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Demonstration of Output Weighting in MIMO Control

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ABSTRACT
Multiple-input/multiple-output (MIMO) vibration control often relies on a least-squares solution utilizing a matrix pseudo-inverse. While this is simple and effective for many cases, it lacks flexibility in assigning preference to specific control channels or degrees of freedom (DOFs). For example, the user may have some DOFs where accuracy is very important and other DOFs where accuracy is less important. This paper shows a method for assigning weighting to control channels in the MIMO vibration control process. These weights can be constant or frequency dependent functions depending on the application. An algorithm is presented for automatically selecting DOF weights based on a frequency-dependent data quality metric to ensure the control solution is only using the best, linear data. An example problem is presented to demonstrate the effectiveness of the weighted solution.

Keywords: MIMO, random vibration, vibration testing, weighted least squares

1 INTRODUCTION

In a typical multiple-input/multiple-output (MIMO) random vibration test, a controller determines inputs to match specified outputs as closely as possible. This is often accomplished using a direct inverse solution of the test system frequency response function (FRF) matrix, which relates the test system inputs and outputs. While this direct inverse solution is often sufficiently accurate, it lacks flexibility in terms of tailoring the inputs and outputs as the direct inverse solution is a least-squares regression and there are not settings or user-controllable parameters.

An ideal test would be perfectly accurate at all outputs locations or degrees of freedom (DOFs). However, it is often not possible to obtain perfect control accuracy at all output DOFs due to test setup constraints or mismatches between the specification response and the test system dynamics. In those cases, the test engineer is limited to either controlling to all available output DOFs or removing poorly performing DOFs. Neither of these choices are desirable as controlling to poorly performing DOFs or not having enough DOFs may introduce errors which affect the accuracy across the test system. Instead, it would be best to apply weighting to the output DOFs to best utilize both good and bad DOFs in the control solution.

This paper presents a straightforward method for introducing output DOF weighting in the MIMO control solution by scaling up or down entries of the specification cross-power spectral density (CPSD) and FRF matrices which correspond to output DOFs. Scaling up entries makes the output DOF more important and scaling down entries makes the output DOF less important. This simple scaling works because the direct inverse solution in MIMO control is effectively a least-squares solution across all output DOFs. Scaling up entries for a DOF makes its contribution to the

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total error larger while scaling down entries for a DOF makes its contribution to the total error smaller. Higher weighted DOFs will therefore be more accurately matched than lower weighted DOFs because the least-squares solution will determine inputs which better match the high weight DOFs than the low weight DOFs.

As MIMO random vibration control is evaluated frequency line by frequency line, the DOF weighting can be made frequency dependent. Choosing weights for each DOF as a function of frequency can be manually specified or automated. Here, an automatic weight selection method is presented which chooses weights based on linearity of each output DOF to the inputs, which effectively ensures that the highest quality (most linear) responses are emphasized in the control solution. This helps MIMO control by not allowing poor responding or nonlinear responding DOFs to contaminate the control solution. This is just one possible weight selection method – many others are possible and could be implemented in a similar way in this DOF weighting framework.

This weighting technique is not novel or unique as there are examples in various mathematics papers, texts and even websites, for example [1, 2, 3]. However, there is not much in the MIMO vibration literature regarding weighted solutions. As such, the purpose of this paper is to show, in a clear and simple way, how flexibility can be added to MIMO control solutions by using weighting of the output DOFs. This paper is laid out as follows. Section 2 presents the MIMO control and weighted least-squares theory. Section 3 provides step-by-step implementation details for weighted MIMO control. Section 4 discusses automatic weight selection methods. Section 5 shows results of a model-based demonstration of weighted MIMO control, where a structural dynamics model is used to compare standard and weighted MIMO control and demonstrate how automatic DOF weighting can help suppress errors in the control solution.

