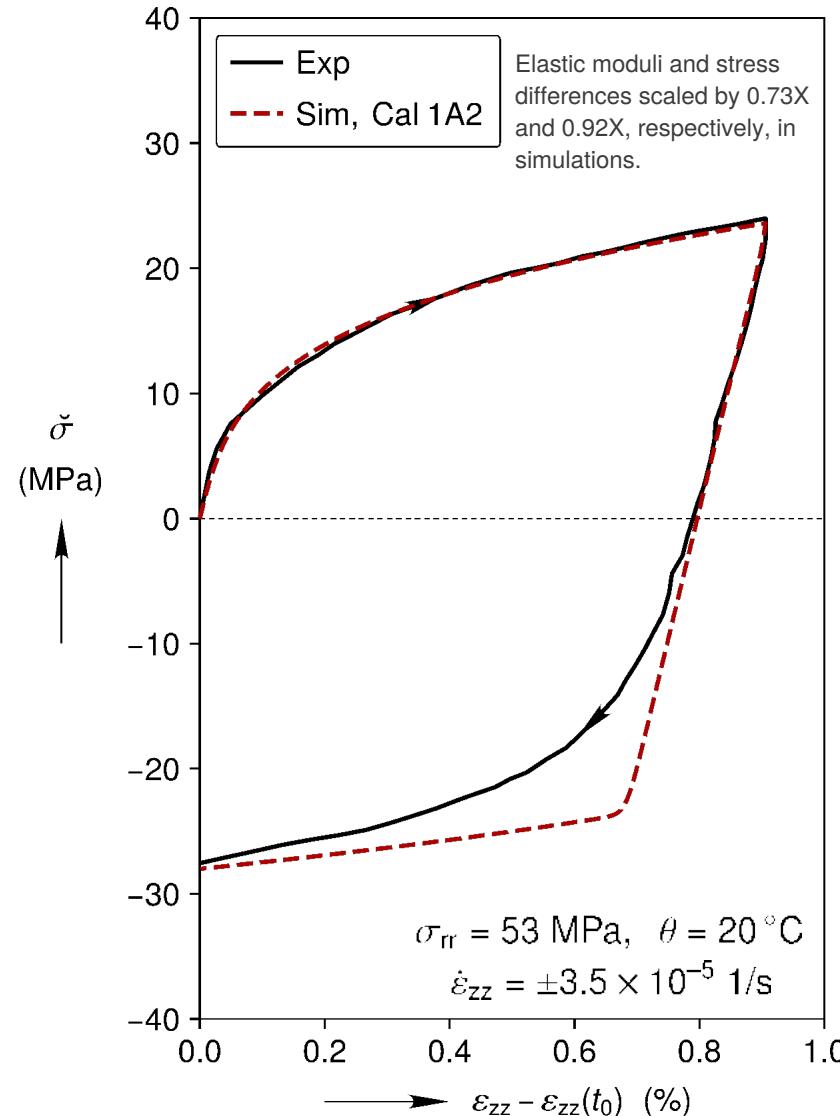
Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)

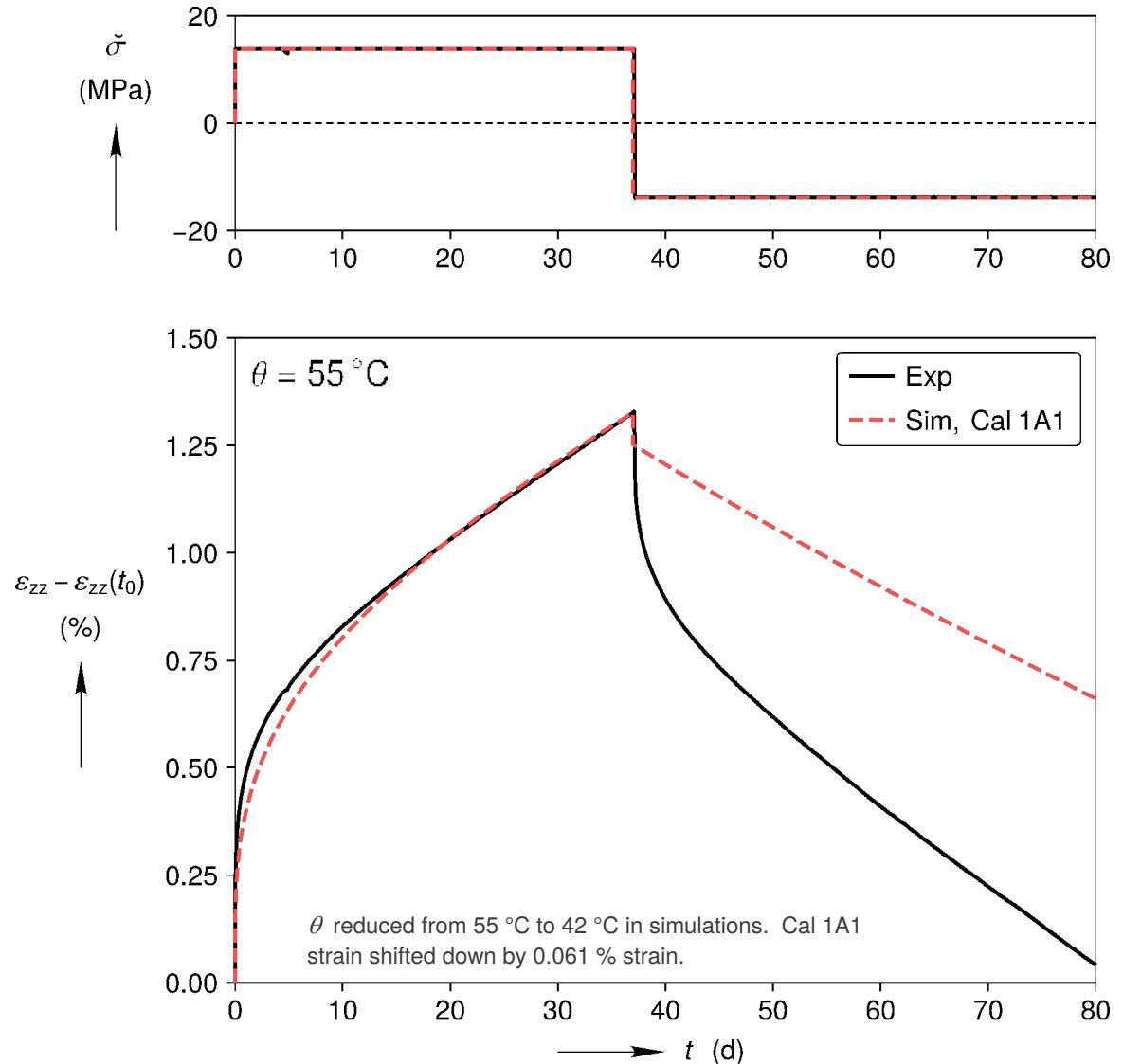
Re-hardening during Non-Monotonic Loading

Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

Multi-Stage Constant Stress Test on Cayuta Salt

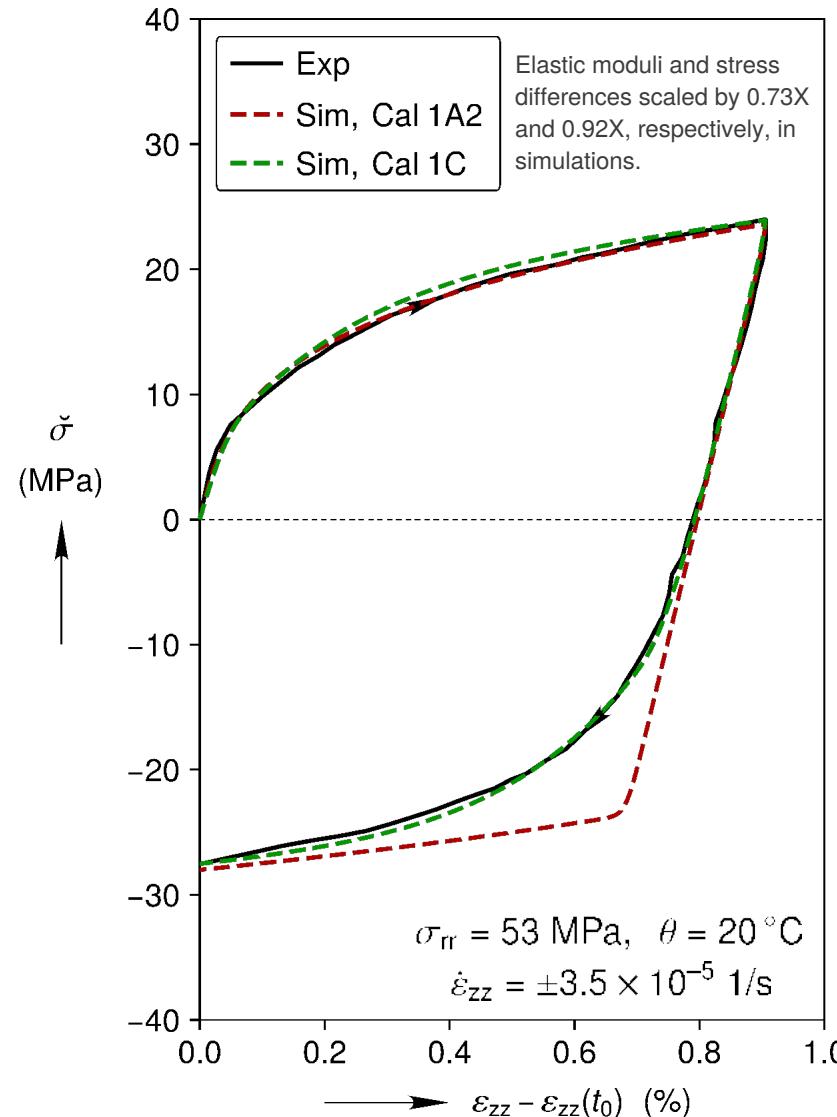


Experimental measurements from: Mellegard et al. (2007)

Re-hardening during Non-Monotonic Loading

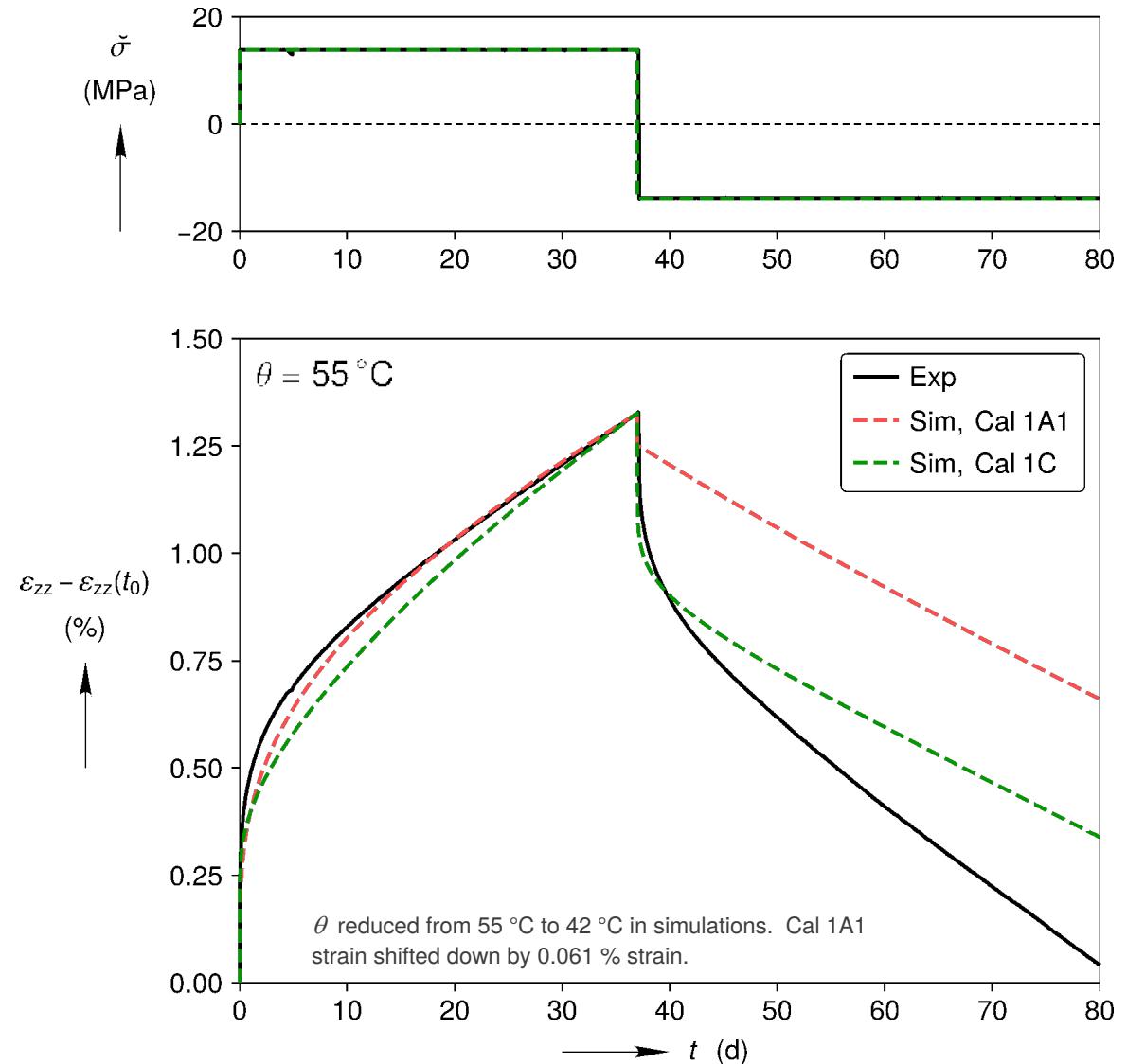


Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)

Summary & Future Work

Summary & Future Work



1. Summary

1. Largely phenomenological model, but key decisions were motivated by micro-physical observations.
2. Pressure solution and dislocation glide branches
3. Combined drag and back stress hardening enables one to capture hardening at low, medium, and high strain rates, as well as re-hardening behavior after non-monotonic loading.

2. Future work

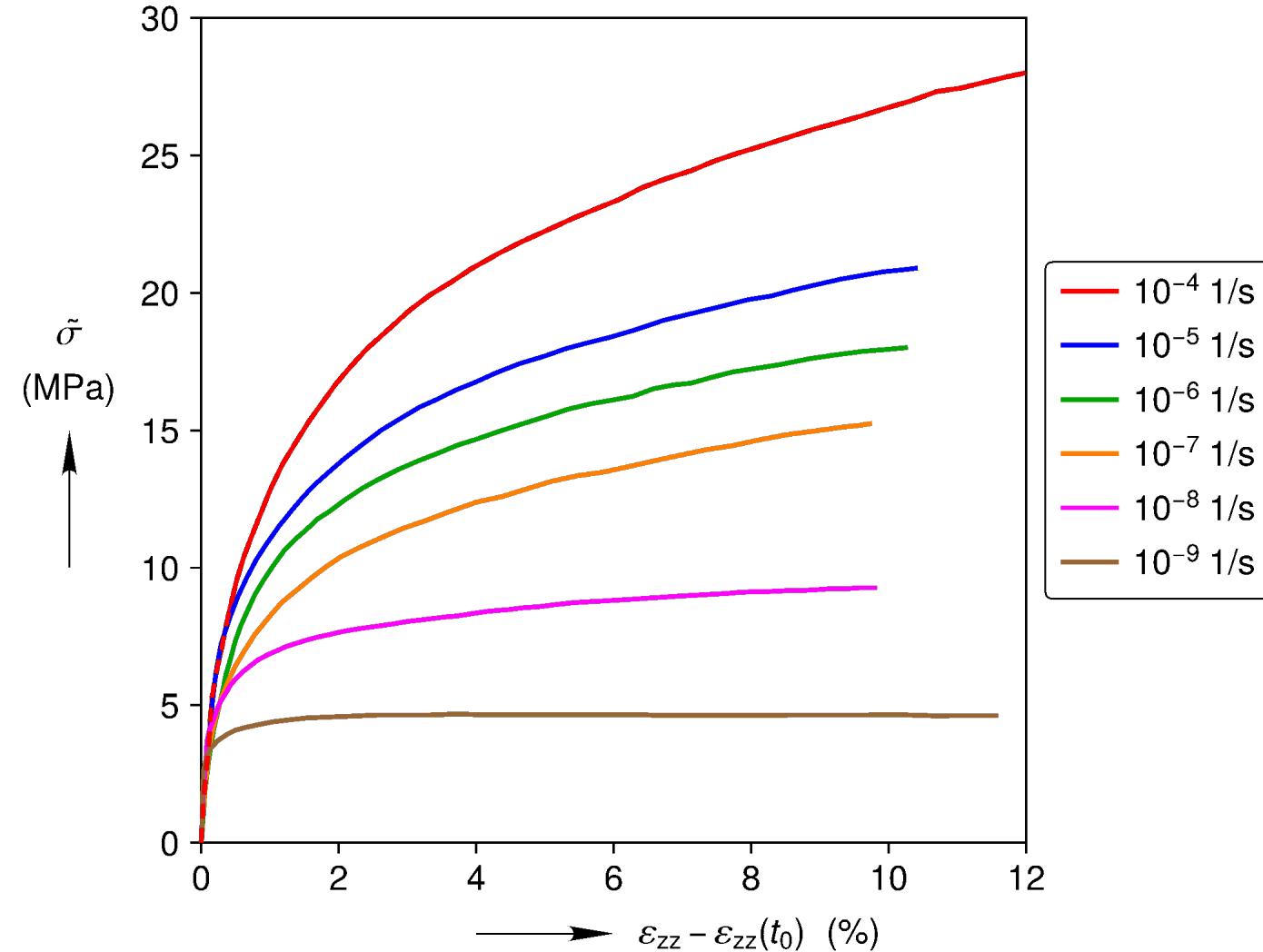
1. Polish numerical implementation
2. Simulate more underground structures
3. Add damage and healing

Extra Slides

Hardening Transition from Medium to High Strain Rates (Stresses)



