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Optimal Mobile Energy Storage Pre-Placement for Black-Start Restoration

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***Abstract*—This paper studies a novel mixed-integer linear programming (MILP) formulation on the pre-blackout placement of mobile energy storage (MES) for black-start (BS) restoration of a transmission network. The formulation is a stochastic program that, rather than re-energization over a multi-interval horizon, for each scenario considers the final energization states of generators in a steady-state power flow. If a scenario of the event inducing the blackout cuts off an island from the network, the island is re-energized following the blackout only if it contains a generator with either BS capability or a pre-placed MES. Besides the model, this paper also explores the novel analytical concept of a discretized expected value realization.**

***Index Terms*—Energy storage, power outages, power system restoration, standby generators.**

NOMENCLATURE

Sets	
$\mathcal{S}$	Set of scenarios
$\mathcal{I}$ / $\mathcal{L}$ / $\mathcal{G}$	Set of buses/branches/generators
$\mathcal{I}_G$	Set of generator buses
$\mathcal{P}_{i_1 i_2}$	Set of paths from bus i_1 to bus i_2
$\mathcal{L}_p$	Set of branches in path p
$\mathcal{G}_i$	Set of generators at bus i
Parameters	
M	Large positive number for big-M constraints
E	Allowance for pre-blackout MES placement
$\overline{\theta}/\underline{\theta}$	Max./min. limit for angle at any bus [rad]
Υ^s	Probability of scenario s
$O_i^s/O_l^s/O_g^s$	1 if bus i / branch l / generator g is disabled in scenario s; o.w. 0
$\overline{F}_l/\overline{F}_g$	Capacity of branch l / generator g [MW]
$VoLL_i$	Value of lost load at bus i [\$/MWh]
D_i	Load demand at bus i [MW]
B_l	Susceptance of branch l [pu]
C_g	1 if generator g is BS-capable; o.w. 0
Variables	
e_i	1 if an MES is placed at bus i; o.w. 0
c_g	1 if generator g can start without power from grid; o.w. 0
n_i^s/n_g^s	1 if bus i / generator g can originate BS restoration in scenario s; o.w. 0
$\Psi_{i_1 i_2}^s/\Psi_{il}^s$	1 if bus i_1 has connectivity to bus i_2 / branch l in scenario s; o.w. 0
$v_{i_1 i_2}^s/v_{il}^s/v_{ig}^s$	1 if bus i_1 can re-energize bus i_2 / branch l / generator g in scenario s; o.w. 0
$r_i^s/r_l^s/r_g^s$	1 if bus i / branch l / generator g is re-energized in scenario s; o.w. 0
θ_i^s/h_i^s	Voltage angle [rad] / load shed [MW] at bus i in scenario s
f_l^s/f_g^s	Flow in branch l / output of generator g in scenario s [MW]
Other notation	
$o(l)/d(l)$	Origin/destination bus of branch l
$i(g)$	Bus hosting generator g

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I. INTRODUCTION

Black-start (BS) generators, which can restart to originate grid re-energization without an external power source, are instrumental to rapid restoration of a transmission network following a complete blackout [1]. Furthermore, utility-scale energy storage (ES) has demonstrated the ability to "black-start" a generator by supplying its cranking power [2]. This paper thus examines the optimal pre-blackout placement of utility-scale mobile energy storage (MES) for BS restoration.

Previous works (e.g., [3]–[5]) propose mixed-integer linear programming (MILP) models on optimal BS restoration. While these models account for a multi-interval horizon, the model in [6] assesses still-intact conducting paths to directly consider the final energization states of generators in a steady-state power flow. Another recently created model [7] integrates MES into BS restoration, yet like others the model supposes a horizon requiring costly integer variables for each interval.

This paper makes two novel contributions.

(I) A stochastic programming, MILP model is developed for pre-blackout placement of MES for BS restoration. To focus on benefits of pre-placement, the steady-state power flow of the model does not account for relocation or other activity of MES over a multi-interval horizon. Instead of relying on costly integer variables as in [6], the constraints that assess conducting paths in the model employ McCormick relaxation to keep variables continuous.

