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Nonlinear Material Characterization in Dynamic Testing: Part III – Uncertainty Quantification

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1. ABSTRACT

Los Alamos National Laboratory (LANL) and Sandia National Laboratories (SNL) engaged in a collaboration to investigate and develop testing, analysis, and uncertainty quantification techniques to capture the relevant physics resulting from nonlinear sources and their corresponding influence on the structural response. The present work is Part III of a three-part series that discusses the inclusion of uncertainty quantification (UQ) approaches taken to excite and identify the properties of a nonlinear, compliant material. The UQ part of the series focuses on the development of a Bayesian model-updating framework for an aluminum and foam stackup with uncertain model parameters for the nonlinear material model. A surrogate model is derived to relate the material parameters to both the linear modes and the nonlinear normal mode backbone curves of the system. The framework establishes an efficient means to sample from the posterior distributions of the material parameters required by the Bayesian approach to update the model with both linear and nonlinear modal data obtained from the experiments.

Keywords: Parametric Studies, Surrogate Modeling, Uncertainty Quantification, Model Validation, Nonlinear Dynamics

2. INTRODUCTION

In an effort to produce representative physics-based models of dynamic systems, sources of nonlinearities can cause challenges in both test and modeling pursuits. One of these key challenges is quantifying the uncertainty associated with test results for the purposes of model updating. In particular, nonlinear materials can strongly influence structural dynamics and present unique challenges when it comes to quantifying model parameters. Los Alamos National Laboratory (LANL) and Sandia National Laboratories (SNL) engaged in a collaboration to investigate and develop testing, analysis, and uncertainty quantification techniques to capture the relevant physics resulting from nonlinear sources and their corresponding influence on the structural response. This extended abstract is Part III of a three-part series, which discusses the UQ aspect of this work.

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Applying Bayesian inference methods to update models with nonlinear materials has been studied; however prior work has primarily focused on using only linear modal properties as data features [1]–[5]. Song et al. [6], [7] have implemented Bayesian model updating using nonlinear normal modes to update the nonlinear parameters of nonlinear models, both numerically and using experimental data for structures with local nonlinearities. This work investigates the development of a Bayesian model updating framework for a structure with uncertain model parameters for the nonlinear material model. Building on the approach from Song et. al, this work develops a surrogate model to map the uncertain model inputs to the nonlinear quantities of interest (e.g., energy-dependent frequency and shapes of the target modes) to efficiently sample from posterior distributions. Here, the developed tools are exercised on the approximately linear quantities obtained from low-level modal tests. These tools can be applied to each energy level to capture the nonlinear effects present in the force-appropriation tests to update the nonlinear model parameters.

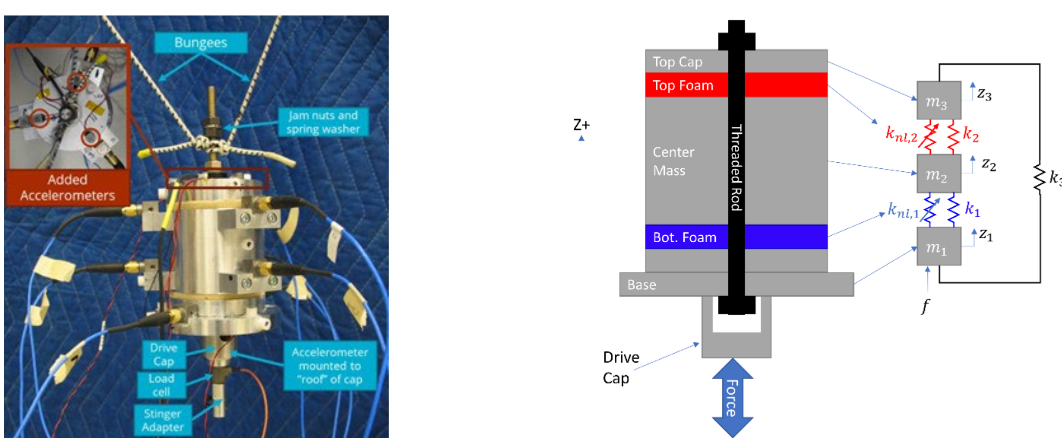
3. SURROGATE MODELING

Bayesian inference relies on Bayes' Theorem, which states that the posterior distribution of a set of unknowns is proportional to the prior distribution modified by the likelihood function. This likelihood function contains measurements that were not used when obtaining the prior distributions. For nonlinear model updating, the quantities contained in the likelihood function include the error between the measured and simulated energy-dependent frequencies and deflection shapes for the modes of interest [7]. Furthermore, sampling from the posterior distribution can be cumbersome, especially considering repeated evaluations of nonlinear forced responses at multiple levels to identify backbone curves.

