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The High-Resolution Wavelet Transform: A Generalization of the Discrete Wavelet Transforms

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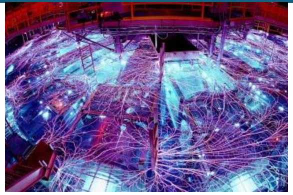

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Bio

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- Main area of research: Fast protection for power distribution systems
  - Designing signal-processing techniques for Traveling Wave detection and data extraction
  - Training Machine-Learning/ Deep-Learning (ML/DL) models for fast fault location

Outline

- Overview
- Introduction
- Background
- The concept behind the HRWT
- Results
- Conclusions

Overview – Heilmeier Catechism

What are you trying to do?

- Decompose a timeseries signal into the energy of each frequency at each time point (time-frequency decomposition)

How is it done today, and what are the limits of current practice?

- There are generally two approaches: 1) methods like FFT that provide high resolution of energy at each frequency but no time component, or 2) methods like wavelet transform that do time-frequency decomposition with less frequency resolution for each time point

What is new and innovative in your approach?

- Ability to achieve finer frequency resolution with more narrow frequency bands in the time-frequency decomposition

What difference will it make?

- Time-frequency is used in a lot of different fields, such as seismic studies and ECGs. Finer resolution provides additional details about what is happening at each frequency band

Introduction

- The High-Resolution Wavelet Transform:
  - Modifies current Wavelet Transform (WT) algorithms in order to achieve a finer time-frequency decomposition

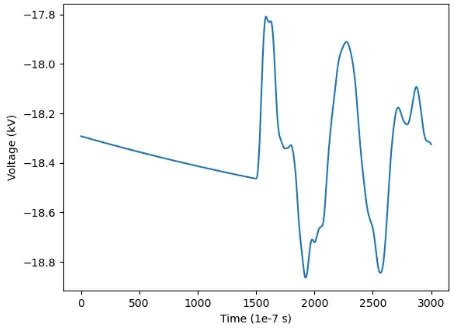
- Status of the idea:
  - Background theory is fully developed. It is based on generally-known WTs related concepts, and expands them to achieve a new level of performance
  - The idea can be potentially used in any field that requires time-frequency decomposition

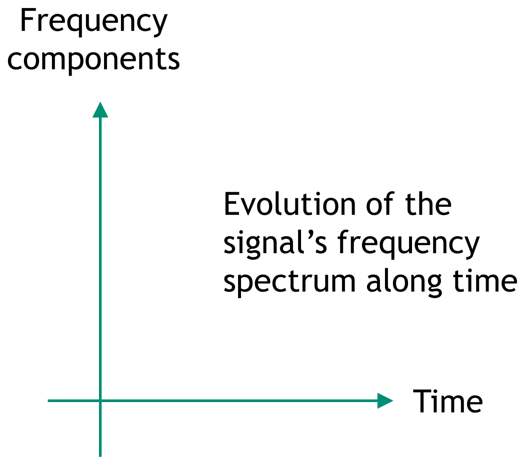
Introduction

- Time-frequency decomposition is a common tool for signals analysis
- Potential fields of application:
  - Medical, such as electroencephalogram (EEG) or electrocardiogram (ECG) analysis
  - Power quality analysis, such as harmonics detection
  - Geophysics, such as earthquakes detection or study
  - Fluids/turbulences analysis, such as for detection of airplanes or submarines
  - Image processing and data compression

What is time-frequency decomposition?

1-D time domain signal





Introduction

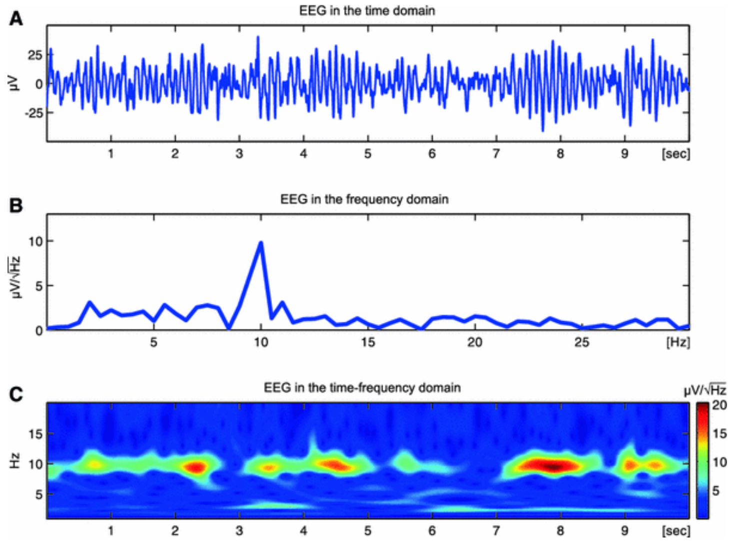
What is time-frequency decomposition?