(II) The analytical concept of a discretized expected value realization is devised to provide a realizable surrogate for the expected value of an integer-valued random parameter vector in the context of stochastic programming.

II. Mathematical Optimization Model

This section presents the stochastic program on optimal pre-blackout placement of MES for steady-state grid operation after BS restoration. The power and energy ratings of each MES are assumed sufficient to supply the start-up sequence of any generator. For further simplification, the model contains a DC power flow to consider only real, not complex, power.

A. Formulation

The stochastic program developed for optimal pre-blackout placement of MES for BS restoration is

$$\min \sum_{s\in\mathcal{S}} \Upsilon^s \sum_{i\in\mathcal{I}} VoLL_i h_i^s \quad (1)$$

subject to

$$\sum_{i\in\mathcal{I}_G} e_i \le E, \quad (2)$$

$$1 - c_g = (1 - C_g)(1 - e_{i(g)}) \quad \forall g \in \mathcal{G}, \quad (3)$$

$$n_g^s = (1 - O_g^s)c_g \quad \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \quad (4)$$

$$1 - n_i^s = \prod_{g\in\mathcal{G}_i} (1 - n_g^s) \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}_G, \quad (5)$$

$$v_{ii}^s = (1 - O_i^s)n_i^s \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}_G, \quad (6)$$

$$v_{i_1 i_2}^s = \Psi_{i_1 i_2}^s v_{i_1 i_1}^s \quad \forall s \in \mathcal{S}, \forall i_1 \in \mathcal{I}_G, \forall i_2 \in \mathcal{I} | i_2 \ne i_1, \quad (7)$$

$$v_{il}^s = \Psi_{il}^s v_{ii}^s \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}_G, \forall l \in \mathcal{L}, \quad (8)$$

$$v_{i_1 g}^s = (1 - O_g^s)v_{i_1 i_2}^s \quad \forall s \in \mathcal{S}, \forall i_1 \in \mathcal{I}_G, \forall i_2 \in \mathcal{I}_G | i_2 \ne i_1, \forall g \in \mathcal{G}_{i_2}, \quad (9)$$

$$\Psi_{ii}^s = 1 \quad \forall s \in S, \forall i \in \mathcal{I}_G, \quad (10)$$

$$1 - \Psi_{i_1 i_2}^s = \prod_{p\in\mathcal{P}_{i_1 i_2}} \left(1 - (1 - O_{i_2}^s) \prod_{l\in\mathcal{L}_p} (1 - O_l^s)\right) \quad \forall s \in \mathcal{S}, \forall i_1 \in \mathcal{I}_G, \forall i_2 \in \mathcal{I} | i_2 \ne i_1, \quad (11)$$

$$\Psi_{il}^s = (1 - O_l^s)\Psi_{i,o(l)}^s \Psi_{i,d(l)}^s \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}_G, \forall l \in \mathcal{L}, \quad (12)$$

$$1 - r_{i_1}^s = \prod_{i_2\in\mathcal{I}_G} (1 - v_{i_2 i_1}^s) \quad \forall s \in \mathcal{S}, \forall i_1 \in \mathcal{I}, \quad (13)$$

$$1 - r_l^s = \prod_{i\in\mathcal{I}_G} (1 - v_{il}^s) \quad \forall s \in \mathcal{S}, \forall l \in \mathcal{L}, \quad (14)$$

$$1 - r_g^s = (1 - n_g^s) \prod_{i\in\mathcal{I}_G} (1 - v_{ig}^s) \quad \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \quad (15)$$

$$f_l^s \le -B_l(\theta_{o(l)}^s - \theta_{d(l)}^s) + M(1 - r_l^s) \quad \forall s \in \mathcal{S}, \forall l \in \mathcal{L}, \quad (16)$$

