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Using Modal Acceleration to Compare Two Environments of an Aerospace Component

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Abstract

Engineers are interested in the ability to compare dynamic environments for many reasons. Current methods of comparing environments compare the measured acceleration at the same physical point via a direct measurement during the two environments. Comparing the acceleration at a defined point only provides a comparison of response at that location. However, the stress and strain of the structure is defined by the global response of all the points in a structure. This paper uses modal filtering to transform a set of measurements at physical degrees of freedom into modal degrees of freedom that quantify the global response of the structure. Once the global response of the structure is quantified, two environments can be more reliably and accurately compared. This paper compares the response of an aerospace component in a service environment to the response of the same component in a laboratory test environment. The comparison first compares the mode shapes between the two environments. Once it is determined that the same mode shapes are present in both configurations, the modal accelerations are compared in order to determine the similarity of the global response of the component.

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Modal Filter, Modal Projection Error, Modal Acceleration, Environment, Aerospace

Introduction

A dynamic environment is defined in this paper as the response of a structure to a dynamic load. Dynamic environments are of interest to designers and engineers because these environments can cause large stresses in their structures and cause them not to function as intended. Defining, comparing, and reproducing dynamic environments has a long history well laid out by Daborn [2]. Although there are many advances in the past century on defining, comparing, and reproducing dynamic environments, all common techniques compare dynamic response at a point or sometimes a limited set of points. This limited set of information is not sufficient to determine the strain field since strain is the relative displacement between two adjacent points. Therefore, global response information is needed to quantify the damage of a structure.

If mechanical failure of the structure is the concern with respect to the environment, mechanical strain is the quantity of interest. Since mechanical strain is the relative displacement of two points on a structure, rigid body motion does not cause strain within the structure. However, the acceleration due to rigid body motion is captured during environments and is included in the environments definition that is subsequently used in a laboratory test. This extra excitation is unnecessary and could be removed to increase the capacity of the testing facilities.

Instead of using the response at a single or small subset of points to characterize the response of a structure, this paper demonstrates how the environment can be characterized in the modal domain and the corresponding benefits. Transforming physical response into modal response makes use of a modal filter. Modal filters have been used in modal theory for techniques such as modal parameter estimation, force reconstruction, and expansion [1][4][5]. This transformation has errors associated with how it fits mode shapes to the measurements. This paper uses the modal projection error to quantify the error in the modal filtering process [7].

There are many ways to use a characterized environment as previously stated. This paper focuses on comparing two environments, specifically, the environment of an aerospace structure while it is mounted in its service configuration and the environment of the same aerospace structure when attached to an electrodynamic shaker in a laboratory configuration. This paper utilizes the modal domain responses from each of the environments, quantifies the global response, and compares the global response of the two environments.