10 seconds of EEG in time domain

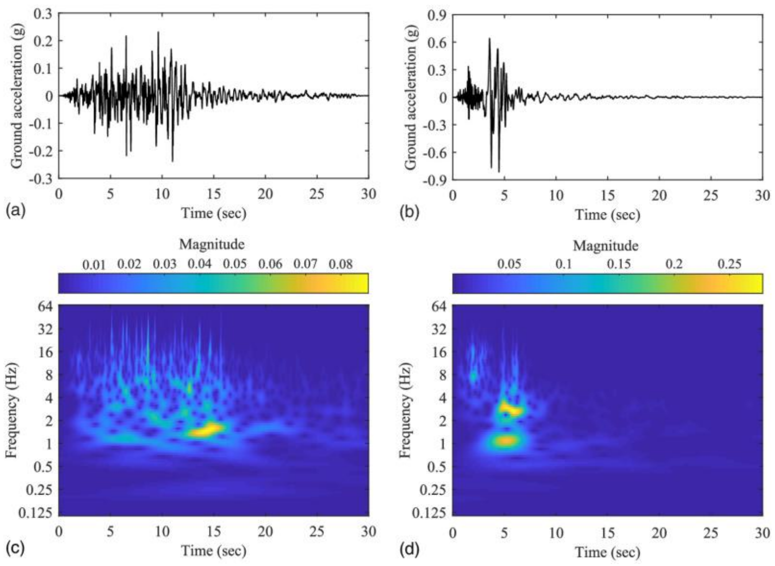
Same 10 seconds of EEG in frequency domain

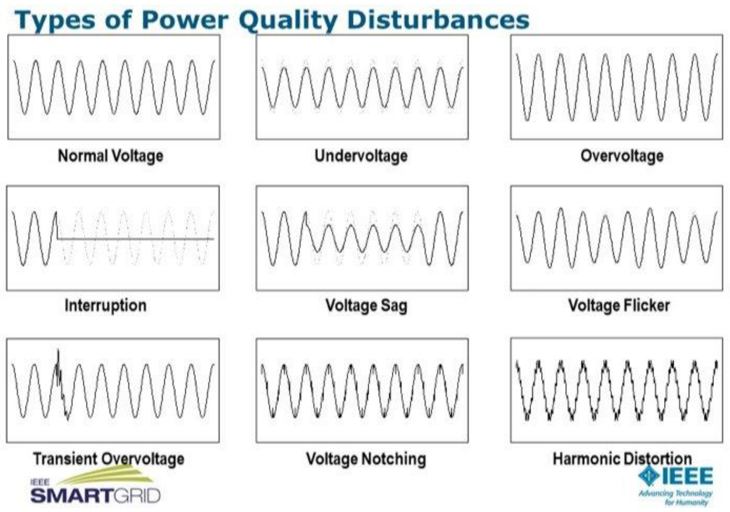
NOTE: Normal brain operation should have oscillations between 8-12 Hz

This is the result of a technique used Short-Time Fourier Transform (STFT)



For seismic studies…





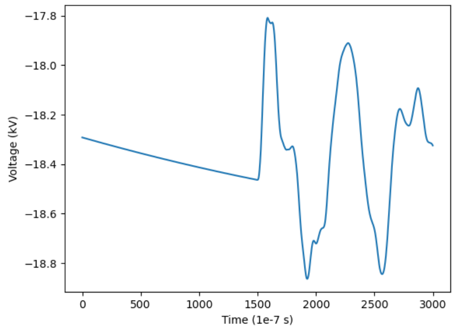
Introduction

- Time-frequency decomposition is critical to understand events and disturbances!

- Methods for time-frequency decomposition:
  - Short-Time Fourier Transform (STFT)
  - Wigner-Ville Distribution (WVD)
  - Stockwell Transform (S-T)
  - **Wavelet Transforms (WTs)**

- WTs have several advantages:
  - They are suitable for non-periodic signals (STFT is not!)
  - Computationally efficient (WVD and S-T are not!)
  - Easier interpretation (WVD is not!)