$$f_l^s + M(1 - r_l^s) \ge -B_l(\theta_{o(l)}^s - \theta_{d(l)}^s) \quad \forall s \in \mathcal{S}, \forall l \in \mathcal{L}, \quad (17)$$

$$-\overline{F}_l r_l^s \le f_l^s \le \overline{F}_l r_l^s \quad \forall s \in \mathcal{S}, \forall l \in \mathcal{L}, \quad (18)$$

$$\underline{\theta} \le \theta_i^s \le \overline{\theta} \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}, \quad (19)$$

$$(1 - r_i^s)D_i \le h_i^s \le D_i \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}, \quad (20)$$

$$0 \le f_g^s \le \overline{F}_g r_g^s \quad \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \quad (21)$$

$$\sum_{l\in\mathcal{L}|o(l)=i} f_l^s - \sum_{l\in\mathcal{L}|d(l)=i} f_l^s = \sum_{g\in\mathcal{G}_i} f_g^s - (D_i - h_i^s) \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I}. \quad (22)$$

The objective (1) minimizes the expected cost, in \$/h, of unserved load. Assuming that MES does not relocate following the blackout, the expected cost is per unit of time after islands have already re-energized to the greatest extent possible.

Constraint (2) limits the total number of MES resources placed, before the blackout, at generator buses throughout the transmission system. Constraint (3) reflects that a generator can start without power from the grid if and only if (i) the generator is BS-capable or (ii) an MES resource is placed at the bus of the generator. Constraint (4) indicates that a generator can originate BS restoration if and only if (i) the generator is not disabled and (ii) the generator can start without power from the grid. Constraint (5) reflects that a generator bus can originate BS restoration if and only if the bus hosts at least one generator that can originate BS restoration.

Without delving into details, constraints (6), (7), (8), and (9) concern whether a generator bus can re-energize itself, another bus (not necessarily a generator bus), a branch, and a generator, respectively, with connectivity as a common requirement for re-energization. Constraints (10), (11), and (12) concern connectivity of a generator bus to itself, another bus (not necessarily a generator bus), and a branch, respectively. Constraints (13), (14), and (15) concern re-energization of a bus, a branch, and a generator, respectively. Constraints (16) and (17) capture real power flow in a branch in terms of the voltage angles at the branch's origin and destination buses. Constraint (18) enforces thermal limits on the flow in a branch. Constraints (19), (20), (21), and (22) concern aspects of bus operation: voltage angle limits, load shed limits, generator output limits, and power balance, respectively.

B. Linearization

Although the optimization model contains some nonlinear constraints, these can be linearized such that the model is recast as a two-stage MILP stochastic program with integer (i.e., binary) variables only in the first stage. A nonlinear equality constraint involving a product of binary-valued variable expressions, such as constraint (5), is linearized exactly as multiple inequality constraints by McCormick relaxation. All second-stage variables are continuous, but ones that the nomenclature identifies as taking on binary values are bounded within $[0, 1]$ and can trace their integrality back to binary-valued parameters and binary first-stage variable e_i.

Additionally, since constraints (11) and (12) involve random parameters but not variables, these constraints are included (in the form of linear inequality constraints from McCormick relaxation) to determine the values of variables $\Psi_{i_1 i_2}^s$ and Ψ_{il}^s only in cases where the formulation is adapted for the expected value and discretized expected problems described in Section III. In any other case, all instances of $\Psi_{i_1 i_2}^s$ and Ψ_{il}^s are regarded as parameters because they can have their values fully determined during preprocessing that occurs before the formulation instance is constructed. The ability to determine the values during preprocessing is noteworthy; while the number of paths $|\mathcal{P}_{i_1 i_2}|$ from one bus to another could be exponential in the number of branches, preprocessing would not exacerbate the exponential complexity as optimization might. On the contrary, determination of values by preprocessing would lend itself to parallelization if needed, which would in general be harder to achieve with optimization.

III. Modeling Uncertainty

This section provides definitions related to stochastic programming that motivate the numerical results in Section IV.

