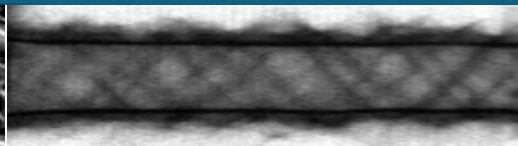
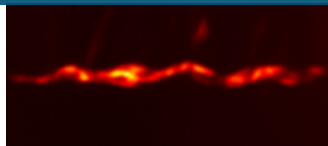
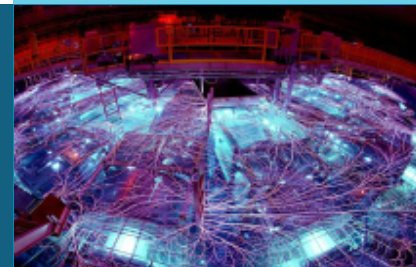




Sandia  
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# *Magnetized Liner Inertial Fusion (MagLIF): Developing a data-driven understanding of magnetic flux compression from computation through experiment*



PRESENTED BY

William Lewis

APS DPP  
10/20/22

P.F. Knapp, S.A. Slutz, P.F. Schmit\*, G.A. Chandler, M.R. Gomez, A.J. Harvey-Thompson, M.A. Mangan, D.J. Ampleford, and K. Beckwith

For new results that will be presented, **special thanks** to O.M. Mannion, C.A. Jennings, D.E. Ruiz,

P.F. Knapp, M.R. Gomez, and A.J. Harvey-Thompson

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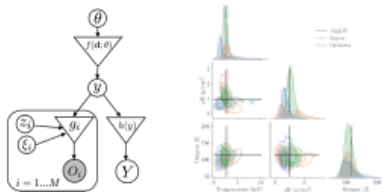
SAND2022-#### C

# the design and discovery cycle at Sandia's Z pulsed power facility.



## Optimization of Diagnostic Configurations in the Presence of Uncertainty Using Bayesian Inference and Optimization

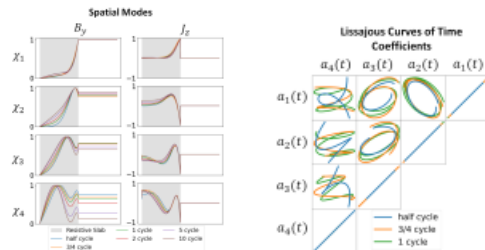
P. Knapp *et al.* **GO06.00005** on 10/17/22



P.F. Knapp *et al.* J. Plasma Phys. (Under review).

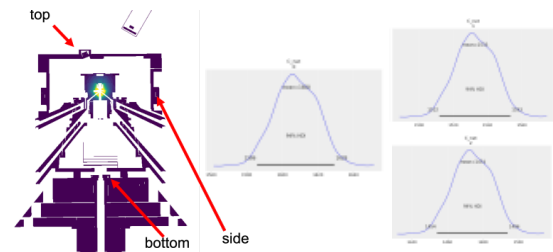
## Identifying Governing ODEs in Irregular Physical Domain with Diffusion

G. Vasey *et al.* **NP11.00069** on 10/19/22



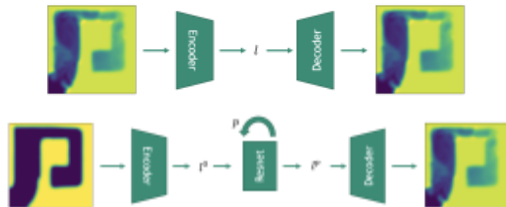
## Bayesian Analysis Applied to Neutron Activation Diagnostics Measurements of MagLIF

M. Mangan *et al.* **TP11.00084** on 10/20/22



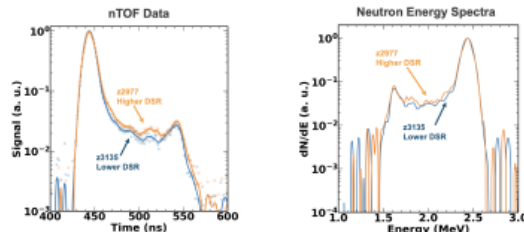
## An autoencoder based reduced order model of low density plasma for optimal experimental design

R. Patel *et al.* **CO05.00011** on 10/17/22



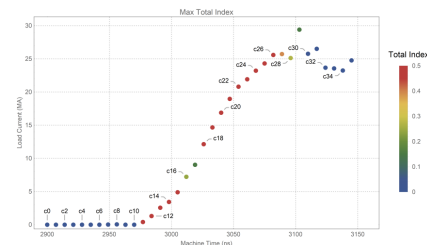
## Analysis of the scattered neutron energy spectrum from magnetized liner inertial fusion implosions on Z

O. Mannion *et al.* **NO04.00007** on 10/19/22



## Quantifying data constraints for velocimetry-based load current determination for inertial confinement fusion targets

A. Porwitzky *et al.* **YO04.00006** on 10/21/22



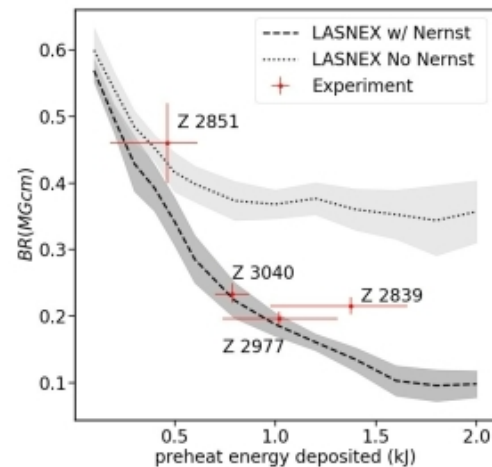
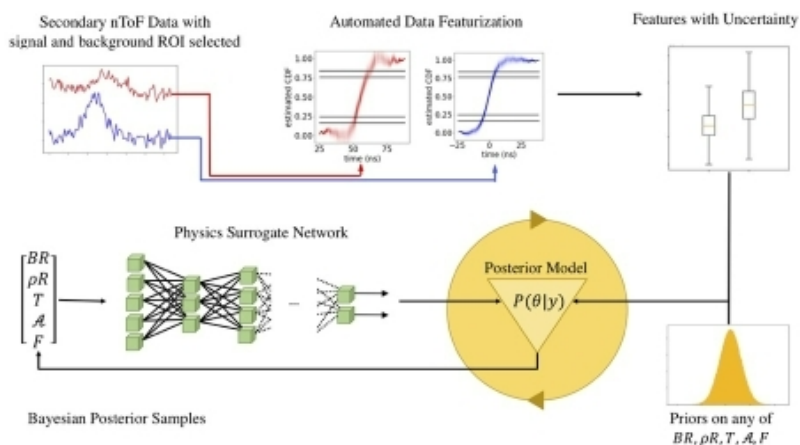
the design and discovery cycle at Sandia's Z pulsed power facility.



**This talk** W.E. Lewis *et al.*, Phys. Plasmas (2021). Selected as an Editor's Pick

### • Assessment of fuel magnetization in MagLIF

- First-ever systematic study of magnetic confinement properties of any neutron producing magneto-inertial fusion platform
  - Enabled by deep-learning and Bayesian inference
  - Experimental evidence for the role of Nernst advection in flux loss



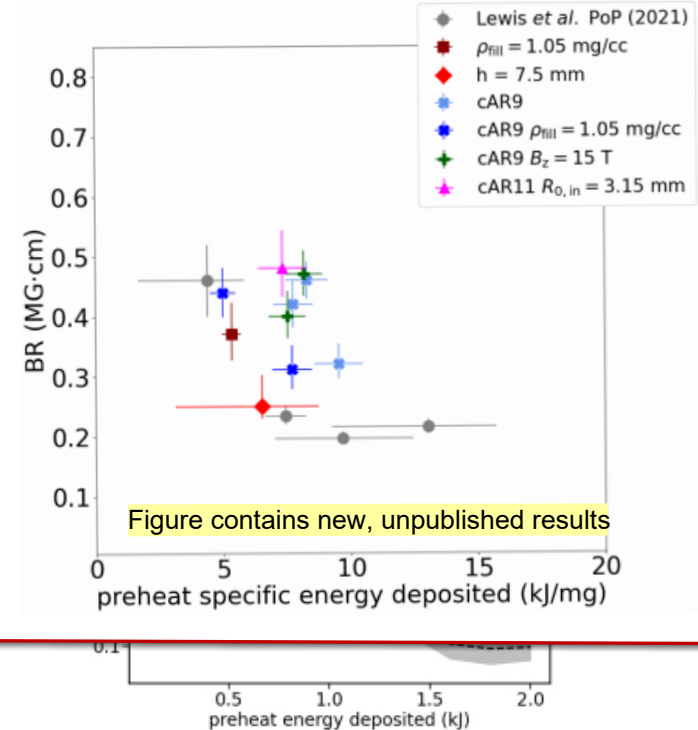
W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701 (2021).

We are applying modern data analysis methods to accelerate the design and discovery cycle at Sandia's Z pulsed power facility.



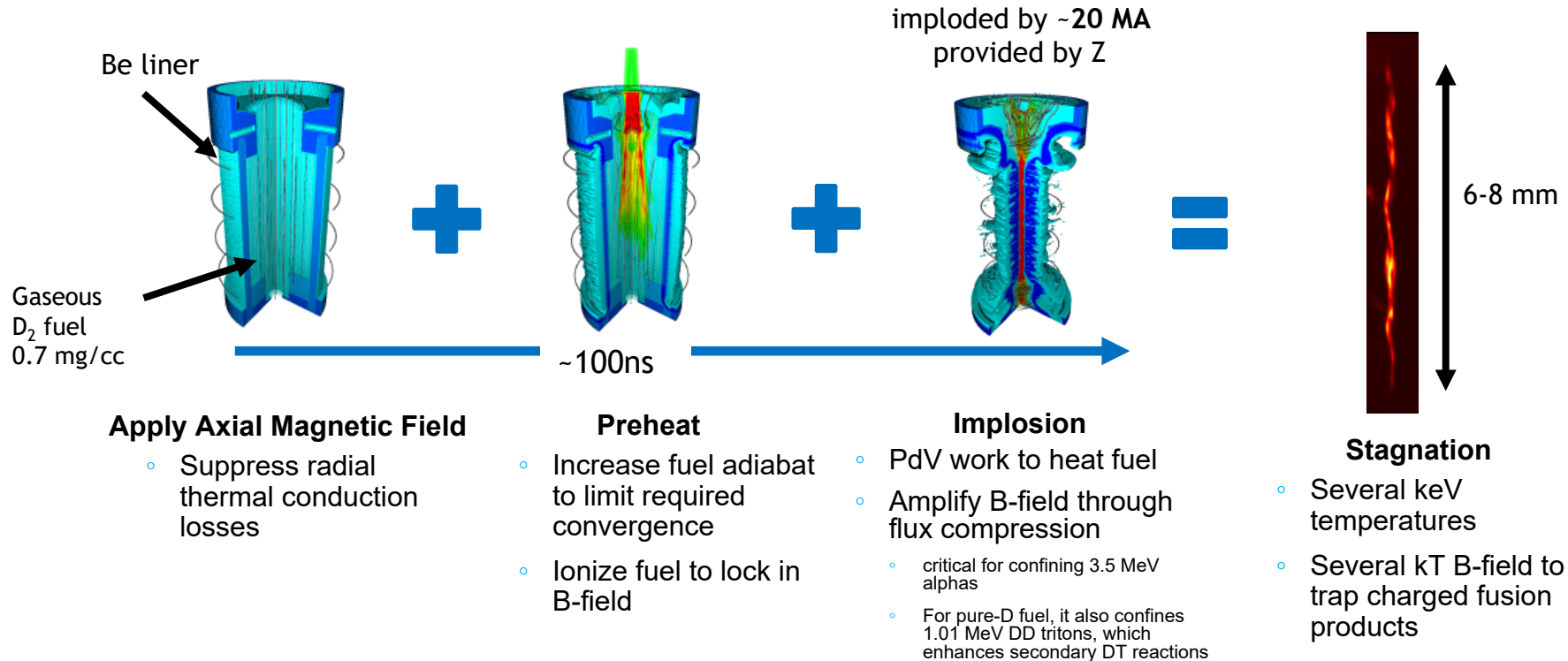
This talk W.E. Lewis *et al.*, Phys. Plasmas (2021). Selected as an Editor's Pick

How does changing other experimental inputs impact flux loss?