Not suitable for transients, such as…



WT is the way to go and it is chosen in most recent research papers!

Introduction

- There are two main types of WTs:
  - Continuous WT (CWT)
  - Discrete WTs:
    - Discrete WT (DWT)
    - Stationary WT (SWT) ← A modification of the DWT

Different calculation procedures!

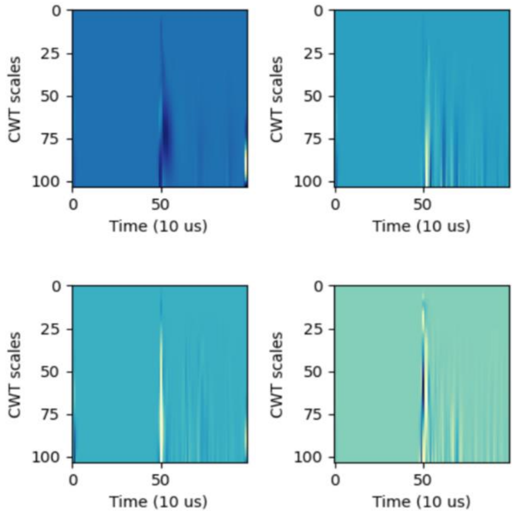
CWT: $$X_w(a,b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} x(t)\overline{\psi}\left(\frac{t-b}{a}\right) dt$$

DWT: $$f(t) = \sum_{j,k} a_{j,k} 2^{j/2} \psi(2^j t - k)$$

- Advantages of Discrete WTs over CWT:
  - No redundant information
  - Faster execution

Discrete WTs are a more advance time-frequency decomposition tool
They are commonly used in many research papers!

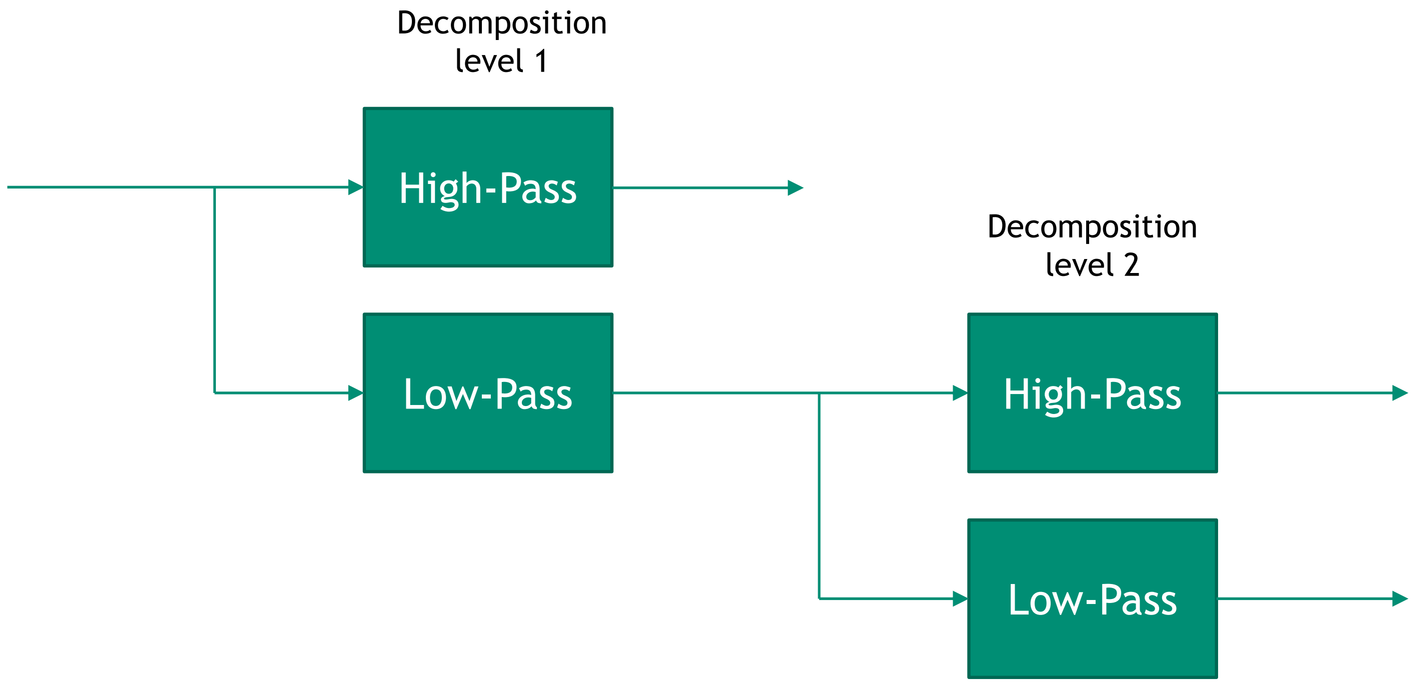
Example of a CWT



Background

Discrete WTs are efficiently implemented using Multi-Resolution Analysis (MRA):