A. Standard Form of Linear Stochastic Program

As a two-stage linear stochastic program, the optimization model in Section II adheres to standard form [8]

$$\begin{aligned} z^* = \min \quad & \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \mathbb{E}[Z(\boldsymbol{x},\boldsymbol{\xi})] \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^{N_1}, \end{aligned} \tag{23}$$

where z^* is the optimal objective value of the stochastic program, $\mathcal{X}$ is a feasible set, and $Z(\boldsymbol{x},\boldsymbol{\xi})$ is the optimal objective value of the second-stage problem

$$Z(\boldsymbol{x},\boldsymbol{\xi}) = \min \quad \boldsymbol{q}(\boldsymbol{\xi})^{\mathrm{T}}\boldsymbol{y} \tag{24a}$$

$$\text{s.t.} \quad \boldsymbol{y} \in \mathbb{R}_+^{N_2}$$

$$\mathbf{T}(\boldsymbol{\xi})\boldsymbol{x} + \mathbf{W}(\boldsymbol{\xi})\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{\xi}). \tag{24b}$$

The first-stage and second-stage decisions are captured in $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, and the first-stage decisions from solving (23) can be denoted as $\boldsymbol{x}^*$. It is assumed that the possible realizations of random parameter vector $\boldsymbol{\xi}$ reside within support Ξ. Technically, the parameter vectors $\boldsymbol{q}(\boldsymbol{\xi})$, $\boldsymbol{h}(\boldsymbol{\xi})$ and parameter matrices $\mathbf{T}(\boldsymbol{\xi})$, $\mathbf{W}(\boldsymbol{\xi})$ can have an affine dependence on $\boldsymbol{\xi}$. In practice, $\boldsymbol{q}$ and $\mathbf{W}$ are frequently constant in $\boldsymbol{\xi}$; the condition of $\mathbf{W}$ being constant in $\boldsymbol{\xi}$ is known as *fixed recourse*.

B. Sample Average Approximation

Often the probability distribution of $\boldsymbol{\xi}$ is either continuous or defined over an astronomical number of countable realizations. Then, for computational tractability, a common approach is *sample average approximation* (SAA) [8]: given a sample $\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^N$ of N realizations of $\boldsymbol{\xi}$, the problem solved in place of the original stochastic program (23) is

$$\min_{\boldsymbol{x}\in\mathcal{X}} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \frac{1}{N}\sum_{n=1}^{N} Z(\boldsymbol{x},\boldsymbol{\xi}^n). \tag{25}$$

C. Wait-and-See Problem

Typically, a major obstacle to finding a satisfactory solution to the stochastic program is that different first-stage decisions cannot be made for each realization. If they could, then (23) would become the *wait-and-see problem* [8] as follows:

$$WS = \mathbb{E}[\min_{\boldsymbol{x}\in\mathcal{X}} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + Z(\boldsymbol{x},\boldsymbol{\xi})], \tag{26}$$

and then it would clearly hold that $WS \leq z^*$ (i.e., WS would be a lower bound on the stochastic program's optimal objective value). The difference $z^* - WS$ is known as the *expected value of perfect information*, or $EVPI$.

D. Expected Value Problem

While the wait-and-see problem provides a lower bound, it ultimately does not yield a single set of first-stage decisions $\boldsymbol{x}$ that attempts to account for the many possible realizations of $\boldsymbol{\xi}$. If $\boldsymbol{q}$, $\boldsymbol{W}$ are constant in $\boldsymbol{\xi}$, an alternative to the wait-and-see problem (26) is the *expected value problem* [9] as follows:

$$EV = \min_{\boldsymbol{x}\in\mathcal{X}} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + Z(\boldsymbol{x},\boldsymbol{\mu}), \tag{27}$$