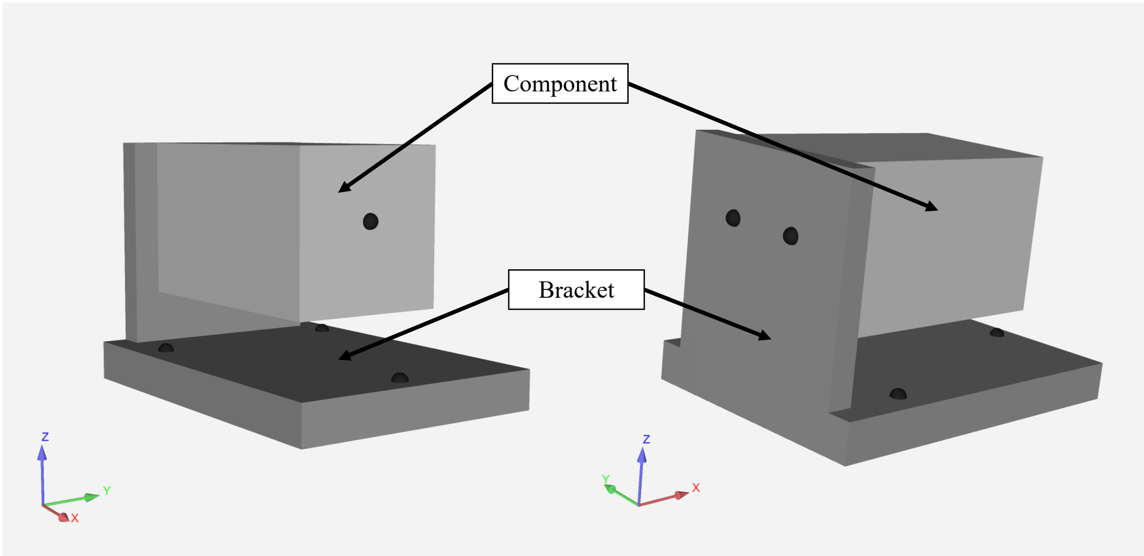


Figure 1: Finite element model of the component and its bracket. Triaxial accelerometers used are indicated by black dots.

The Aerospace Structure

The structure of interest is a relatively small component in a larger system. The component is bolted to a bracket in a cantilevered configuration. For the service configuration, the bracket is bolted to another component in a large assembly. The laboratory configuration has the same component and bracket assembly bolted rigidly to a single axis shaker system. A finite element model of this structure and its bracket is shown in Figure 1. Representations of the service and laboratory configurations of the component is shown in Figure 2.

Computing Modal Acceleration

Computing the modal acceleration of a structure produces an understanding of the global response of the structure. This global understanding is obtained because the modal acceleration is uniquely tied to a single mode shape of the structure. Modal acceleration is defined in this paper as the coefficients that scale the mode shapes to approximate the physical accelerations of a structure shown as

$$\ddot{x} \approx \boldsymbol{\phi}\ddot{q}, \tag{1}$$

where $\ddot{x}$ is the vector of accelerations at the measured degrees of freedom, $\boldsymbol{\phi}$ is the mode shape matrix at the measured degrees of freedom and $\ddot{q}$ is the modal acceleration for each corresponding mode. The linear combination of modes in Eqn 1 is approximately the acceleration due to modal truncation.

In order to transform the physical accelerations into modal accelerations, the measured accelerations are projected on to the mode shapes by using the pseudoinverse of ϕ,

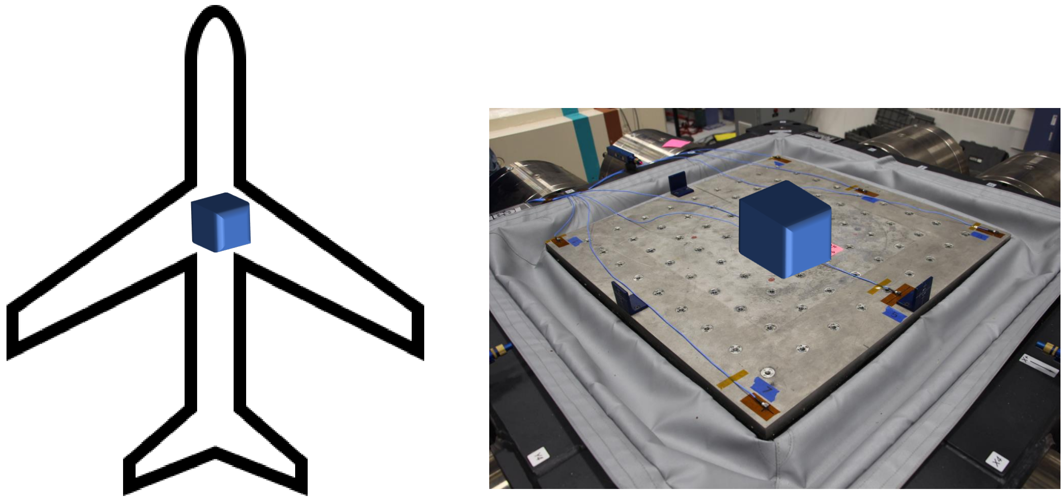


Figure 2: Representation of the component in its service environment (left) and its laboratory environment (right)

$$\boldsymbol{\phi}^{+}\ddot{x} \approx \ddot{q}, \tag{2}$$

where the $^{+}$ superscript denotes the pseudoinverse of a matrix. Equation 1 and 2 are written generally as the shapes used in the pseudoinverse can be from test data or a finite element model.

