Logical Majorana Fermions for Fault-Tolerant Quantum Simulation

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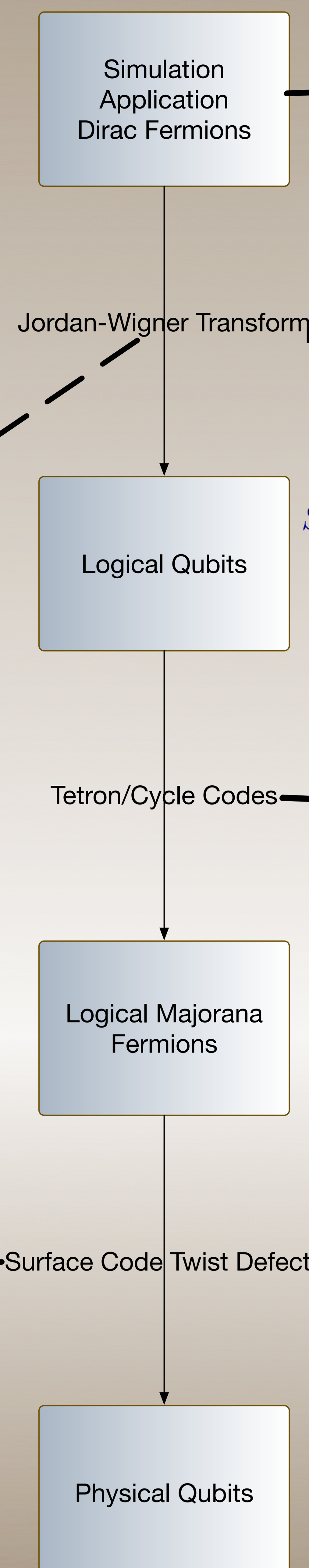
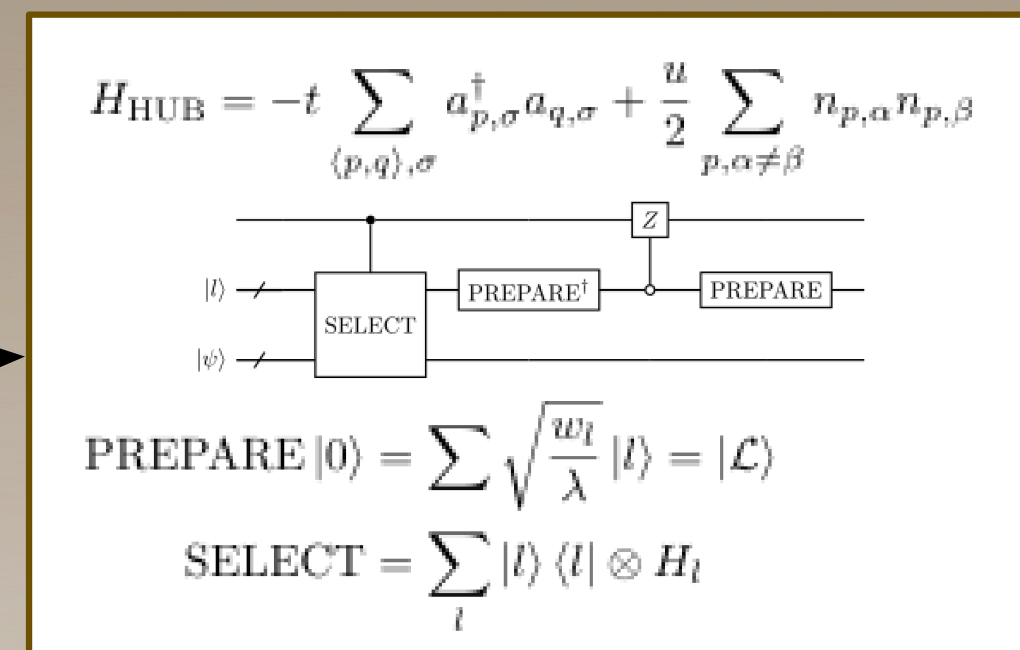
Abstract