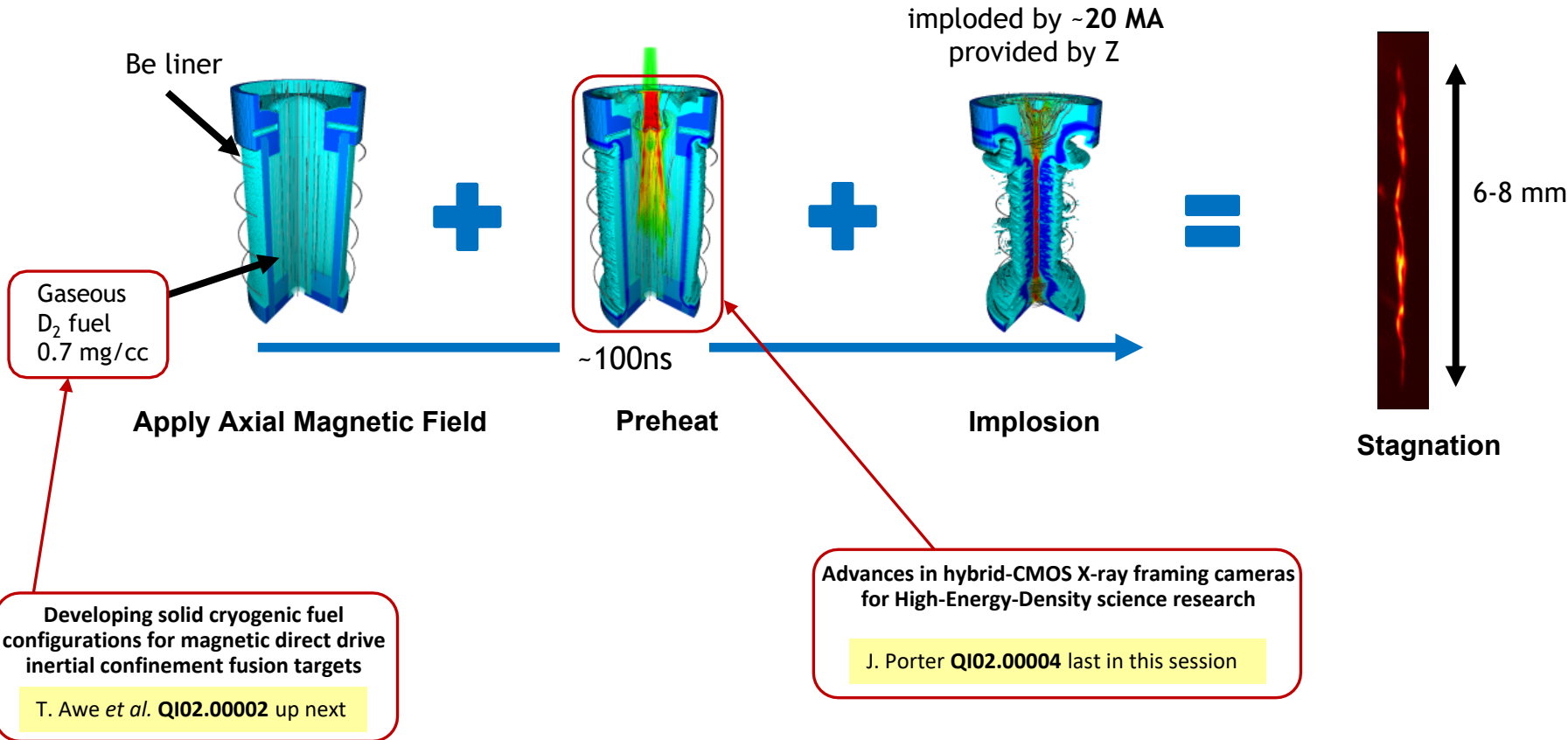


Bayesian Posterior Samples

Priors on any of  
 $BR, \rho R, T, A, F$



Magnet relies on three stages to reach fusion relevant conditions.

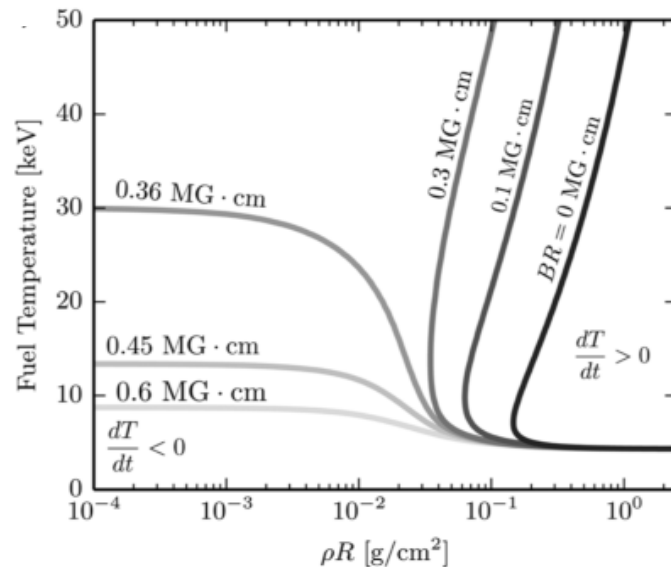


Magnetization, or magnetic field-fuel radius product (BR), is a critical confinement parameter for MagLIF.



Effective BR for  $B_z(r)$  profile:

$$\overline{BR} = \frac{\Phi_R}{\pi R} = \frac{2}{R} \int_0^R r B_z(r) dr$$



P.F. Knapp et. al., PoP (2015)

$\overline{BR}$  determines trapping of fast charged particles:

trapping condition for particles born at  $r=0$

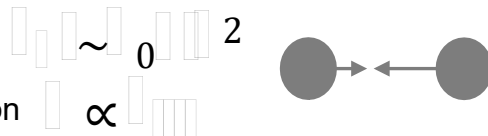
$$\overline{BR} \geq \sqrt{\frac{8mE_{\perp}}{q^2}}$$

A variety of plasma transport effects will modify the flux compression process. Measuring BR could provide insights into these effects.

- Ideal flux compression  $\sim 1000\times$  B-field amplification
  - trapping of fusion products and reduction of electron heat conduction
- Physical mechanisms leading to flux loss
  - Resistive diffusion
  - Nernst

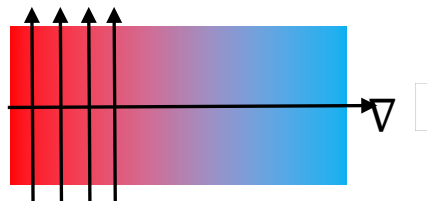
### Resistive diffusion

- Current disrupted by collisions
  - Allows magnetic field diffusion

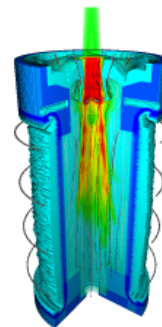


### Nernst effect

- B-field locked into warm electrons
- Thermal transport perpendicular to B transports flux



$$v_{Nernst} = \frac{\beta_{\Lambda} \nabla_{\perp} T_e}{eB}$$



- Increased preheat  $\propto \nabla_{\perp} T_e$  increases Nernst
- Increased  $B_z$  decreases Nernst
- What about geometry?
- Fill density?
- Impact of mix throughout implosion
- Measurements needed to study effects
  - **can't do proton deflectometry/radiography**
    - **O(50 MG) fields driving Z-pinch!**

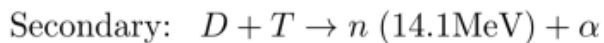
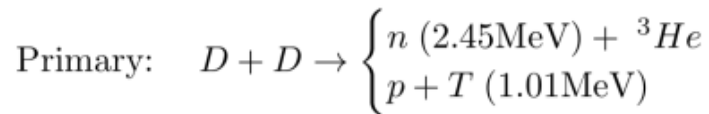


# Radially and axially viewed secondary DT neutron spectra and yield ratio $\bar{Y} = Y_{DT}/Y_{DD}$ are sensitive to fuel magnetization.

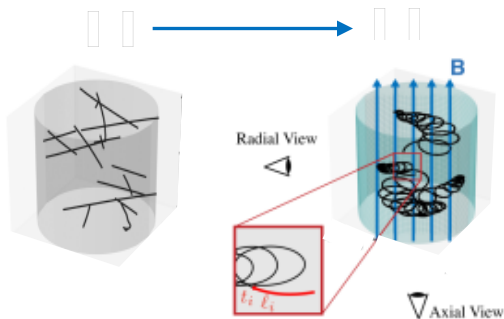


- Pure Deuterium fuel

- ~1.01 MeV tritons produced by DD fusion



$$\mathcal{P}_{DT} \propto \langle \rho_D \ell \rangle \sigma_{DT} \xrightarrow{\text{Magnetized}} \ell \propto \left( \frac{\mu_0 \mu_B}{k_B T} \right)$$

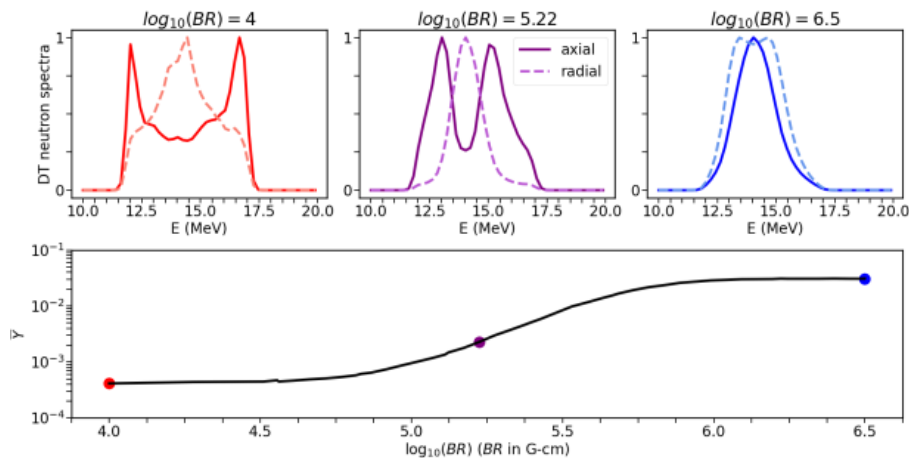


P.F. Schmit *et al.*, PRL (2014)  
P.F. Knapp *et al.*, PoP (2015)

- Surrogacy of tritons for  $\alpha$ 's

- similar Larmor radius
  - 3.5 MeV  $\alpha$  stopping length  $\sim 0.5 \times 1.01$  MeV tritons

Increase in  $P_{DT}$  increases  $\bar{Y}$



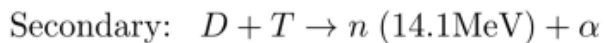
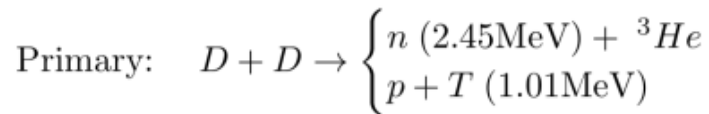
W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701 (2021)

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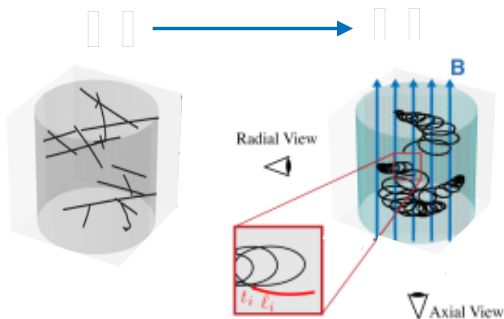


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$$\mathcal{P}_{DT} \propto \langle \rho_D \ell \rangle \sigma_{DT} \xrightarrow{\text{Magnetized}} \ell \propto \left( \frac{\mu_0}{B} \right)$$



P.F. Schmit *et al.*, PRL (2014)  
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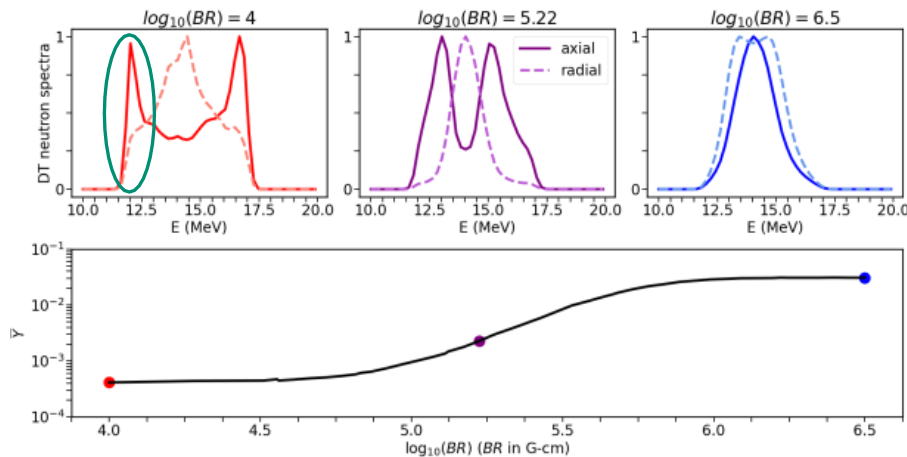
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14 MeV in CM  
Doppler down shift