For the structure in Figure 1, the measured accelerations in both the service and laboratory environments are transformed using mode shapes generated from a finite element model to acquire the modal accelerations. Because there are two sets of data being transformed into the modal domain, Eqn 2 is executed once per set of data. In order to compare the modal accelerations between the two environments, the same set of mode shapes must be used. However, the same degrees of freedom do not need to be measured. Being able to have two sets of instrumentation is especially important because it is common for sets of data to not measure exactly the same degrees of freedom. This paper uses data where one of the triaxial accelerometers is in a different location between the two sets.

Equation 2 provides an estimate of the modal acceleration as there are sources of error that influence the computation. The first source of error is in the measured accelerations. There is standard measurement error and location error. The location error manifests itself when the location of the accelerometer is thought to have a certain coordinate location but the measurement actually happens at an alternate location. This results in mismatched degrees of freedom in the mode shapes. The magnitude of the location error is dependent on the gradient of the mode shape at the measured degree of freedom and the magnitude of the response. The standard measurement error comes from the inherent error in the accelerometer after calibration. This error is minimized through the least squares fitting of the random measurement error to the mode shapes.

Another large source of error to the computation in Eqn 2 is in the projection of the measured accelerations onto the mode shape space. In order to understand this error, it is important to discuss the projection of the test data onto the mode shapes as a curve fitting process. The mode shapes are scaled independently in order to best fit the data in a least squares sense. The scale factors are the modal accelerations. The least squares fitting is all done through the computation of the pseudoinverse of the mode shape matrix.

The error of the curve fitting process can be quantified by use of the Modal Projection Error (MPE)[6]. The MPE is analogous to the coefficient of determination and can provide insight into how well a set of basis vectors can be combined to reproduce a set of data. The MPE, $\Psi^2(t)$, is computed by

$$\Psi^2(t) = 1 - \ddot{x}(t)^+ \boldsymbol{\phi}\boldsymbol{\phi}^+ \ddot{x}(t). \tag{3}$$

In order to to obtain a least squares estimation of the modal accelerations, the mode shapes need to be independent and the matrix needs to be rectangular with more rows than columns representing more degrees of freedom than modes. The more rectangular the mode shape matrix is, the more reliable the MPE is in being a metric for the success of the modal filter. In order to make the mode shape matrix more rectangular, only the necessary modes are included in the pseudoinverse. Greedy algorithms are used to iteratively down-select mode shapes until a satisfactory minimum number of modes are selected with a satisfactory MPE.

Another important aspect is the independence of the mode shapes relative to each other. There are many ways to determine if two shapes or vectors are independent, but this paper uses the Modal Assurance Criteria (MAC). If there is a high MAC value between two shapes, then they will counteract each other and give artificially high magnitudes in the projection using Eqn 2.

Comparing Service and Laboratory Environments

The component shown in Figure 1 has a service and laboratory configuration. There are two service environments each excited in the X, Y, and Z directions. Service environment 1 is a low level environment while service environment 2 is a high level environment. Additionally, there is a laboratory configuration where the component in Figure 1 is attached directly to an electrodynamic shaker. This laboratory environment is excited in the X, Y, and Z directions. All of the service and laboratory environments are considered to have random vibration excitation. The modal accelerations for all environments and excitation directions are calculated per Eqn 2 using the mode shapes from the finite element model. The mode shapes from the finite element model are used because they are the only set of shapes that had all of the degrees of freedom from both configurations.

Six rigid and four elastic modes from the finite element model are used to represent the motion of the component in all of the environments compared. The four elastic modes are not the four modes with the lowest natural frequencies, but the four modes that minimize the MPE along with being independent with respect to the other basis shapes. This combination provides a mode shape matrix that includes 18 degrees of freedom measured and 10 modes.

