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# Calibration and Uncertainty Propagation of Multiaxially Loaded Threaded Fasteners

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## Motivation

An essential part of Sandia's mission is predicting through analysis the performance of complex systems and structures subjected to various normal and abnormal environments.

Fasteners are an integral connector in many of these system and structures, but there are limitations to conventional fastener modeling approaches.

### **Challenges (Solid Mechanics):**

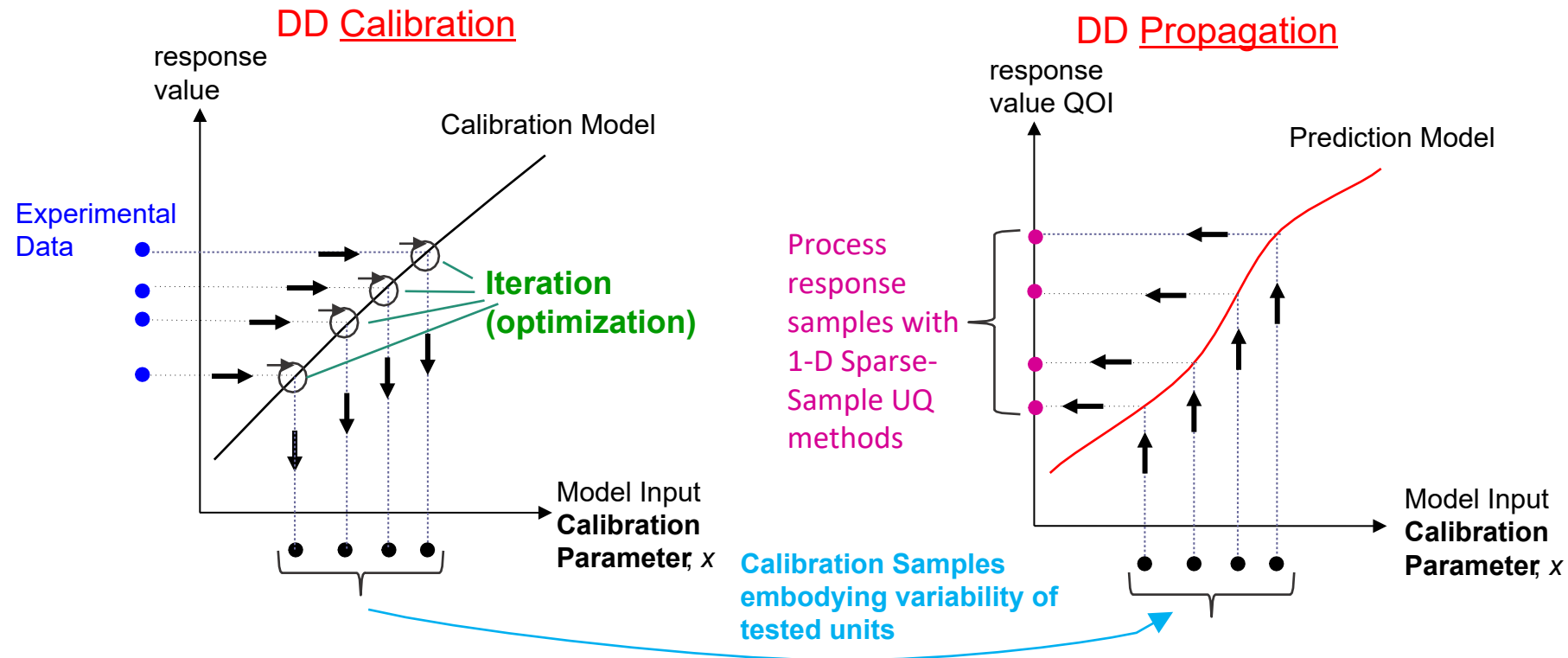
- Modeling fidelity requirements of system-level models are restrictive and create challenges for capturing relevant behavior while maintaining feasibility of the larger simulation.
- It is infeasible to test all fasteners in all environments to obtain expected behavior.
  - Different fastener materials, sizes, loadings, etc.
  - Uncertainty manifests throughout this process!