▽ Axial View

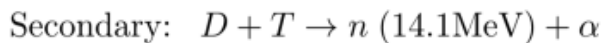
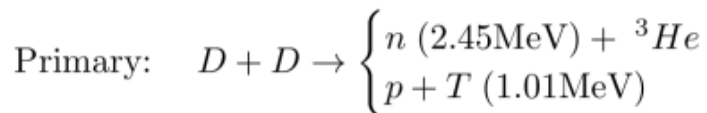


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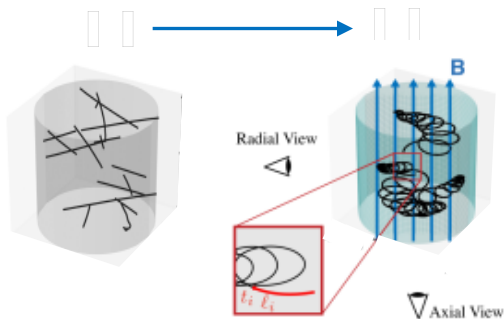


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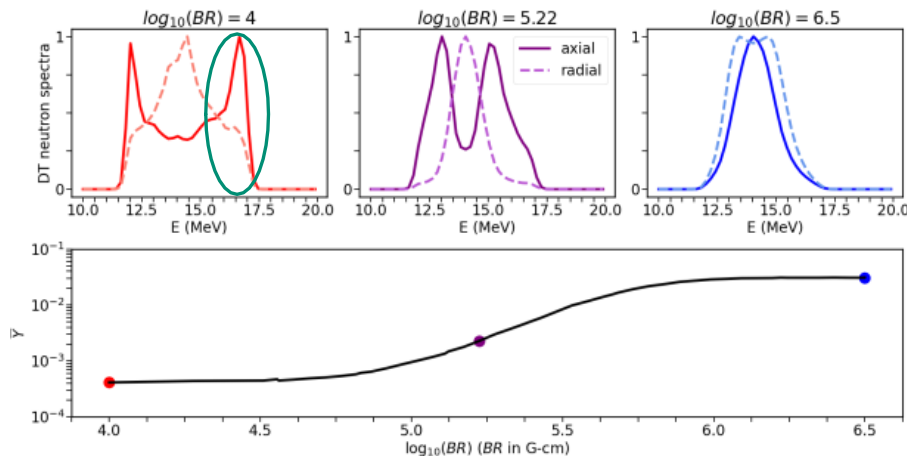
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Doppler up shift

▽ Axial View

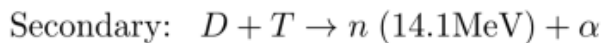
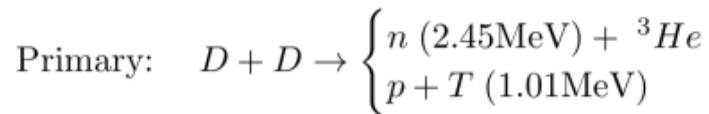


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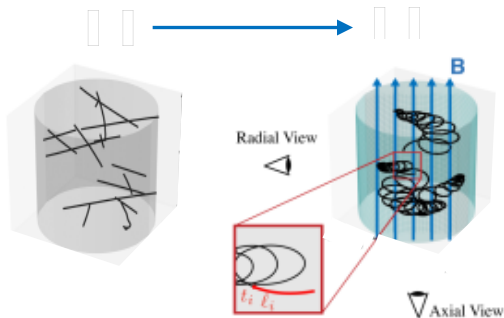


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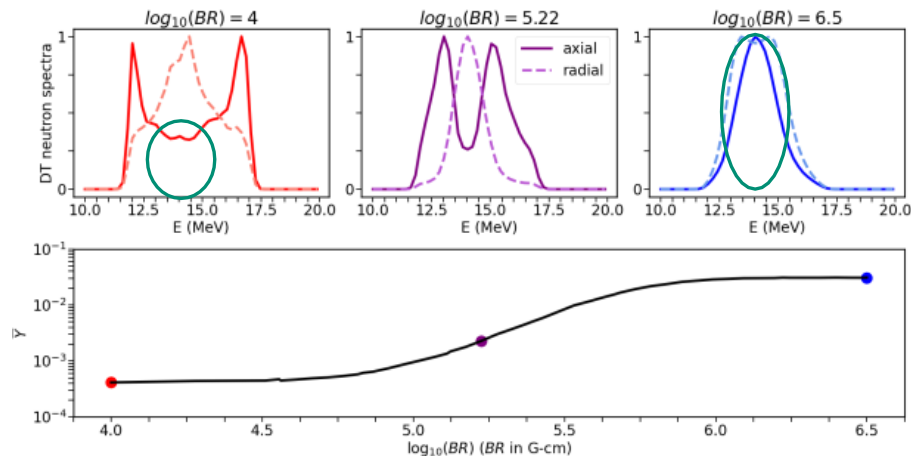


14 MeV in CM  
No Doppler shift  
 $B_z \sim$  probability

Low probability

▽ Axial View

High probability

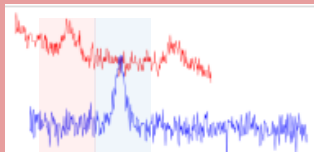


W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701 (2021)

Our analysis is based on a Bayesian inference which makes use of NN surrogate for speedup of physics model.

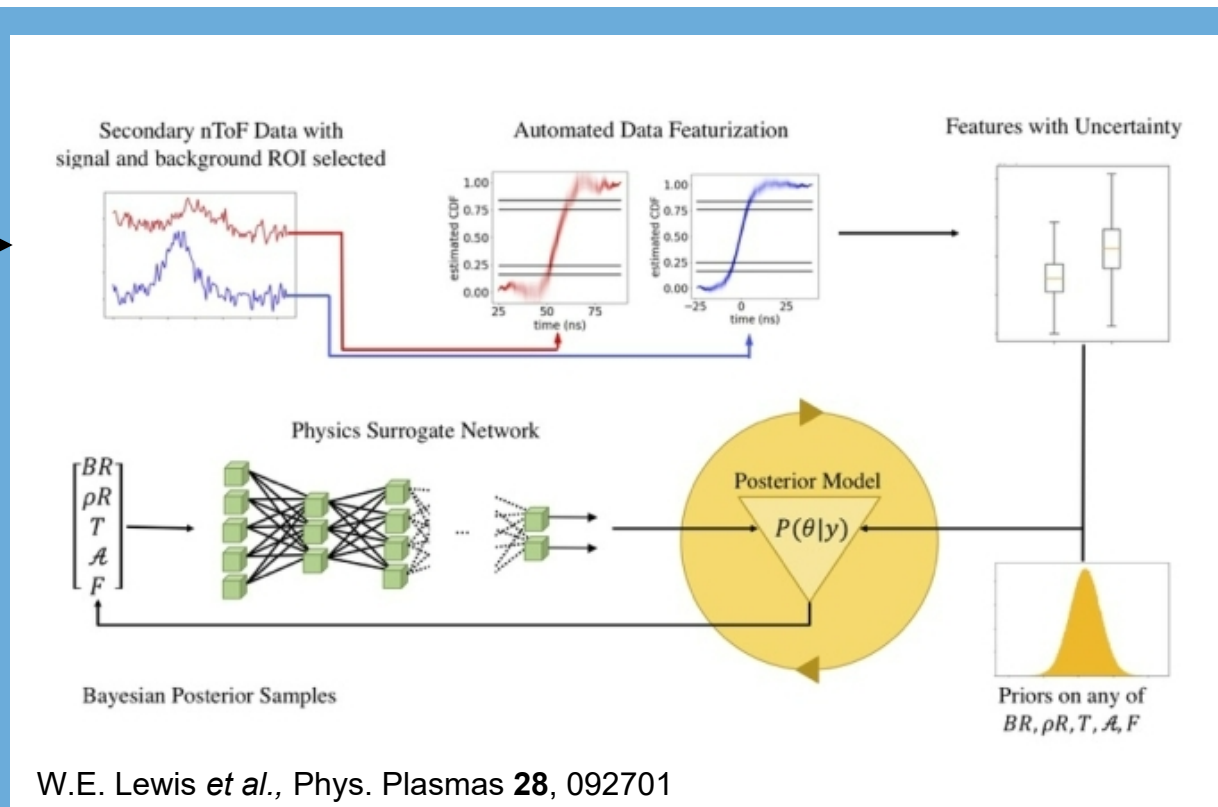


Obtain secondary neutron  
Time of Flight (nToF)



Select ROIs

Provide DT and DD yields  
with uncertainty

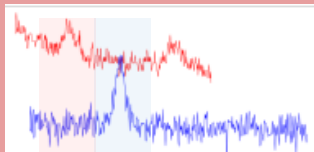


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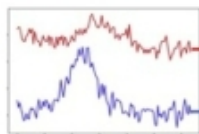
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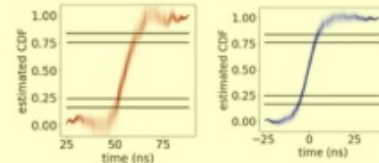
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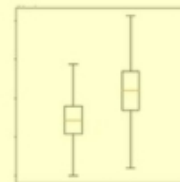
Secondary nToF Data with  
signal and background ROI selected



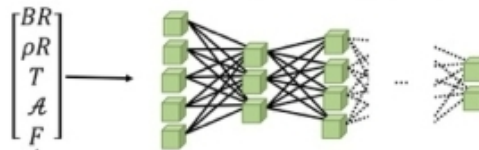
Automated Data Featurization



Features with Uncertainty



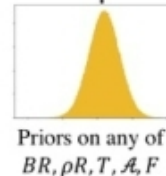
Physics Surrogate Network



Bayesian Posterior Samples

Posterior Model

$P(\theta|y)$

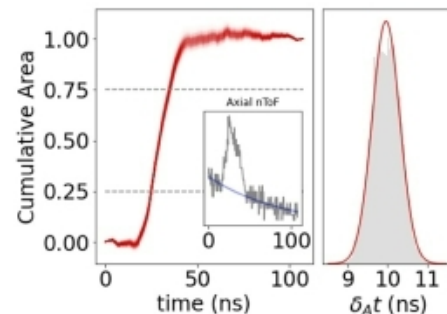
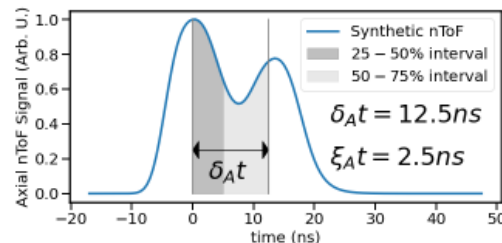
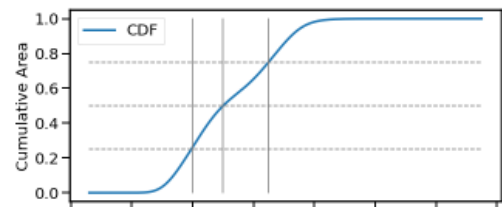


W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701  
(2021).

We wish to “featurize” the nToF data collected experimentally to reduce dimensionality while retaining relevant information.



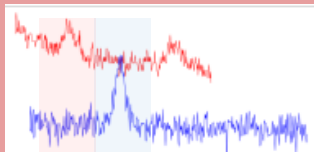
- Width and asymmetry features with uncertainty
  - percentiles of nToF signals
    - integration smooths noise
    - error from Bayesian fitting
- nToF avoids e.g.
  - unavailable timing fiducial
  - ill-posed instrument response deconvolution



Our analysis is based on a Bayesian inference which makes use of NN surrogate for speedup of physics model.



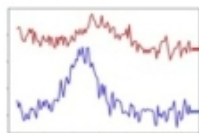
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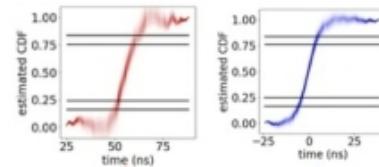
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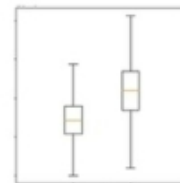
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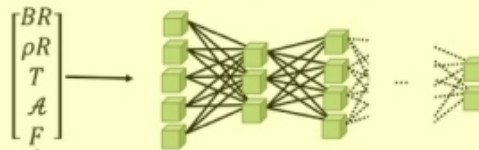
Automated Data Featurization



Features with Uncertainty



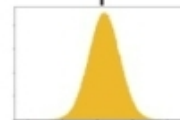
Physics Surrogate Network



Bayesian Posterior Samples

Posterior Model

$P(\theta|y)$



Priors on any of  
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W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701  
(2021).



