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Bayesian Analysis Applied to Neutron Activation Diagnostics Measurements of MagLIF Experiment

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Abstract: Neutron activation diagnostics are commonly used to infer neutron yields in inertial confinement fusion experiments (ICF). At Sandia's Z-Facility, Inertial Confinement Fusion (ICF) experiments using the Magnetized Linear inertial Fusion (MagLIF) concept are being conducted and activation diagnostics are employed to infer neutron yields. To infer neutron yields from the activation measurements, radiation transport modeling is relied upon to correct for scattering and attenuation present in the experiment that modifies the activation. To understand the likely error involved in those corrections Bayesian methods are used on inferred neutron yield data from a recent MagLIF experiment.

The problem to be analyzed: The primary neutron yield for a deuterium fuel MagLIF shot arises from one neutron producing path of the two equal probability reactions given below:

(Eq. 1)
$$ {}^{2}_{1}D + {}^{2}_{1}D = \begin{cases} {}^{3}_{1}T(1.01\ MeV) + p^{+}(3.02\ MeV) \\ {}^{3}_{2}He(0.82\ MeV) + n^{0}(2.45\ MeV) \end{cases} $$

The neutron yield is inferred using activation diagnostics, using the relationship below:

(Eq. 2)
$$ Y_{DD} = \frac{4\pi c_{net}\langle d^2{}_i\rangle}{m_i h_{i,DD} F_{(n,n'),DD}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)} $$

where the quantities: c_{net} : number of gammas counted minus the background; t_1 and t_2:times of start and stop of the activation measurement; m: mass of the sample; d: distance of sample in the experiment; F: the detector calibration are measured directly but the quantity, h, that accounts for scattering and attenuation effects in the experiment is found through radiation transport modeling (with MCNP.)

In this, we are going use Bayesian methods to see how well we can reproduce the observable, C_{net} (the measured number of counts minus the background due to activity of the activation sample) using reasonable expected errors for our directly measurable quantities to determine error ranges one might expect in the correction factor, h.

(Eq. 3)
$$ c_{net} = \left\{ Y \frac{mhF}{4\pi\langle d^2\rangle}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) \right\}_{DD} $$

Bayes Theorem: Given 2 events A and B, the conditional probability of A given that B is true is expressed by:

$$ P(A|B) = \frac{P(B|A)P(A)}{P(B)} $$

where: A=proposition, B=observations, P(A)=prior probability, P(B|A)=likelihood function (the probability of the evidence B given the A is true.) And P(A|B)=post probability (the probability of the proposition A after taking the evidence B into account) The probability of the evidence P(B) can be calculated using the law of total probability:

$$ P(B) = \sum_i P(B|A_i)P(A_i) $$

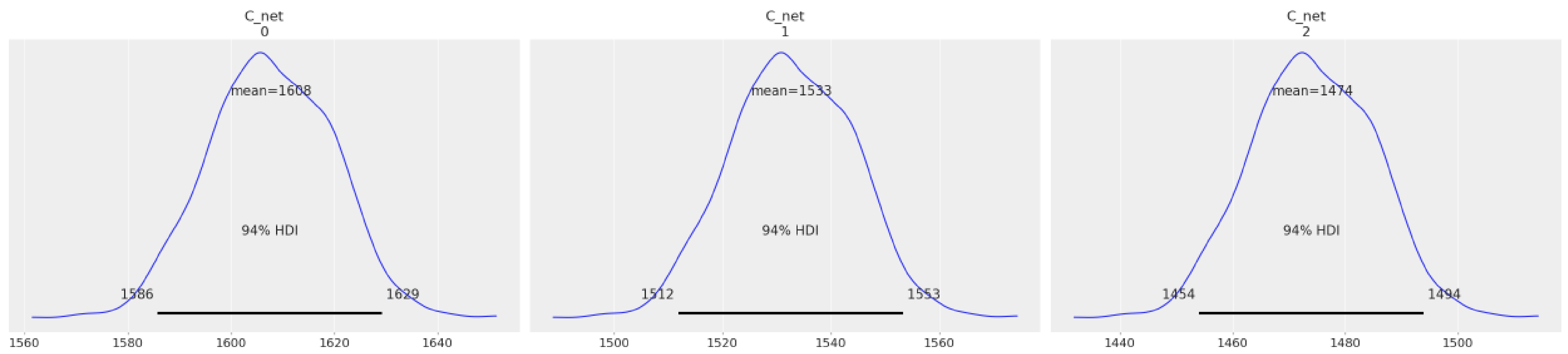
Methodology: We use Python with the library pymc3 [1] to facilitates the analysis. Consider the expression given in Eq. 3. The observed random variable is Cnet. The remaining variables are treated as free random variables and are expressed as distributions. In the analysis for the most likely outcomes from the model where the free variable distributions are used to determine the most likely outcome of the observed variable. The quantities measured have fairly well understood uncertainties. The modeled correction factor, h, has a poorly understood error that we would like to better understand likely bounds.