2 THEORY

A multiple-input, multiple-output linear system can be represented in the frequency domain as

$$Y = H_{yx}X \tag{1}$$

where X are the inputs, Y are the outputs, and H_{yx} is the FRF matrix which relates the inputs to the outputs. Note that Equation 1 is evaluated at each frequency line of interest. For ease of notation, the frequency dependence has been omitted in this and all subsequent equations. In MIMO random vibration, the inputs and outputs are typically represented as CPSD matrices and Equation 1 becomes

$$S_{yy} = H_{yx}S_{xx}H_{yx}^{H} \tag{2}$$

where S_{xx} is the input CPSD matrix, S_{yy} is the output CPSD matrix, and H_{yx} is the same FRF matrix from above [4, 5, 6]. The superscript $\cdot^{H}$ denotes a conjugate transpose.

In MIMO random vibration control, the direct inverse, open-loop solution determines inputs to best match some desired or specified response, $S_{yy,spec}$ by multiplying Equation 2 by the pseudo-inverse of the FRF matrix, H_{yx}^{+}:

$$S_{xx} = H_{yx}^{+}S_{yy,spec}H_{yx}^{+H} \; . \tag{3}$$

These inputs would then be applied to the test article to run the MIMO random vibration test. It should be noted that this pseudo-inverse results in a least-squares solution where the inputs are determined to minimize the squared error in the outputs. As such, outputs with large responses or high errors have relatively large influence on the results. Outputs from this test can be predicted using the FRF matrix using Equation 2. This simple CPSD representation of the linear system and control solution is used in this work to derive inputs and predict responses.

Weighting of output DOFs can be accomplished in the MIMO random vibration control solution (i.e. Equation 3) by applying a diagonal matrix W to both the FRF matrix and the specification CPSD matrix as [1, 2]:

$$\hat{H}_{yx} = WH_{yx} \, , \tag{4}$$

$$\hat{S}_{yy,spec} = WS_{yyx}W \; . \tag{5}$$

Then, the control solution is evaluated using the weighted CPSD and FRF matrices:

$$S_{xx} = \hat{H}_{yx}^{+}\hat{S}_{yy,spec}\hat{H}_{yx}^{+H} \; . \tag{6}$$

Expanding this out, we get:

$$S_{xx} = \left(WH_{yx}\right)^{+}\left(WS_{yy,spec}W\right)\left(WH_{yx}\right)^{+H} \; . \tag{7}$$

The weights apply to the rows of the FRF matrix, which correspond to the output DOFs, and to both the rows and columns of the output CPSD matrix as that matrix has output DOFs on both the rows and columns. Note that the values in the weight matrix can be any positive real number, and the relative weight between DOFs is only determined by the ratio of weights of those DOFs, not the overall level of the weights. For example, with three DOFs, the weights [10, 2, 1] and [100, 20, 10] would have the same effects.

Scaling up or down the values in the specification CPSD and FRF matrices associated with output DOFs changes the relative contributions of errors in the least-squares solution. For example, if DOF 1 is weighted by 100 and DOF 2 is weighted by 1, the error in DOF 1 becomes much larger in the least-squares solution compared with DOF 2, so inputs estimated with Equation 6 will be biased to minimize error on DOF 1. As these expressions are evaluated frequency line by frequency line, the weight matrix could be frequency dependent to emphasize or ignore different DOFs at different frequencies in the test bandwidth.

3 IMPLEMENTATION DETAILS

Implementing weighted MIMO control for random vibration is straightforward and outlined in the steps below.

1. Choose weights for each output DOF and form the weight matrix
2. Perform a system identification test and form the FRF matrix
3. Apply the weight matrix to the FRF and specification CPSD matrices
4. Estimate the input CPSD matrix which best matches the weighted specification CPSD matrix
5. Predict response with the estimated inputs and verify the quality of the results
6. Adjust weights if the effects of the weights are not achieved in the predictions
7. Run the test with those inputs

This weighted MIMO control solution fits nicely into typical MIMO control methods and can be easily implemented into existing controller methods. One thing to note is that because weights are applied to both the specification CPSD matrix and the FRF matrix, no scaling is needed to “un-do” the effects of the weights on the input CPSD.

Determining weight values which provide the desired effects, for example emphasizing or ignoring specific DOFs, may not be straightforward, so some iteration on choosing weights and predicting results is recommended. In the course of this work it was found that a 100x ratio in weights was effective in emphasizing or ignoring DOFs.