### **Questions:**

- How do we go beyond one calibration, one nominal answer, and try to quantify our uncertainty?
- How do we practically do this in the context of large-scale FEA (limited time and analysis cycles)?



# "Discrete Direct" (DD) Model Calibration and Uncertainty Propagation

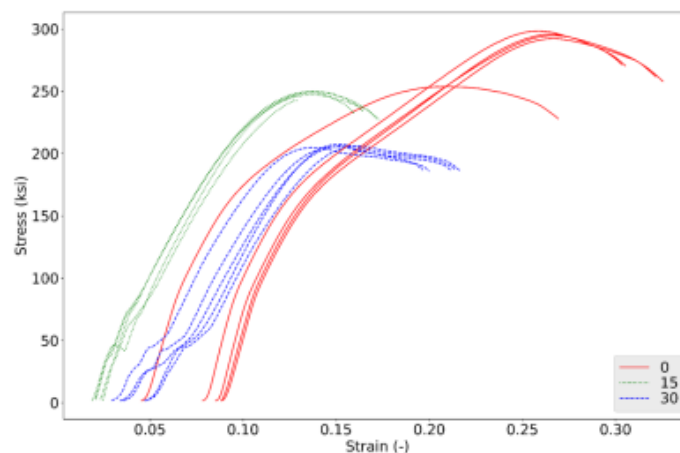
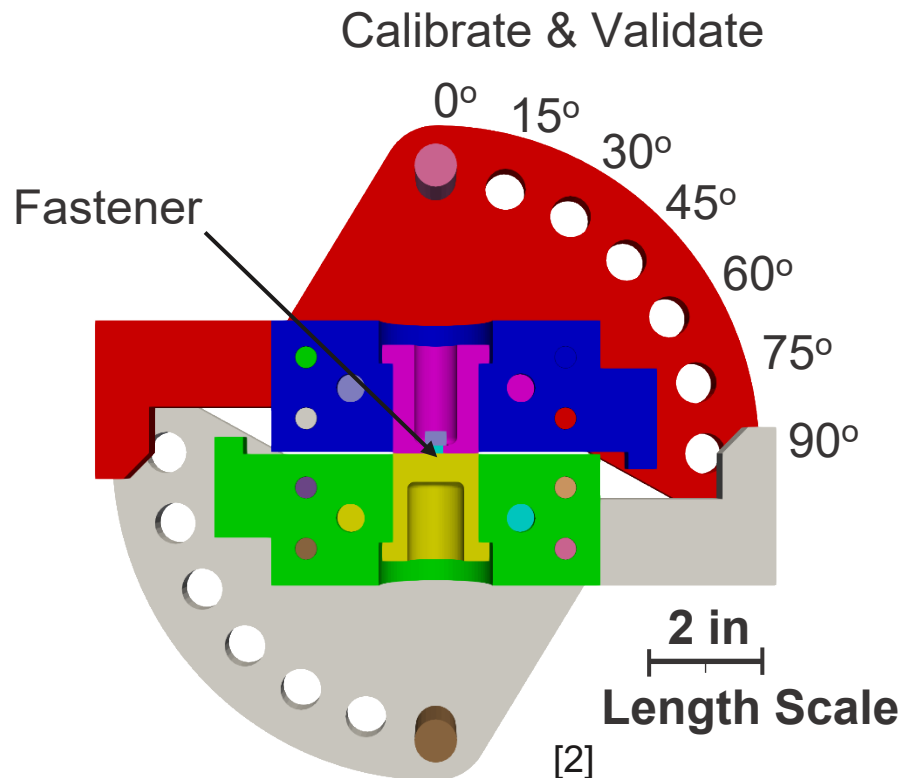


- Propagate the discrete values of the calibration parameters
- Straightforwardly extends to problems with multiple calibration parameters
- $N$  runs of model to propagate  $N$  param. values or sets from  $N$  calibration experiments
- Simple to update w/new experiments/data that may become available (w/out Bayes' rule & machinery)