We directly measure the number of counts, C_{net}, mass and times of the measurement, t_1 and t_2, in the experiment. The mean distance squared of the sample location <d²>, is determined by the sample thickness (measured) and the locations distance from the target, determined from CAD drawings of the experiment. The calibration factors, F, are determined experimentally, and have a well defined error. Through neutron transport modeling, we determine the relation between our detector calibration and Z's complex scattering environment, h. The observed random variables are represented by prior distributions, in this case uniform distributions with bounds given by:

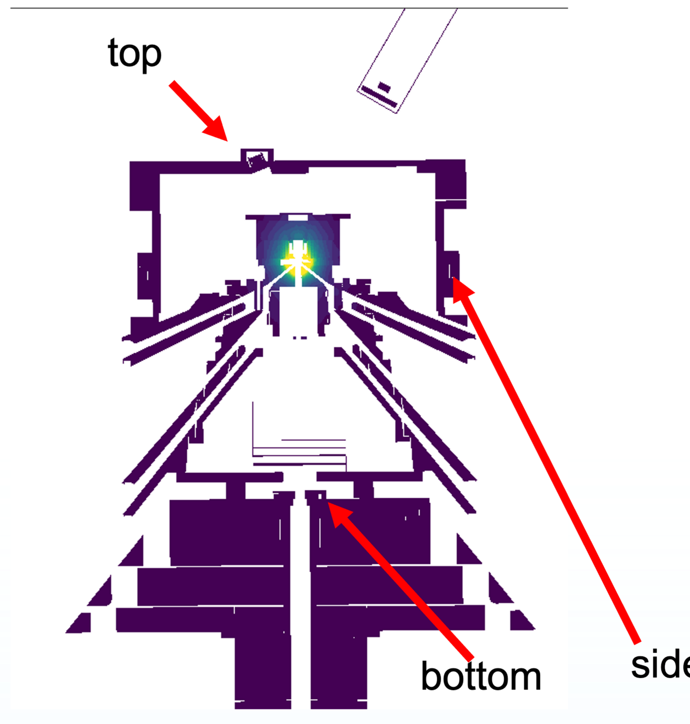
Y +/- 27%
d +/- 15%
m +/-5%
F +/- 20%
h +/- 50%
σ = 5 (half-normal distribution)

For our unobserved variable, c_{net}, we represent it by a likelihood distribution. In this case a normal distribution.

Preliminary Results: We have determined the 'accepted' yield to be the mean of the yields calculated from all sample locations on the blast shield: 3 on the top, 3 on the side and 3 on the bottom. Shown below is the modeled results for the bottom location for z3501 where the measured values are 1518,1522 and 1583.



We use the relation of the modeled counts compared to the actual measured counts as a metric of the 'goodness' of the model results. We can then compare the MCNP determined correction, h, and compare against mean value of the models most likely determined h value to get bounds on the errors of the MCNP correction values.



Considering the positional data for z3501, where the model works well. Here it appears the bounds on h are around +/-10%.

		z3501
top	Cnet1	5.9 %
	Cnet2	0.7 %
	Cnet3	-6.9 %
	h	9.5 %
side	Cnet1	3.4 %
	Cnet2	2.7 %
	Cnet3	-6.7 %
	h	2.1 %
bottom	Cnet1	5.9 %
	Cnet2	0.7 %
	Cnet3	-6.9 %
	h	-0.9 %

Comparing the bottom activation results for shots: z3500, z3501, and z3587 the apparent bounds on error in h increase to +/-13%.

		z3500	z3501	z3587
bottom	Cnet1	2.5 %	5.9 %	-2.2 %
	Cnet2	2.8 %	0.7 %	-1.5 %
	Cnet3	-7.1 %	-6.9 %	4.8 %
	h	-13.0 %	-0.9 %	-0.8 %

References:

[1] Salvatier J., Wiecki T.V., Fonnesbeck C. (2016) Probabilistic programming in Python using PyMC3. PeerJ Computer Science

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NNSA National Nuclear Security Administration
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