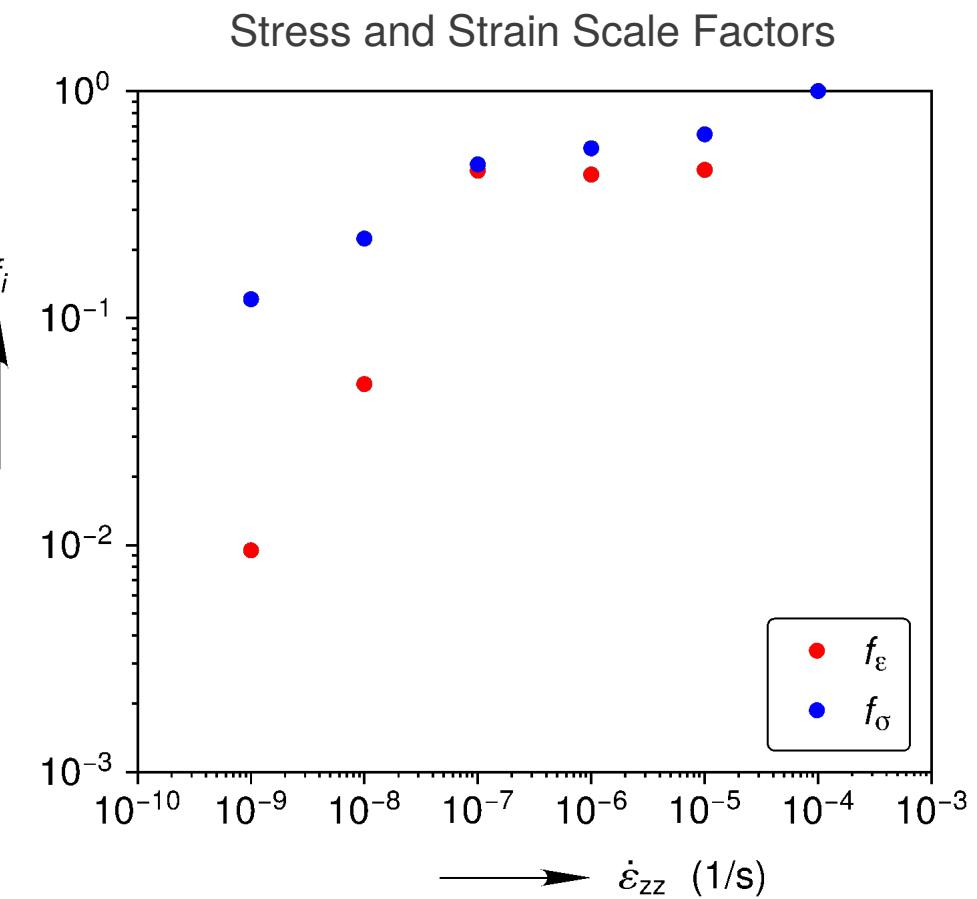
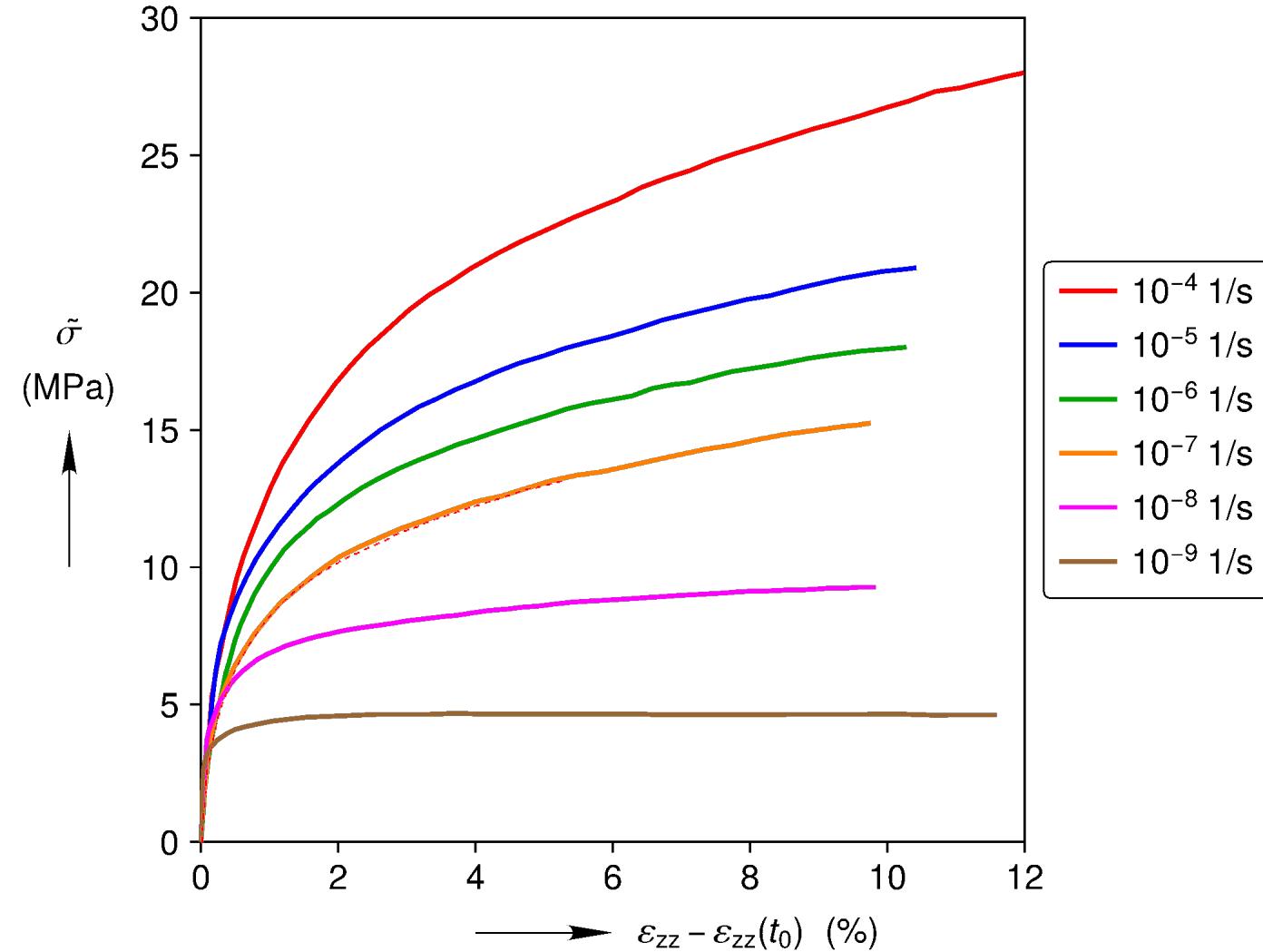
Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$



Hardening Transition from Medium to High Strain Rates (Stresses)



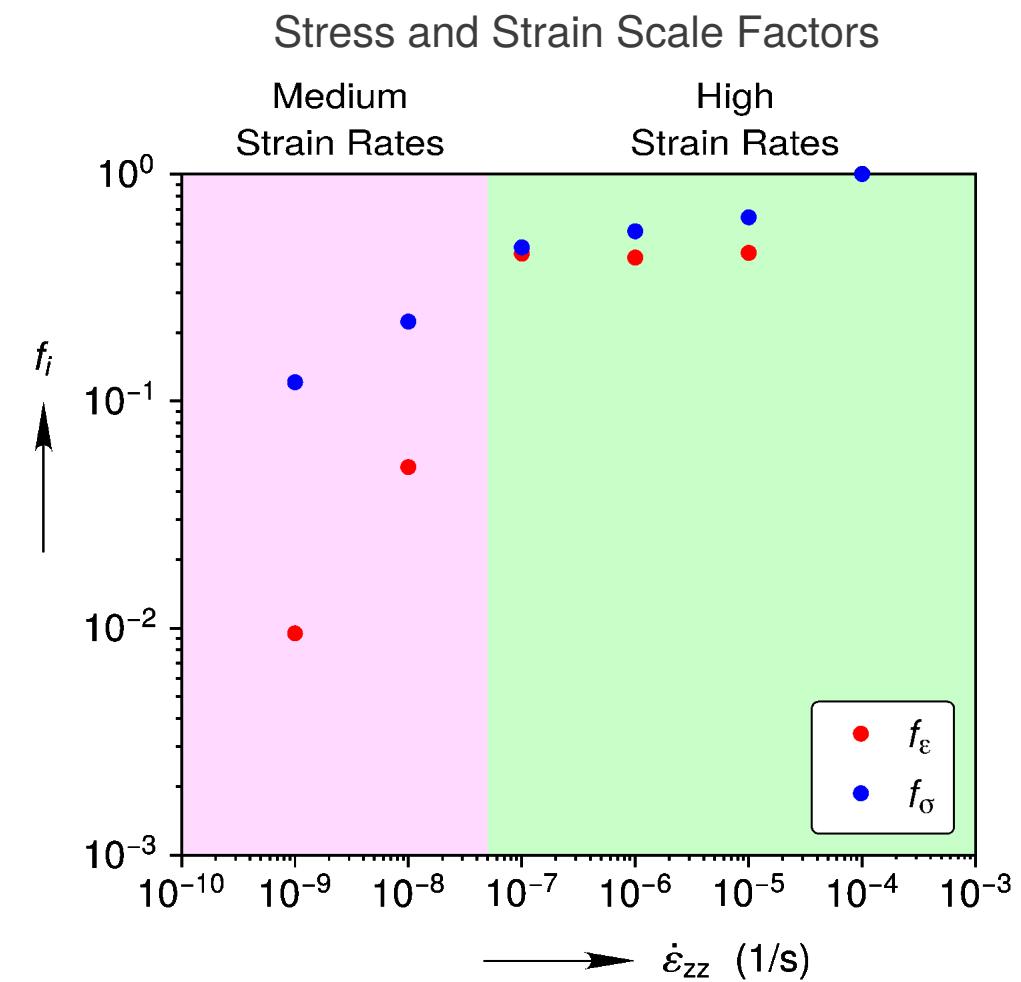
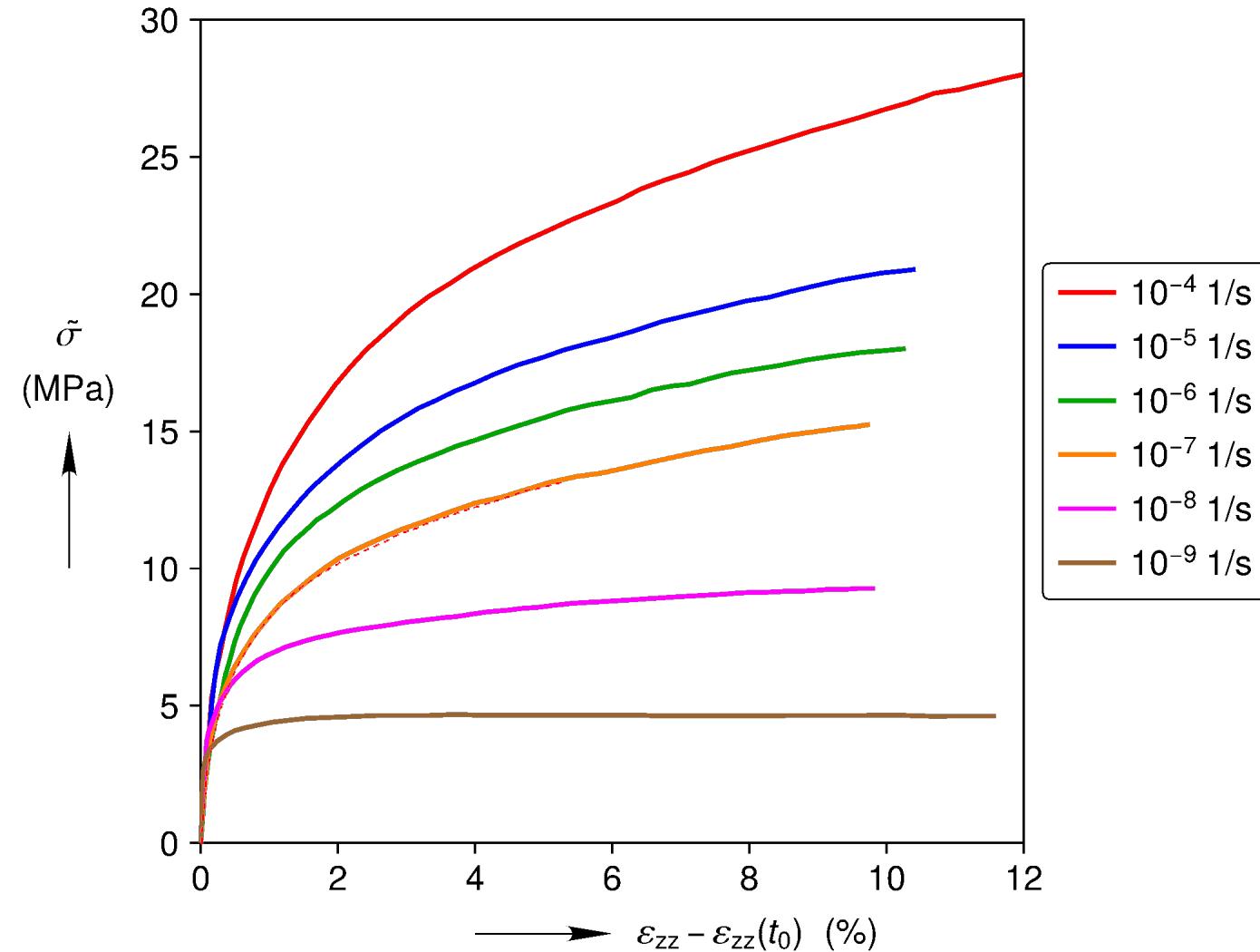
Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$



Hardening Transition from Medium to High Strain Rates (Stresses)



Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$

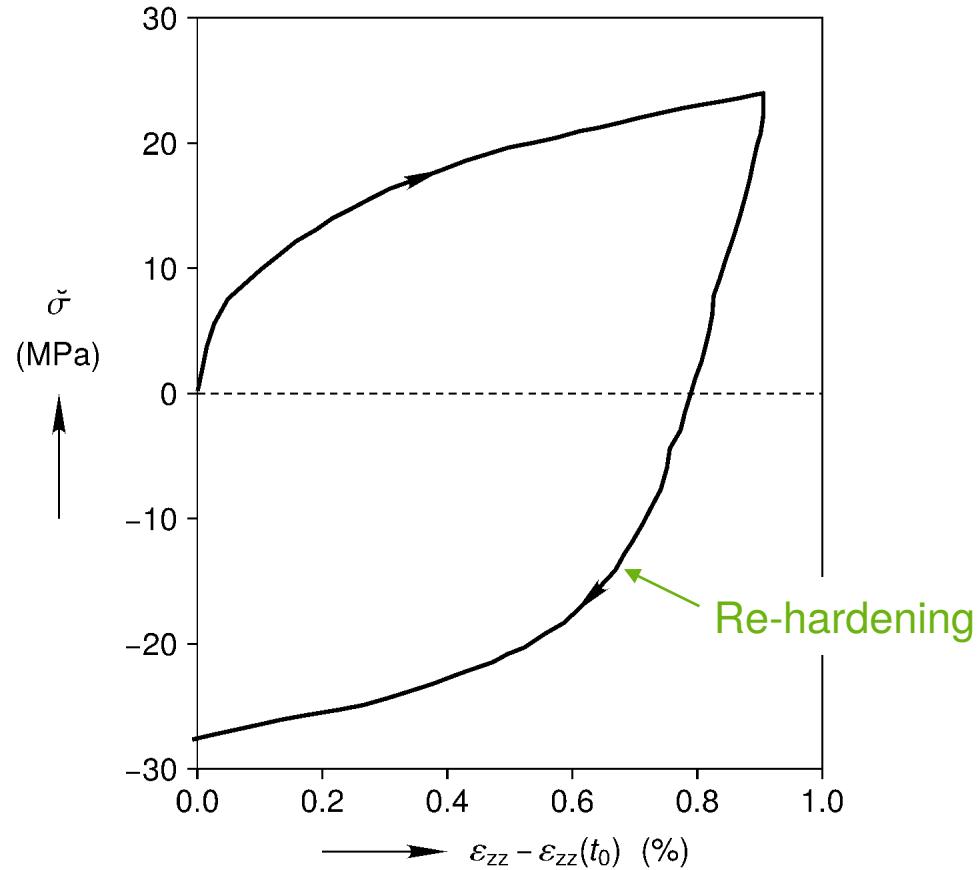


Re-hardening During Non-Monotonic Loading



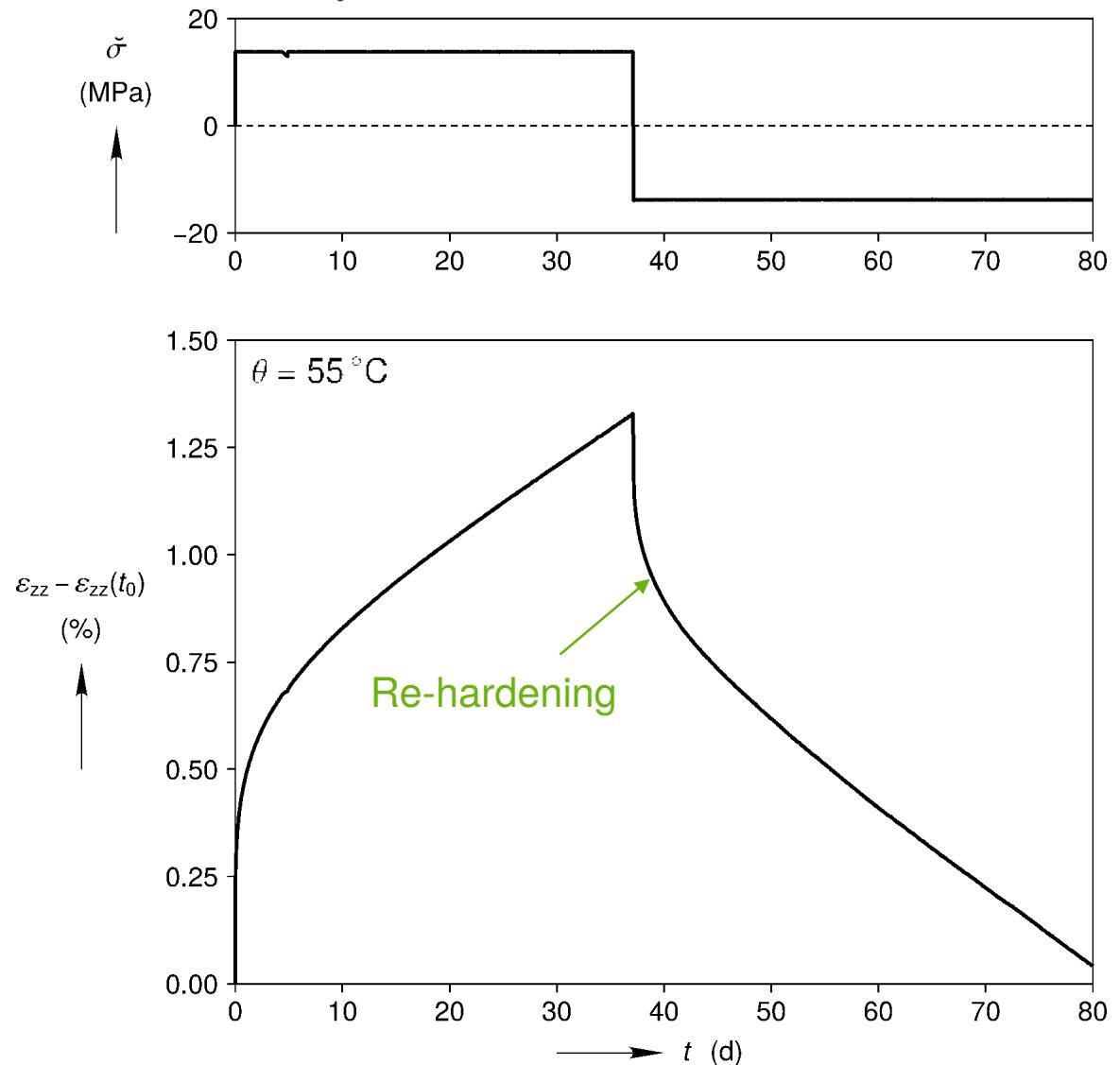
Artificial Salt, Stress vs. Strain Curve

$$\sigma_{rr} = 53 \text{ MPa}, \theta = 293 \text{ K}, \dot{\varepsilon}_{zz} = \pm 3.5 \times 10^{-5} \text{ 1/s}$$



Experimental measurements from Aubertin et al. (1999)

Cayuta Salt, Stress and Strain Histories



Experimental measurements from: Mellegard et al. (2007)