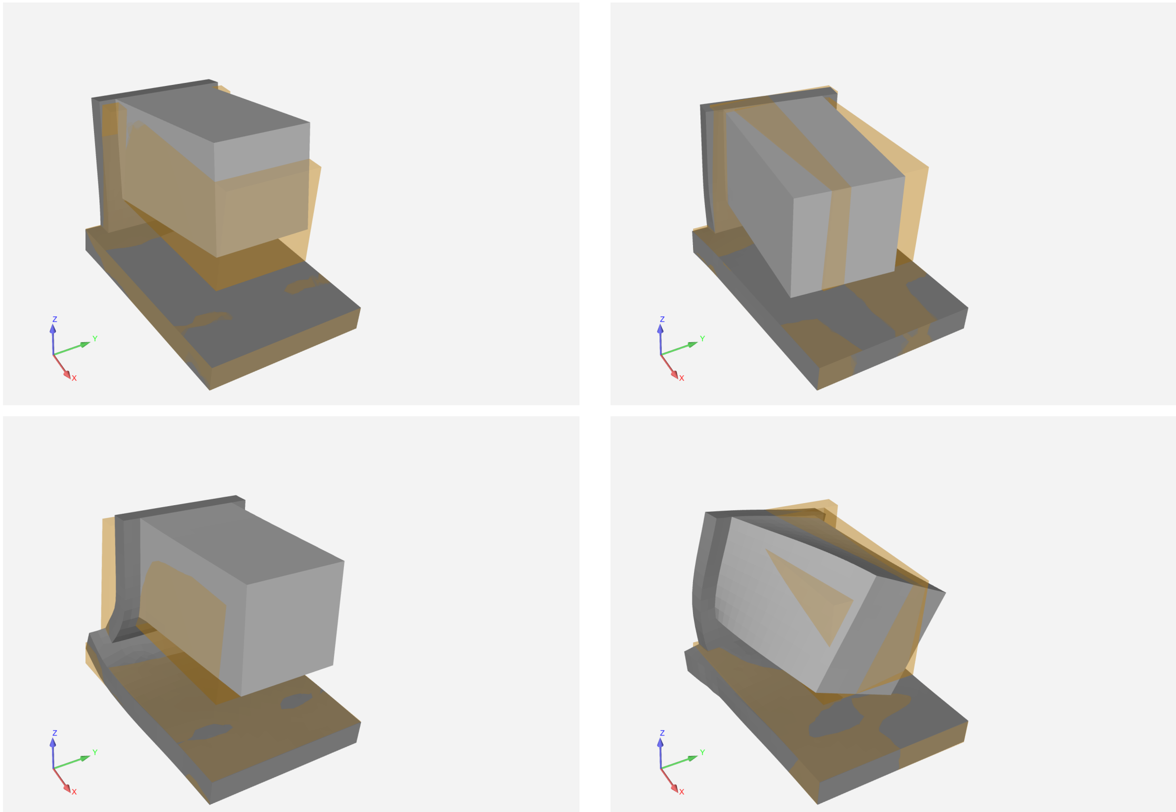


Figure 3: Deformed (grey) and undeformed (orange) shapes of the 1st (upper left), 2nd (upper right), 3rd (lower left), and 4th (lower right) mode shapes used in the modal filter

The pseudoinverse of the mode shape matrix of these dimensions provides an overdetermined estimation of the modal accelerations when computing the pseudoinverse. Only the modal accelerations of the elastic modes are used to define the response of the environment because rigid body modes do not induce stresses in the structure. The four elastic modes that this analysis uses are shown in Figure 3.

The MPE is calculated for every environment at each time step. The average MPE over the time history of each environment is in Table 1. The significance of the MPE is relative to the ratio of degrees of freedom to the number of modes used in the pseudoinverse as shown in [6] and to what degrees of freedom are chosen. In an overdetermined pseudoinverse calculation, values under 0.02 typically result in excellent projections [6]. The low values of MPE provide evidence of how well the ten finite element modes fit the test data. This low error provides confidence that the ten modes acting as the basis vectors for the test data are accurate representations of the modes of the structure in both service and laboratory configurations. It also shows no other modes of the structure are excited in any of the environments. This low error for both environments indicates that the test fixture attaching the component to the electrodynamic shaker produces the same mode shapes for the structure of interest in the laboratory configuration as in the service configuration, which is necessary for a successful laboratory test.