# Costly analysis results in the need for data-driven approaches to routinely invert experimental data for BR.

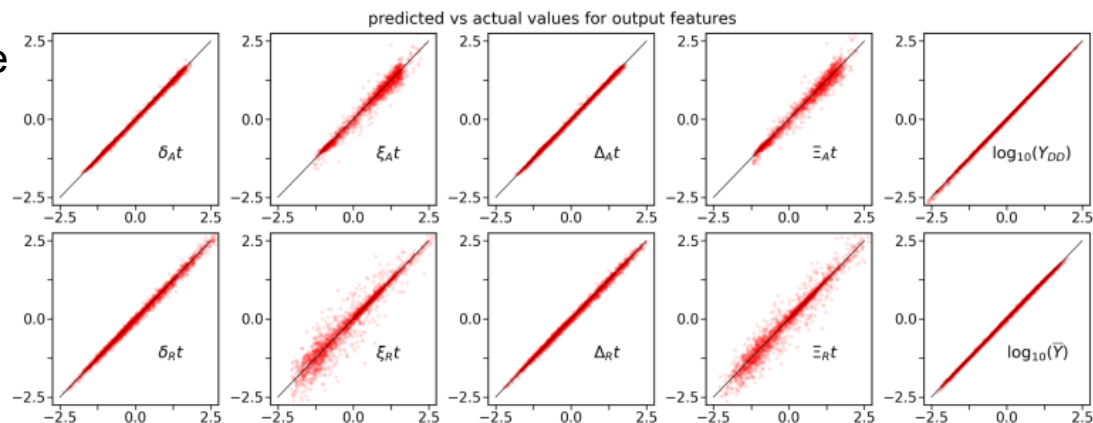
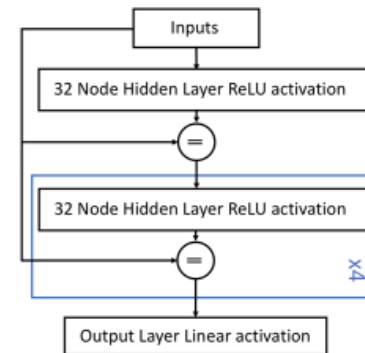


- Previously, one experiment analyzed for BR in the literature:
  - P.F. Schmit *et al.*, Phys. Rev. Lett. (2014)
  - P.F. Knapp *et. al.*, Phys. Plasmas (2015)
- Computational cost of forward physics model
  - O(10-100) CPU hours evaluation on a high-performance cluster
  - 10k-100k + evaluations per experiment for inference and uncertainty quantification
- Use a **surrogate model/fast emulator**
  - Rigorous UQ with **Bayesian statistics** feasible with

neural network surrogate, which drastically improves evaluation times.



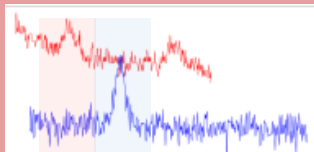
- ~65k simulation samples
  - 95%-4%-1% train-validation-test split
- neural network with skip connections
  - about 5.5k fit parameters
- Validation data used to estimate error
  - propagate uncertainty of surrogate



Our analysis is based on a Bayesian inference which makes use of NN surrogate for speedup of physics model.



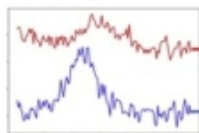
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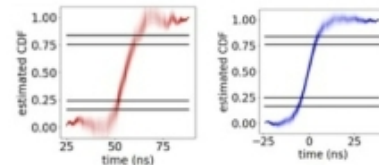
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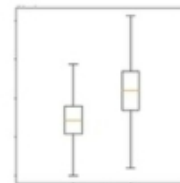
Secondary nToF Data with  
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Automated Data Featurization



Features with Uncertainty



Physics Surrogate Network



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Priors on any of  
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W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701  
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Bayes theorem allows us to incorporate multiple sources of data and rigorously define statistical data models for UQ.



- Sources of uncertainty considered:
  - Experimental data (multivariate normal)
  - Surrogate model (multivariate normal)

Bayes theorem allows us to incorporate multiple sources of data and rigorously define statistical data models for UQ.



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  - Experimental data (multivariate normal)
  - Surrogate model (multivariate normal)

We can encode assumptions about statistics in Bayesian framework with a bit of maths.



## Bayes theorem and manipulations:

Posterior distribution for parameters ( $\mathbf{x}$ ) given data ( $\mathbf{y}$ )

Prior information on parameters

$$p(\mathbf{x}|\mathbf{y}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x})/p(\mathbf{y})$$

Likelihood function describing distribution of data around model

Parameter independent normalization (unimportant)

Introduce latent variable  $\mathbf{z}$  to track uncertainty from surrogate

$$\longrightarrow p(\mathbf{x}|\mathbf{y}) = \int p(\mathbf{x}, \mathbf{z}|\mathbf{y}) d\mathbf{z}$$

Bayes theorem for data including surrogate model  $\longrightarrow p(\mathbf{x}, \mathbf{z}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x}, \mathbf{z})p(\mathbf{z}|\mathbf{x})p(\mathbf{x})$

## Our data models:

Physics model value normally distributed about NN prediction with OOS estimate of covariance

$$p(\mathbf{z}|\mathbf{x}) \sim \mathcal{N}(f_{NN}(\mathbf{x}), \Lambda_{NN})$$

Assumed independence of different measured quantities

$$\begin{aligned} p(\mathbf{y}|\mathbf{x}, \mathbf{z}) &= p(\mathbf{y}|\mathbf{z}) \\ &= p(\mathbf{y}_{\text{ntoF}}|\mathbf{z}_{\text{ntoF}})p(y_Y|z_Y)p(y_{\bar{Y}}|z_{\bar{Y}}) \end{aligned}$$

Observations normally distributed about “latent model”

$$p(y_Y|z_Y) \sim \mathcal{N}(z_Y, \Lambda_Y)$$

$$p(y_{\bar{Y}}|z_{\bar{Y}}) \sim \mathcal{N}(z_{\bar{Y}}, \Lambda_{\bar{Y}})$$

$$p(\mathbf{y}_{\text{nToF}}|\mathbf{z}_{\text{nToF}}) \sim \mathcal{N}(\mathbf{z}_{\text{nToF}}, \Lambda_{\text{nToF}})$$



## Advanced Data Analysis in Inertial Confinement Fusion and High Energy Density Physics

P. F. Knapp<sup>1</sup> and W. E. Lewis<sup>1</sup>

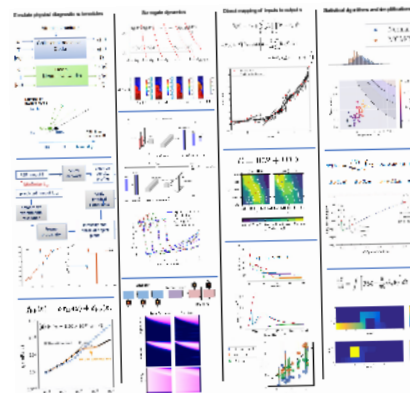
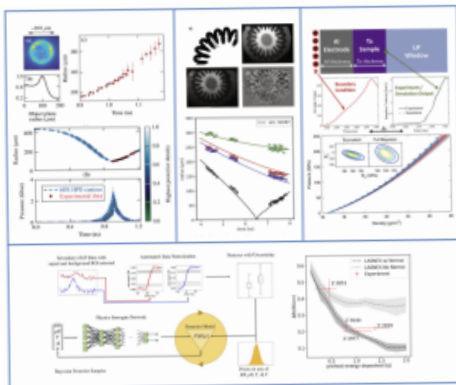
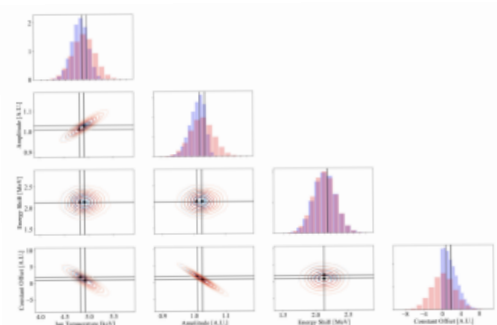
*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

Tutorial on Bayesian inference with code

Extensive ICD/HEDP literature review of:

Bayesian inference

Applied ML methods



Submitted to special issue of Rev. Sci. Instrum.

$$p(\mathbf{y}_{\text{nToF}} | \mathbf{z}_{\text{nToF}}) \sim \mathcal{N}(\mathbf{z}_{\text{nToF}}, \Lambda_{\text{nToF}})$$

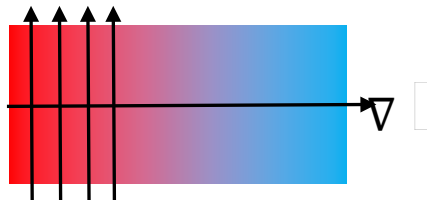
deposited in the fuel show a trend consistent with Nernst effect.



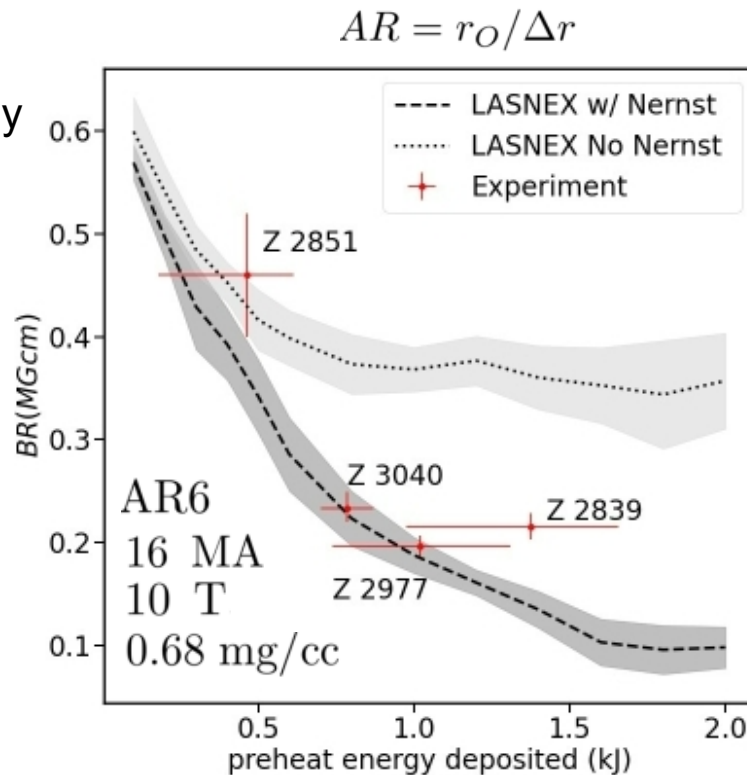
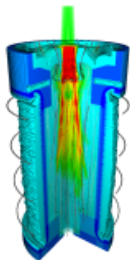
- Evaluated fast-particle confinement parameter
  - across family of experiments varying preheat energy
  - rigorously defined UQ
- Consistent with flux loss via Nernst.

#### Nernst effect

- B-field locked into warm electrons
- Thermal transport perpendicular to B transports flux



$$v_{Nernst} \sim \frac{\nabla_{\perp} T_e}{eB}$$

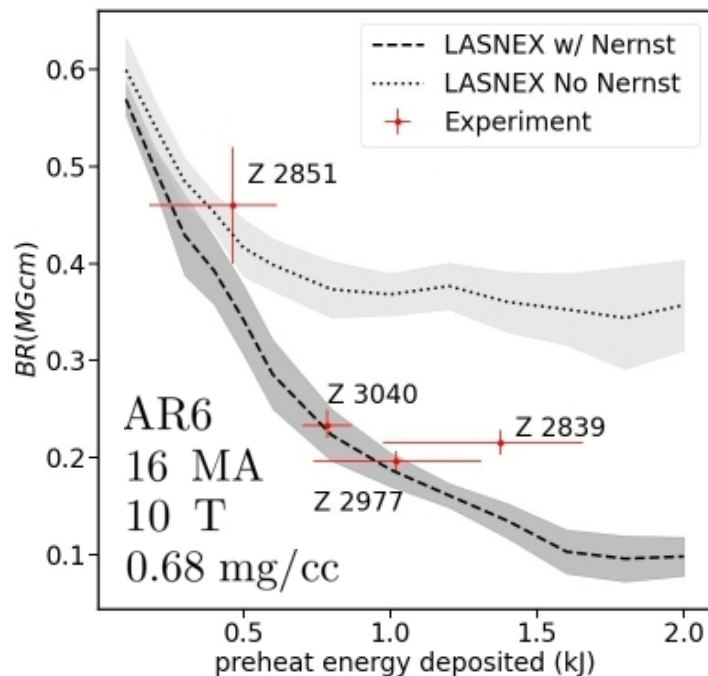




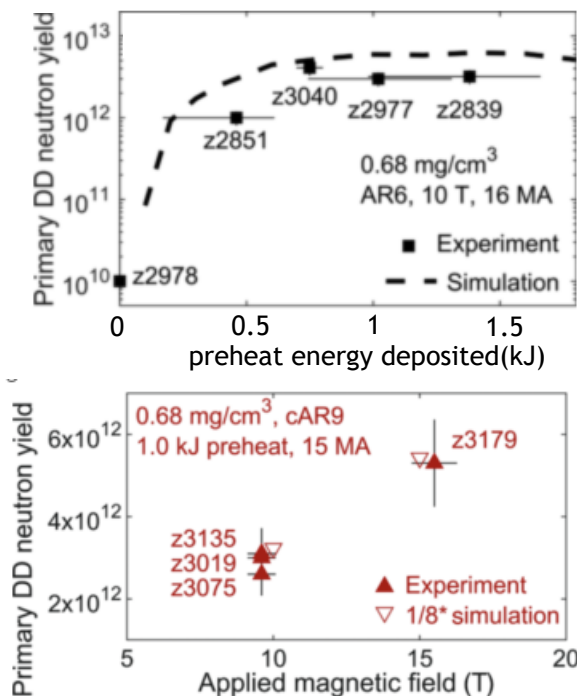
# Nernst is integral to performance scaling in MagLIF.