- MRA consists of a series of low-pass and high-pass filters organized in several decomposition levels



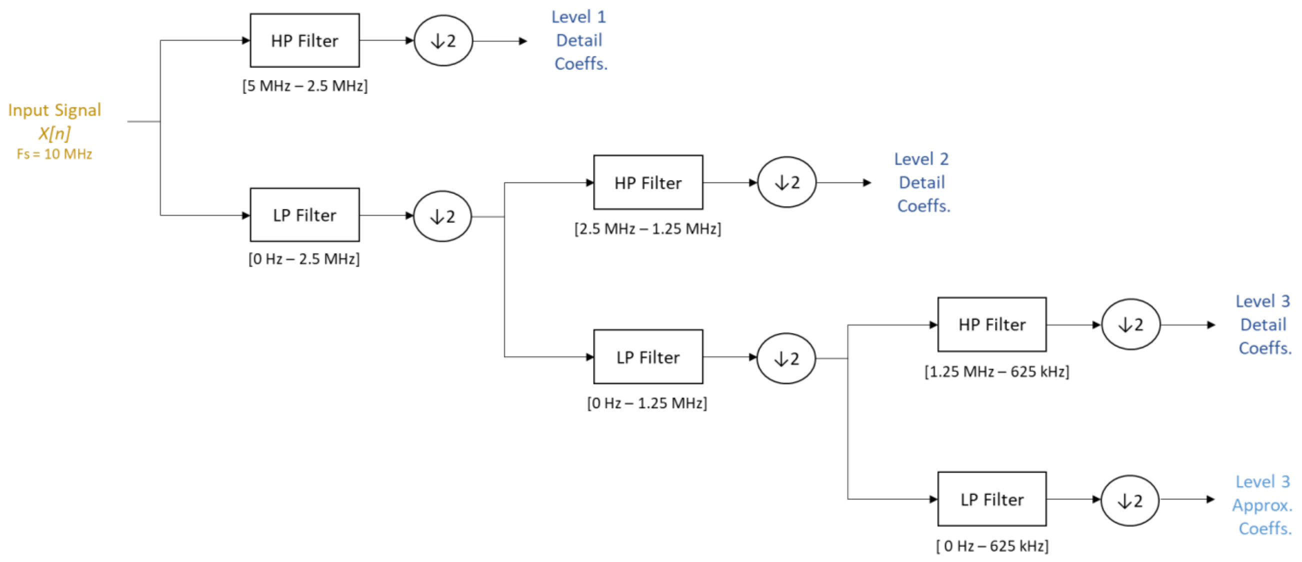
High-pass filter: Only higher-frequencies go through

Low-pass filter: Only lower-frequencies go through

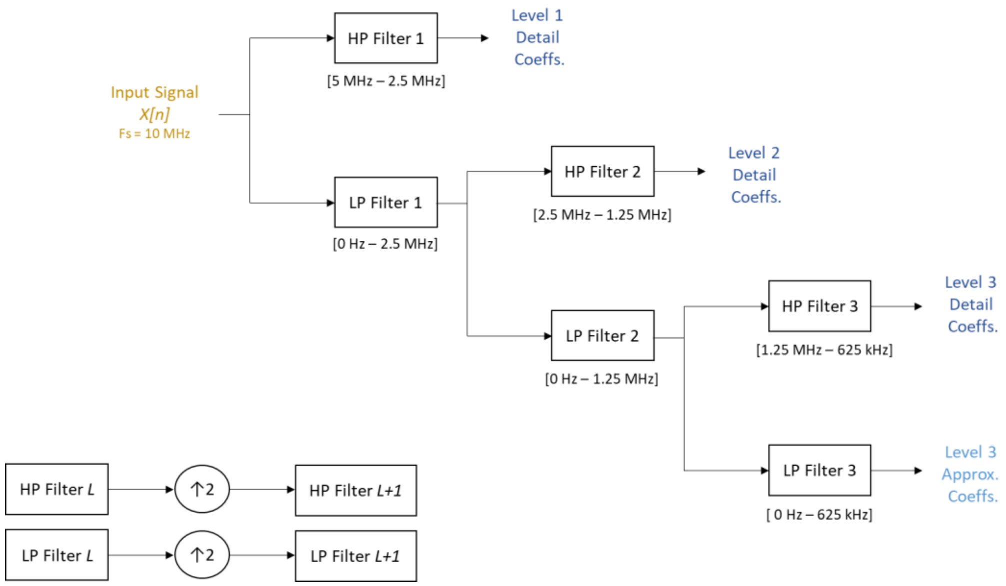
Background

The difference between DWT and SWT is:

- Instead of down-sampling the output of every filter, the filter coefficients are up-sampled in every decomposition level
- This avoids losing one half of the filtered signal coefficients in every decomposition level

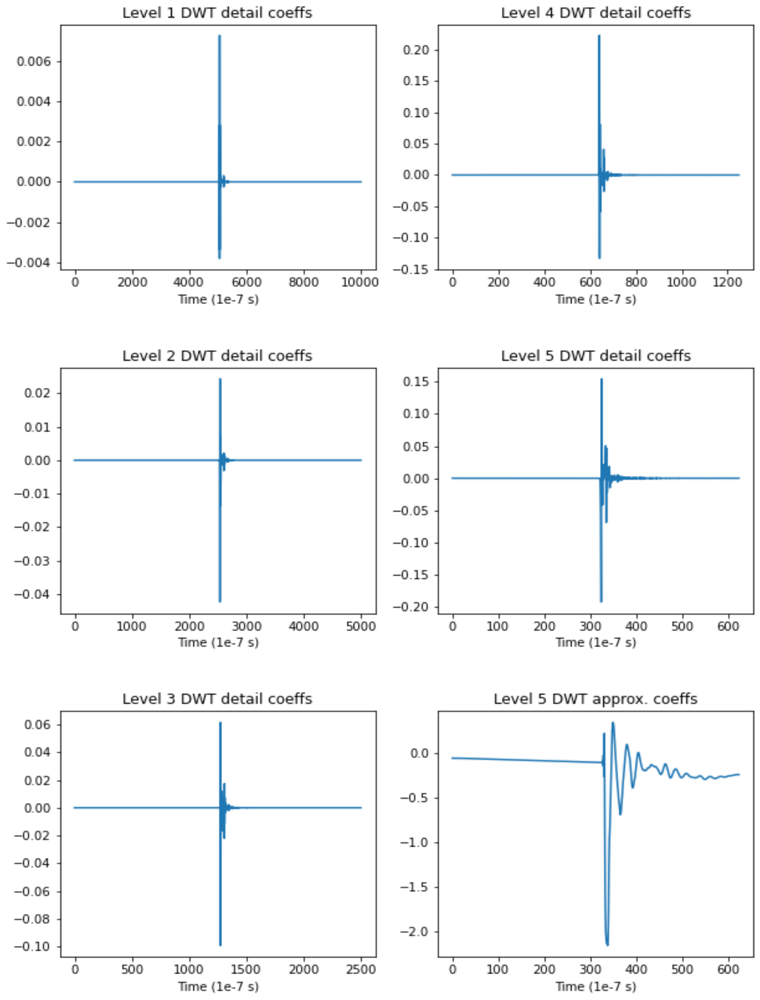


DWT workflow



SWT workflow

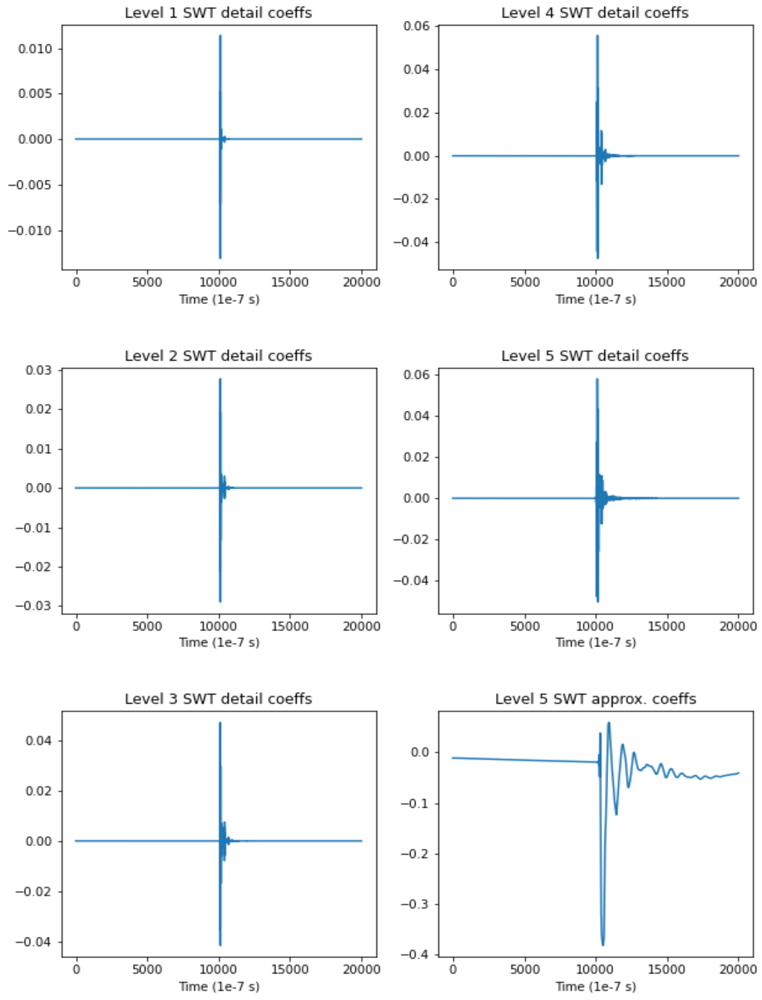
Background



There is a certain loss of detail in the DWT

The more decomposition levels, the fewer coefficients in them

The SWT is preferred over the DWT



Background

By applying MRA, the output of each level is a set of coefficients that describes the signal in a certain range of frequencies ("frequency bands")

Note that the width of these bands decreases exponentially

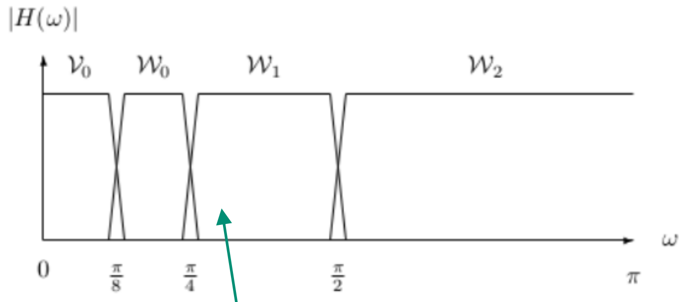


Figure 5. Frequency Bands for the Analysis Tree