We show how to absorb fermionic quantum simulation's expensive fermion-to-qubit mapping overhead into the overhead already incurred by surface-code-based fault-tolerant quantum computing. The key idea is to process information in surface-code twist defects, which behave like logical Majorana fermions. Our approach encodes Dirac fermions, a key data type for simulation applications, directly into logical Majorana fermions rather than atop a logical qubit layer in the architecture. Using the 2D Fermi-Hubbard model as an exemplar, we show two applications of our approach that yield improvements in algorithms. First, by preserving the locality of fundamental fermionic operations, we can reduce the asymptotic circuit depth of a Trotter-Suzuki expansion of the time evolution operator. Second, by working in the paradigm of the Majorana fermion data type, we were able to obtain a *T*-count reduction for the block-encoding SELECT oracle that can be applied even without the use of the twist-defect/logical Majorana architecture described here.



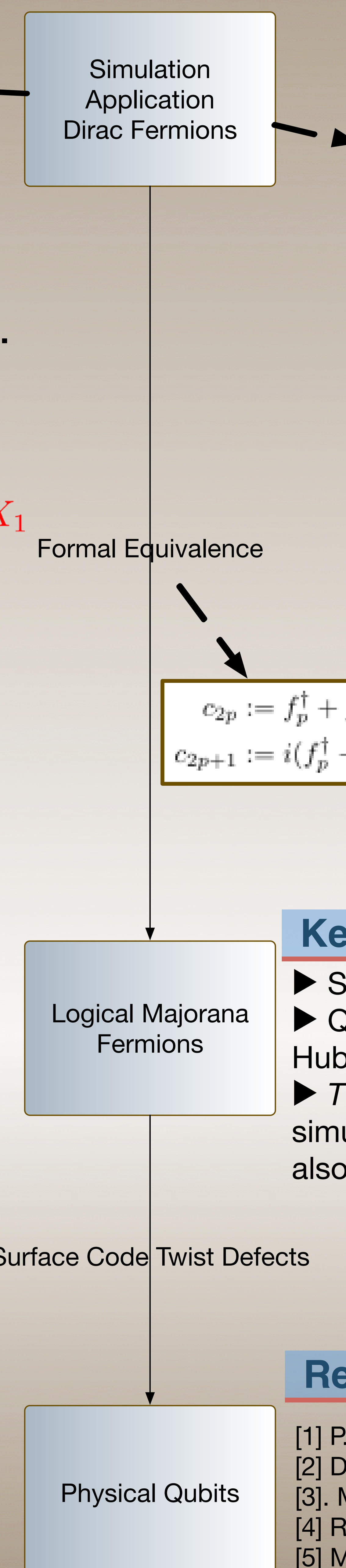


$$H_{\mathrm{HUB}} = -t \sum_{\langle p,q\rangle,\sigma} a^\dagger_{p,\sigma} a_{q,\sigma} + \frac{u}{2} \sum_{p,\alpha\neq\beta} n_{p,\alpha} n_{p,\beta}$$