A surrogate model can help reduce the computational burden by identifying the mapping between the unknown model inputs to the target variables, which consist of energy-dependent frequencies and deflection shapes for the modes of interest. The current approach utilizes a unique surrogate model generated with Gaussian Process Regression (GPR) for each energy level. For the energy-dependent frequencies, the surrogate model generates a direct mapping from the model inputs; however, the shape matrix will have as many rows as number of measurements, which can result in a large number of target variables. Principle component analysis (PCA) can reduce the number of target variables associated with the shape matrix to a significantly smaller number of weights. A sampling of the input space (e.g., using a space-filling design, such as Latin Hypercube Sampling) can generate the data necessary to optimize the parameters required by the GPR surrogate model.

4. APPLICATION TO EXPERIMENTAL TESTBED

This work assumes a simplified model of the experimental structure to capture the salient dynamics. Figure 1 shows the testbed and its corresponding simplified, lumped-parameter model, which consists of three degrees of freedom (DOFs). The three masses represent the aluminum blocks, and linear and nonlinear stiffness elements couple these masses. The stiffness elements coupling the top and bottom masses to the middle mass represent the foam samples, whereas a third linear stiffness element coupling the top and bottom masses represent the threaded rod. For updating the model, the masses were measured, and the unknowns included the linear and nonlinear stiffness elements.

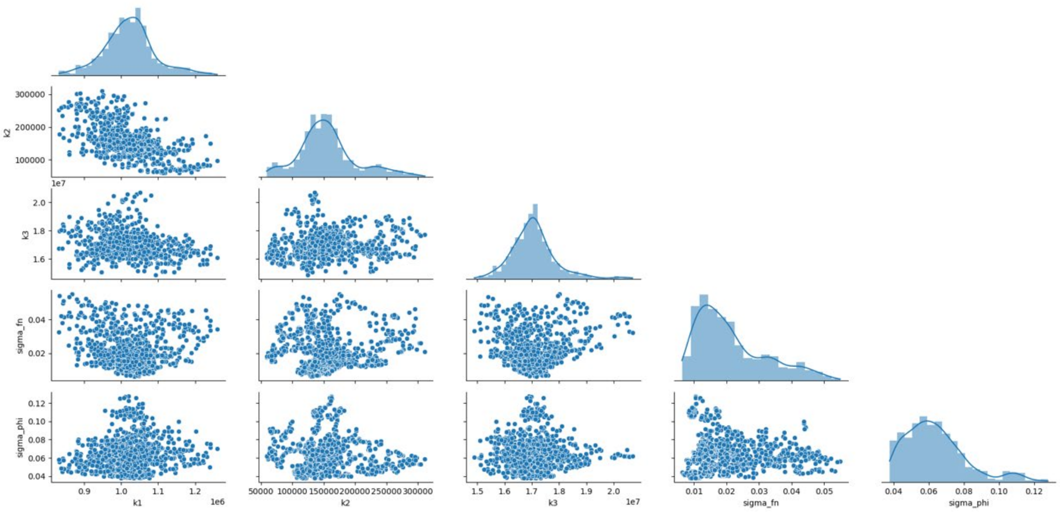


(a) (b)

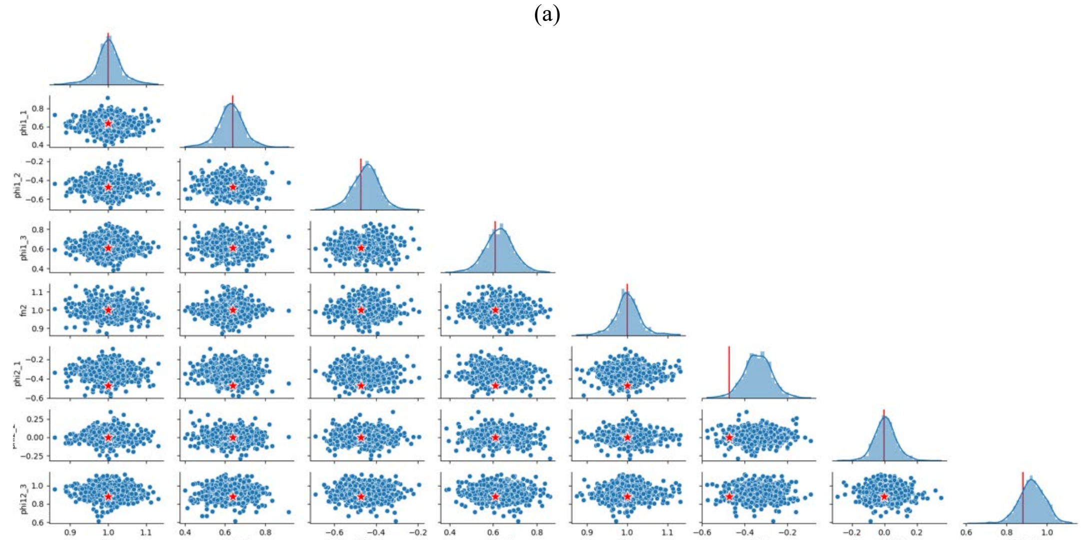
Figure 1. Testbed hardware showing (a) experimental setup and (b) simplified lumped-parameter model.