An optional step is to normalize the specification CPSD and FRF matrices to unity auto-power spectral density (APSD) levels prior to applying weights. This forces the specification amplitudes to be the same, unity, for all DOFs at all frequency lines. This avoids any biasing due to relative response amplitudes and ensures that the chosen weights are implemented as desired.

4 AUTOMATIC WEIGHT SELECTION METHODS

In some cases, it may be useful for DOF weights to be determined automatically on a frequency-by-frequency basis. For example, weights could be determined based on response amplitude, linearity, or data quality. As MIMO random vibration control (at least as described above) is based on linear system theory, utilizing output DOFs which have good linear response to inputs is critical.

If the linearity between outputs and inputs is poor, the estimated inputs and resulting responses may be poor as well. As such, it is desirable to not utilize DOFs with poor linearity. However, there may be cases where DOFs cannot be simply removed from the control set due to having limited control DOFs or wanting to utilize as much specification data as possible. A weighted MIMO solution can allow DOFs with marginal or poor linearity to still be used in the control solution but be de-emphasized by using smaller weights.

Linearity between an output and multiple inputs can be quantified with the multiple coherence, which takes values between zero and one, with zero indicating no linear relationship and one indicating a strong linear relationship. Multiple coherence is a function of frequency and output DOF, so it can be leveraged to create a set of weights as

$$W_i = MCOH_i^{\alpha} \,, \tag{4}$$

where W_i is the weight for the i-th DOF which has multiple coherence $MCOH_i$. The factor α is used to exaggerate the difference between high and low multiple coherence. In this work, an α of 8 was used and provided good results, allowing DOFs with multiple coherence near one to have weights near one but DOFs with lower multiple coherence to have very small weights. Figure 1 demonstrates how α affects the MCOH to weight relationship.

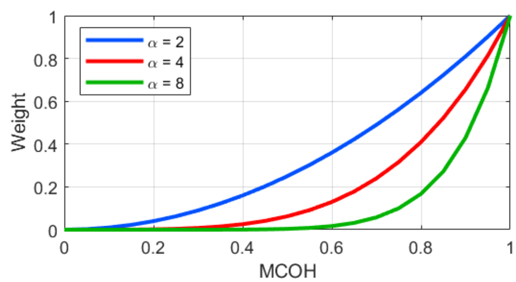

Figure 1: Weight vs MCOH using different α values

One can envision a similar method applied to signal-to-noise ratios or some other data quality metric as a way to de-emphasize poorly responding DOFs. Similarly, weighting could be determined based on DOF response accuracy, where the weight is determined by the predicted response accuracy using an un-weighted solution. DOFs with good accuracy could be assigned large weights and DOFs with poor accuracy could be assigned smaller weights. This may help account for location-specific errors (e.g. remove effects of one inaccurate gauge or gauges on a component that cannot be properly excited). Different weighting approaches could also be combined by simply multiplying them together, for example the total weight could be a product of multiple coherence weights and control accuracy weights. In short, there are many ways to create weights that would improve MIMO control solutions and result in the best balance between data quality, available specification data, and control accuracy.

5 DEMONSTRATION OF WEIGHTED MIMO CONTROL

To demonstrate how weighted MIMO control works, a model-based example is created. This example system is first subjected to one set of inputs which represent loads in a field or service environment. The response from this field configuration becomes our MIMO specification. Next, inputs are moved to different locations to represent a laboratory (lab) test configuration. This input DOF change is utilized to demonstrate a common challenge in MIMO testing where the true load paths are unknown or not available in the lab test, and this mismatch of input DOFs creates a non-trivial control solution. The lab system FRFs are used to predict the inputs and responses of a lab test and these responses are compared with the specification to assess test accuracy. Weights are then applied to demonstrate how output DOF weighting can be used to affect test results.