where $\boldsymbol{\mu}$ is the expected value (i.e., mean) of $\boldsymbol{\xi}$ in view of its probability distribution, and the first-stage decisions from solving (27) can be denoted as $\boldsymbol{x}_{\mathrm{EV}}$. In exchange for the accuracy of (25), problem (27) offers swift solution. It can be proved [9] by Jensen's inequality that $EV \leq z^*$ (i.e., EV is, like WS, a lower bound on the stochastic program's optimal objective value). Moreover, the optimal objective value from fixing $\boldsymbol{x}$ to $\boldsymbol{x}_{\mathrm{EV}}$ in (23) is referred to as the *expected cost of expected value solution*, or EEV. Certainly, it holds that $EEV \geq z^*$, as no first-stage decisions can achieve a lower objective value for (23) than $\boldsymbol{x}^*$ does. The difference $EEV - z^*$ is the *value of stochastic solution*, or VSS.

Furthermore, it has been shown [9] that if parameter matrix $\mathbf{T}$ is constant in $\boldsymbol{\xi}$ and $\mathcal{X}$ is continuous, then $EV \leq WS$.

E. Discretized Expected Value

As is the case for the model in Section II, a two-stage linear stochastic program may have the following characteristics:

- All second-stage variables $\boldsymbol{y}$ continuous,
- Parameter vector $\boldsymbol{q}$ and matrices $\mathbf{T}$, $\mathbf{W}$ constant in $\boldsymbol{\xi}$,
- Random parameter vector $\boldsymbol{\xi}$ with nonnegative, integer-valued (e.g., binary-valued) support Ξ, where nonzero values for components represent adverse system conditions (e.g., damages) hindering (24a).

Nevertheless, since the expected value $\boldsymbol{\mu}$ generally does not have integer-valued components, it may cause a stochastic program with these characteristics to exhibit unintended behavior. Therefore, in addition to the optimization model for pre-blackout MES placement, this paper introduces and studies the novel concept of a *discretized expected value realization* $\boldsymbol{\xi}_{\mathrm{DEV}}$. For a stochastic program with the aforementioned characteristics, the $\boldsymbol{\xi}_{\mathrm{DEV}}$ is found by solving the MILP

$$\min \quad \|\boldsymbol{\xi} - \boldsymbol{\mu}\|_1 \tag{28a}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X}, \boldsymbol{y} \in \mathbb{R}_+^{N_2}, \boldsymbol{\xi} \in \Xi \subseteq \mathbb{Z}_+^K$$

$$\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{q}^{\mathrm{T}}\boldsymbol{y} \leq WS \tag{28b}$$

$$\mathbf{1}^{\mathrm{T}}\boldsymbol{\xi} \geq \mathbf{1}^{\mathrm{T}}\boldsymbol{\mu} \tag{28c}$$

$$\mathbf{T}\boldsymbol{x} + \mathbf{W}\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{\xi}), \tag{28d}$$

a search problem that, given $\boldsymbol{\mu}$, identifies the nearest integer-valued realization of $\boldsymbol{\xi}$ satisfying requirements encoded in the constraints. Then, if the *discretized expected value problem*

$$DEV = \min_{\boldsymbol{x}\in\mathcal{X}} \quad \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + Z(\boldsymbol{x},\boldsymbol{\xi}_{\mathrm{DEV}}) \tag{29}$$

is solved, it holds that $DEV \leq WS \leq z^*$ because the inequalities $DEV \leq WS$ and $WS \leq z^*$ hold individually. The first-stage decisions from solving (29) can be denoted as $\boldsymbol{x}_{\mathrm{DEV}}$, and the optimal objective value from fixing $\boldsymbol{x}$ to $\boldsymbol{x}_{\mathrm{DEV}}$ in (23) can be called the $EDEV$.

Typically, realizations of $\boldsymbol{\xi}$ within a support Ξ vary in severity, as reflected in the extent to which they hinder the second-stage objective (24a). If a single realization is to be selected to represent the probability distribution of $\boldsymbol{\xi}$, then two extremes should be avoided: (i) excessive severity, especially where no first-stage decisions $\boldsymbol{x}$ can appreciably relieve the impact of the realization, and (ii) trivial severity, especially where no first-stage decisions $\boldsymbol{x}$ are adequately warranted. While the inequality $EV \leq z^*$ holds naturally, one of its side effects is that the severity of $\boldsymbol{\mu}$ tends not to be excessive in view of the distribution, though it may still be trivial.