For a sampling frequency of 10 MHz

Level	Lower bound frequency	Upper bound frequency
1	2.5 MHz	5 MHz
2	1.25 MHz	2.5 MHz
3	625 kHz	1.25 MHz
4	312.5 kHz	625 kHz
5	156.25 kHz	312.5 kHz
6	78.125 kHz	156.25 kHz

Why the HRWT?
These bands should be as fine as possible for a more detailed analysis!

The concept behind the HRWT

How to increase the number of frequency bands in the HRWT?

From

Up-sampling the filter coefficients by 2 in each decomposition level (as in the SWT)

→

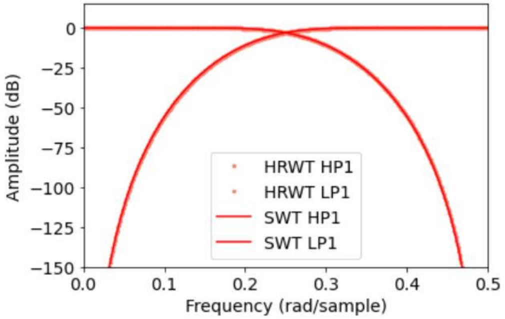
To

Increasing the number of inserted zeroes in a linear way (or in any other arbitrary way)

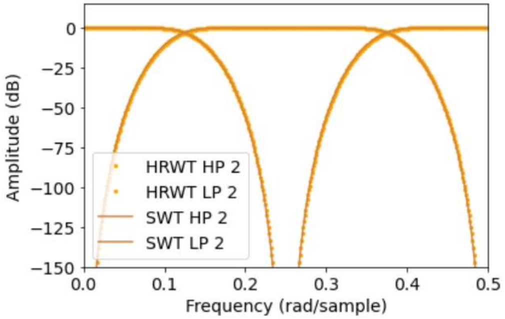
SWT Level	Num 0s SWT	HRWT Level	Num 0s HRWT
1	0	1	0
2	1	2	1
3	3	3	2
		4	3
4	7	5	4
		6	5
		7	6
		8	7
5	15	9	15