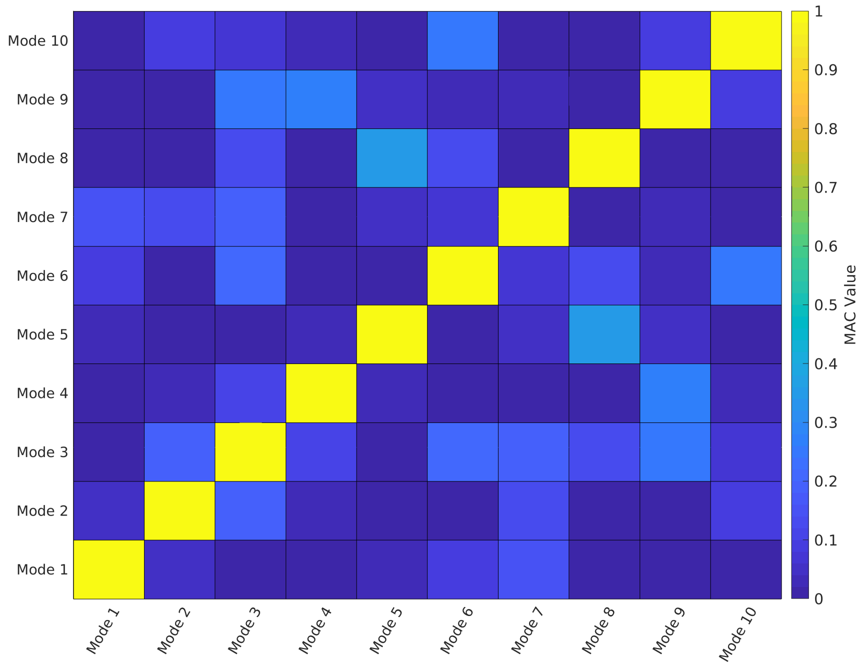


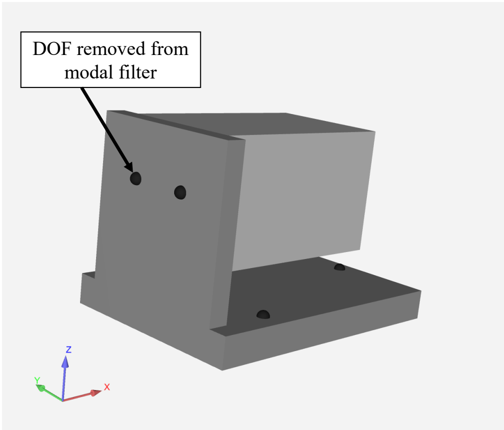
Figure 4: Self Modal Assurance Criteria plot of the mode shapes used in the modal filter

The independence of the mode shapes as basis vectors is critical in using a modal filter. If the basis vectors are similar to each other, then the pseudoinverse will use what small differences there are between the similar vectors to fit noise or other data not represented by the basis vectors. It will also inflate the magnitude of the modal accelerations. The self MAC of the mode shape matrix verifies the independence of the mode shapes and is shown in Figure 4.

Another check on the validity of Eqn 2 is to use the modal accelerations for modal expansion. This process removes one of the degrees of freedom from the mode shape matrix and then uses the full finite element shapes to calculate the acceleration at that degree of freedom and compare to the measurement. This process is done for the Y direction degree of freedom at

Table 1: Average Modal Projection Errors for the Modal Filter per Environment

Environment	Direction	Avg MPE
Service 1	X	12.6e-3
Service 1	Y	19.7e-3
Service 1	Z	14.3e-3
Service 2	X	8.6e-3
Service 2	Y	10.6e-3
Service 2	Z	9.2e-3
Laboratory 1	X	4.4e-3
Laboratory 1	Y	4.4e-3
Laboratory 1	Z	4.7e-3