Within the search problem (28), constraints (28b) and (28d) are feasible for a given $\boldsymbol{\xi}_{\mathrm{DEV}}$ if and only if $DEV \leq WS$

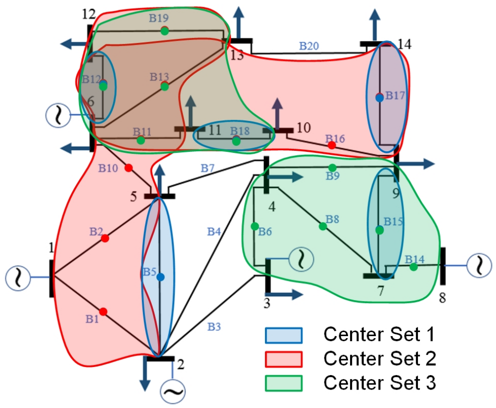

Fig. 1. IEEE 14-Bus System and its centers for correlated branch outages.

holds. The inequality $DEV \leq WS$ mirrors $EV \leq WS$. Then, the inequalities $DEV \leq WS$ and $WS \leq z^*$ together imply that $DEV \leq z^*$, which not only mirrors $EV \leq z^*$ but likewise has the effect of preventing the severity of $\boldsymbol{\xi}_{\text{DEV}}$ from being excessive. (If $\mathcal{X}$ is not continuous—it is not for the model in Section II—a $\mathbf{T}$ constant in $\boldsymbol{\xi}$ does not ensure $EV \leq WS$, but $EV \leq z^*$ still holds.) Similarly, constraint (28c) intends to reallocate damages in $\boldsymbol{\mu}$ to constitute integer-valued components in $\boldsymbol{\xi}_{\text{DEV}}$, preventing the severity from being trivial. Configurable tolerances and factors are avoided in (28) so as to limit arbitrariness in the definition of $\boldsymbol{\xi}_{\text{DEV}}$.

IV. Numerical Results

This section describes experiments by which the formulation was validated and analyzes results from the experiments.

A. Experiment Setup

The experiments were conducted in a variant of the IEEE 14-Bus System. Compared to the MATPOWER version [10] of the network, this variant maintained a total load demand of 259 MW but, for a more realistic reserve margin, halved the real generation capacities to sum to 386.2 MW. Because the system contained just five generators as seen in Fig. 1, only the one at Bus 2 was considered BS-capable, and only one MES resource could be placed before the blackout. Additionally, value of lost load was assumed uniform throughout the system so that the objective function effectively became system load shed.

For the experiments, realizations $\boldsymbol{\xi}$ within a probability distribution differed in what branches they disabled but did not disable any buses or generators. Six distributions for $\boldsymbol{\xi}$ were assessed. To achieve correlated branch outages, each distribution supposed a set of centers for branch outages, where each center consisted of one or more branches. Within a distribution's set of centers, the occurrences of the centers were regarded as mutually exclusive, equiprobable events with total probability of 1. However, the centers were permitted to overlap as only one could occur at a time. Then, the process for generating a scenario from a distribution was as follows.

(I) From the distribution's set of centers for branch outages, randomly select one center to occur.

(II) Given a selected center, consider all branches in the center to be disabled. Moreover, perform the following steps for each branch not in the selected center.

(A) Consider $\gamma^{(\delta/\Delta)}$ the conditional probability that the non-center branch is disabled, where γ is a fixed fractional decay rate, δ is the distance (on the one-line diagram) of the branch's midpoint from the nearest midpoint of any branch in the selected center, and Δ is a fixed distance.

(B) With the conditional probability from (A), randomly decide whether the branch is disabled.