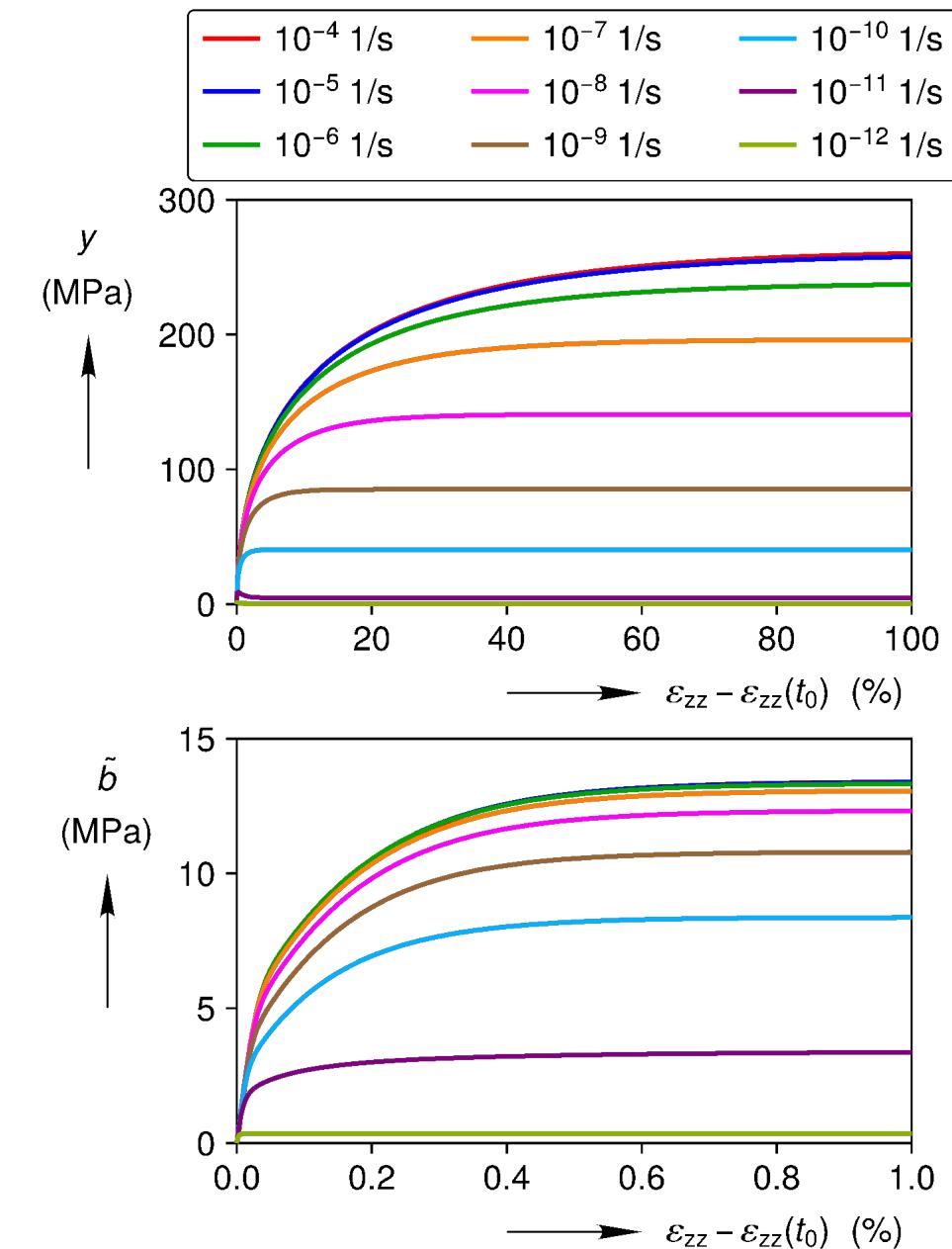
Dislocation Glide Hardening

Drag Stress Evolution Equation

$$\dot{y} = Y_1 \left(\frac{Y_1}{y} \right)^{Y_2} \left(1 - \frac{y}{\bar{y}} \right) \dot{\varepsilon}^{\text{dg}}$$

Back Stress Evolution Equation
(proportional, monotonic, loading)

$$\dot{\tilde{b}} = B_1 \left(1 - \frac{\tilde{b}}{\bar{b}} \right) \dot{\varepsilon}^{\text{dg}}$$



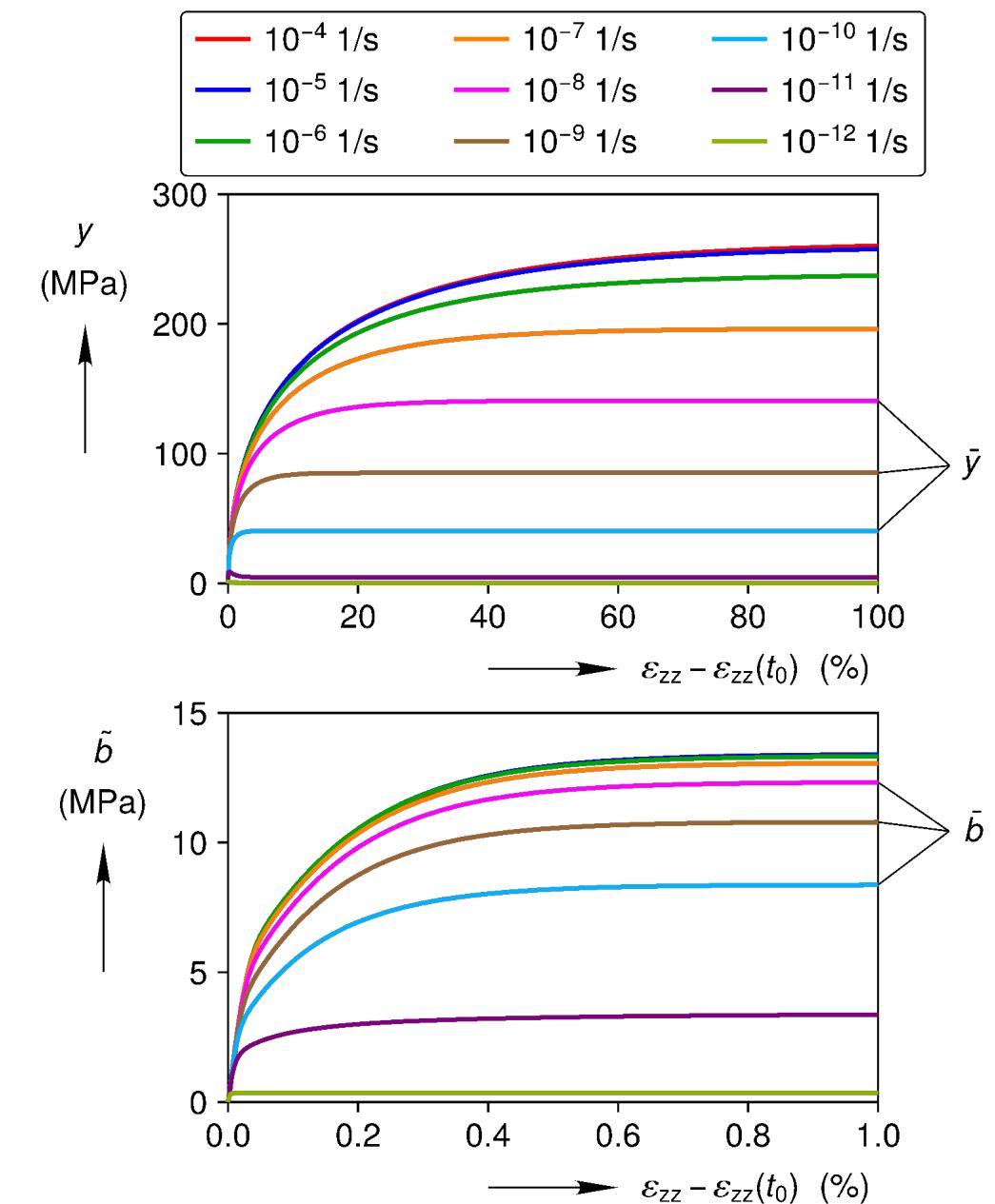
Dislocation Glide Hardening

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Back Stress Evolution Equation
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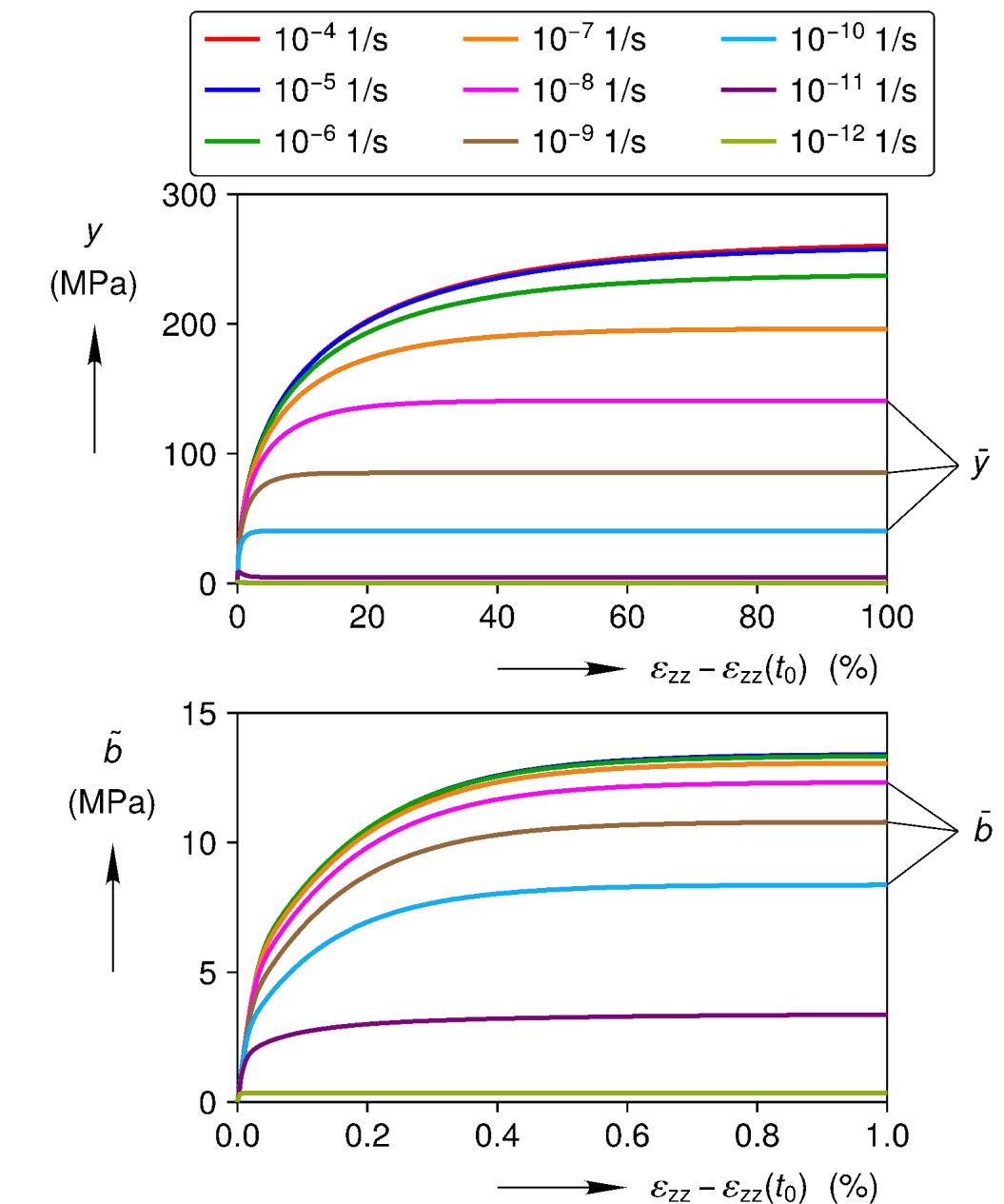


Dislocation Glide Hardening

Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

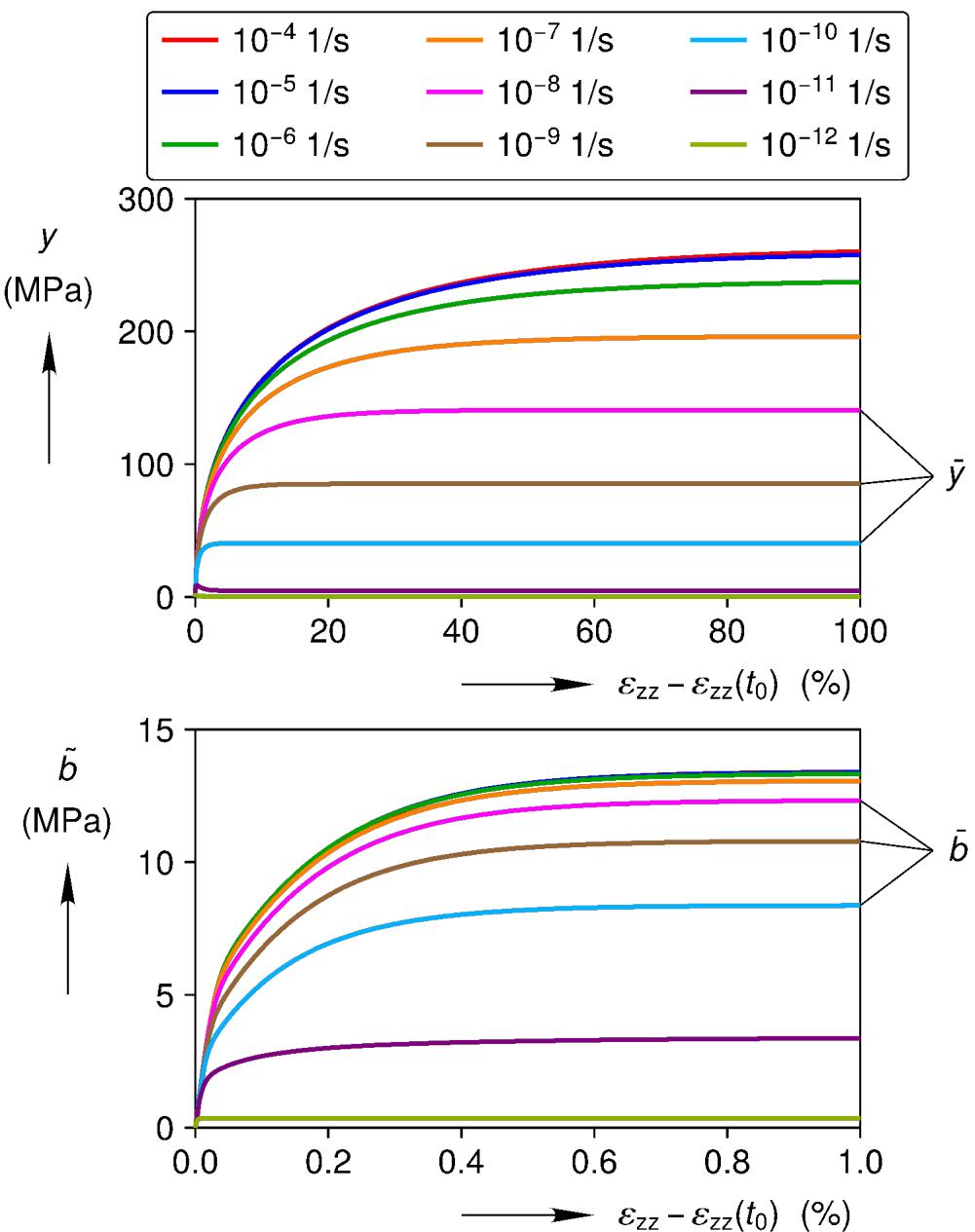


Dislocation Glide Steady-State

Steady-State Strain Rates

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \bar{b}}{\bar{y}}\right) \right]^{G_3}$$

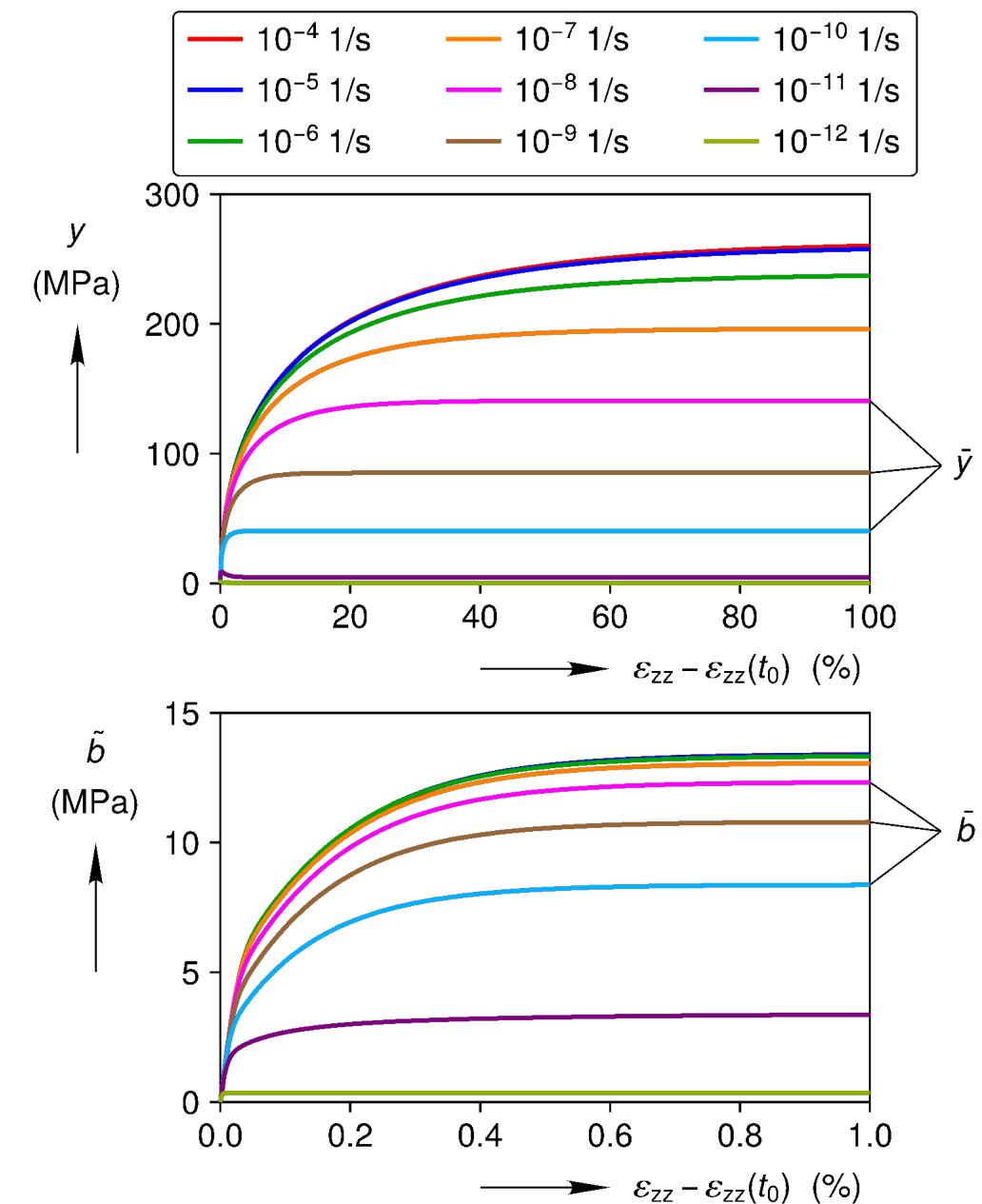


Dislocation Glide Steady-State

Steady-State Strain Rates

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = Y_3 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{Y_4}\right)\right]^{Y_5}$$