Results

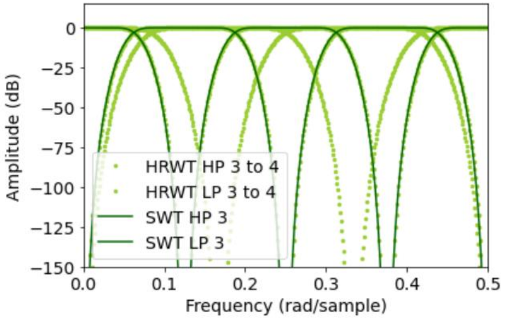
Using the HRWT, after decomposition level 2, former filters' frequency bands are multiplexed into additional bands



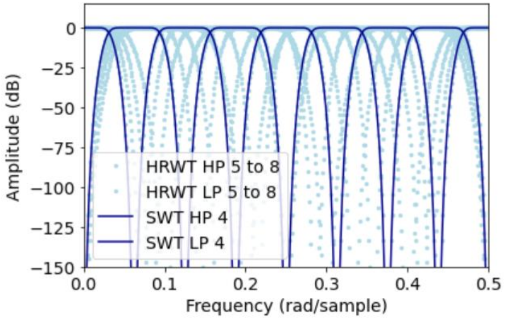
(a) Equivalent SWT Level 1



(b) Equivalent SWT Level 2



(c) Equivalent SWT Level 3

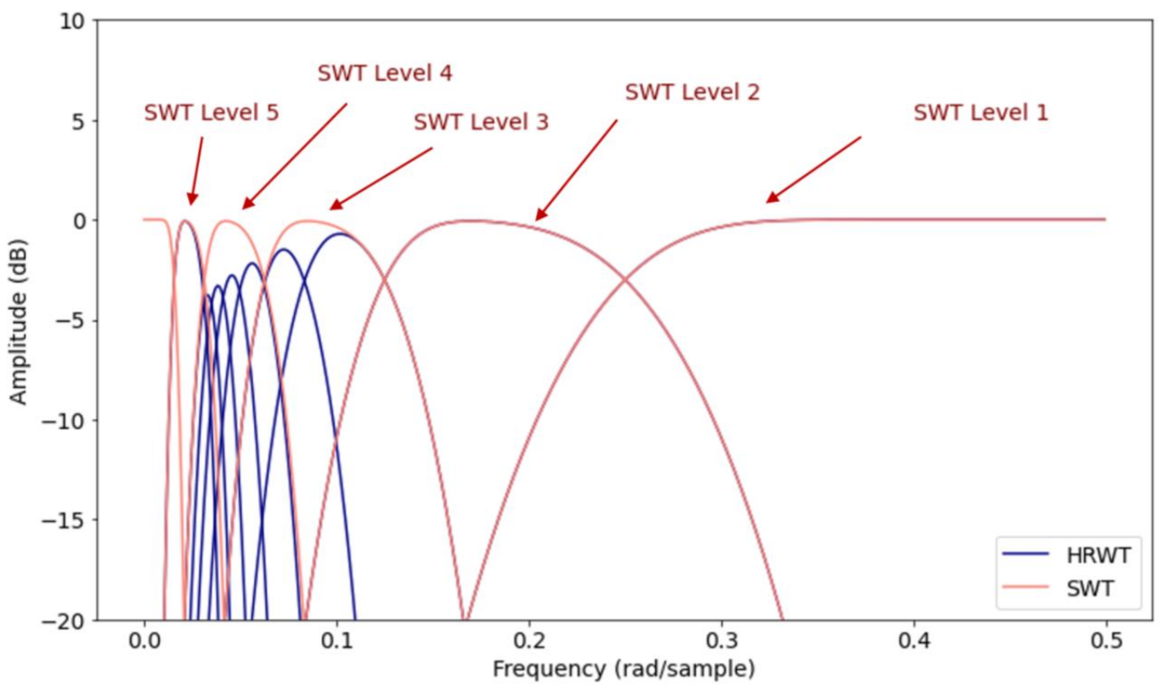


(d) Equivalent SWT Level 4

Results

The HRWT effectively allows to get finer decomposition frequency bands

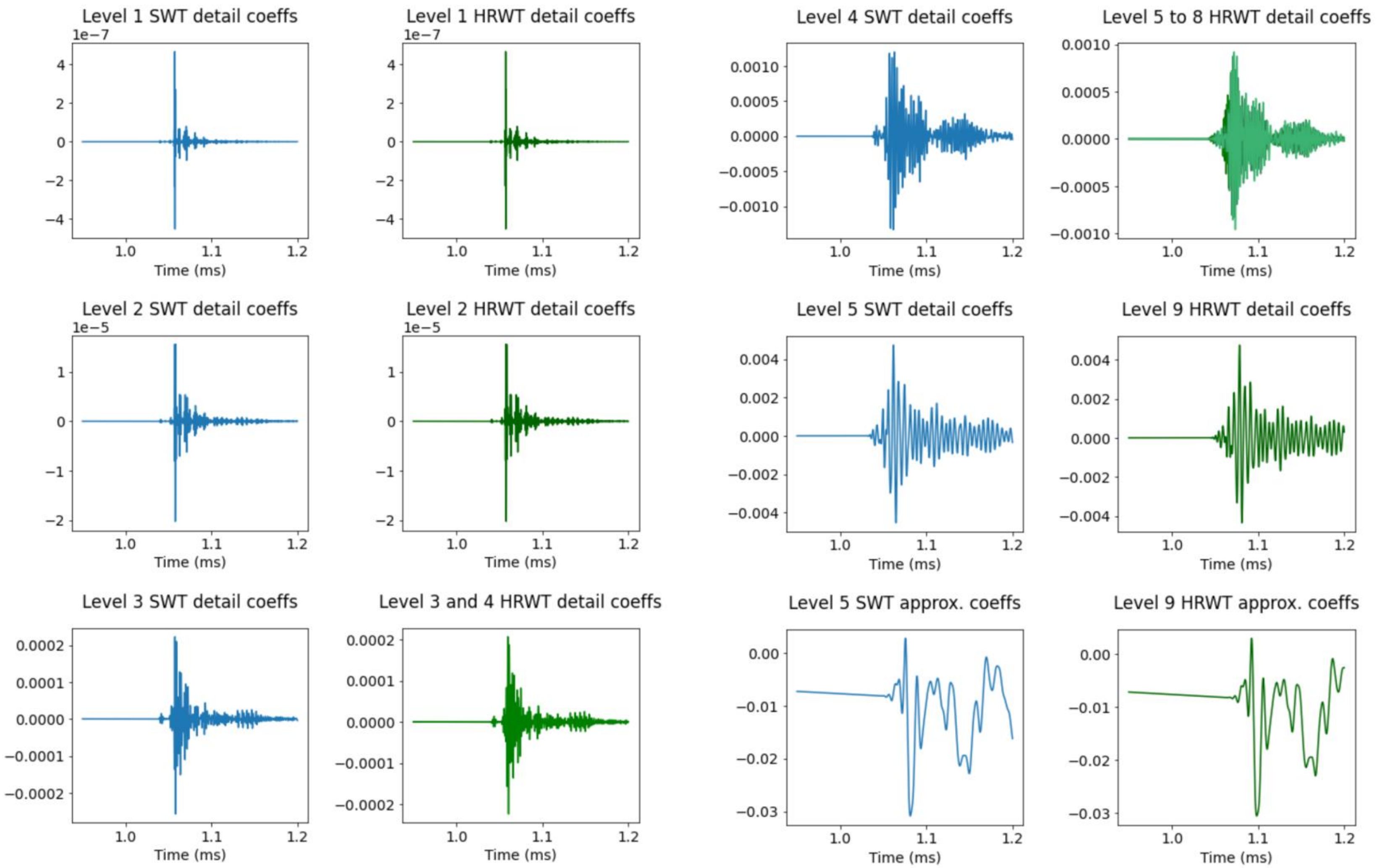
The proposed HRWT implementation allows to come back to normal resolution if desired



Results

Compared to the SWT, the HRWT provides additional levels of decomposition in former SWT levels

A finer resolution allows to pin-point more accurately the underlying frequencies



Conclusion

- Discrete WTs (and the SWT in special) are a more advance time-frequency decomposition tools, with multiple advantages regarding efficiency, applicability to many types of signals, and interpretability
- The HRWT generalizes the SWT and leads to more flexibility in the design of filters' decomposition frequency bands
- Main advantages brought by the HRWT:
  - The HRWT maximizes the number of frequency bands in the former SWT decomposition levels, which leads to finer resolution
  - Finer resolution allows a better understanding of the frequency components in the studied phenomenon
- Potential applications:
  - It can be applied for time-frequency decomposition as any other WT