- Nernst limits the gains by increasing preheat alone
- $\rho_{fill}$ ,  $B_z$ ,  $I_{max}$  etc. must be improved to enable performance gains



W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701



M.R. Gomez *et al.*, PRL **125**, 155002

# We can use this tool to let our data tell us the larger story behind the physics of magnetic confinement in MagLIF

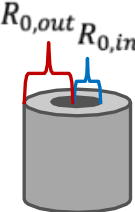


## • How do other experiment inputs impact flux loss?

- fill density/target height
- aspect ratio
- inner liner radius
- mix
- $B_z$

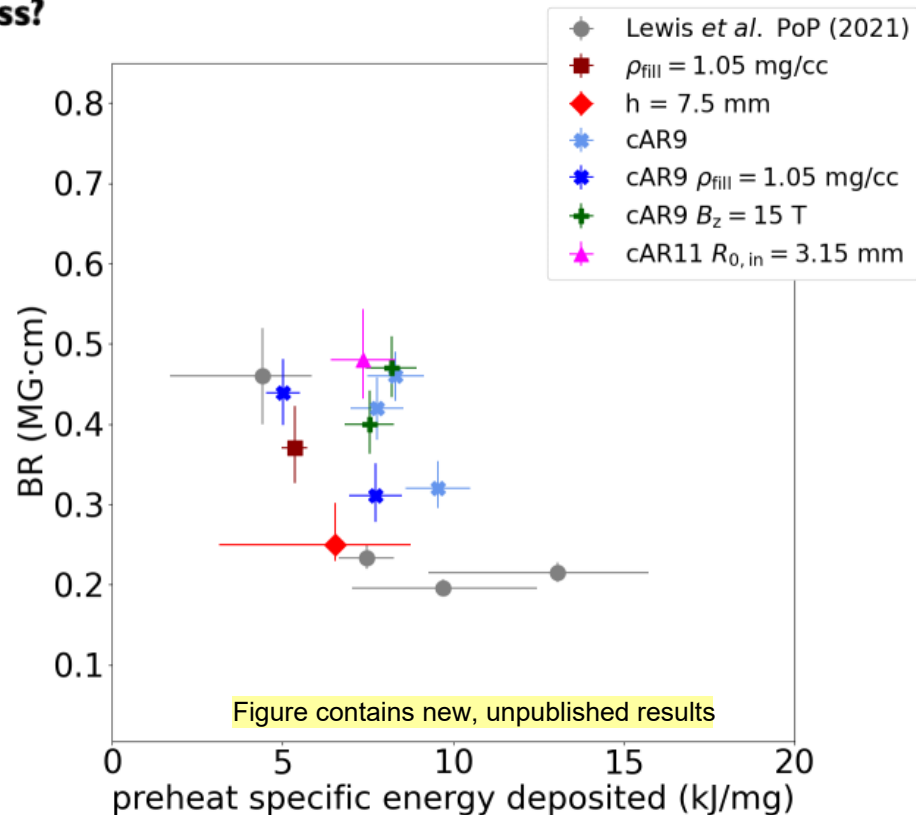
## • Gray points reference cases PoP 2021:

- $AR = 6$ ,  $B_z = 10$  T, and  $\rho_{fill} = 0.7$  mg/cc

$$AR = \frac{R_{0,out}}{R_{0,out} - R_{0,in}}$$


## • 1D resistive radMHD code Kraken\* for comparison

- C.A. Jennings implementation of GORGON system of MHD equations



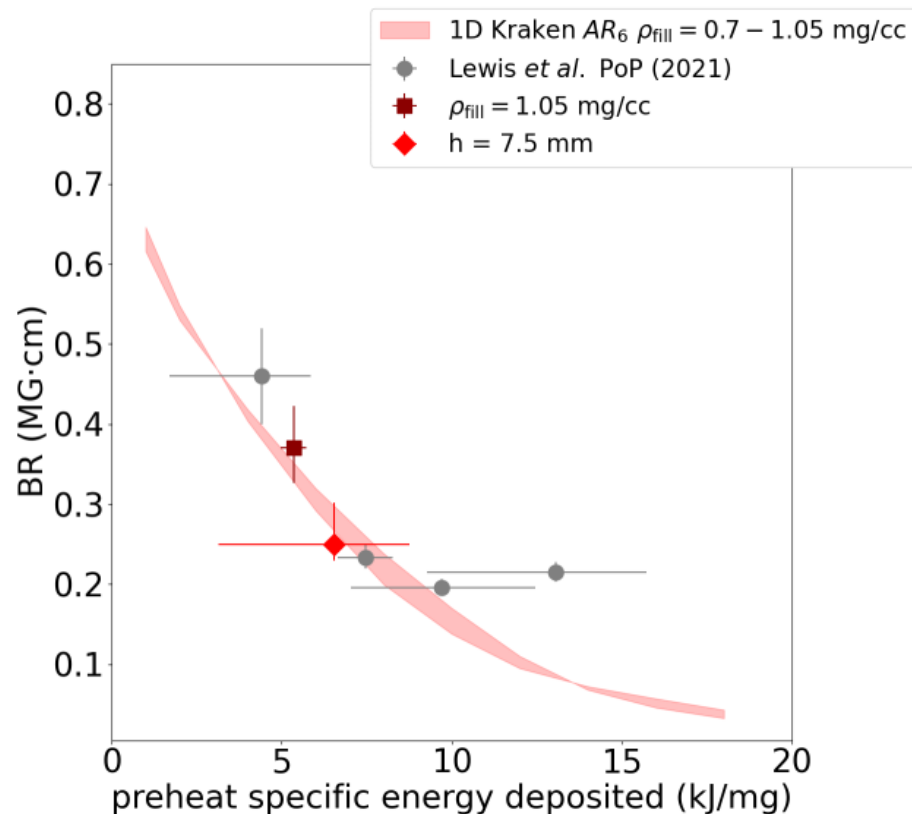
# impact of Nernst vs PSE is nearly independent of fill density

- $T_{\text{preheat}}$  appears to dominate scaling
  - Flux loss relatively insensitive to
    - $\rho_{\text{fill}}$
    - target height
- Bands show  $\rho_{\text{fill}} = 0.7 - 1.05 \text{ mg/cc}$  range

$$PSE = \frac{PE \text{ deposited}}{\text{fuel mass}}$$

$$\langle T_{\text{preheat}} \rangle \propto PSE \quad x_e \propto \frac{T_e^{3/2} B}{\rho} \sim B/\rho \text{ at fixed PSE}$$

$$v_{\text{Nernst}} = \frac{\beta_{\perp}(x_e) \nabla_{\perp} T_e}{eB}$$



# Increasing AR increases BR, largely through increased compression of fuel (CR) and hence magnetic flux

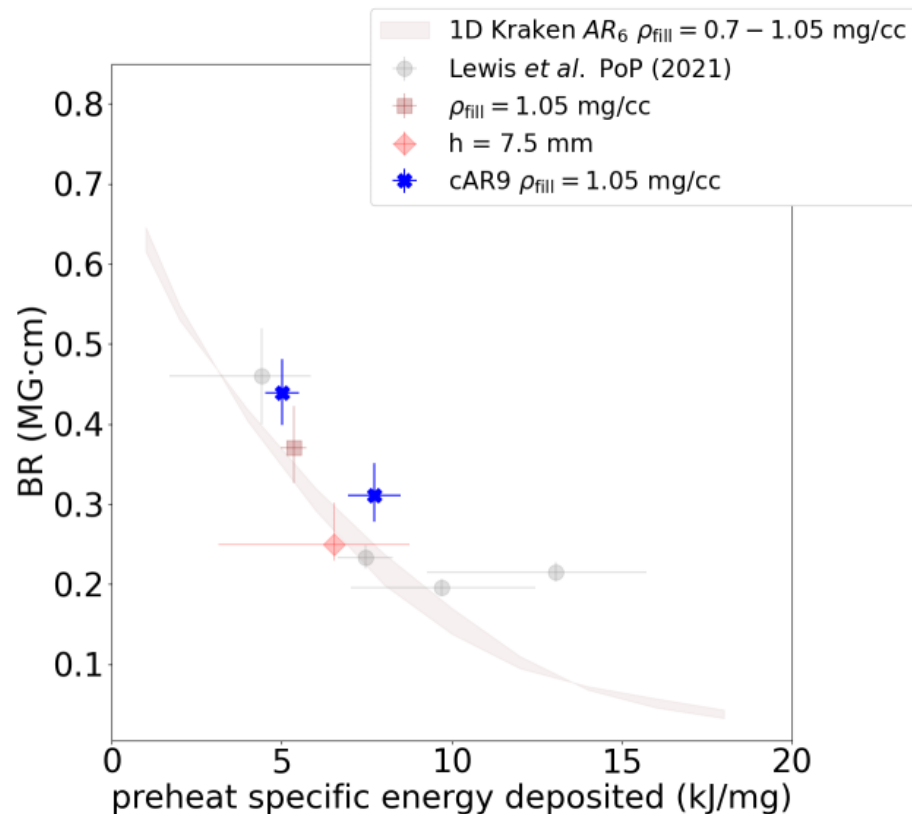


Increasing AR



- Less mass easier to compress

$$CR = \frac{R_{in,0}}{R_{in,f}} \quad (BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$



# Increasing AR increases BR, largely through increased compression of fuel (CR) and hence magnetic flux



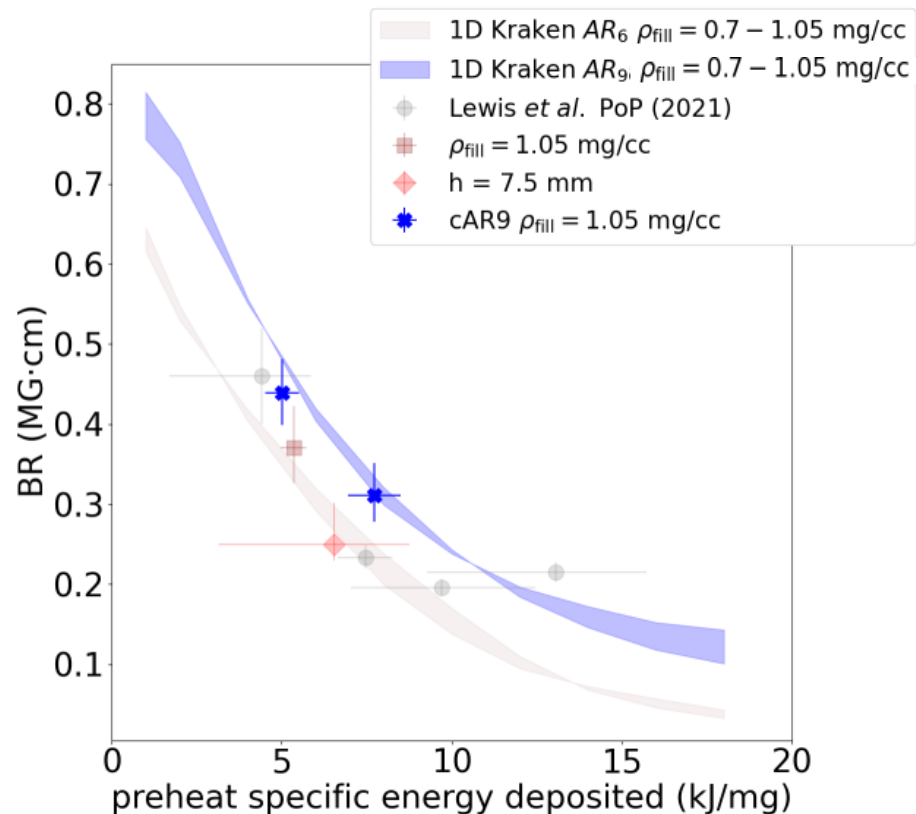
Increasing AR



- Less mass easier to compress

$$CR = \frac{R_{in,0}}{R_{in,f}} \quad (BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$