Dislocation Glide Hardening



Equivalent Dislocation Glide Strain Rate

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}^{\text{dg}}}{y}\right) \right]^{G_3}$$

Isotropic Dislocation Glide Hardening



Equivalent Dislocation Glide Strain Rate

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{y}\right)\right]^{G_3}$$

Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$

Flow Rule

$$\dot{\boldsymbol{\varepsilon}}^{\text{dg}} = \dot{\tilde{\varepsilon}}^{\text{dg}} \frac{\partial f_i}{\partial \boldsymbol{\sigma}}$$

Isotropic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_i = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

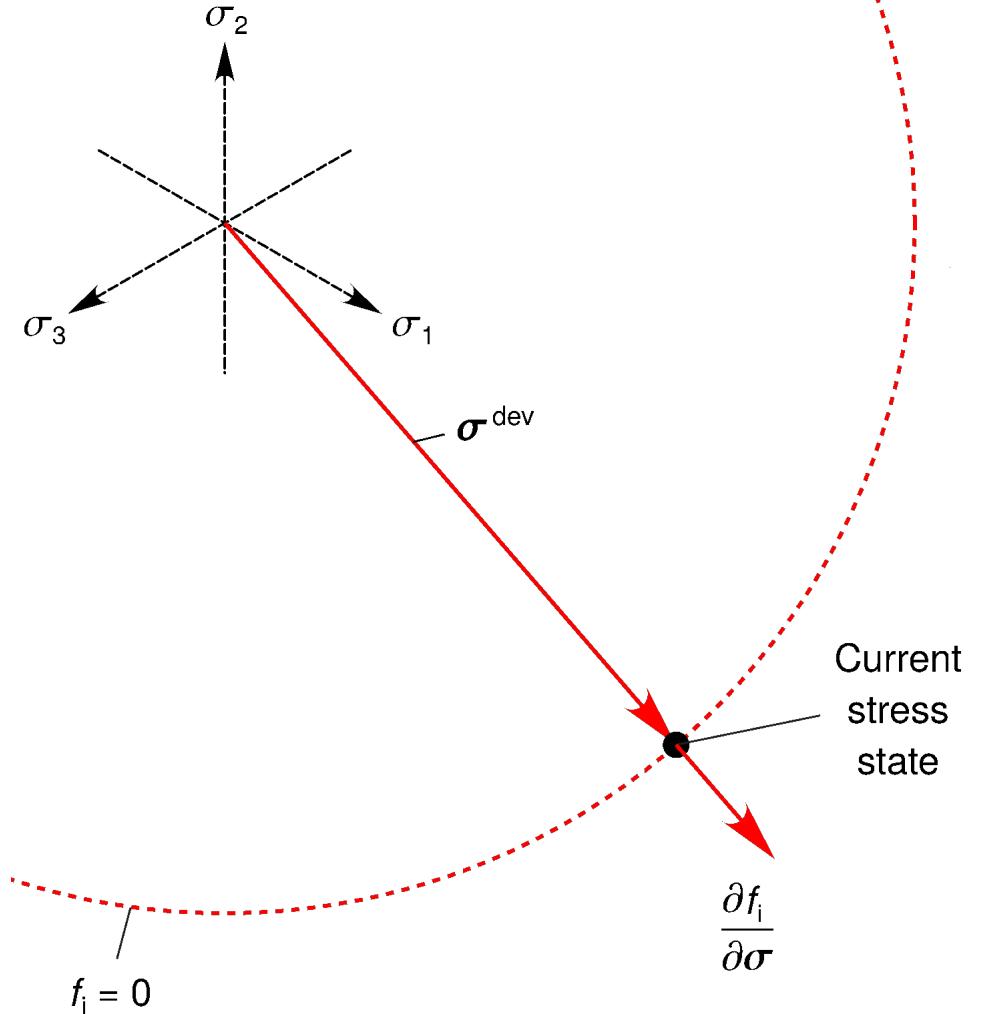
Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$

Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_i}{\partial \sigma}$$

Deviatoric Plane



Isotropic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_i = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

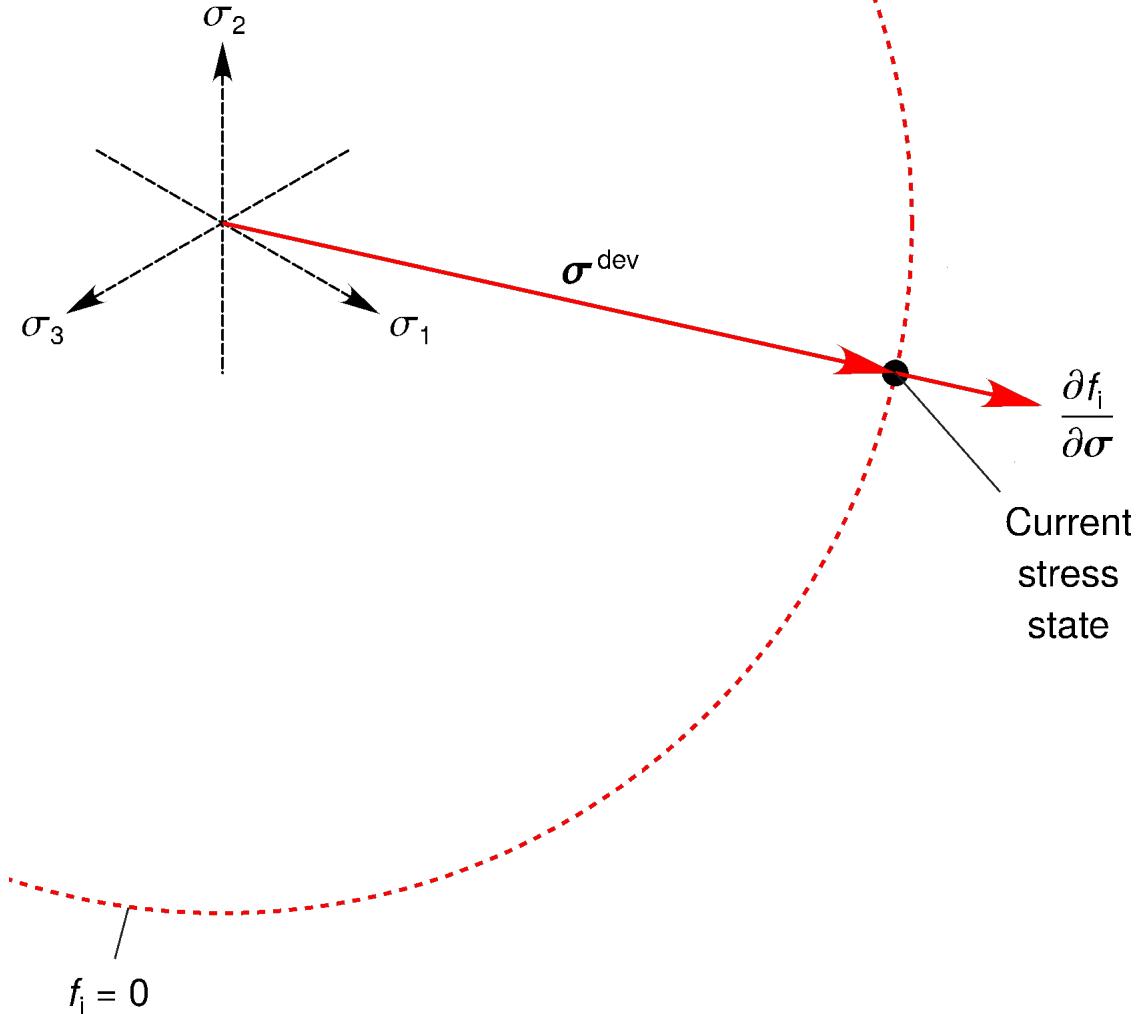
Equivalent Dislocation Glide Stress

$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$

Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_i}{\partial \sigma}$$

Deviatoric Plane



Isotropic and Kinematic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Dislocation Glide Stress

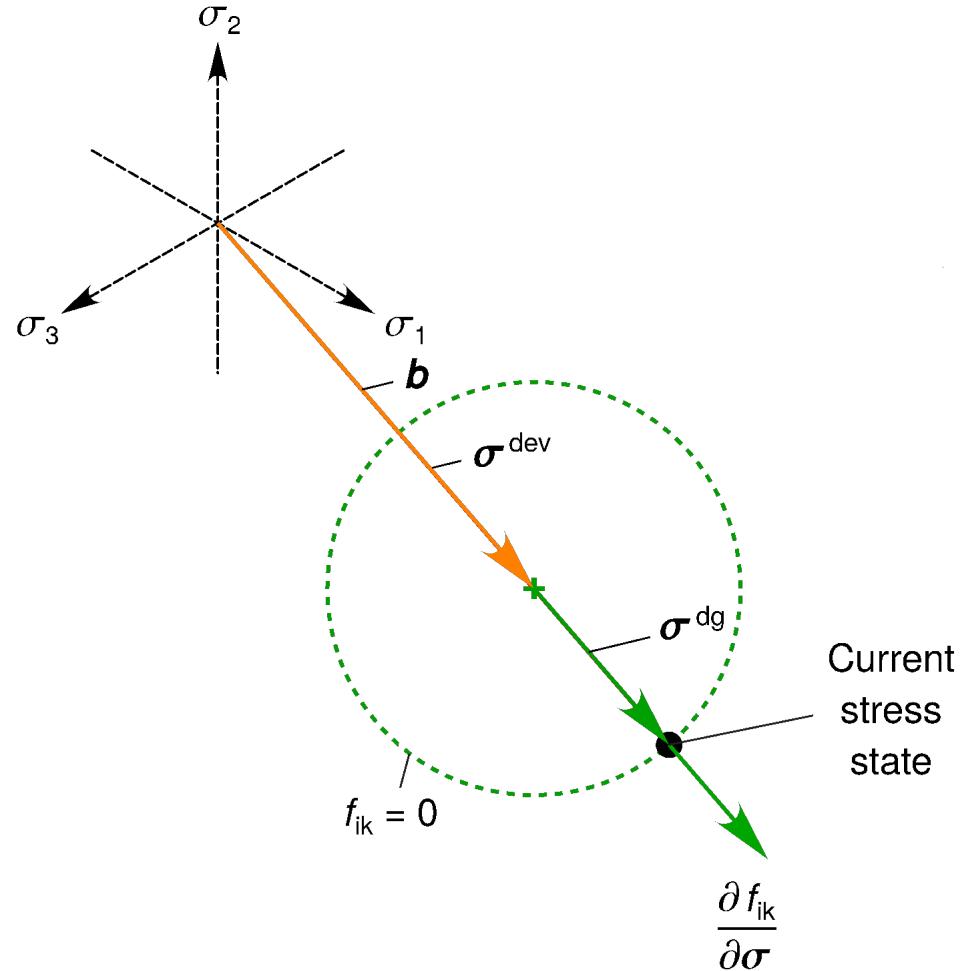
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_{ik}}{\partial \sigma}$$

Deviatoric Plane



Isotropic and Kinematic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Dislocation Glide Stress

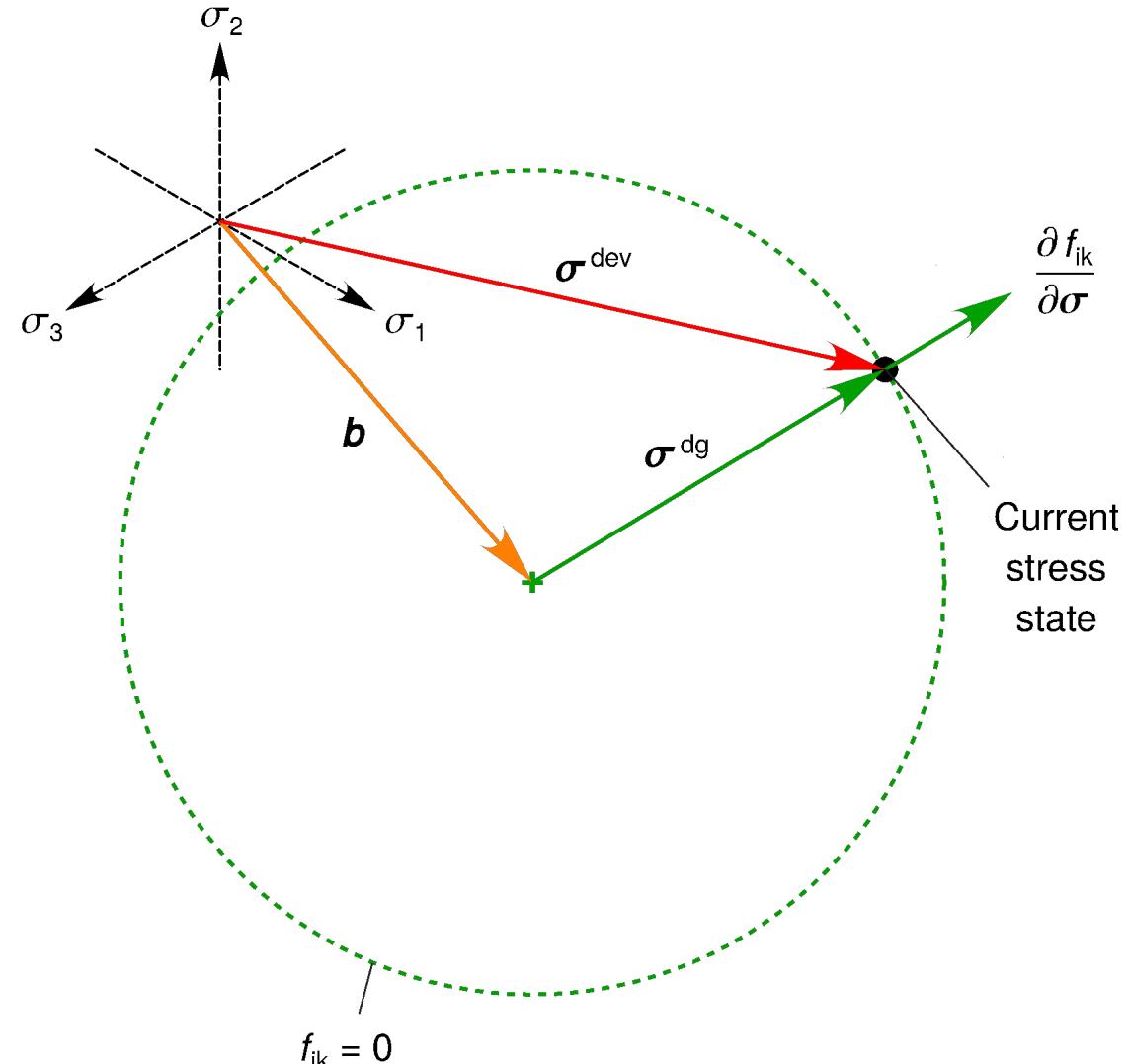
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_{ik}}{\partial \sigma}$$

Deviatoric Plane



Isotropic and Kinematic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Dislocation Glide Stress

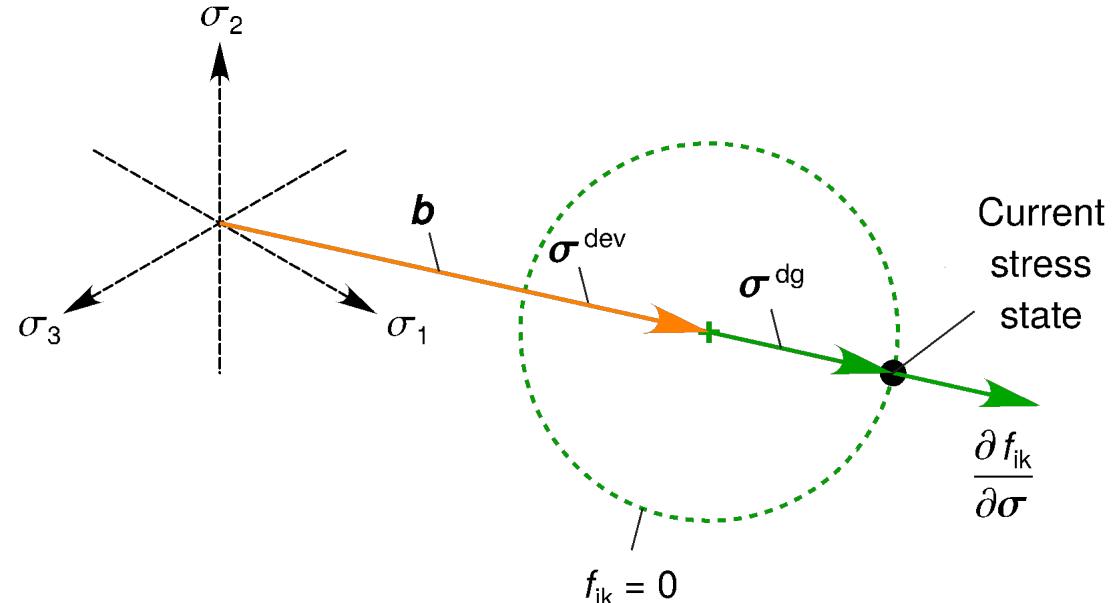
$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_{ik}}{\partial \sigma}$$

Deviatoric Plane



Isotropic and Kinematic Dislocation Glide Hardening



Dynamic Yield Surface

$$f_{ik} = \tilde{\sigma}^{dg} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