5.1 EXAMPLE SYSTEM

The example system is shown in Figure 2 below. A finite element model of this system was used to compute the modes and then those modes were used in modal transient simulations to get acceleration response due to force inputs. Two configurations are shown, the field configuration and the lab configuration. The system is the same in each configuration but the input locations and directions change. In the field configuration, uncorrelated force inputs are applied in the X, Y, and Z directions. Response time histories at the output DOFs (two locations labeled 101 and 304, three directions each) are processed into the specification CPSD matrix. FRFs for the lab configuration are determined using the responses at the output DOFs due to uncorrelated force inputs at four locations in the Y direction at the bottom corners of the system. Noise can be added to the time histories to simulate some data contamination effects.

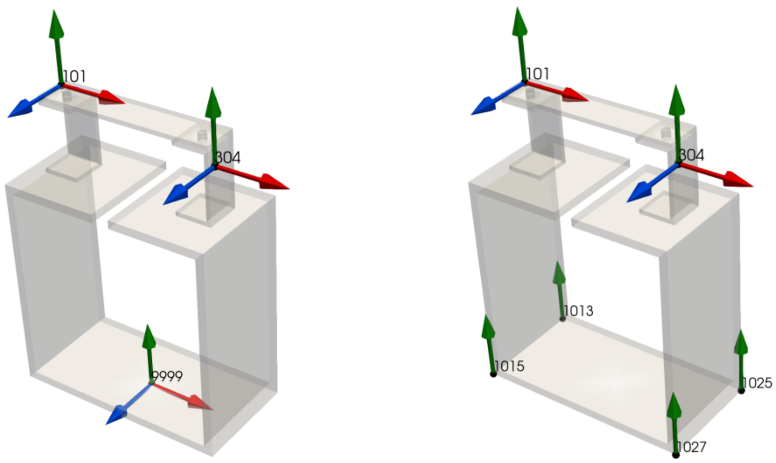

Figure 2: Example system with outputs (labeled 101, 304) and inputs for both field (left, labeled 9999) and lab (right, labeled 1013, 1015, 1025, 1027). Directions indicated by arrow color with red = X, green = Y, blue = Z.

5.2 WEIGHTED MIMO, CONSTANT WEIGHTS

First, an example with constant weights is provided. Here the Y-direction DOFs (101Y+ and 304Y+) have high weights (1.0) and the X- and Z-direction DOFs (101X+, 101Z+, 304X+, and 304Z+) have low weights (0.01). The weights are constant for all frequency lines. Figure 3 shows the APSD response predictions compared with the specification APSD using un-weighted (standard MIMO) and weighted MIMO control solutions. Figure 4 shows the APSD response in terms of decibel (dB) error with respect to the specification APSD. Figure 5 shows the RMS of the dB errors to provide a single error value for each DOF. These results clearly show how applying weights causes the Y DOFs to have nearly perfect control, though at the expense of accuracy at the X and Z DOFs. Intuitively, changing the relative weights, either higher or lower, changes the balance of accuracy between the DOFs (Figure 6).