Supposing a one-line diagram taken from [11], Fig. 1 shows three sets of centers for branch outages. Of the six distributions assessed, Distr. A through C supposed center sets 1 through 3, respectively, and a fractional decay rate of $\gamma = \frac{1}{3}$ for conditional probability. Distr. D through F likewise supposed center sets 1 through 3, respectively, but $\gamma = \frac{1}{5}$. The dimensions of the one-line diagram of Fig. 1 were around 170×210, so all distributions supposed $\Delta = 100$.

For each of the six distributions, two samples of 2000 scenarios each were generated: a training sample and a validation sample. The following experiments were then conducted.

(I) For the training sample, solve an SAA instance (25). Then, fixing the first-stage decisions $\boldsymbol{x}$ to those from (25), solve (24) for each scenario in the validation sample.

(II) To estimate WS, solve an approximation of (26) using the validation sample.

(III) Solve (27), yielding objective value EV and first-stage decisions $\boldsymbol{x}_{\text{EV}}$. Then, to estimate EEV, fix $\boldsymbol{x}$ to $\boldsymbol{x}_{\text{EV}}$ and solve (24) for each scenario in the validation sample.

(IV) Solve (28) for $\boldsymbol{\xi}_{\text{DEV}}$, as well as (29) for $\boldsymbol{x}_{\text{DEV}}$ and DEV. Then, to estimate $EDEV$, fix $\boldsymbol{x}$ to $\boldsymbol{x}_{\text{DEV}}$ and solve (24) for each scenario in the validation sample.

Whenever the validation sample was used, the 2000 second-stage objective values were then taken to produce a 95% confidence interval, the midpoint of which was a sample mean estimate. In place of the true value of WS, (28) used the bottom of the confidence interval for WS.

B. Analysis and Observations

The optimization model instances for the experiments were solved to zero optimality gap using Gurobi 9.1 [12] on a desktop computer with a 3.70-GHz, 8-core processor and 16 GB of RAM. For any distribution, the SAA instance was the single most computationally expensive model instance to solve. Notwithstanding, serial execution of preprocessing (e.g., determination of values for instances of $\Psi^s_{i_1 i_2}$ and Ψ^s_{il}) for the SAA instance of each distribution required less than five minutes, and solving the SAA instance once preprocessing was completed required no more than 12 seconds.

Fig. 2 displays objective (i.e., system load shed) values from the experiments for each distribution. Table I contains key qualitative results. Stated under "MES Bus" are the pre-blackout MES placements prescribed by (25), (27), and (29), respectively. Reported under "Discretized Expected Value" are the 1-norm distance from $\boldsymbol{\mu}$ to $\boldsymbol{\xi}_{\text{DEV}}$, the amount by which that distance exceeded the distance from $\boldsymbol{\mu}$ to the binary-valued realization simply rounding each component of $\boldsymbol{\mu}$, and the indices of branches disabled in $\boldsymbol{\xi}_{\text{DEV}}$.

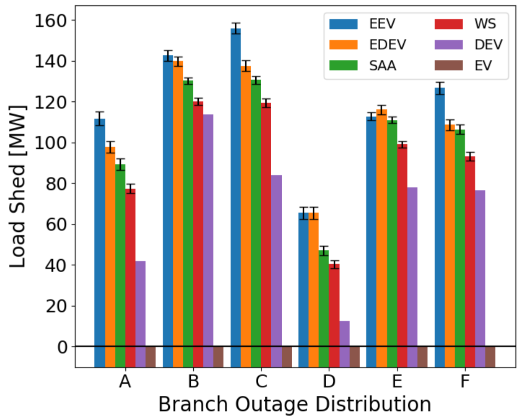

Fig. 2. System load shed values for various representations of uncertainty.