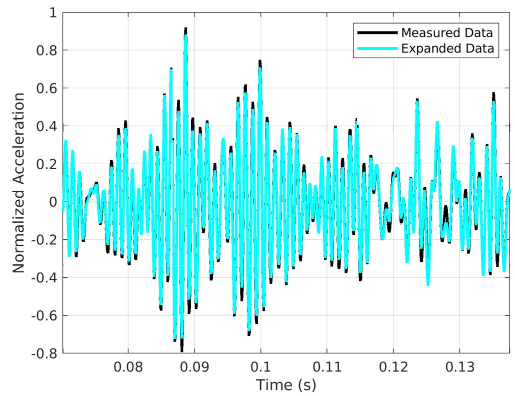


Figure 5: Reproduction of the acceleration response through SEREP expansion not utilized in the expansion technique

the point indicated in Figure 5. Although this method of validating the modal filter appears intuitive, it is only useful if the modal accelerations are overdetermined as removing a degree of freedom from the modal filter can cause errors in the pseudoinverse. The reconstruction of that degree of freedom response is shown in Figure 5 and has an average point by point error of 5% over the entire time history.

The modal filter in Eqn 2 uses the 10 basis modes and calculates the modal accelerations. The root mean squared values of the modal acceleration time histories are calculated and normalized for each environment. The root mean squared values of the modal accelerations for the elastic modes are shown in Figure 6 for each environment. Being able to examine just the elastic motion removes any rigid body motion from the response. The rigid body response does not induce any stress but can account for a lot of the acceleration in any structure. Examination of the root mean squared values of each of the modal accelerations provides additional information that cannot be determined from comparing physical accelerations. Because the modal acceleration is a measure of the global response, the modal acceleration provides a better estimate of the energy in the system when comparing the energy at a single degree of freedom.

The service and laboratory environments shown in Figure 6 are typical in that they are executed with a single degree of freedom shaker in three orthogonal directions, X, Y, and Z. Although the structure of interest is excited in orthogonal directions, the four elastic modes of the structure are excited in each direction. By examining the response in the modal domain, the assumptions of only X degrees of freedom being excited during an X direction laboratory test can be abandoned. This revelation is important because current testing assumes that testing in the X, Y, and Z direction separately provides orthogonal response and that the damage can be superimposed, which is false.

The modal accelerations alone cannot be used to state which modes are the most damaging as each mode will scale to stress at different locations on the structure, however, each mode can be directly compared to the same mode in different environments. It is observed