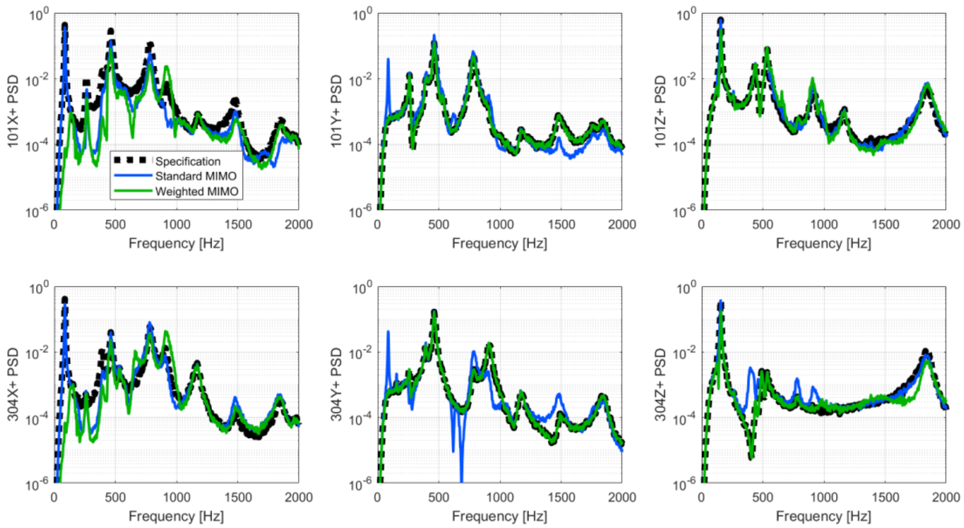

Figure 3: APSDs for each of the six output DOFs comparing the field response (specification, black dotted line) and MIMO test predictions using un-weighted (standard, blue line) and weighted (green line) MIMO solutions

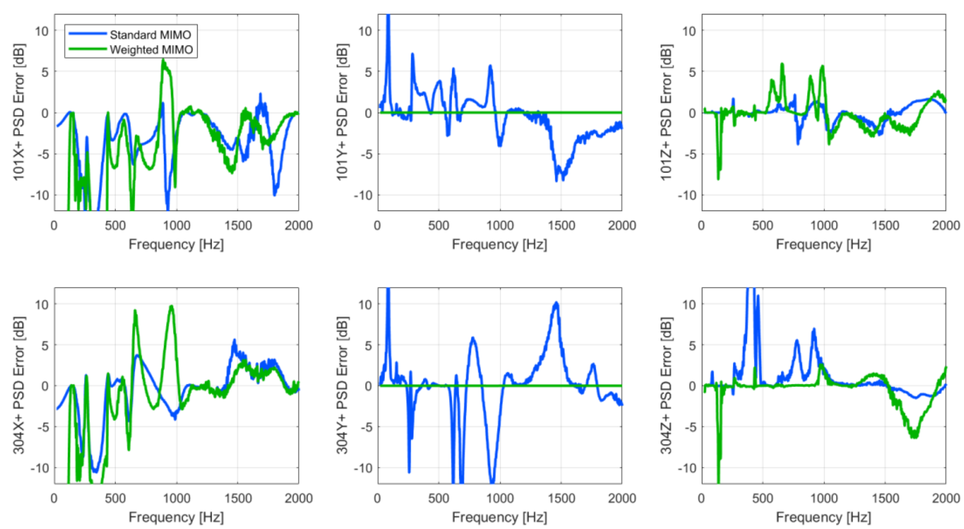

Figure 4: dB error in APSDs with respect to the field response comparing un-weighted (standard, blue line) and weighted (green line) MIMO solutions

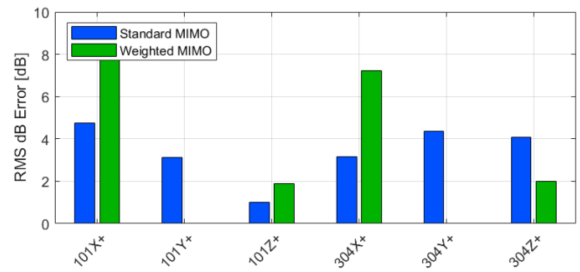

Figure 5: RMS of the dB error comparing un-weighted (standard, blue) and weighted (green) MIMO solutions

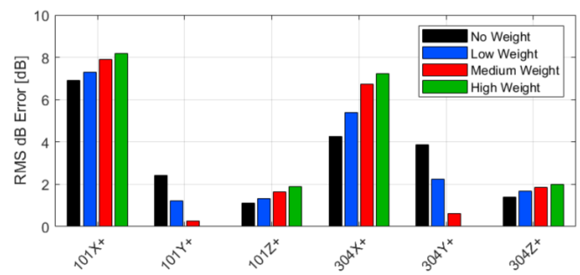

Figure 6: RMS of the dB error for different amounts of weighting between the Y and X, Z DOFs

5.3 WEIGHTED MIMO, AUTOMATICALLY CHOSEN WEIGHTS

To demonstrate how weights can be automatically determined using multiple coherence, first the FRF data needed to be contaminated with noise to cause the coherence to drop. This is akin to what may happen to low-responding channels in a system, where the response is near the noise floor of the sensor. Here the 101 X, Y, and Z DOFs had low noise and 304 X, Y, and Z DOFs had high noise. The multiple coherence computed from this noisy data shows how the noise affects the linear relationship, Figure 7. Next, the multiple coherence is converted to weights for each DOF at each frequency line using an α of 8. Figure 7 shows how the weights are just an exaggeration of the multiple coherence, and how low multiple coherence results in very low weights as desired.