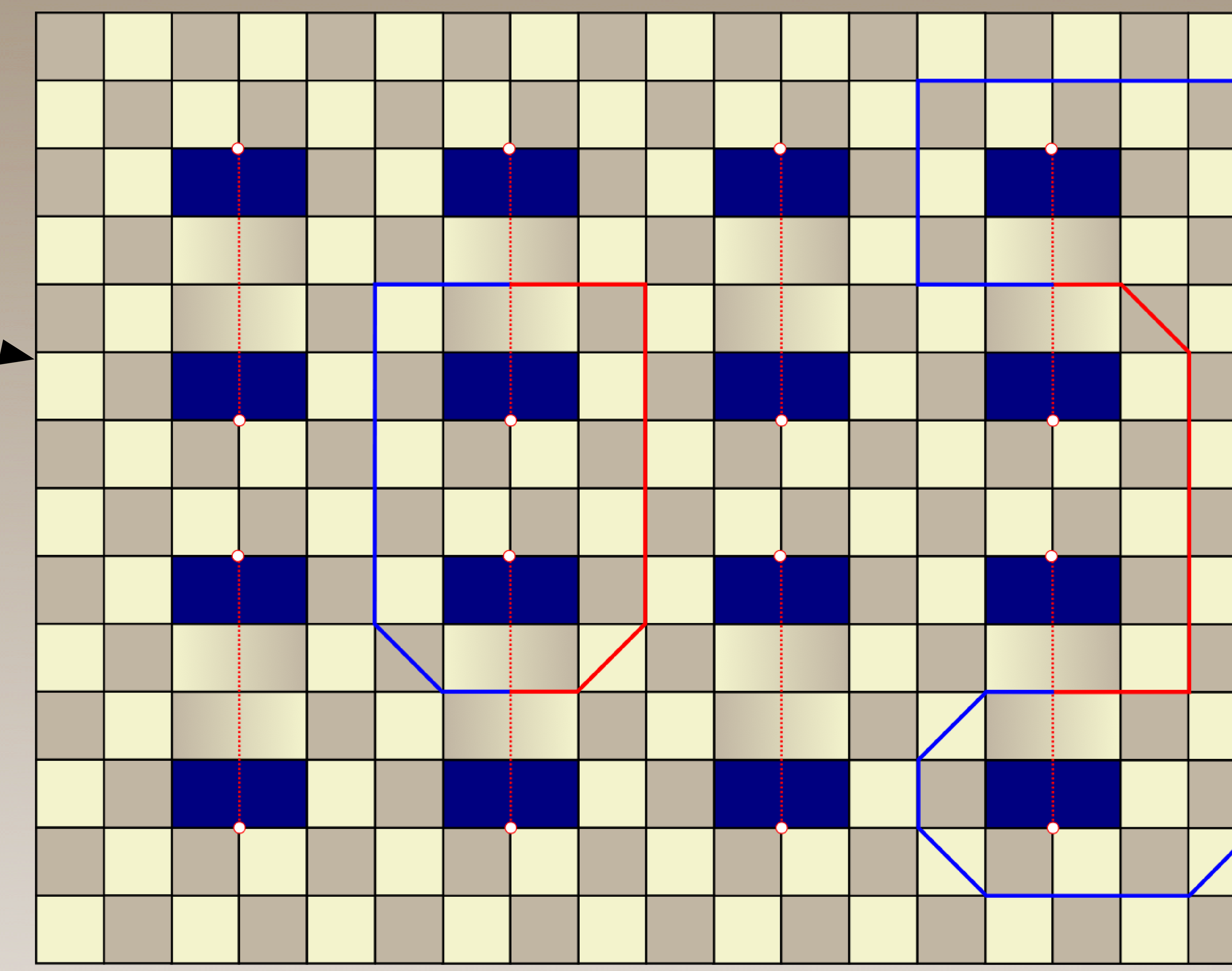
$$\mathrm{PREPARE}\,|0\rangle = \sum \sqrt{\frac{w_l}{\lambda}}\,|l\rangle = |\mathcal{L}\rangle$$

$$\mathrm{SELECT} = \sum_l |l\rangle\langle l| \otimes H_l$$

The Hubbard model Hamiltonian, and the structure of a block-encoded walk operator for qubitized simulation [4].