- Simulations show
  - 20-30% higher CR
  - CR impact dominant over flux loss



through  $(BR)_0$  while CR and total flux loss effects depend on details

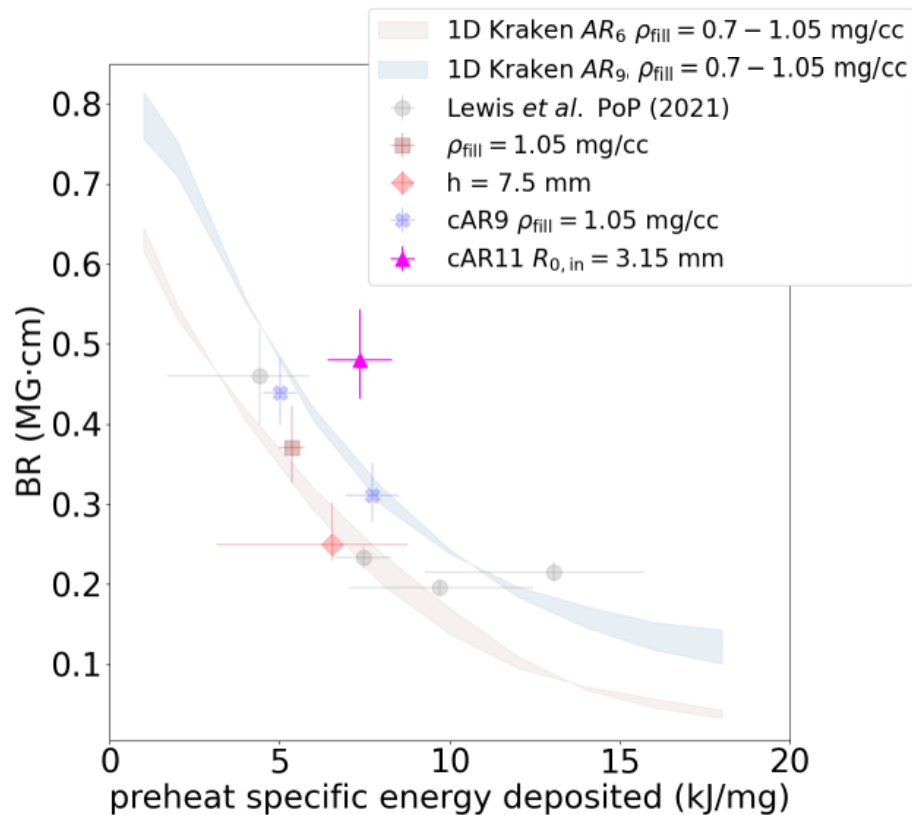


Increasing AR and  $R_{in,0}$



- $(BR)_0$  increases with  $R_{in,0}$ 
  - 35% larger for AR11 target

$$CR = \frac{R_{in,0}}{R_{in,f}} \quad (BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$



through  $(BR)_0$  while CR and total flux loss effects depend on details



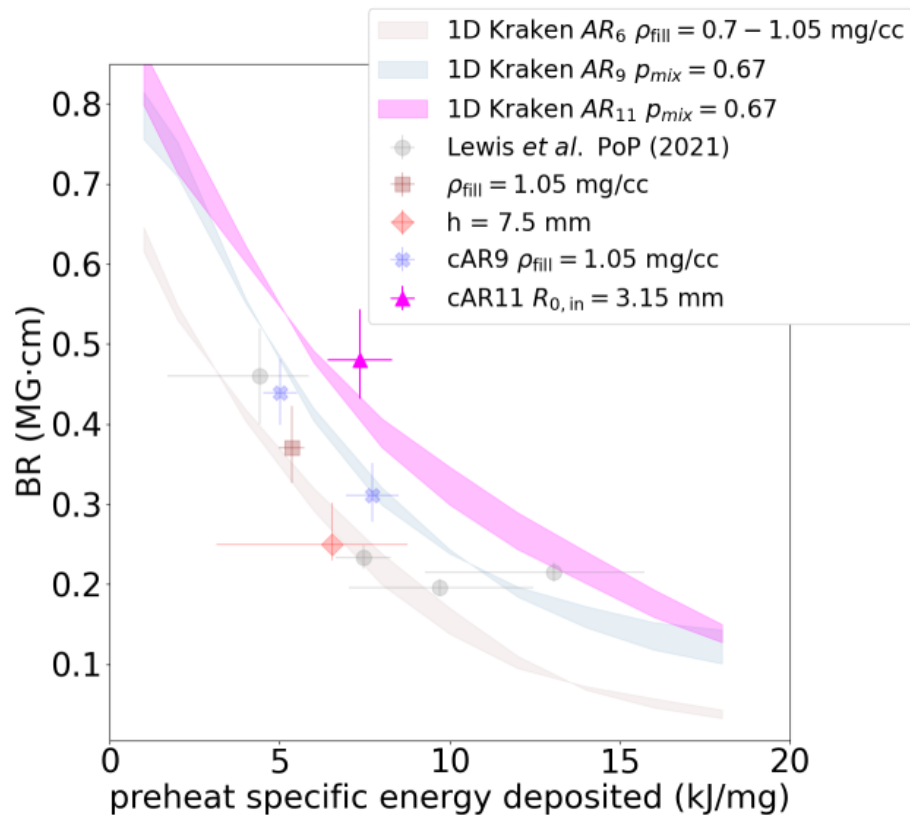
Increasing AR and  $R_{in,0}$



- $(BR)_0$  increases with  $R_{in,0}$ 
  - 35% larger for AR11 target

$$CR = \frac{R_{in,0}}{R_{in,f}} \quad (BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$

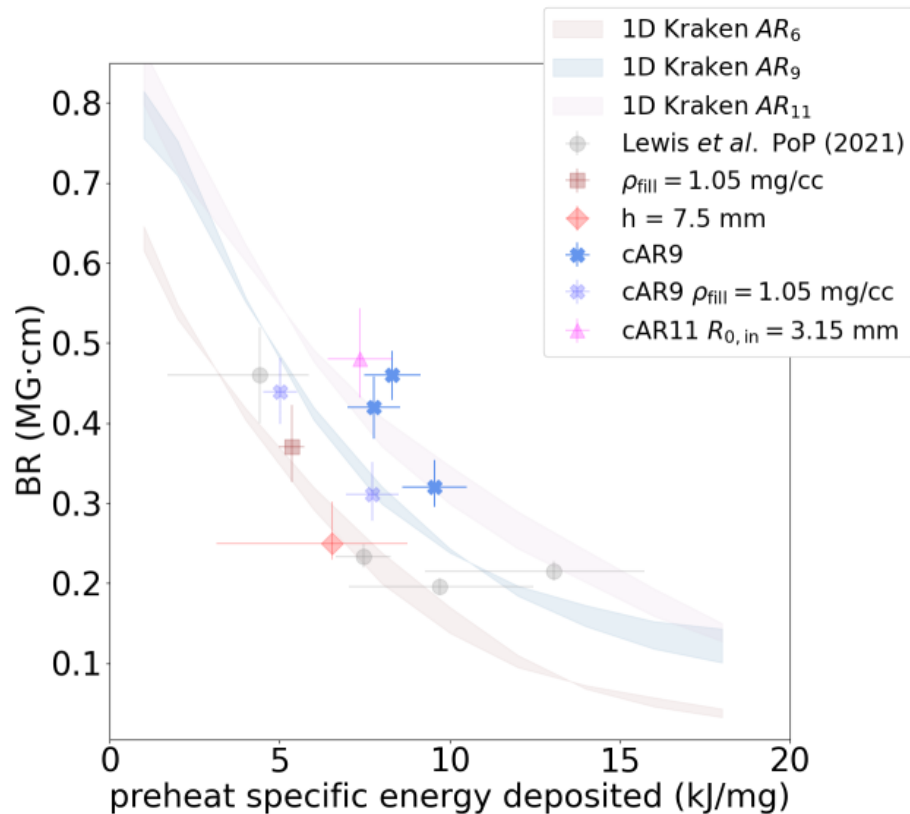
- Simulation shows
  - Similar CR to AR6 target
    - ~35% increase to BR over AR6
    - ~10-20% increase over AR9



# Lower fill density cAR9 shots show anomalously high BR which may be a result of introducing significant mix\*



- Mix enhances radiative losses
  - Can cool the fuel leading to
    - Reduced temperature gradients
    - Higher CR
    - Reduced performance



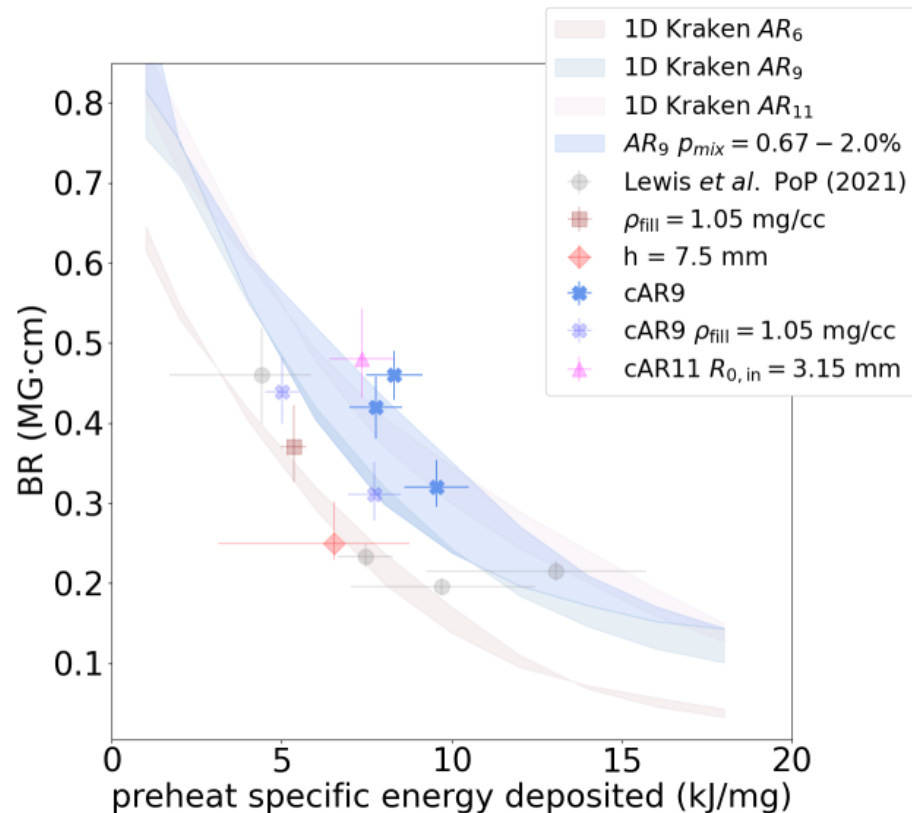
\*Ratio of x-ray to DD neutron yield is consistent with the increased mix interpretation



# Lower fill density cAR9 shots show anomalously high BR which may be a result of introducing significant mix\*



- Mix enhances radiative losses
  - Can cool the fuel leading to
    - Reduced temperature gradients
    - Higher CR
    - Reduced performance
- Results appear consistent with
  - 1D simulations
  - $Y_v / Y_{DD}$
  - Bayesian inference of mix



\*Ratio of x-ray to DD neutron yield is consistent with the increased mix interpretation

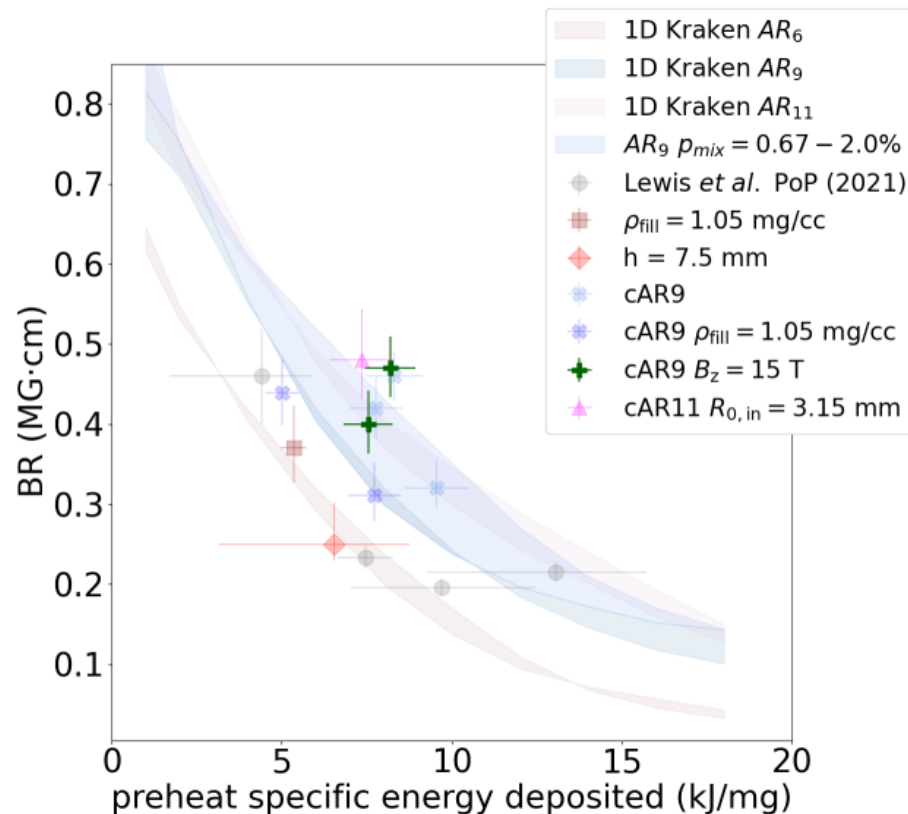
# Increasing $B_z$ is expected to increase $BR$ through $(BR)_0$ but flux loss and CR depend on details