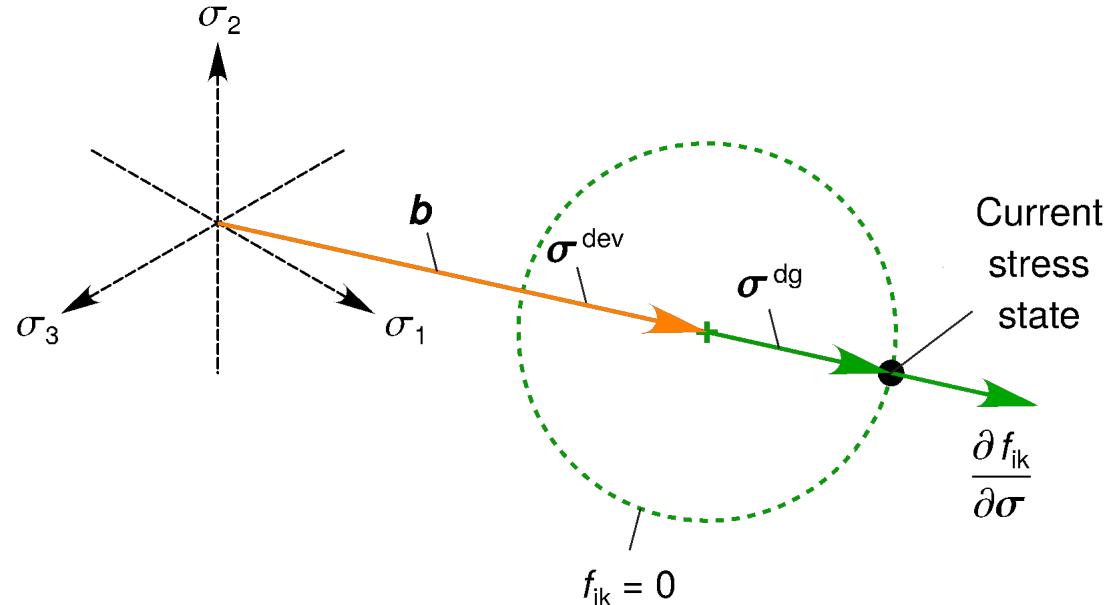
Equivalent Dislocation Glide Stress
(proportional, monotonic, loading, or steady-state)

$$\tilde{\sigma}^{dg} = \tilde{\sigma} - \tilde{b}$$

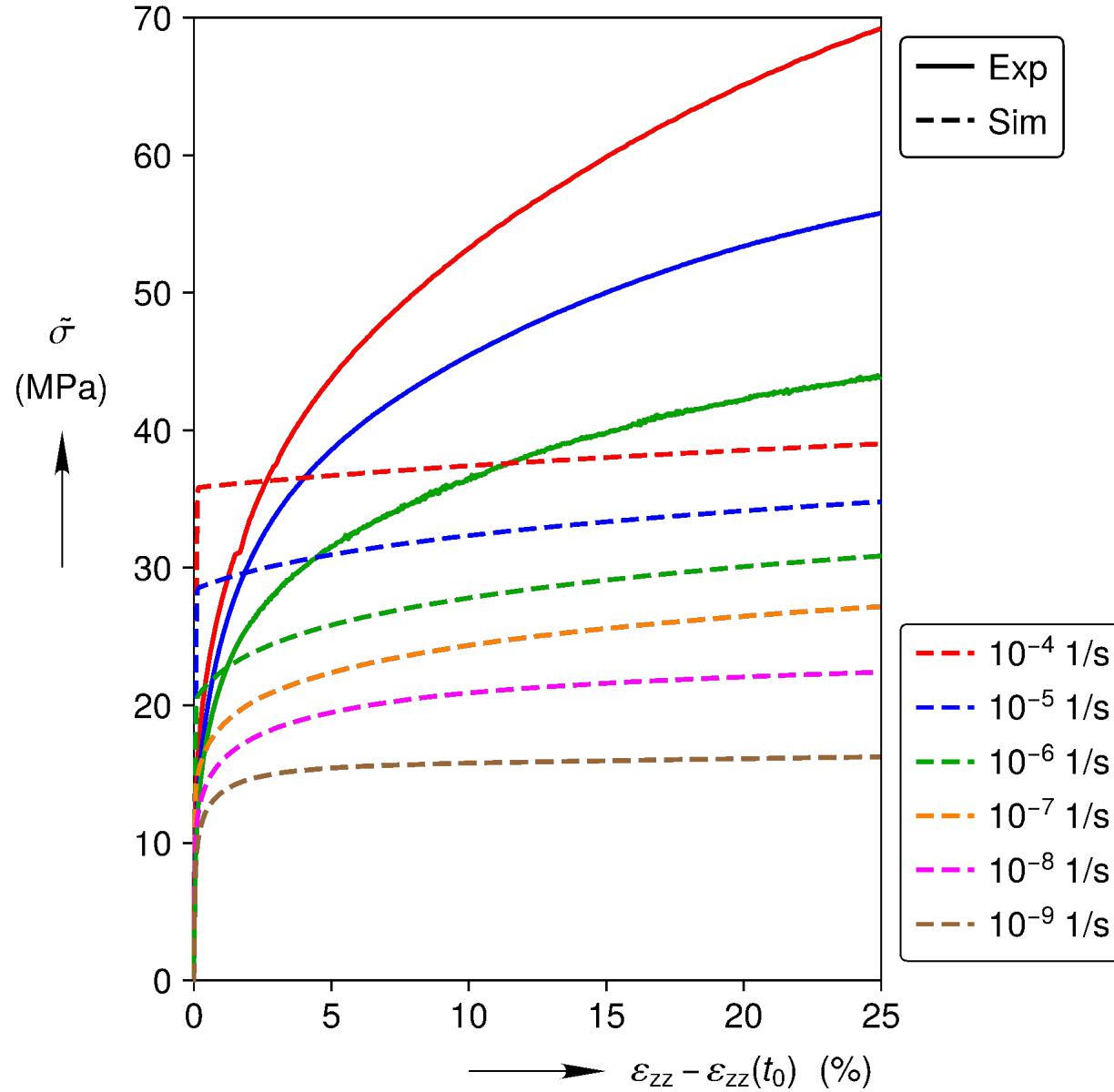
Flow Rule

$$\dot{\varepsilon}^{dg} = \dot{\varepsilon}^{dg} \frac{\partial f_{ik}}{\partial \sigma}$$

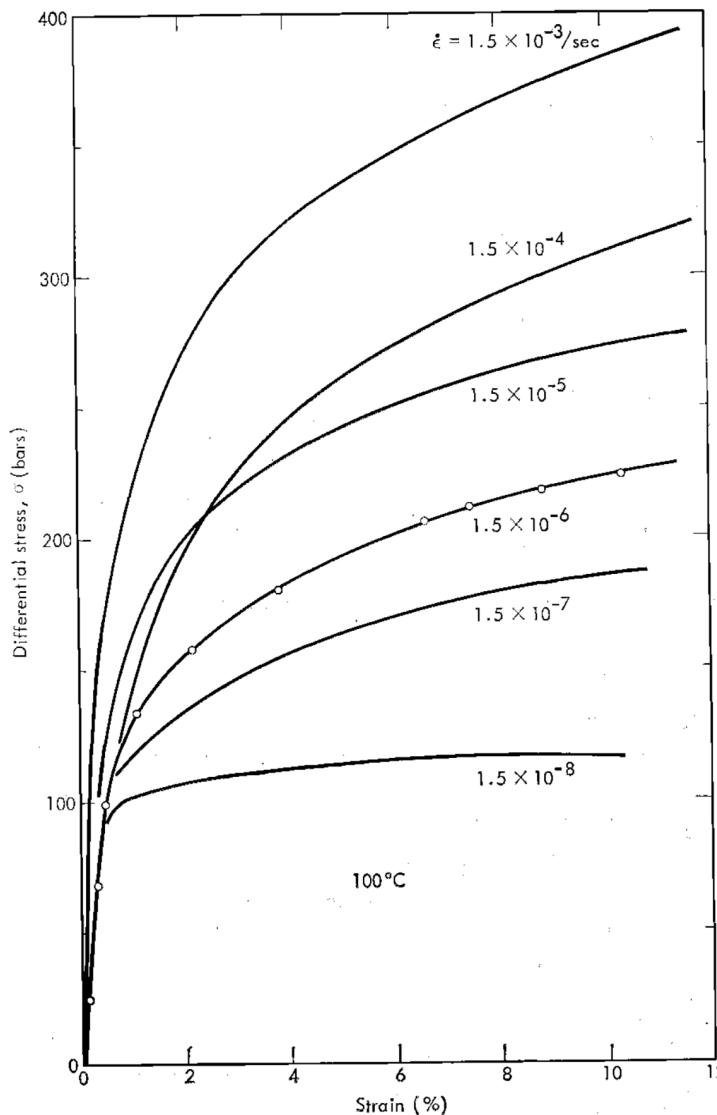
Deviatoric Plane



Munson-Dawson Model Constant Strain Rate Predictions



Constant Strain Rate Tests on Artificial Salt

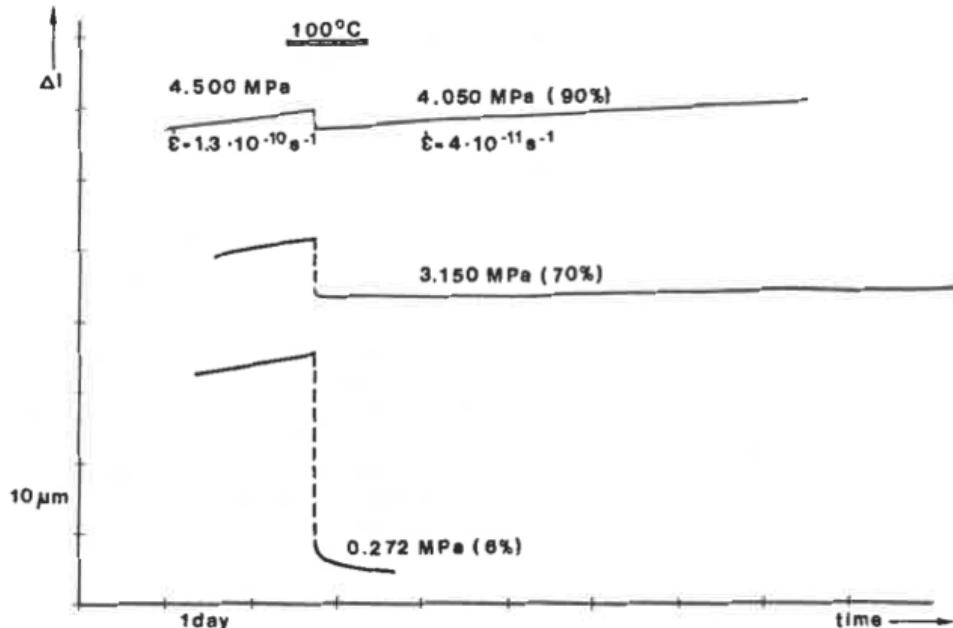


Heard, H. (1972): Steady-state flow in polycrystalline halite at pressure of 2 kilobars. *Flow and fracture of rocks*. Vol 16. pp 191-209.

Fig. 1. Differential stress-strain curves for polycrystalline halite extended at 2 kb, $\dot{\epsilon} = 1.5 \times 10^{-3}$ to $1.5 \times 10^{-8} \text{ sec}^{-1}$, and 100°C.

Back Stress Measurements

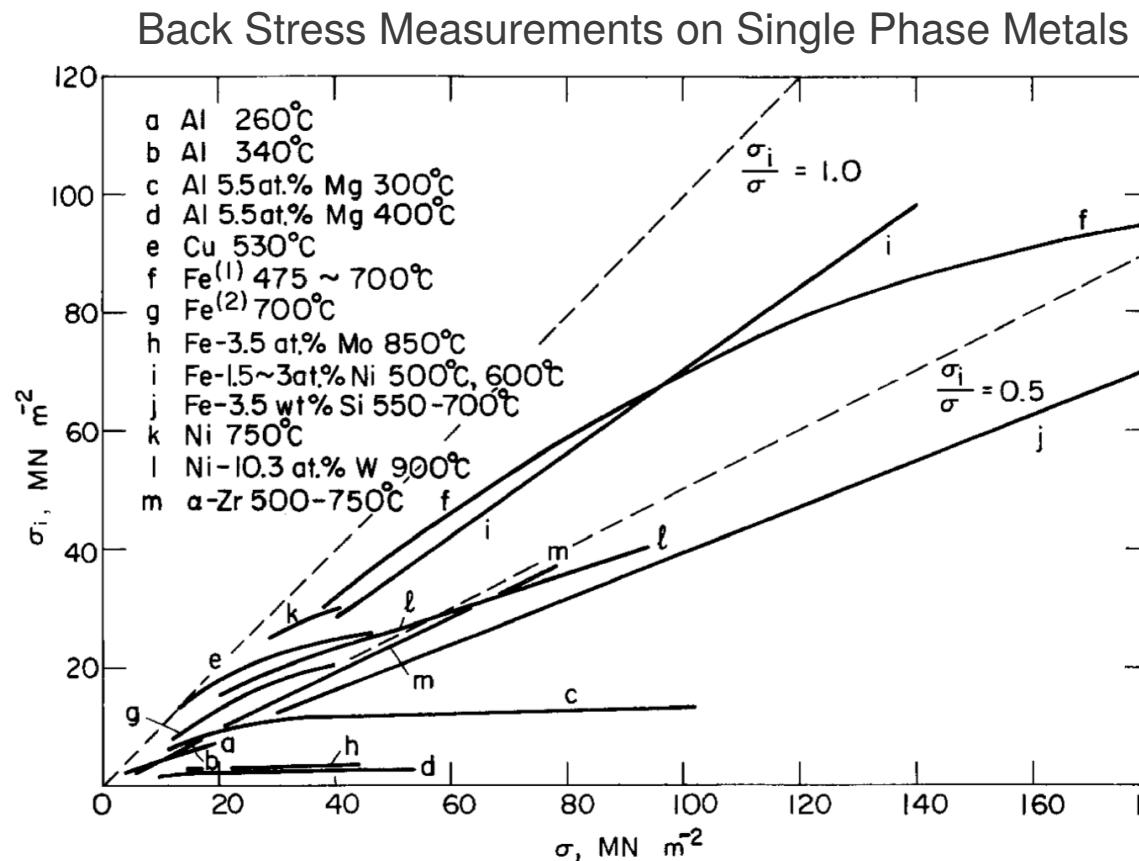
Creep Responses due to Small Stress Changes Asse Rock Salt



Hunsche, U. 1988. Measurement of creep in rock salt at small strain rates. Proceedings of the 2nd Conference on the Mechanical Behavior of Salt. Pg. 187-196

Dislocation Glide Strain Rate (proportional loading)

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{|\check{\sigma} - \check{b}|}{y}\right) \right]^{G_3} \text{sign}(\check{\sigma} - \check{b})$$

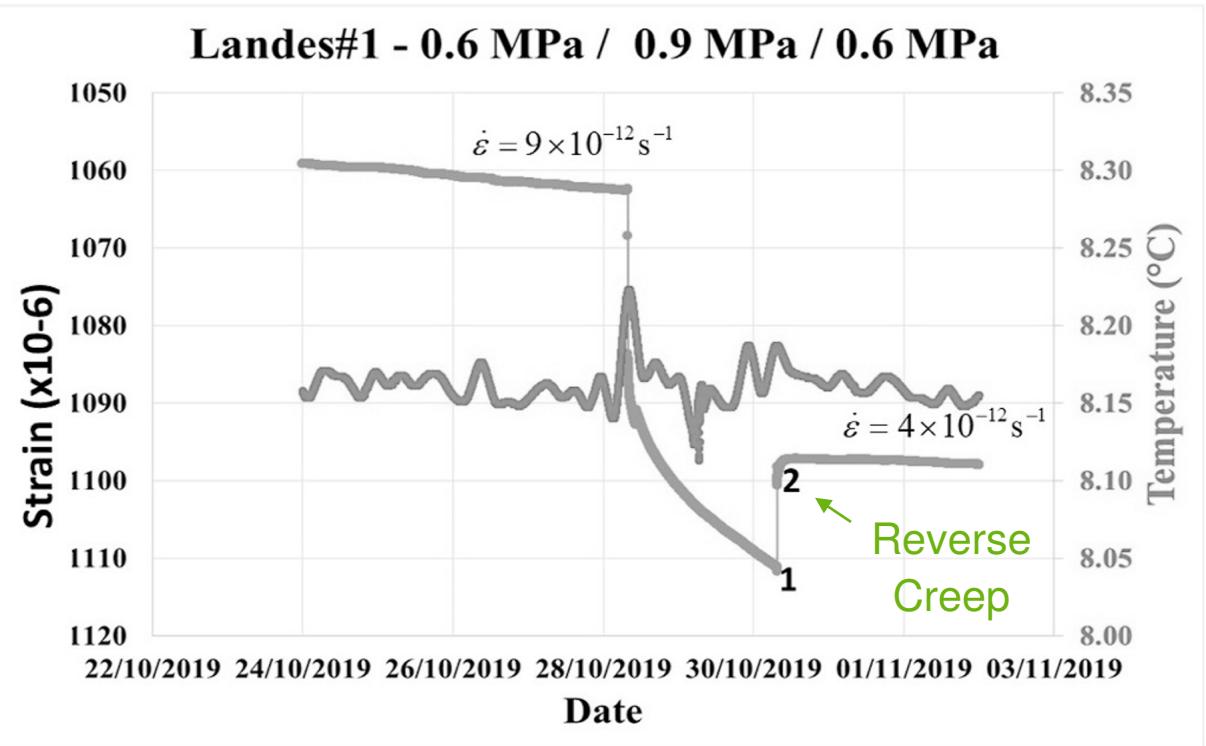


Takeuchi, S. and Argon, A. (1976). "Steady-state creep of single-phase crystalline matter at high temperature". In: Journal of Materials Science 11.8, pp. 1542-1566.

Reverse Creep at Low Stress and Room Temperature

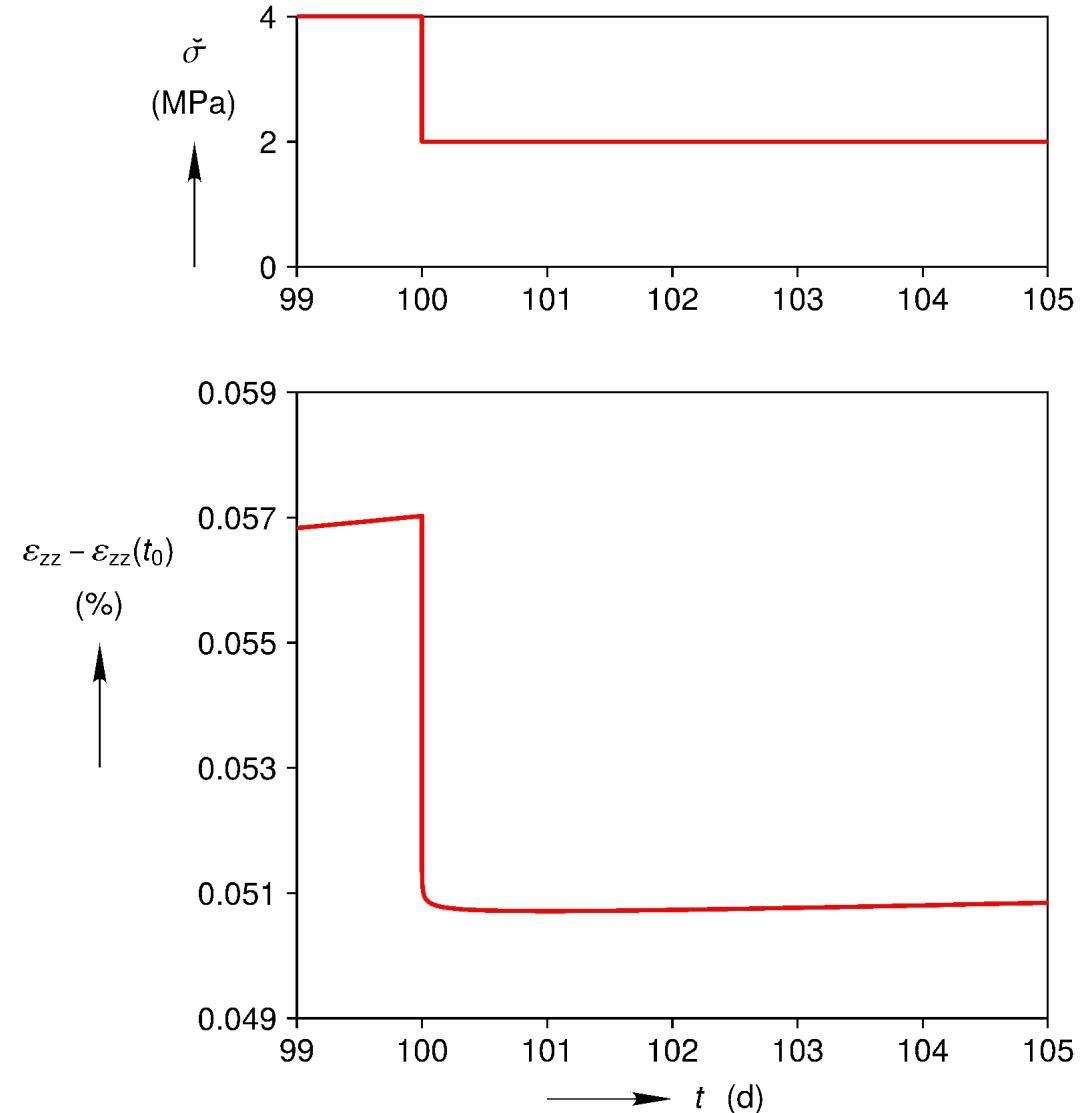


Multi-stage Constant Stress Test on Landes Salt



Gharbi, H., Bérest, P., Blanco-Martín, L., and Brouard, B. (Oct. 2020). "Determining upper and lower bounds for steady state strain rate during a creep test on a salt sample". In: International Journal of Rock Mechanics and Mining Sciences 134, p. 104452