Using the multiple coherence-determined weights is effective in improving control to the 101 DOFs where FRF linearity is good as shown in Figures 8, 9 and 10. Accuracy at the 304 DOFs is not as good but still reasonable. This demonstrates how using a weighted approach is preferable to simply removing the 304 DOFs entirely for two reasons. First, if the 304 DOFs were removed entirely, the problem would go from over-determined to under-determined, which would require a completely different solution method which has its own challenges. Second, by still including the 304 DOFs, some response accuracy can be maintained at that location. If it were completely removed from the solution, the response could become very inaccurate at that location.

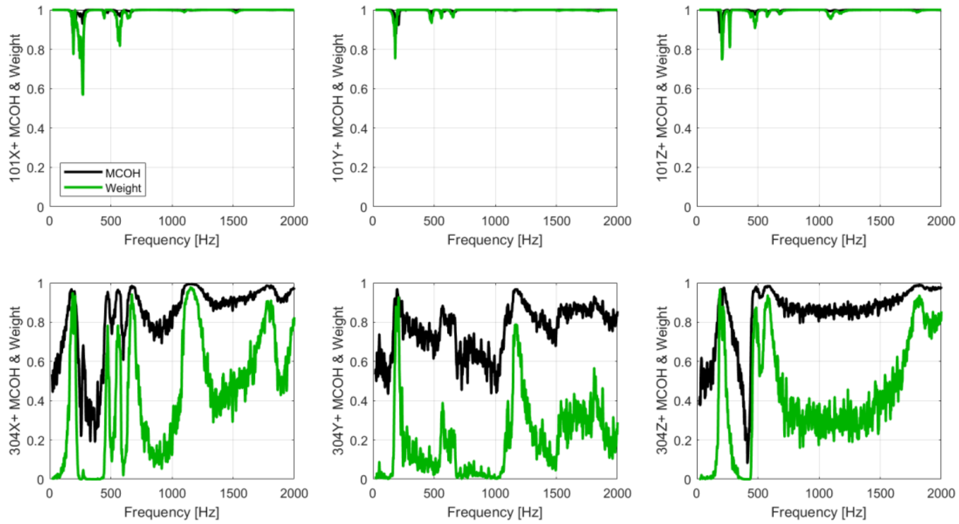

Figure 7: Multiple coherence (black) and weights derived from the multiple coherence (green) at each of the six output DOFs