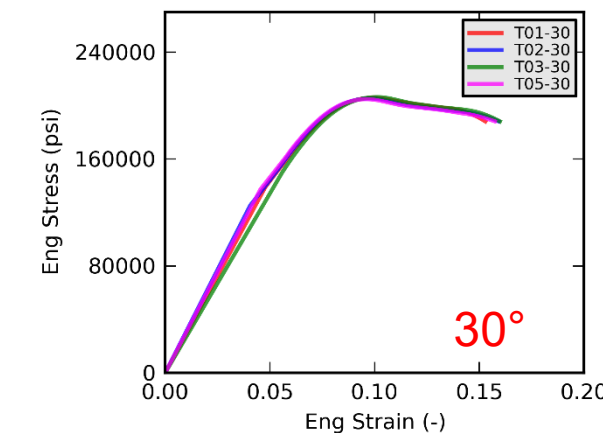
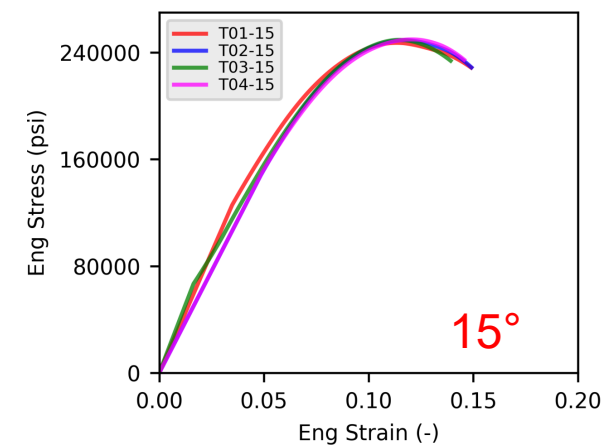
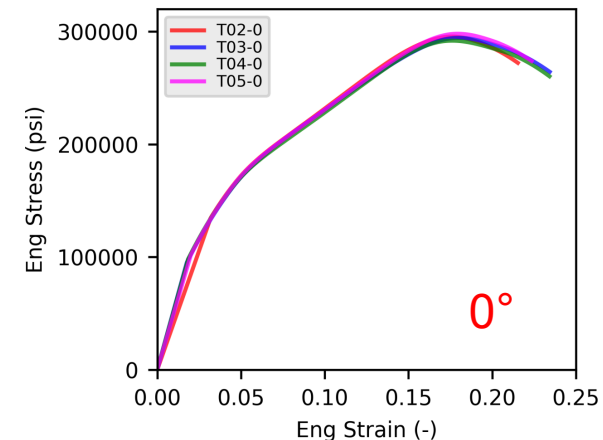


# Experimental Setup

- Multiaxial fastener loading
  - Motivated by NASM-1212-2[1]
  - Fastener material: MP35N
  - Goal is to test fasteners in
    - Tensile
    - Shear
    - Mixed-modes
- Testing angles:
  - $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$
  - More work to be done at higher angles!
- Four repeat tests at each angle
  - More variability observed at higher angles.



Raw Data



Conditioned Data

[1] Aerospace Industries Association, "Fastener Test Methods Method 2 Interaction", National Aerospace Standard, NASM1312-2, 29 August 1997.