Multi-stage Constant Stress Predictions (Cal 1C)
(different stresses than on Landes salt)



Dislocation Glide Hardening Saturation (Steady-State)

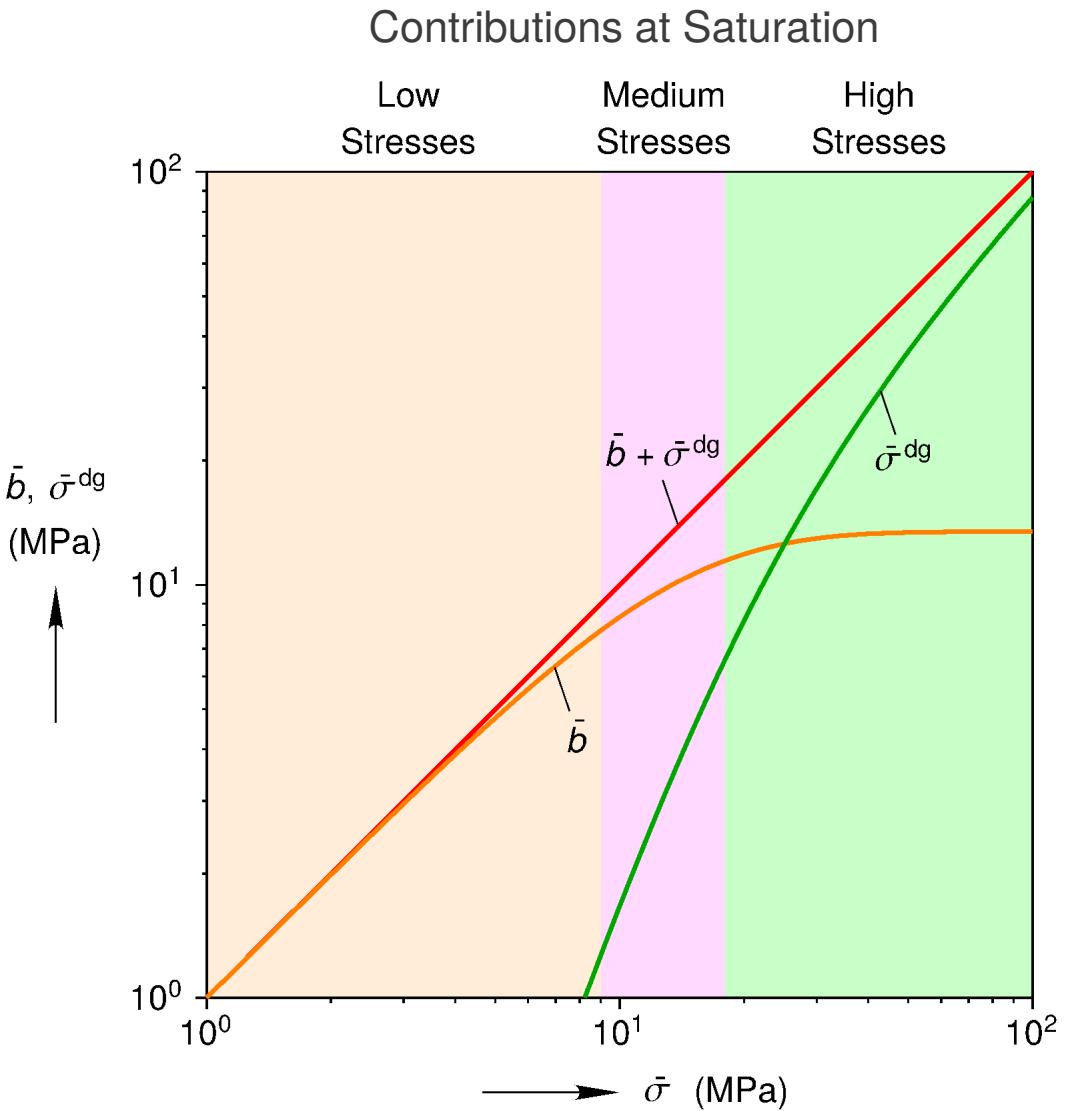


Equivalent Stress Decomposition
(proportional, monotonic, loading)

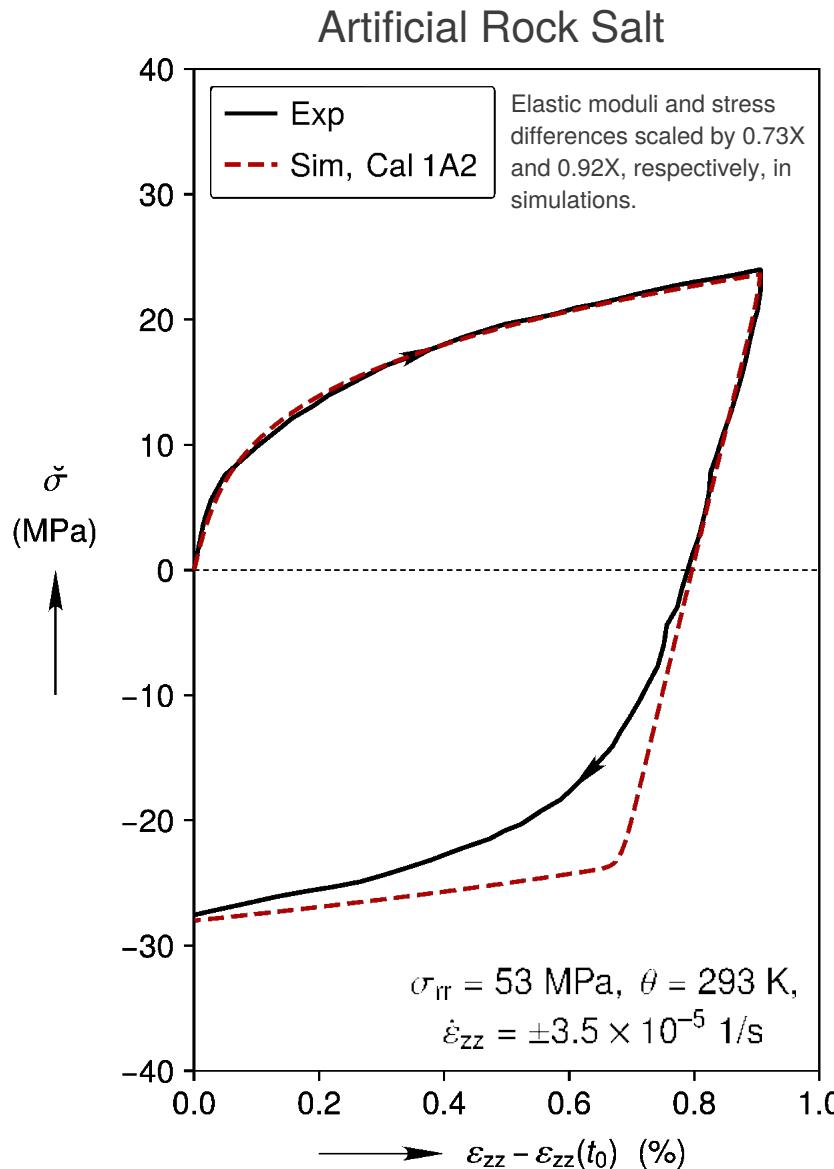
$$\bar{\sigma} = \bar{b} + \bar{\sigma}^{\text{dg}}$$

Drag Stress Contribution

$$\bar{\sigma}^{\text{dg}} = \bar{y} \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$



Bauschinger Effect: Isotropic Hardening Only



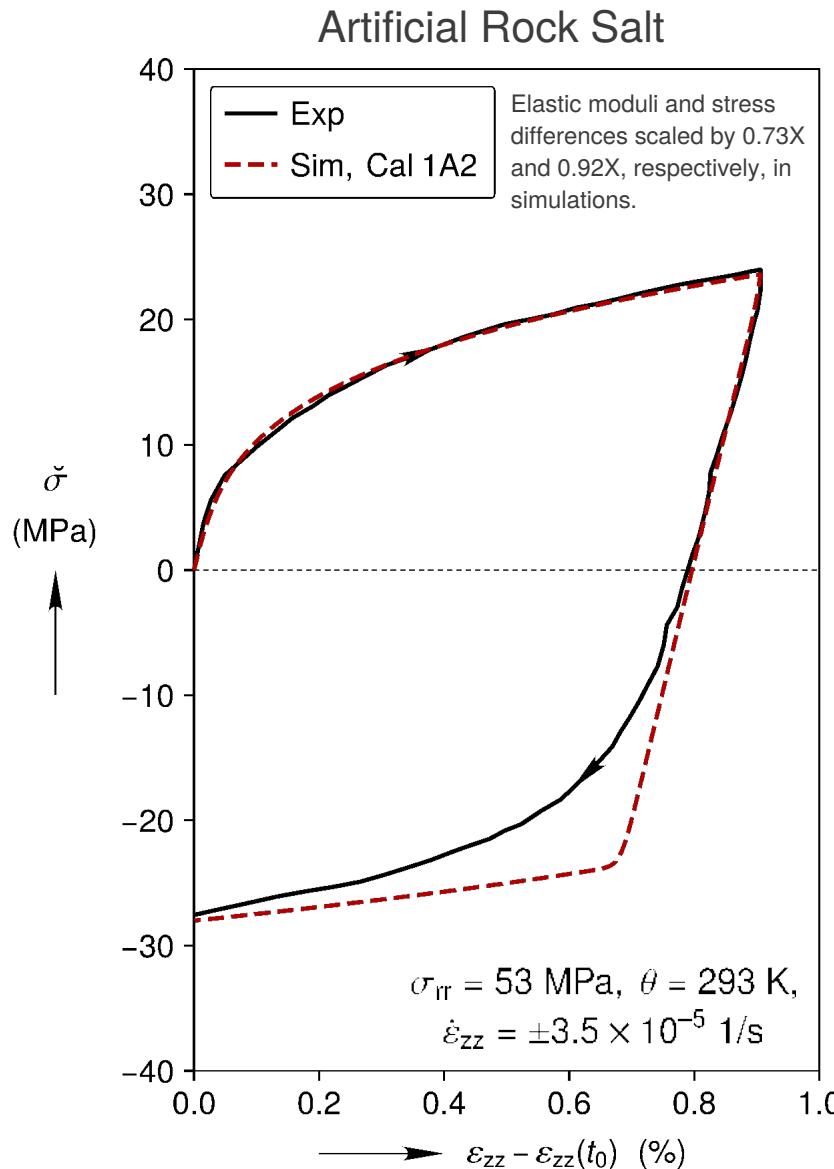
Strain Rate

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\check{\sigma}}{y}\right)\right]^{G_3}$$

Equivalent Stress

$$\check{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$

Bauschinger Effect: Isotropic Hardening Only



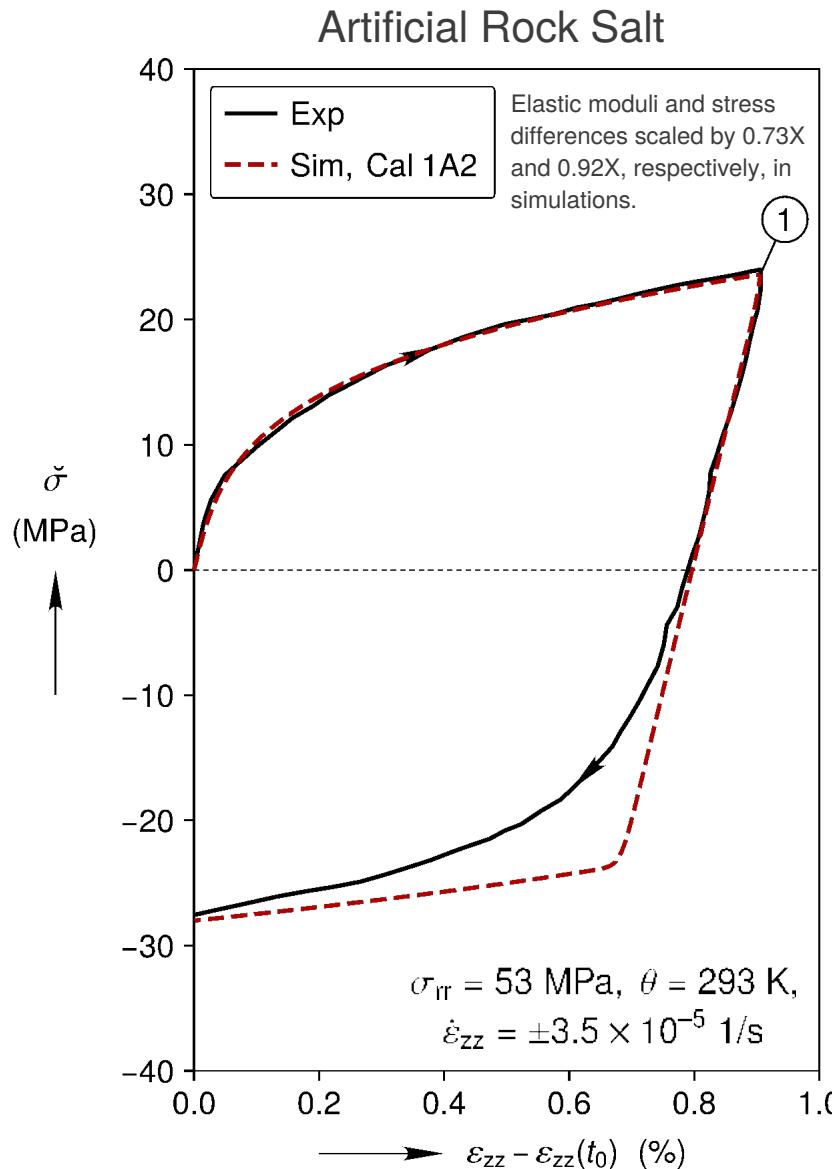
Dynamic Yield Surface

$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[\frac{\dot{\epsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Equivalent Stress

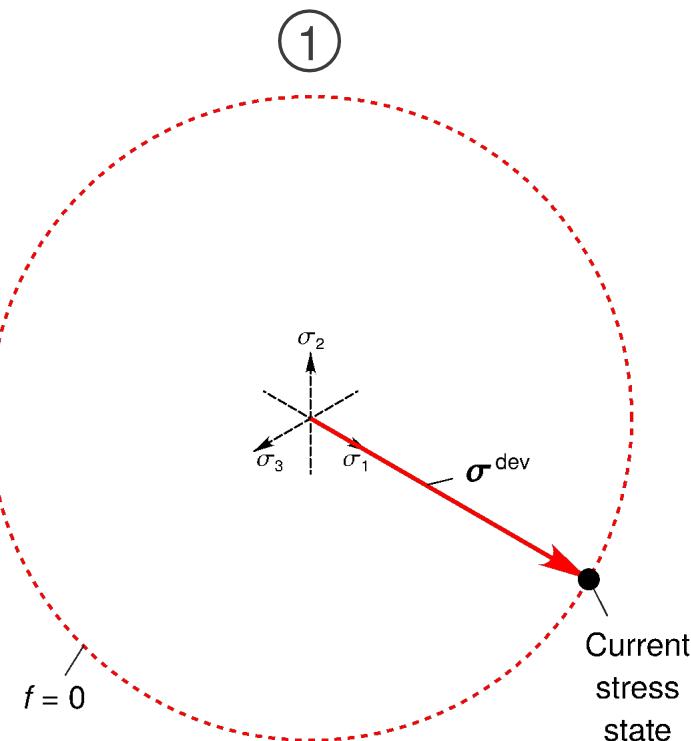
$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$

Bauschinger Effect: Isotropic Hardening Only



Dynamic Yield Surface

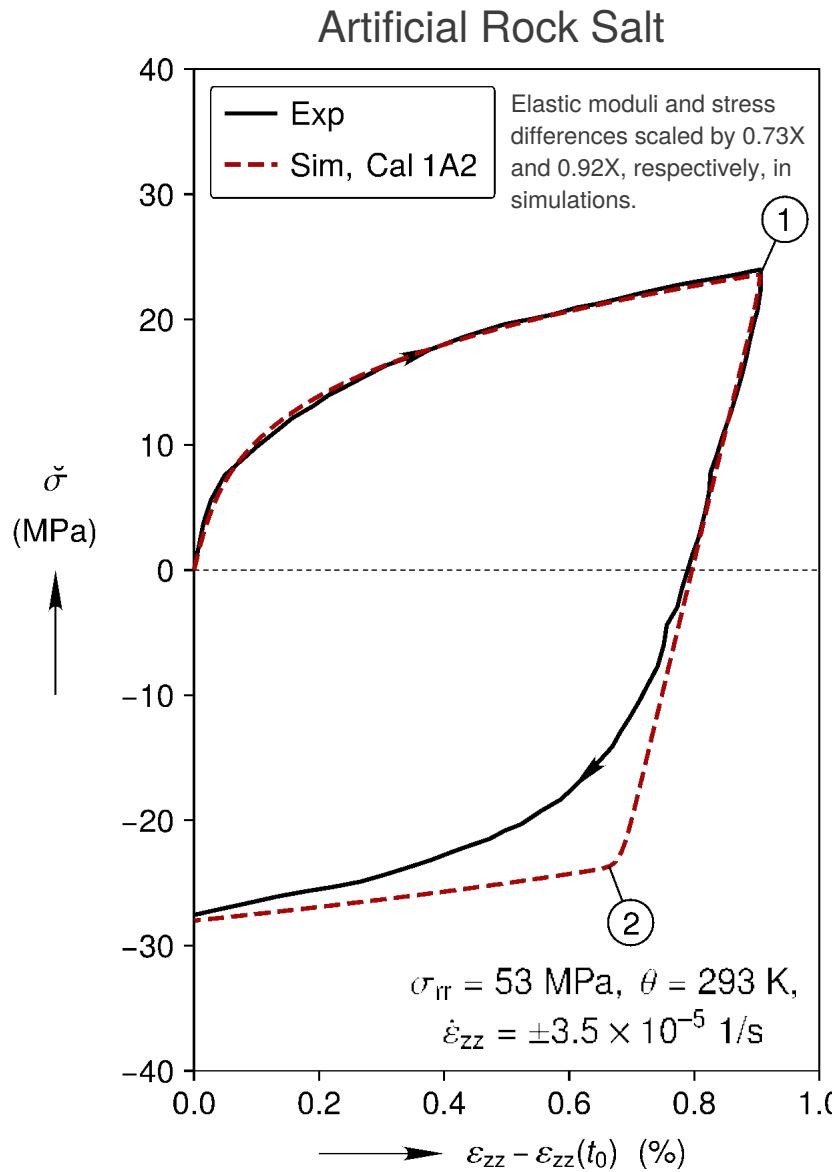
$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$