The analysis presented here utilized the low-level energy data obtained from a modal test (modal frequencies and mode shapes) to first update the three linear stiffness elements. The two target modes included the top and bottom masses moving in phase with each other and out of phase with the middle mass, and the top and bottom masses moving out of phase with each other with minimal motion of the middle mass. The surrogate model was built using the GPyTorch Python library [8] to identify the mapping between the three linear stiffness elements and the frequencies and shapes of these two target modes. The posterior distribution of the unknowns, which include the stiffness elements and error terms for both the frequencies and shapes, was then sampled using the NUTS algorithm available in the Pyro Python library [9].

Figure 2 shows the posterior distributions for both the unknowns and the target variables. Figure 2a shows that the first stiffness element, corresponding to the bottom foam sample, is larger than the second element, corresponding to the top foam sample, indicating that the two foam samples are experiencing unequal levels of compression. Furthermore, the third stiffness is significantly larger than both foam stiffness elements due to the coupling provided by the threaded rod. Figure 2b shows a comparison between the target variables generated using the posterior stiffness samples; it also shows the measured values (red vertical line and star). The top-left and bottom-right quadrants correspond to the frequencies and shapes of the first and second target modes, respectively. For both modes, the generated distributions capture all the measured values.



(a)



(b)

Figure 2. Uncertainty quantification using the low-level modal data showing a) inferred quantities and b) target variables.

5. CONCLUSIONS

This work described an approach to perform Bayesian model updating for a nonlinear system using a surrogate model. It is Part III of a three-part series of collaborative efforts between LANL and SNL to characterize the dynamic response of a stiff component sandwiched between compliant materials with inherently nonlinear material properties. The results presented here only include the low-level measurements to update the linear stiffness elements. This work also extends the surrogate model to include the quantities at several energy levels for updating the nonlinear elements. The nonlinear model update can also leverage the posterior distributions for the linear stiffness elements obtained from the low-level data as prior distributions when considering the full nonlinear model update.

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6. REFERENCES

[1] M. Song, R. Astroza, H. Ebrahimian, B. Moaveni, and C. Papadimitriou, "Adaptive Kalman filters for nonlinear finite element model updating," *Mech. Syst. Signal Process.*, vol. 143, p. 106837, 2020, doi: https://doi.org/10.1016/j.ymssp.2020.106837.

[2] E. N. Chatzi and A. W. Smyth, "The unscented Kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing," *Struct. Control Health Monit.*, vol. 16, no. 1, pp. 99–123, 2009, doi: https://doi.org/10.1002/stc.290.

[3] E. Asgarieh, B. Moaveni, and A. Stavridis, "Nonlinear finite element model updating of an infilled frame based on identified time-varying modal parameters during an earthquake," *J. Sound Vib.*, vol. 333, no. 23, pp. 6057–6073, 2014, doi: https://doi.org/10.1016/j.jsv.2014.04.064.

[4] E. Asgarieh, B. Moaveni, A. R. Barbosa, and E. Chatzi, "Nonlinear model calibration of a shear wall building using time and frequency data features," *Mech. Syst. Signal Process.*, vol. 85, pp. 236–251, 2017, doi: https://doi.org/10.1016/j.ymssp.2016.07.045.

[5] R. Astroza and A. Alessandri, "Effects of model uncertainty in nonlinear structural finite element model updating by numerical simulation of building structures," *Struct. Control Health Monit.*, vol. 26, no. 3, p. e2297, 2019, doi: https://doi.org/10.1002/stc.2297.

[6] M. Song, L. Renson, J.-P. Noël, B. Moaveni, and G. Kerschen, "Bayesian model updating of nonlinear systems using nonlinear normal modes," *Struct. Control Health Monit.*, vol. 25, no. 12, p. e2258, 2018.

[7] M. Song, L. Renson, B. Moaveni, and G. Kerschen, "Bayesian model updating and class selection of a wing-engine structure with nonlinear connections using nonlinear normal modes," *Mech. Syst. Signal Process.*, vol. 165, p. 108337, 2022.

[8] J. R. Gardner, G. Pleiss, D. Bindel, K. Q. Weinberger, and A. G. Wilson, "GPyTorch: Blackbox Matrix-Matrix Gaussian Process Inference with GPU Acceleration," In *NeurIPS*, pp. 7587-7597, 2018.

[9] E. Bingham, J. P. Chen, M. Jankowiak, F. Obermeyer, N. Pradhan, T. Karaletsos, R. Singh, P. Szerlip, P. Horsfall, and N. D. Goodman, "Pyro: Deep Universal Probabilistic Programming," *Journal of Machine Learning Research*, vol. 20, pp. 1-6.