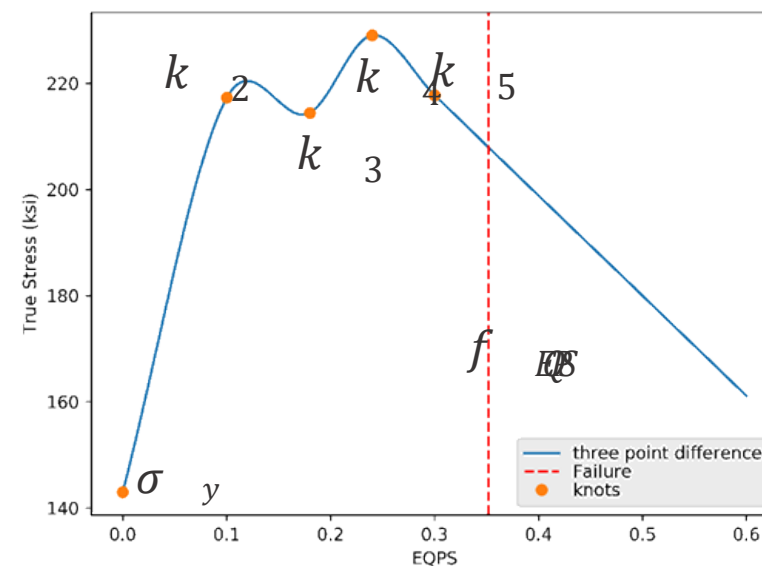
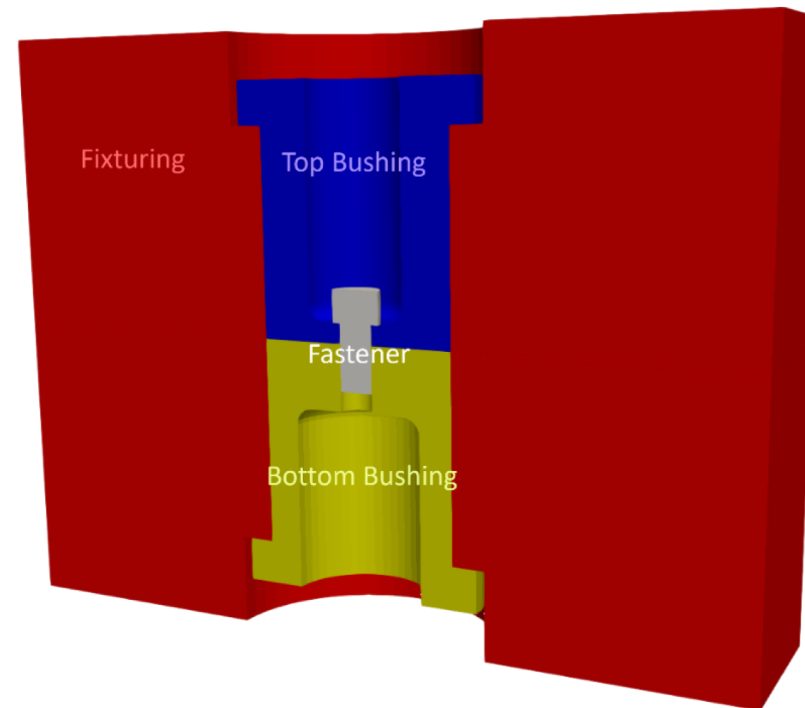
[2] Mersch, J.P., Smith, J.A., Orient, G.E., Grimmer, P.W., Gearhart, J.S., "Calibration Strategies and Modeling Approaches for Predicting Load-Displacement Behavior and Failure for Multiaxial Loadings in Threaded Fasteners," *International Mechanical Engineering Congress and Exposition*, Salt Lake City, UT, 2019. IMECE2019-10521



# Approach

- Low-fidelity fastener model - Plug
- Calibration:
  - Calibrate each test individually.
  - Optimization: Single Objective Genetic Algorithm (SOGA) in Dakota
- Parameters:
  - Yield Stress:  $\sigma_y$
  - Hardening curve: Cubic Spline [3]
    - Knot stress locations:  $k_2, k_3, k_4, k_5$
    - Note: The cubic-spline hardening curve is intentionally flexible as this allows for the calibration to account for unknown model-form errors. Improving constraints on this procedure is left as future work.
  - Failure EQPS:  $f_{EQPS}$

Very different than previous approach! [2]



Example Hardening Curve

[2] Mersch, J.P., Smith, J.A., Orient, G.E., Grimmer, P.W., Gearhart, J.S., "Calibration Strategies and Modeling Approaches for Predicting Load-Displacement Behavior and Failure for Multiaxial Loadings in Threaded Fasteners," *International Mechanical Engineering Congress and Exposition*, Salt Lake City, UT, 2019. IMECE2019-10521

[3] Reedlunn, B., "Cubic Hermite Spline Hardening Model for Plasticity", Sandia Memo, July 6<sup>th</sup>, 2015.