Equivalent Stress

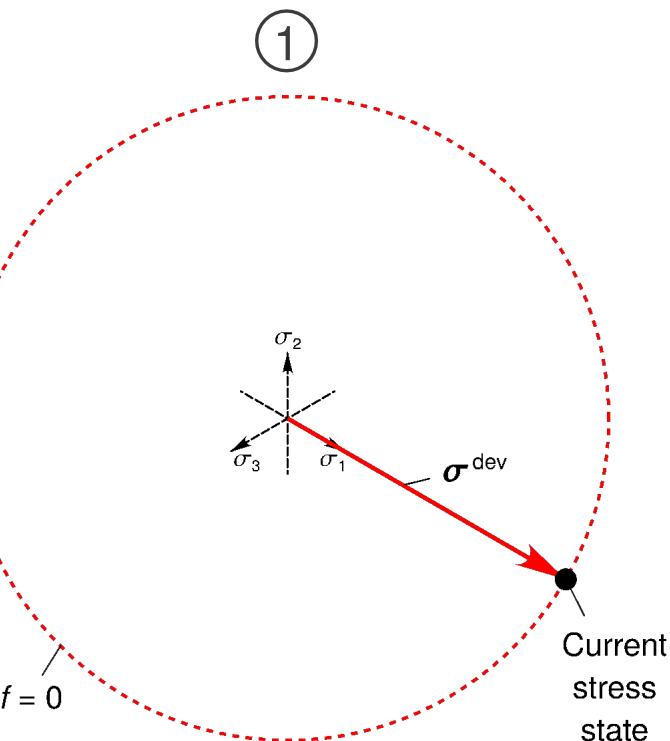
$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$

Bauschinger Effect: Isotropic Hardening Only



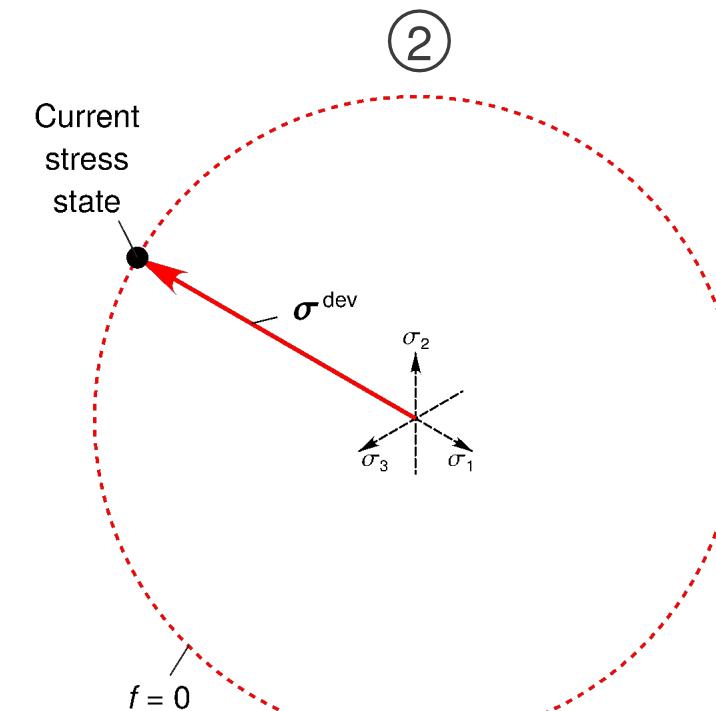
Dynamic Yield Surface

$$f = \tilde{\sigma} - y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

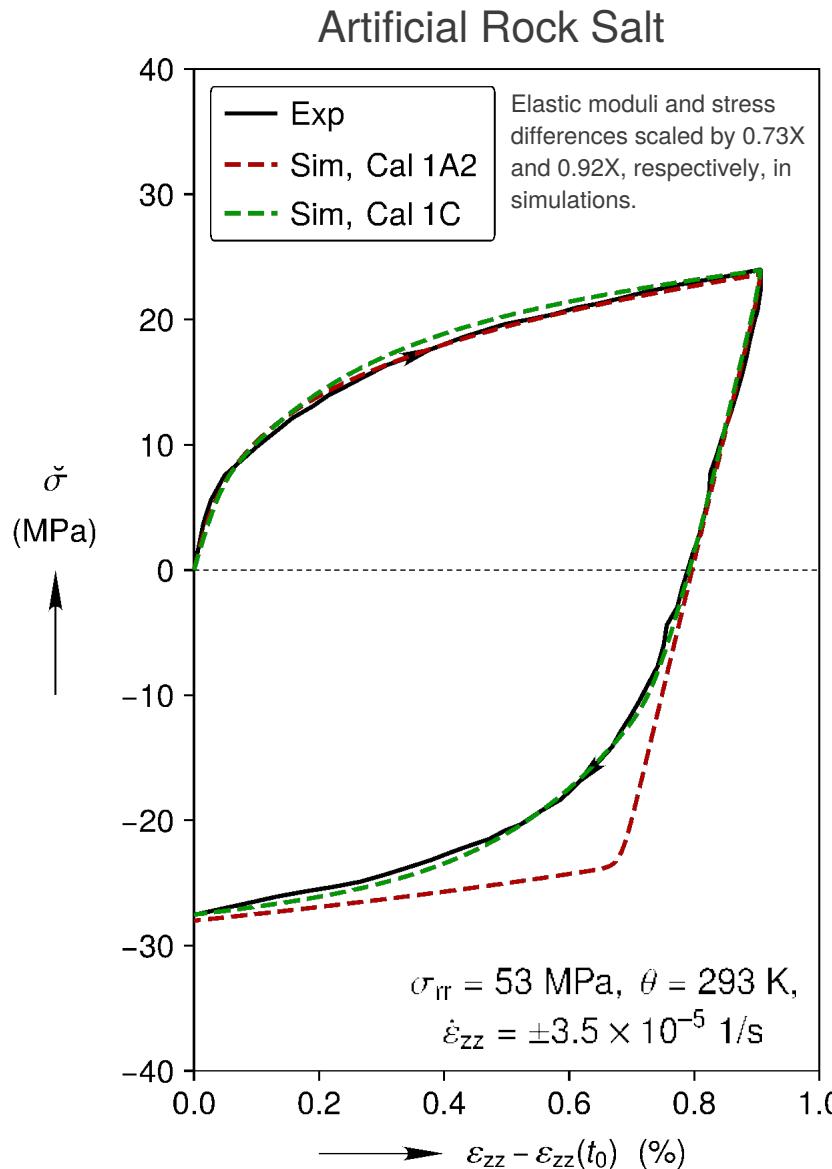


Equivalent Stress

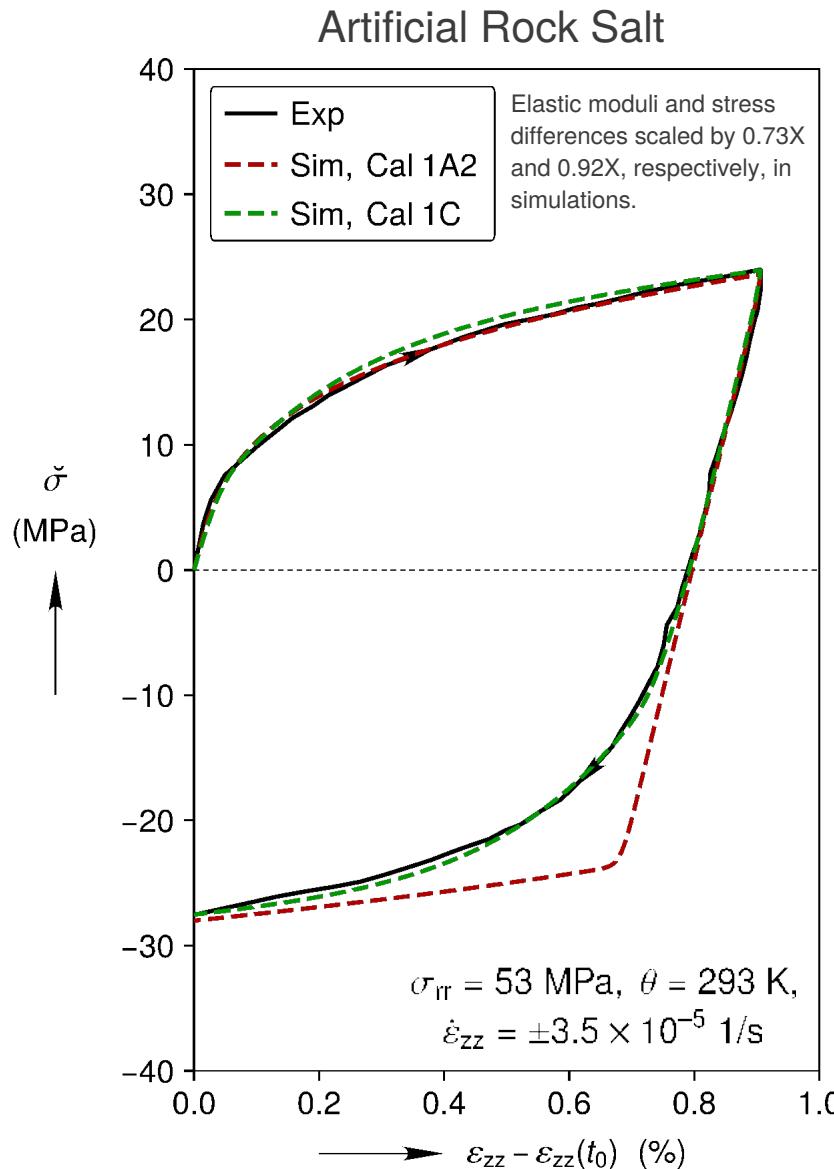
$$\tilde{\sigma} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}}$$



Bauschinger Effect: Isotropic + Kinematic Hardening



Bauschinger Effect: Isotropic + Kinematic Hardening



Dynamic Yield Surface

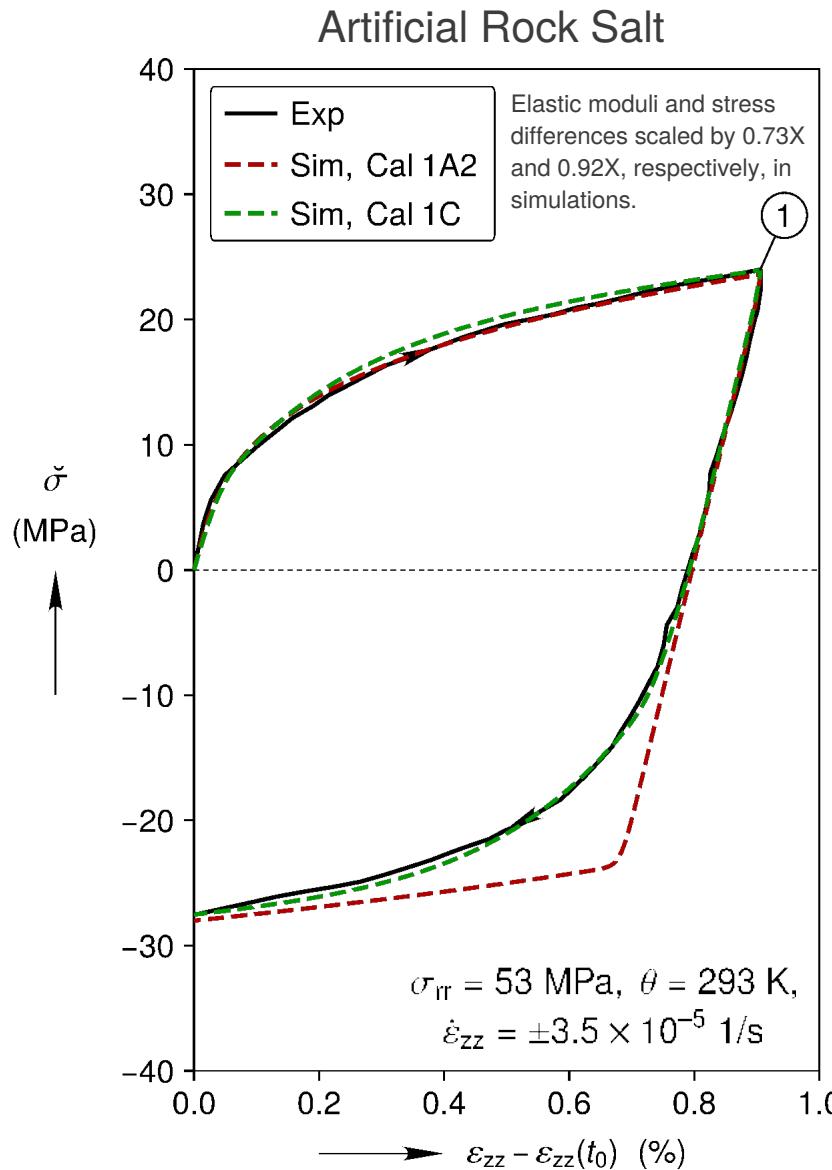
$$f = \tilde{\sigma}^{\text{dg}} - y \sinh^{-1} \left\{ \left[\frac{\dot{\tilde{\epsilon}}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

Equivalent Stress

$$\sigma^{\text{dg}} = \sigma^{\text{dev}} - \mathbf{b}$$

$$\tilde{\sigma}^{\text{dg}} = \sqrt{\frac{3}{2} \sigma^{\text{dg}} : \sigma^{\text{dg}}}$$

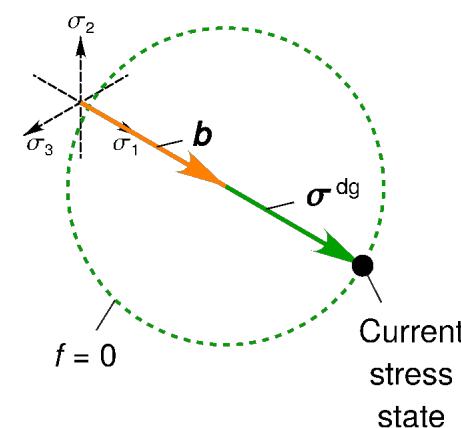
Bauschinger Effect: Isotropic + Kinematic Hardening



Dynamic Yield Surface

$$f = \tilde{\sigma}^{\text{dg}} - y \sinh^{-1} \left\{ \left[\frac{\dot{\epsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

1

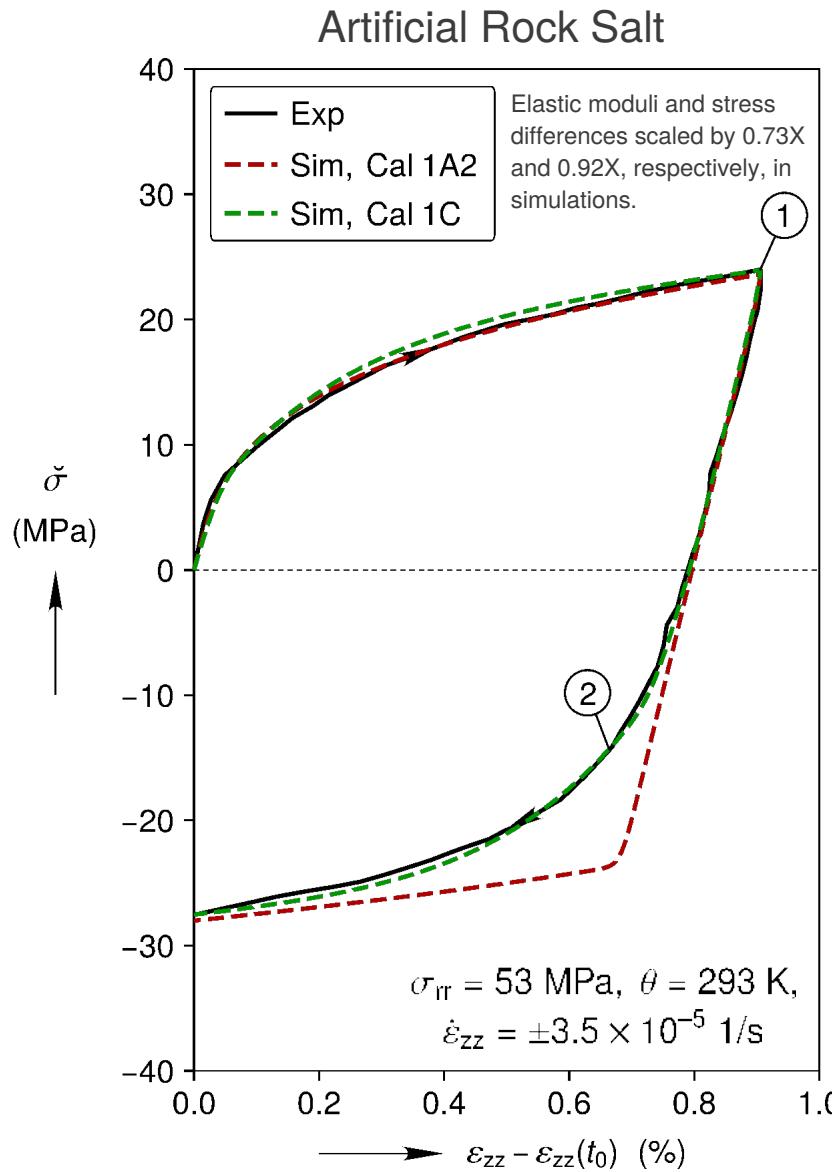


Equivalent Stress

$$\sigma^{\text{dg}} = \sigma^{\text{dev}} - \mathbf{b}$$

$$\tilde{\sigma}^{\text{dg}} = \sqrt{\frac{3}{2} \sigma^{\text{dg}} : \sigma^{\text{dg}}}$$

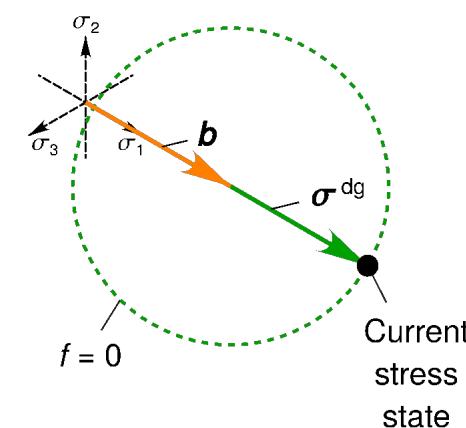
Bauschinger Effect: Isotropic + Kinematic Hardening



Dynamic Yield Surface

$$f = \tilde{\sigma}^{\text{dg}} - y \sinh^{-1} \left\{ \left[\frac{\dot{\epsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\} = 0$$

1



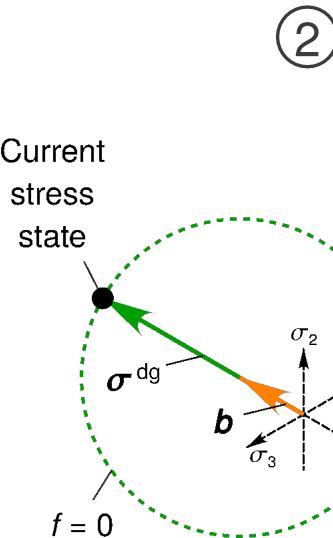
Equivalent Stress

$$\sigma^{\text{dg}} = \sigma^{\text{dev}} - \mathbf{b}$$

$$\tilde{\sigma}^{\text{dg}} = \sqrt{\frac{3}{2} \sigma^{\text{dg}} : \sigma^{\text{dg}}}$$