- $(BR)_0$  increases with  $B_z$  (50% larger)

$$(BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$

- Reduced thermal conduction
  - Possibly changes to temperature gradients
  - May reduce CR



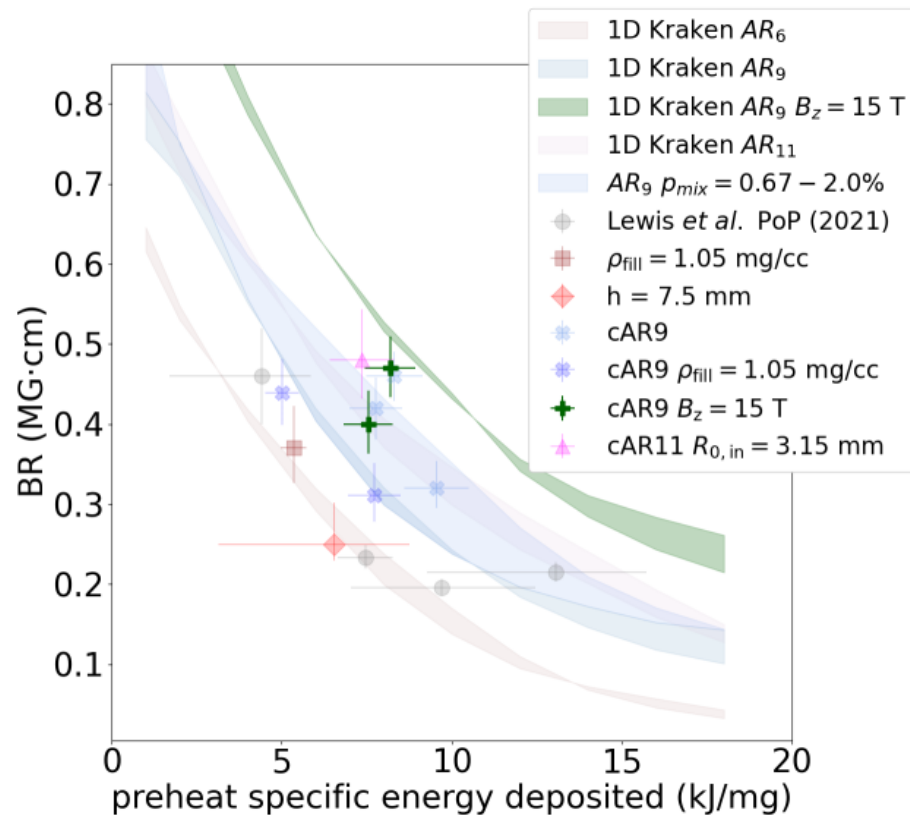
# Increasing $B_z$ is expected to increase BR through $(BR)_0$ but flux loss and CR depend on details



- $(BR)_0$  increases with  $B_z$  (50% larger)

$$(BR)_f = CR \frac{\phi_f}{\phi_0} (BR)_0$$

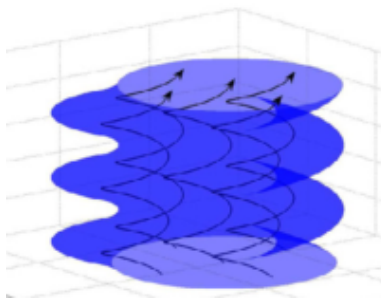
- Reduced thermal conduction
  - Possibly changes to temperature gradients
  - May reduce CR
- Simulation shows
  - ~50% increase to BR over AR9 10T
  - Initial field effect seems dominant
- Laser alignment issue → 3D effects?



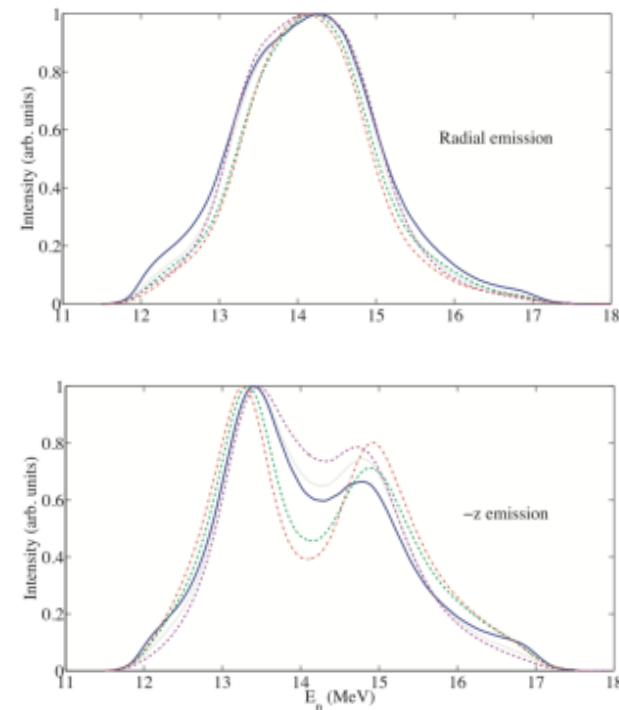
Our model makes simplifying assumptions pointing to interesting open questions, but well reproduced observations encourage its application.



- nToF shape features
- 2 ns Gaussian burn history
- $T_e = T_i$
- 1D power law profile model with
  - $B_z \propto \rho$
  - Axially uniform B-field
- **Unknown impact of 3D effects**



Magnetic field topology alters secondary neutron spectra



simulations that may enable us to better characterize the importance of 3D effects.



### Statistical characterization of experimental magnetized liner inertial fusion stagnation images using deep-learning-based fuel-background segmentation

- Tool to process images

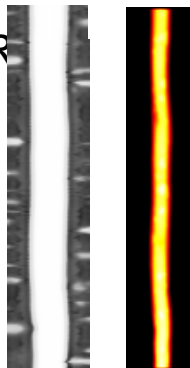


- Characterized noise and background
  - Can be used for UQ

W.E. Lewis *et al.*, J. Plasma Phys. **88**, 895880501 (2022).

- Investigate impact of 3D on BR
  - 3D simulation studies\*
  - Image analysis tools

\*Animations courtesy C.A. Jennings

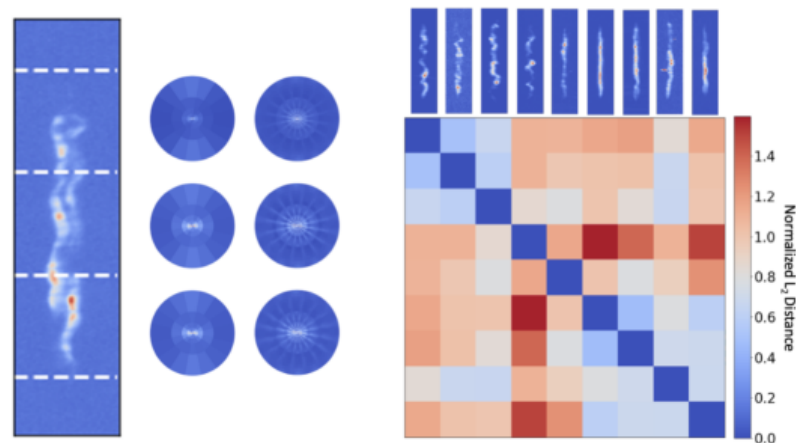


### Invited talk at ICDDPS-4 (Okinawa, JP)

4/22

A framework for experimental-data-driven assessment of Magnetized Liner Inertial Fusion stagnation image metrics\*

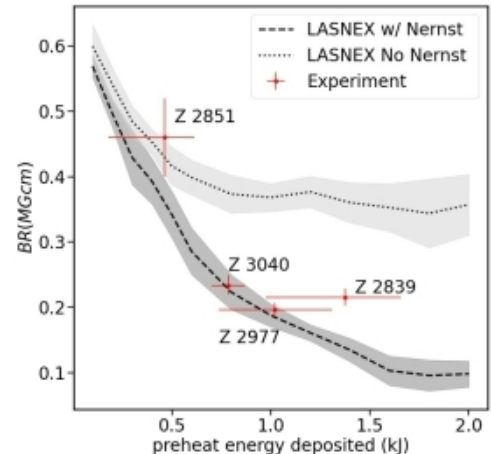
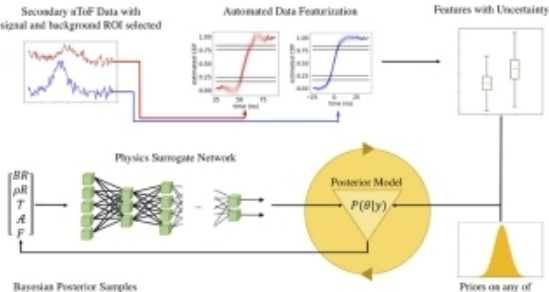
- Framework to understand sensitivity
  - SNR, resolution, registration, *etc.*
- Metrics for image-to-image comparison



W.E. Lewis *et al.*, In Preparation.



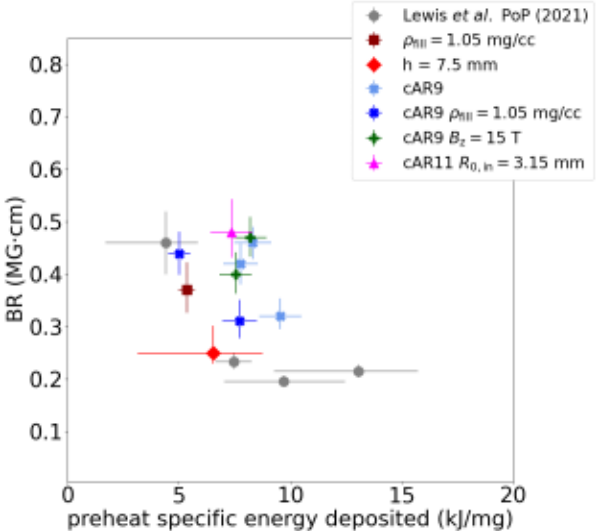
## Method + initial analysis:



W.E. Lewis *et al.*, Phys. Plasmas **28**, 092701 (2021).

## New analysis:

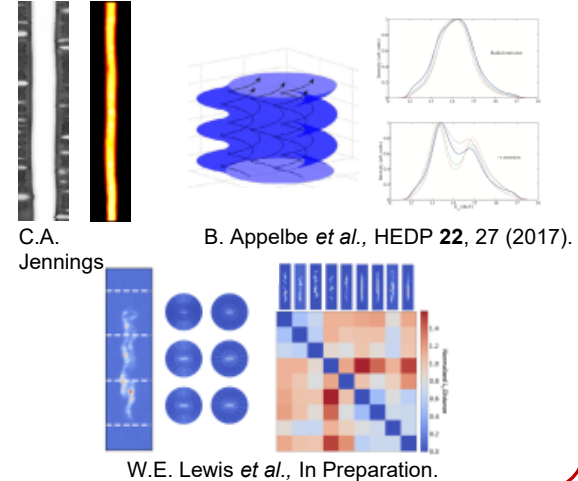
- Scaling of BR/flux compression
  - Fill density, AR,  $R_{in,0}$ , mix,  $B_z$



W.E. Lewis *et al.*, In Preparation.