$$c_{2p} := f^\dagger_p + f_p$$
$$c_{2p+1} := i(f^\dagger_p - f_p)$$

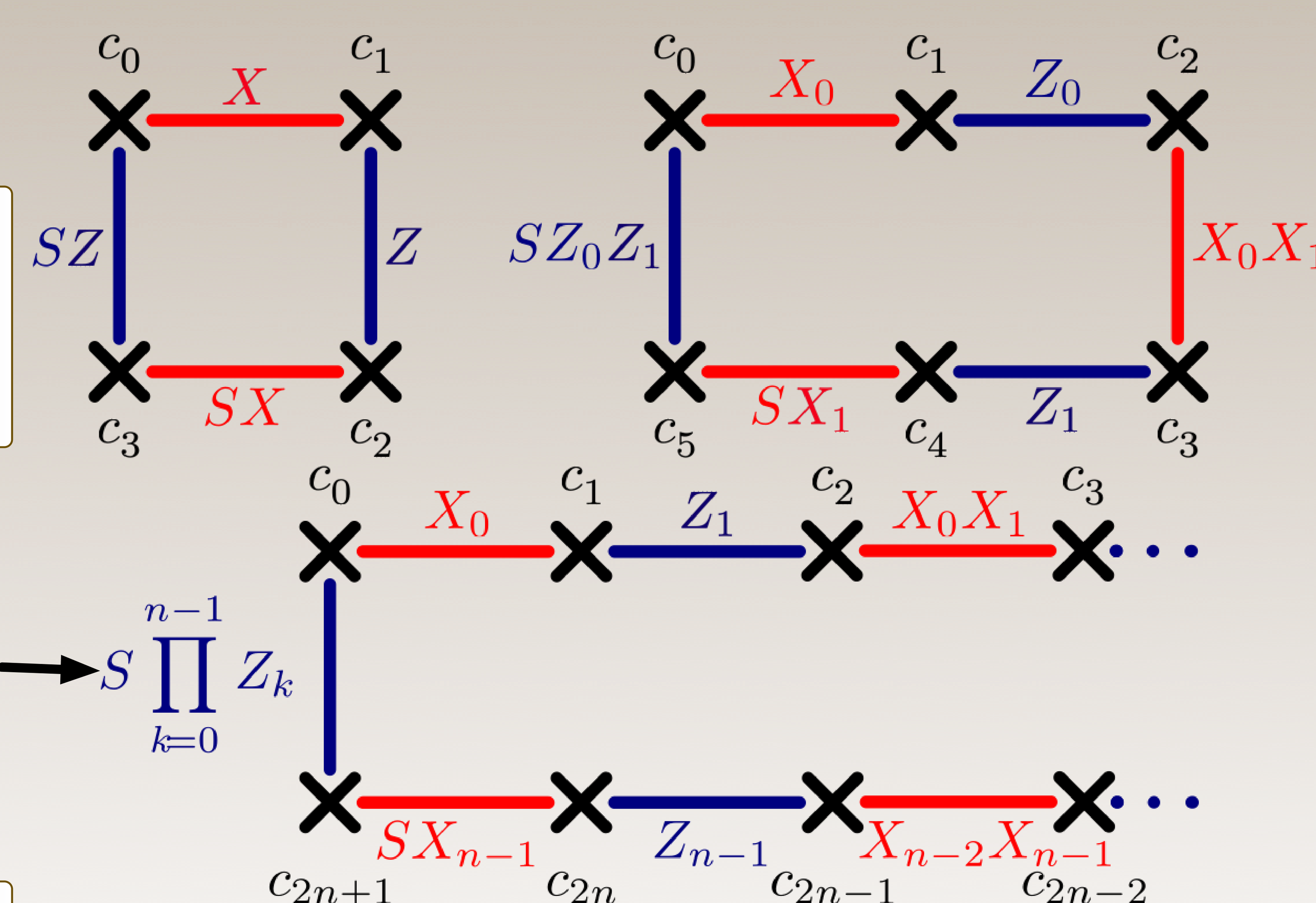


Quadratic logical Majorana operators encoded as physical Pauli strings encircling twist defects.

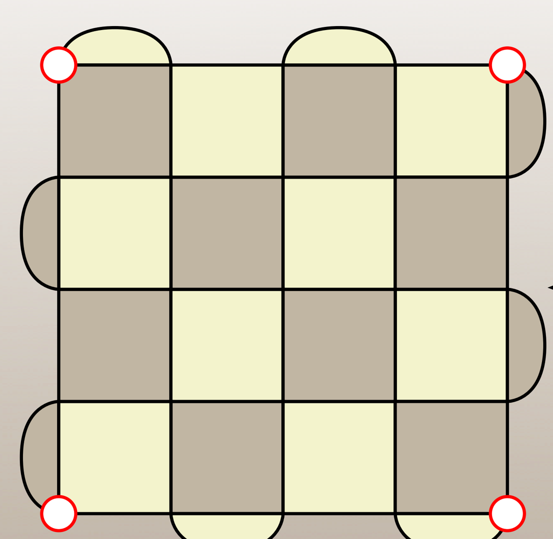
$$f_p = \frac{1}{2}(Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes (X_p + iY_p)$$
$$f^\dagger_p = \frac{1}{2}(Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes (X_p - iY_p)$$
$$c_{2p} = (Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes X_p$$
$$c_{2p+1} = (Z_0 \otimes \cdots \otimes Z_{p-1}) \otimes Y_p$$
$$X_p = (-i)^p c_0 \ldots c_{2p}$$
$$Z_p = -ic_{2p}\, c_{2p+1}$$