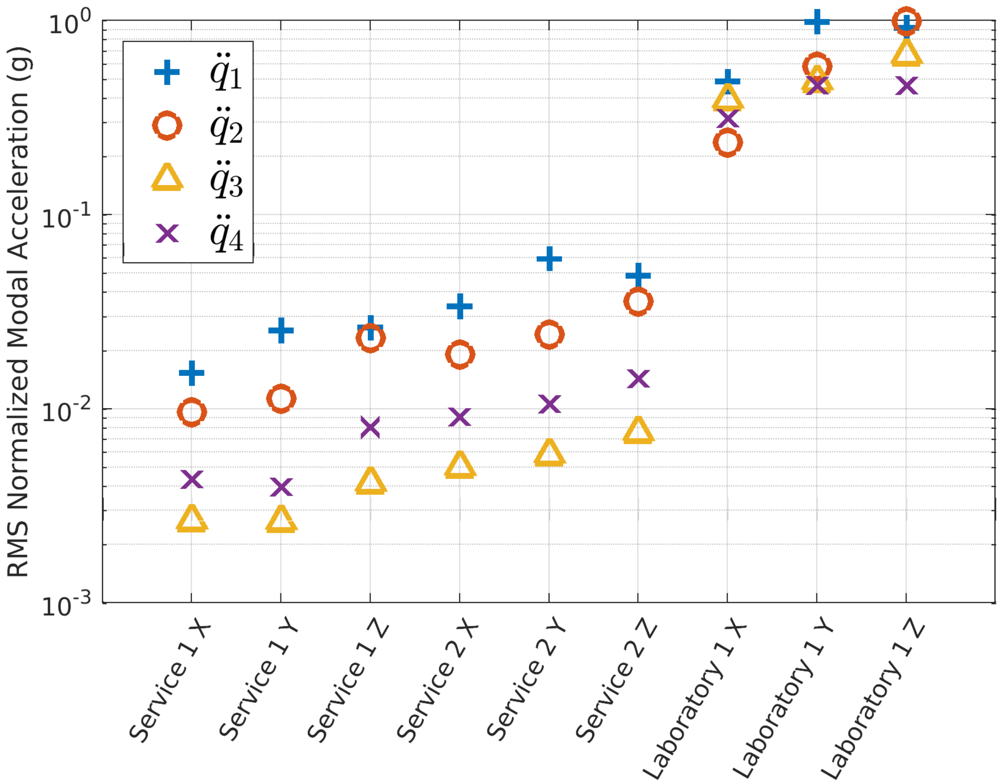


Figure 6: Normalized modal accelerations from service and laboratory environments

that the laboratory tests are exciting the same elastic modes to approximately an order of magnitude more acceleration than the two service environments. It is also observed that the order of the modal accelerations in the laboratory environment does not match the order in the service environments. This order of which mode is excited the most is controlled by the test apparatus or shaker system. An alternate forcing function or multi degree of freedom excitation can gain additional control of the different modes to make the laboratory test behave more like the service environments.

Typically, the purpose of the laboratory test is to determine if the component will survive in the service environment. In that scenario, it can be said with confidence that the stresses in the laboratory environment are greater because the excitation of each of the elastic modes are greater than any of the service environments. In addition, Figure 6 shows that each of the laboratory environments excite all of the modes more than the service environment. Therefore, the traditional test in three orthogonal directions is unnecessary and only one test setup is needed.

These results quantify the difference in damage responses between environments because the amplitude of the same mode shapes excited are compared. However, it is important to address the uncertainties in the analysis. The highest amount of uncertainty in this process of computing and comparing modal accelerations comes from the curve fitting process in the projection of the mode shapes onto the time histories. The finite number of modes included can cause change the calculated modal accelerations and should be examined when computing the modal accelerations. This case study had low errors with respect to the MPE and there is high confidence in the results. This result may not be common for all case studies. Additional work should be done to investigate these sensitivities. This analysis uses random vibration as the excitation. Additional work could utilize this process for a transient response as the computation works on the time domain and should be robust against light nonlinearities due to mode shapes being the basis of the analysis [3].

Conclusion

This paper demonstrates the process of computing modal accelerations and using specific tools to determine the validity of the process. Modal accelerations provide a framework for comparing the global response of a structure in different environments and greatly reduces the errors associated with comparing single physical accelerations between two environments. These errors include differences in physical location of the accelerometer and the inclusion of rigid body acceleration in the comparison. This paper uses an aerospace component and compares its modal accelerations in service and laboratory environments. The results show a transparent and intuitive approach to comparing the elastic response between environments.

The process also provides evidence of the laboratory environment's representation of the service environment through the modal projection error. The low MPE indicates that the same modes existed between the service environments and the laboratory environments because the shapes are independent and the pseudoinverse is overdetermined. Having the same mode shapes in each environment would be impossible if the test fixture did not represent the dynamic impedance of the next level of assembly in the service configuration.

This process comes with challenges. There are many sources of errors when computing the modal accelerations. One challenge is ensuring that there is an appropriate set of basis vectors with enough instrumentation to independently observe them. If the basis set of mode shapes are inadequate, then the values of modal accelerations are meaningless. Use of the MPE, MAC, and making sure that the modal accelerations are overdetermined are critical in gaining confidence in the modal acceleration calculation.

One unknown of this process is the effect of the accelerometer placement in the pseudoinverse calculation. Because the pseudoinverse is a least squares fitting process, if one shape has more measurements participating over other shapes, then the fitted modal accelerations will be weighted toward that shape. Also important is how independent the elastic shapes are with respect to the rigid shapes. Any misfitting of the elastic shapes to the measurements is typically compensated by rigid body modes, which can also skew the results. Additional research in these areas can improve the quality and repeatability of the modal acceleration calculation.

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