## Future directions: *interested in collaborations!*

- Next generation pulsed power
- 3D effects
- Relaxed assumptions
- Current scaling



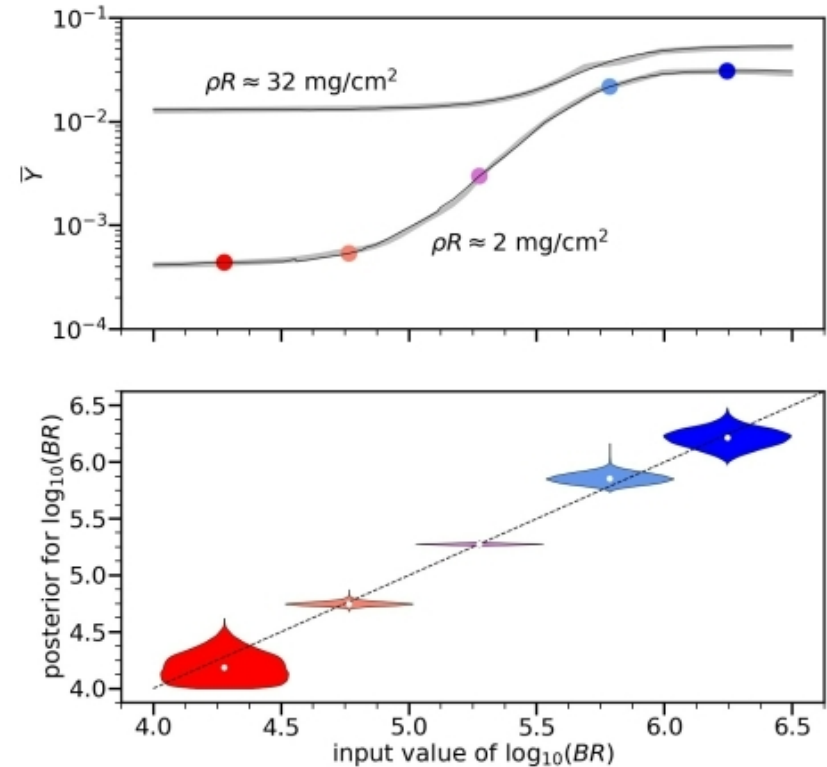


# Backup Slides

# synthetic datasets and the only available previously analyzed experiment.



- Surrogate model quantitatively captures features of physics model
- We have demonstrated that BR inference on noisy synthetic data is robust to S/N ratios comparable to experiment
- Results are consistent with the only available previously analyzed experiment.

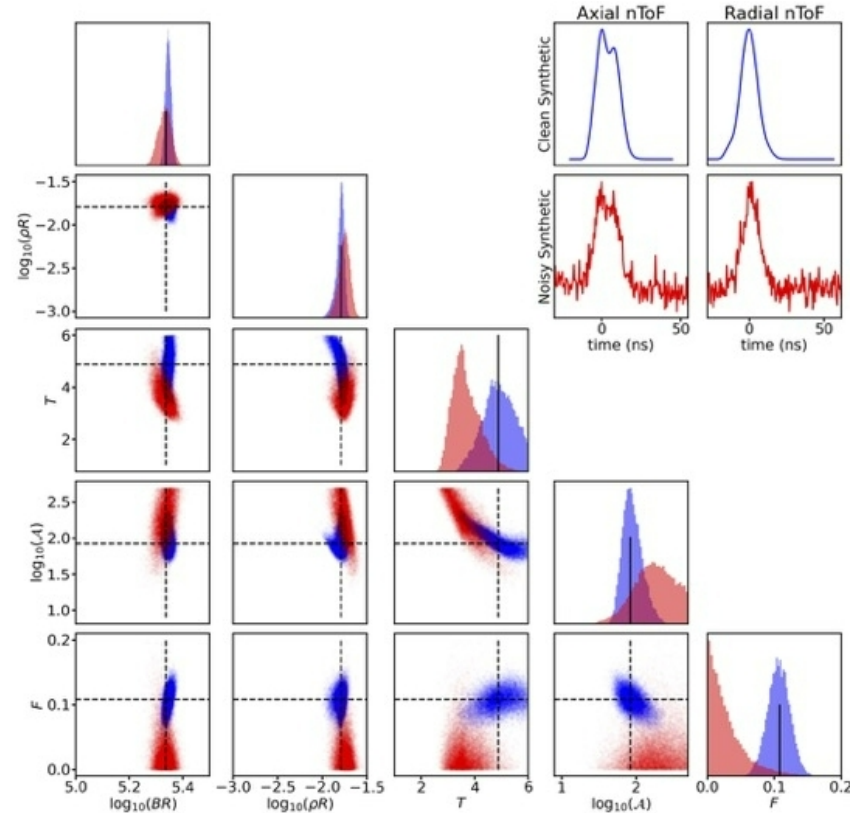




# synthetic datasets and the only available previously analyzed experiment.



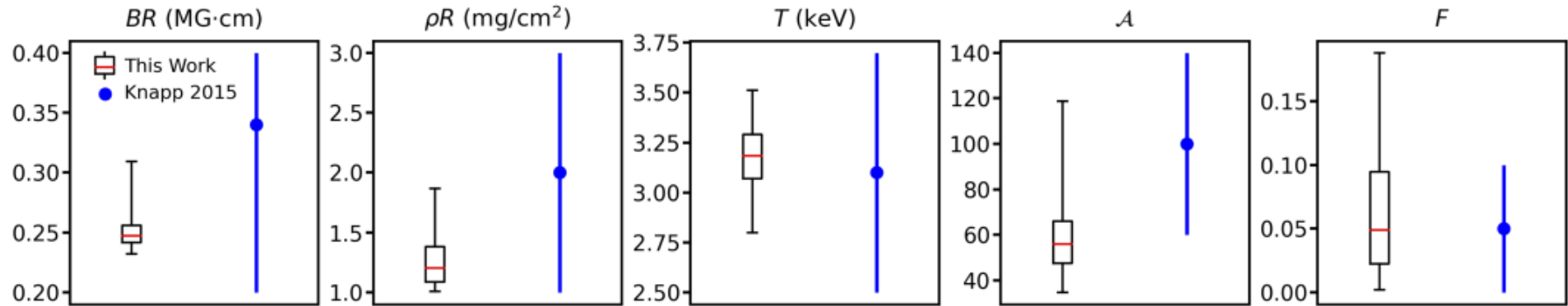
Shape features, DD and DT  
yields sufficient for BR  
recovery from noisy  
synthetic data even when  
other model parameters  
obscured.



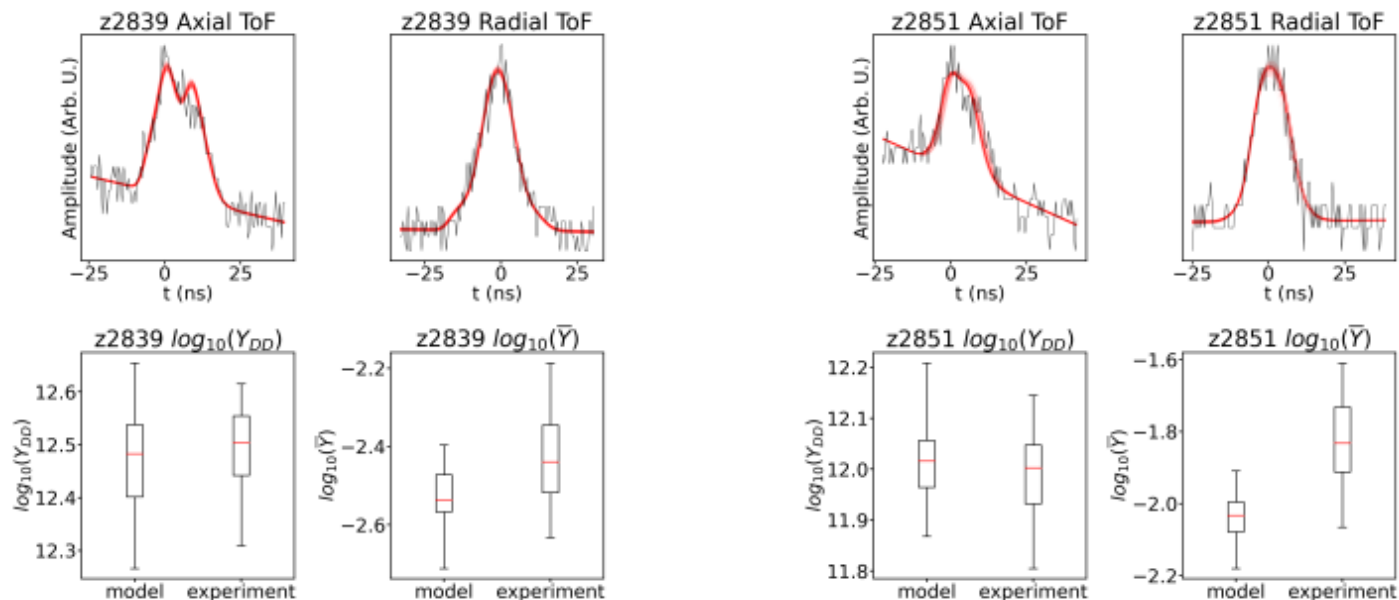
synthetic datasets and the only available previously analyzed experiment.



Results are consistent with previous analysis of z2591



physics model, good agreement with observations is obtained.



The agreement obtained with observations indicates that our results are consistent with what would be obtained were it feasible to conduct a Bayesian analysis using the full physics model.

# Bayes theorem allows us to incorporate multiple sources of data and rigorously define statistical data models for UQ.



## Bayes theorem

Posterior distribution for  
parameters ( $\mathbf{x}$ ) given data ( $\mathbf{y}$ )

Prior information  
on parameters

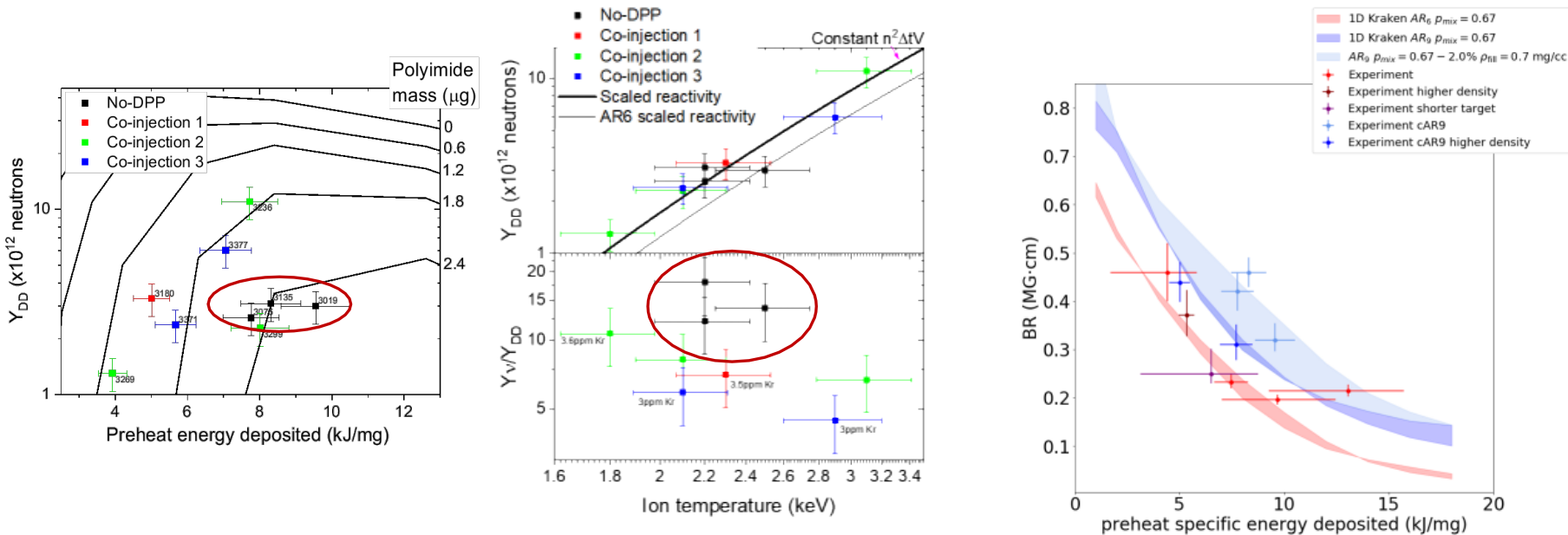
$$p(\mathbf{x}|\mathbf{y}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x})/p(\mathbf{y})$$

Likelihood function  
describing distribution of  
data around model

Parameter independent  
normalization (unimportant)

- Provides a distribution of model parameters most consistent with data
- We incorporate sources of uncertainty from:
  - use of NN surrogate
  - featurizing nToF
  - DD and DT yield measurements

# X-ray to DD yield ratios are consistent with higher mix for the lower fill density coated AR9 experiments analyzed for BR



A.J Harvey-Thompson *et al.* In Preparation.

$$P_{\nu} = A_{ff} 4\pi P_{HS}^2 e^{-\tau_{\nu}^{\ell}} \int_{V_{HS}} \frac{\langle Z \rangle g_{FF}}{(1 + \langle Z \rangle)^2} \sum_i f_i \tilde{j}_i \frac{e^{-h\nu/T_e}}{T_e^{5/2}} dV$$

$$Y_{DD} = \frac{1}{2} P_{HS}^2 \tau_b \int_{V_{HS}} \frac{\langle \sigma v \rangle_{DD}}{(1 + \langle Z \rangle)^2 T_i^2} dV$$

P.F. Knapp *et al.* Phys. Plasmas **26**, 012704 (2019).