Jordan-Wigner mappings [1] for

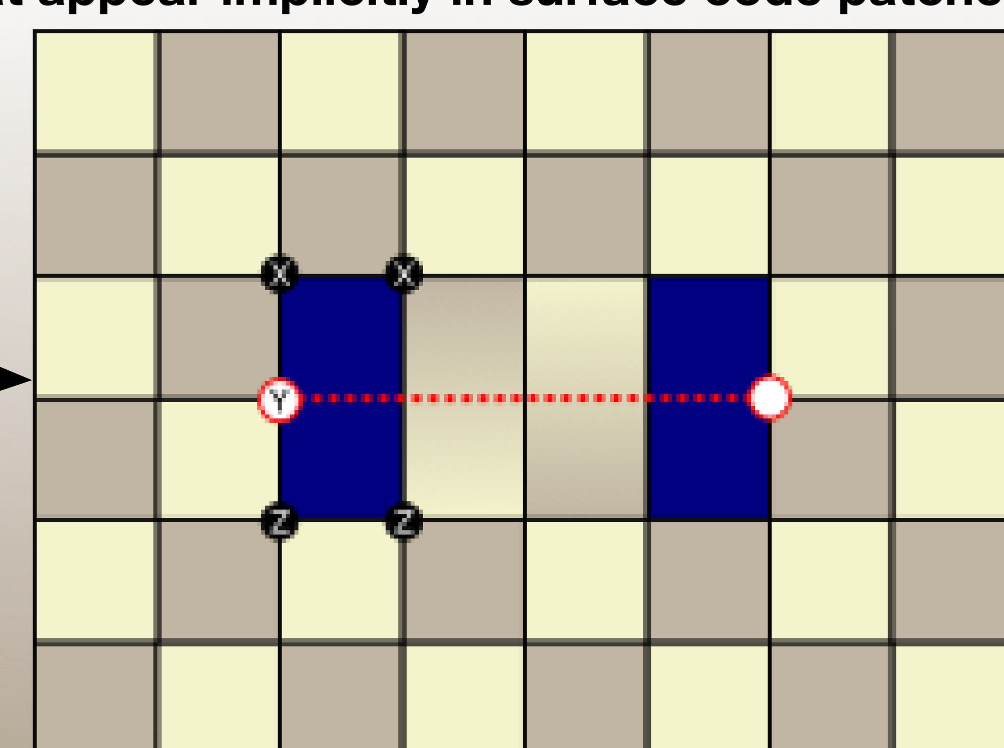
- ▶ **annihilation/creation to Pauli operators**
- ▶ **Majorana to Pauli operators**
- ▶ **Pauli to Majorana operators**



Tetron, hexon, and generalized Majorana cycle codes that appear implicitly in surface code patches [2].



A surface code patch. The color-changes at each corner function as logical Majorana fermions [3], marked with red and white circles.



A pair of logical Majorana fermions encoded in a pair of twist defects in the surface code.

Key Results

- ▶ Simplified architecture stack without logical qubit layer.
- ▶ Quadratic gate depth reduction for Trotter-Suzuki simulation [5] of 2D Hubbard model by exploring locality for parallelism.
- ▶ *T*-count reduction of 20% to leading order over [4] for qubitized simulation of 2D Hubbard model using Majorana-inspired techniques also compatible with logical qubit architectures.

References

[1] P. Jordan and E. Wigner (1928).
[2] D. Litinski and F. van Oppen (2018).
[3]. M. S. Kesselring, F. Pastawski, J. Eisert, and B. J. Brown (2018).
[4] R. Babbush, et al (2018).
[5] M. Suzuki (1990).

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