TABLE I
RESULTS FOR MES PLACEMENT AND DISCRETIZED EXPECTED VALUE

Branch Outage Distr.	MES Bus			Discretized Expected Value		
	SAA	EV	DEV	Dist.	Excess Dist.	Disabled Branches
A	6	None	8	9.25	0.04	4,5,6,7,9,11,13,15,16,17,18
B	3	6	8	5.49	0.05	1,2,4,5,6,7,8,9,10,11,12,13,16,17,18,19,20
C	3	None	6	6.63	0.48	2,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20
D	6	None	None	8.84	1.29	6,7,8,11,15,16,17,18
E	3	8	6	5.91	0.00	1,2,5,7,9,10,11,12,13,16,17,18,19
F	3	None	6	7.28	0.19	6,7,8,9,10,11,12,13,14,15,16,18,19

The fractional component values in $\boldsymbol{\mu}$ caused the linear inequality constraints from McCormick relaxation to behave poorly for (27): even if a branch had a component value above 0.8 in $\boldsymbol{\mu}$, two islands on opposite ends of the branch could still be considered connected through it for energization. Fig. 2 shows that, for each distribution, (27) ultimately saw zero load shed. In the same vein, Table I shows that except for Distr. B and E, which had some binary-valued components in $\boldsymbol{\mu}$, (27) did not prescribe MES placement.

In contrast, each $\boldsymbol{\xi}_{\text{DEV}}$ not only seemed tenable in view of its distribution but also led to a reasonable MES placement. For example, since the branches Distr. A disabled tended to be near the middle of the network, the only branches that the $\boldsymbol{\xi}_{\text{DEV}}$ of Distr. A disabled along the fringes of the network were 15 and 17. The disabling of Branches 15 and 17, in conjunction with branches near the middle, isolated an island consisting of Buses 4, 7, and 8. Thus, (29) prescribed placement of the MES at Bus 8, where the MES could start up the generator that then supplied load at Bus 4. Likewise, the $\boldsymbol{\xi}_{\text{DEV}}$ of Distr. B, which tended to disable branches in the northern and western portions of the network, cut off an island consisting of Buses 7, 8, and 9. Therefore, (29) again prescribed placement of the MES at Bus 8. Unfortunately, as the $\boldsymbol{\xi}_{\text{DEV}}$ of Distr. D only isolated an island that contained Bus 8 but no load, (29) did not prescribe placement of an MES for Distr. D. Nevertheless, constraint (28c) kept the severity of $\boldsymbol{\xi}_{\text{DEV}}$ from being trivial; among the branches disabled were 11, 16, and 18 near the middle of the network, cutting off load buses 10 and 11.

By applying for each distribution the *multiple replication procedure* [13], a method for assessing the quality of a candidate set of first-stage decisions $\overline{\boldsymbol{x}}$ to a stochastic program, it was found safe to practically regard the midpoint of the confidence interval for SAA in Fig. 2 as an estimate of z^*. Then, from the confidence intervals in Fig. 2, it can be said that $DEV \leq WS \leq z^*$ held empirically, as did $EV \leq z^*$. For Distr. A, C, and F, (29) prescribed MES placement while (27) did not, so $EDEV$ was noticeably lower (i.e., better) than EEV and not much greater (i.e., worse) than z^*. For Distr. B and E, (27) somehow prescribed MES placement; EEV was worse than $EDEV$ for Distr. B but better for Distr. E. Distr. D was the only one for which (27) and (29) both did not prescribe MES placement; EEV was clearly equal to $EDEV$. Because (27) only prescribed MES placement for two distributions, overall it seems that $VSS = EEV - z^*$ was substantial. In contrast, (29) reliably prescribed MES placements that were reasonable even if different from those prescribed by SAA, so $EDEV - z^*$ was less sizeable in general. Additionally, it seems that $EVPI = z^* - WS$ was usually moderate in magnitude but not as substantial as VSS.

V. CONCLUSION

This paper has presented a stochastic programming model on optimal pre-blackout MES placement for BS restoration, as well as the novel analytical concept of a discretized expected value realization. Experiments have not only validated the model but also demonstrated the potential of the concept.

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