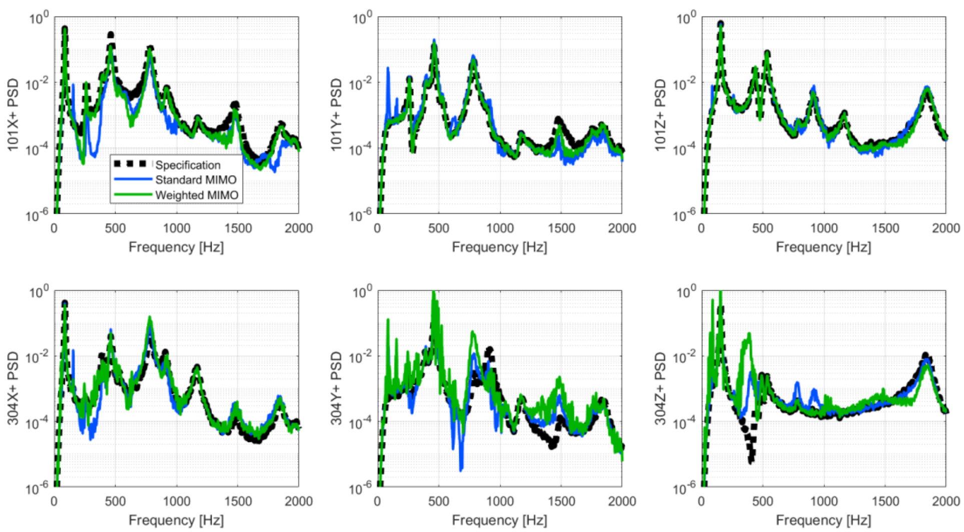

Figure 8: APSDs for each of the six output DOFs comparing the field response (specification, black dotted line) and MIMO test predictions using un-weighted (standard, blue line) and weighted (green line) MIMO solutions. Weighted solution utilizes weights automatically determined based on multiple coherence.

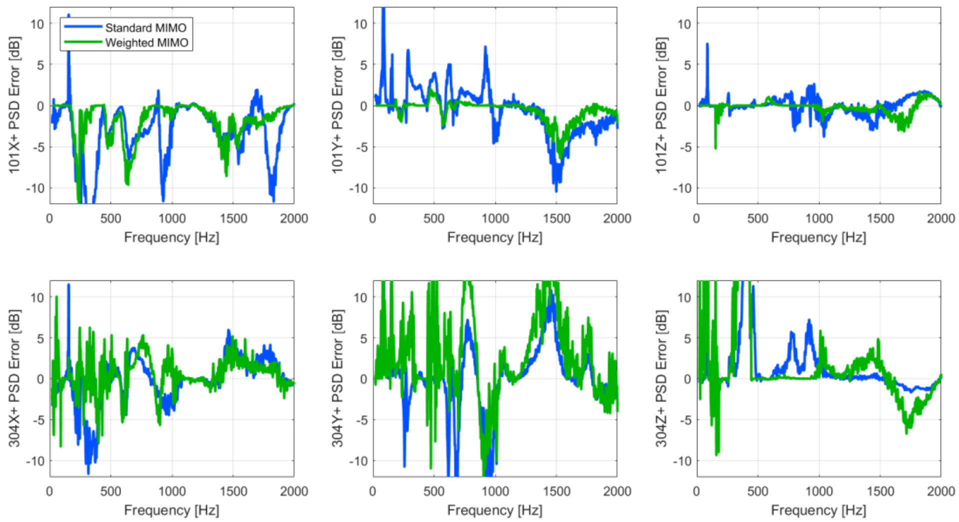

Figure 9: dB error in APSDs with respect to the field response comparing un-weighted (standard, blue line) and weighted (green line) MIMO solutions. Weighted solution utilizes weights automatically determined based on multiple coherence.

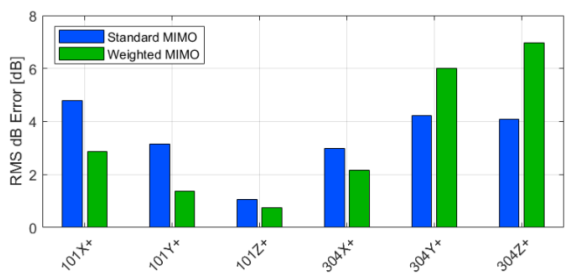

Figure 10: RMS of the dB error comparing un-weighted (standard, blue) and weighted (green) MIMO solutions. Weighted solution utilizes weights automatically determined based on multiple coherence.

6 CONCLUSIONS

Flexibility is useful in MIMO control solutions to enable the test engineer to achieve desired results or make the best use of available data. Weighted MIMO control is one tool that can provide additional flexibility. By simply scaling the output DOF terms in the specification CPSD and FRF matrices, the least-squares solution utilized in MIMO control can be biased to increase or decrease the accuracy of solutions at specific DOFs. Further, because the MIMO control solution is evaluated at each frequency line, frequency dependent weights can be utilized. Here, a method for automatically determining DOF weights is presented. This approach de-weights DOFs with poor linearity, which would otherwise be detrimental to the solution. Many other possible weight selection methods are possible and could be explored in future work. Overall, weighted MIMO control is practical and effective, as demonstrated in a simple model-based example.

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