2

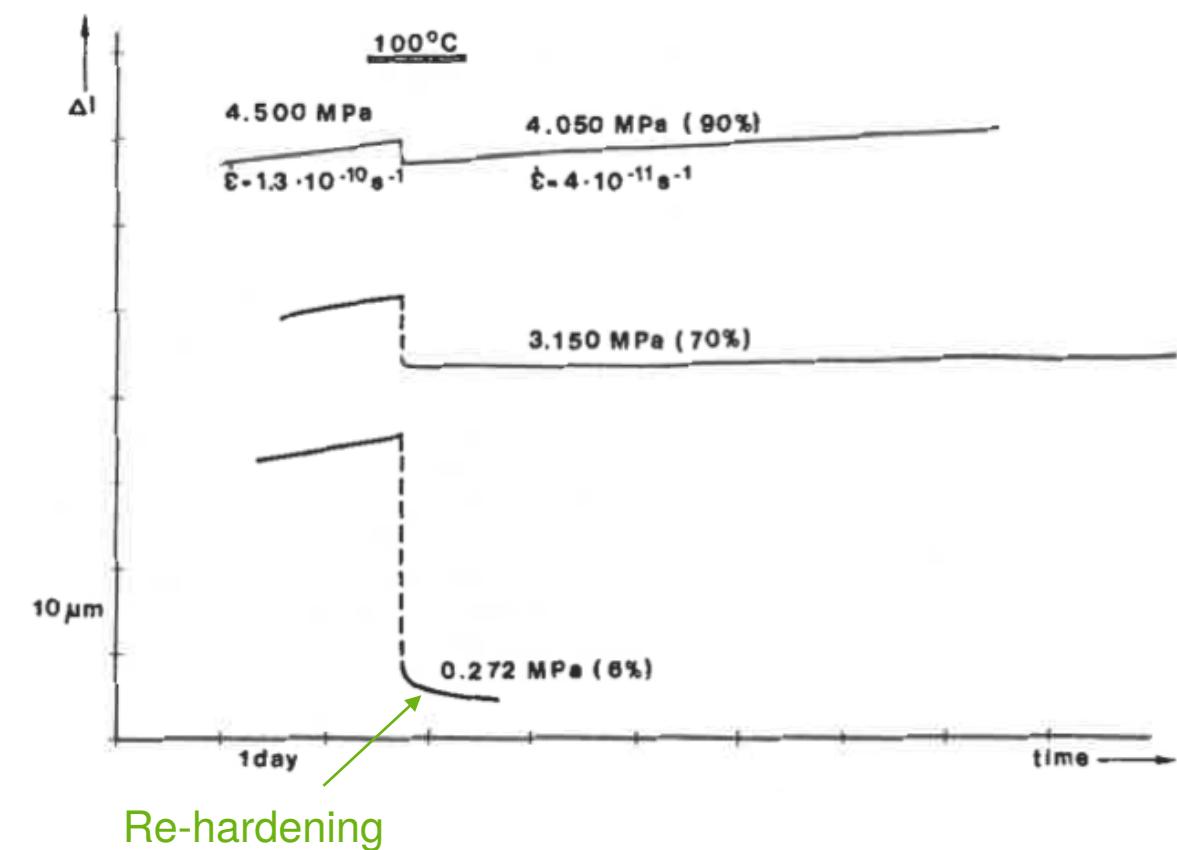
Current
stress
state



Re-hardening During Non-Monotonic Loading

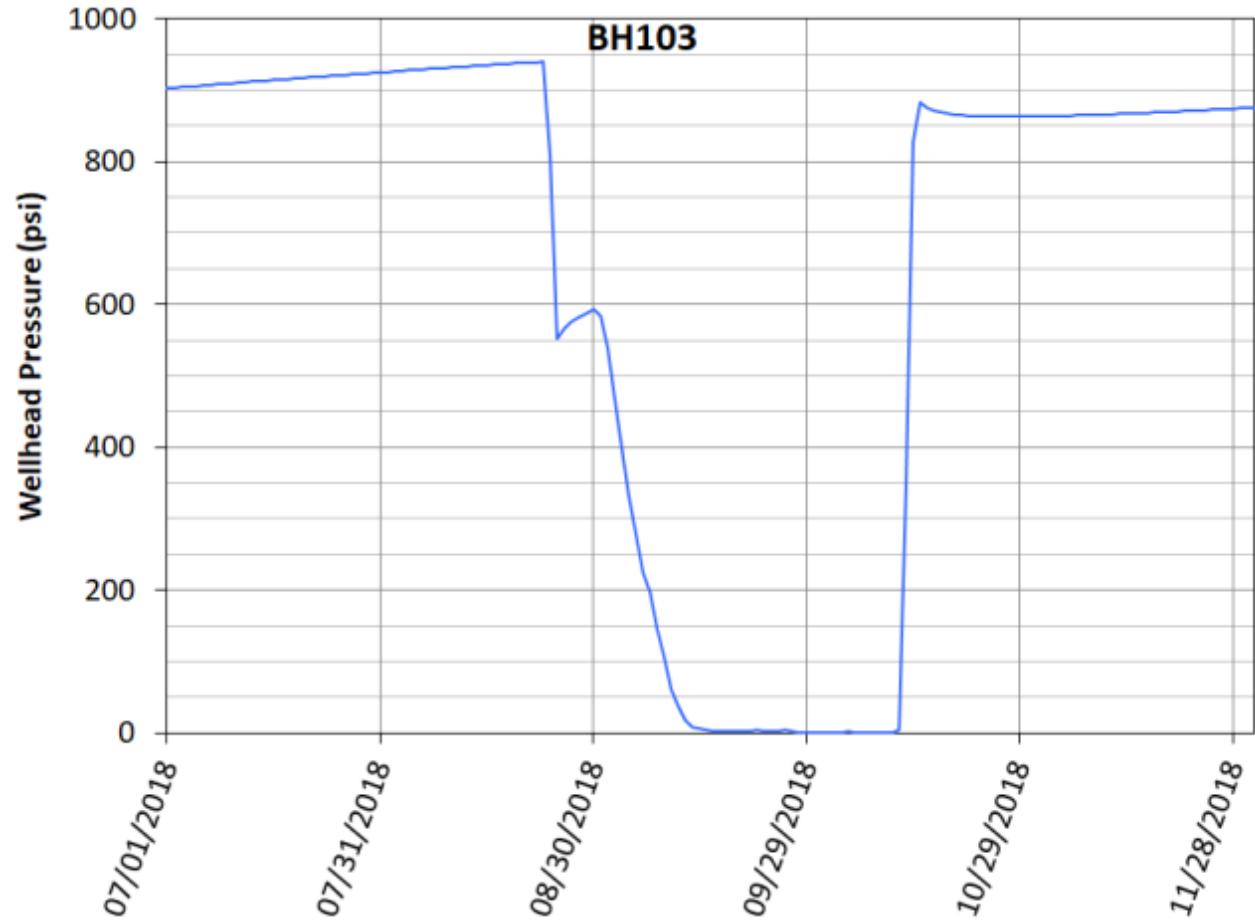


Asse Salt, Strain Histories



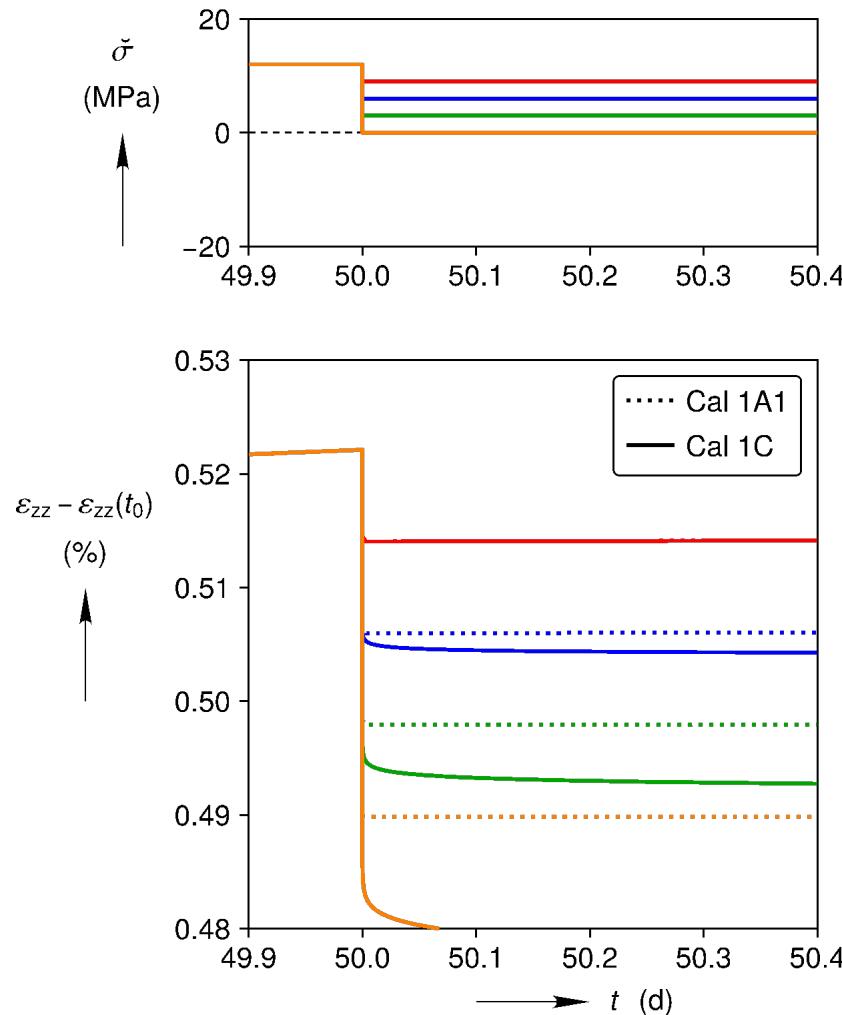
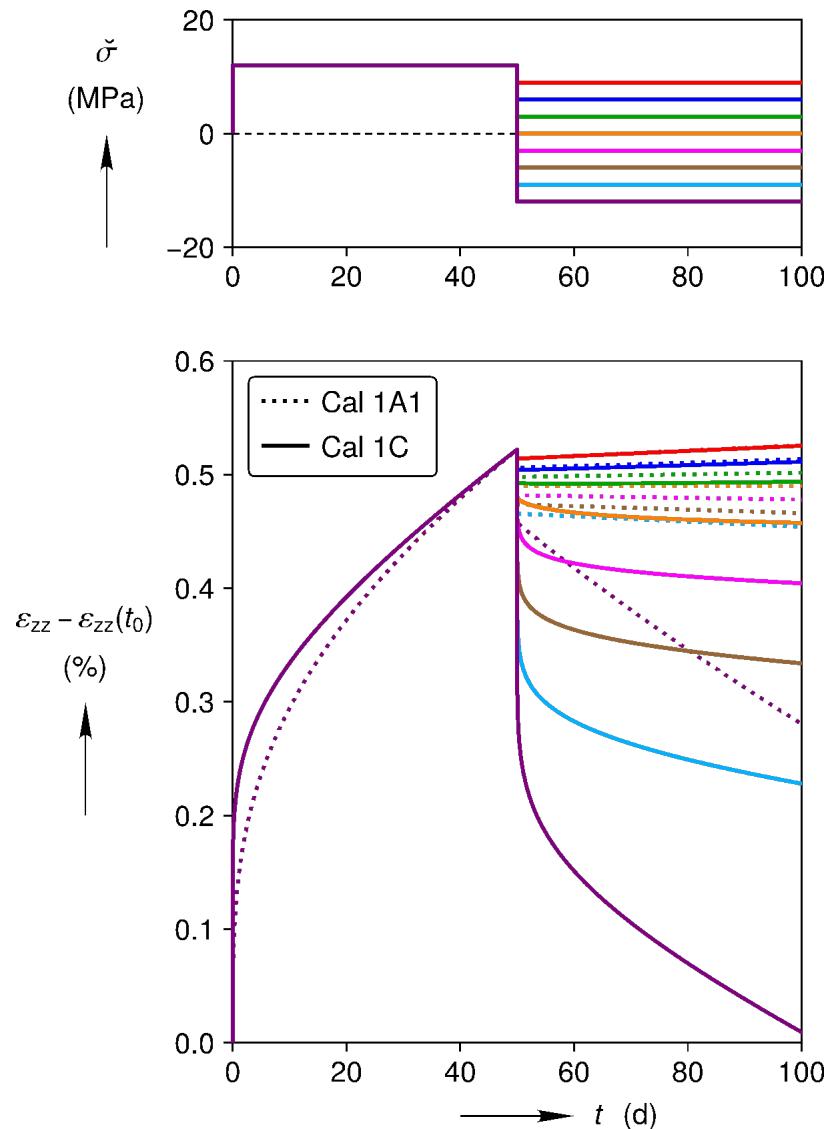
Hunsche, U. 1988. Measurement of creep in rock salt at small strain rates. Proceedings of the 2nd Conference on the Mechanical Behavior of Salt. Pg. 187-196

Wellhead Pressure Before, During, and After a Workover



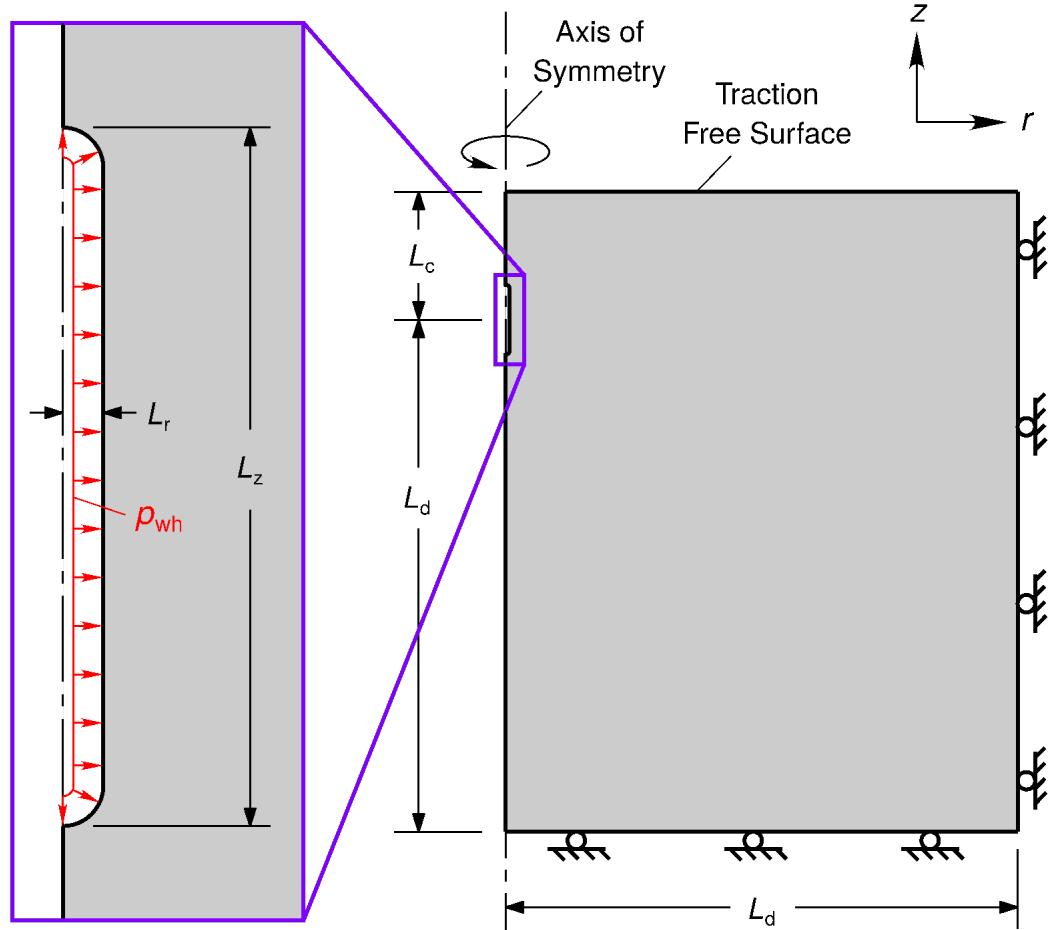
Sobolik, S. 2022. Personal Communication.

Stress Drop Behavior



Cal 1A1 strain shifted down by 0.067 % strain.

Gas Storage Cavern Simulation

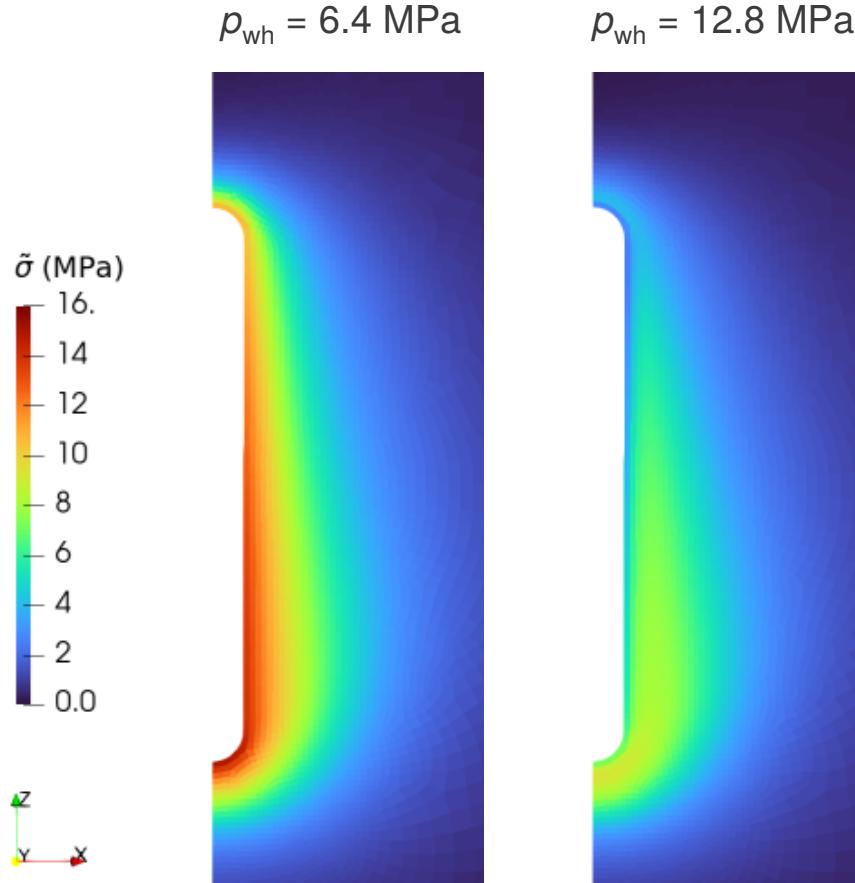


1. Isolated cigar shaped cavern
 1. Axisymmetric analysis
 2. Center of cavern was 1100 m deep
 3. Wellhead pressure held high for 5 years, held low for 3 days, and then cycled 3X.
2. Constitutive model robustness
 1. Model evaluated 2.6×10^9 times
 2. Model failed to converge 8 times
3. Constitutive model adaptive time stepping
 1. Initial time step = 10^{-3} s
 2. Time step after 5 years = 10 d

Gas Storage Cavern Simulation Results



von Mises Stress Distributions



Wellhead Pressure and Cavern Volume Loss Histories