# Two-Step Calibration Process

## Objective:

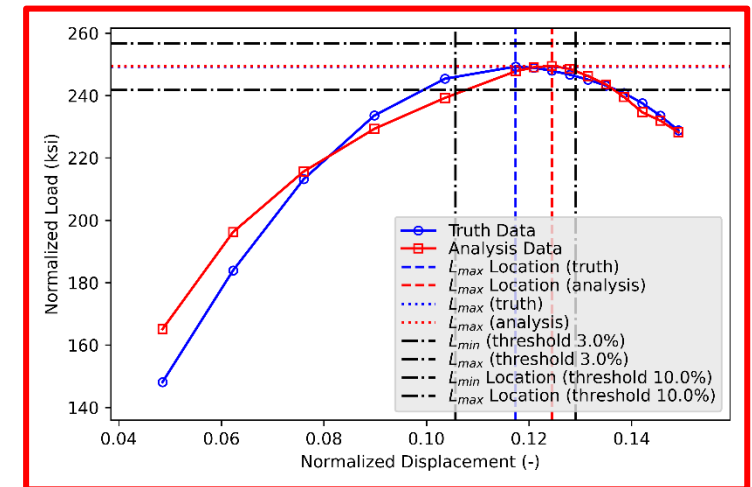
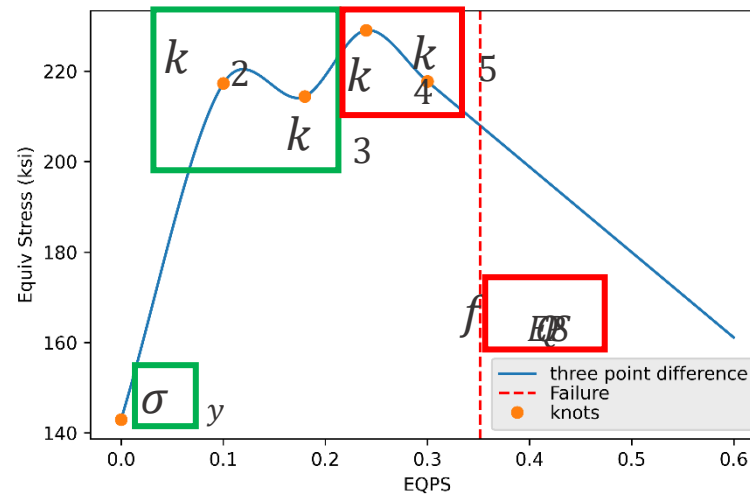
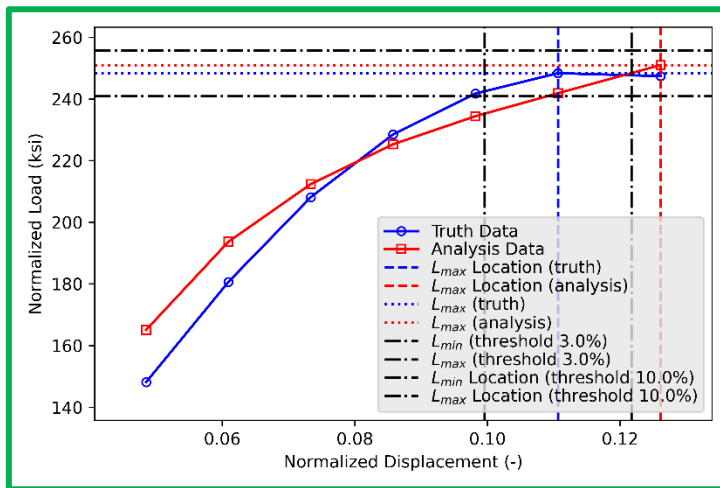
- Minimize sum-of-squares error between model (Analysis Data) and experimental (True Data) results.
- Constrain via "Stress-Strain" window.

### 1. Calibrate up to peak load.

- Material parameters:  $\sigma_y$
- Focus on initial knot points:  $k_2, k_3$

### 2. Constrain parameters and calibrate over entire data set.

- $\sigma_y$  fixed from first step.
- Constrain knots based on 1<sup>st</sup> calibration.
- Hone in on final knot points:  $k_4, k_5$
- Failure EQPS:  $f_{EQPS}$

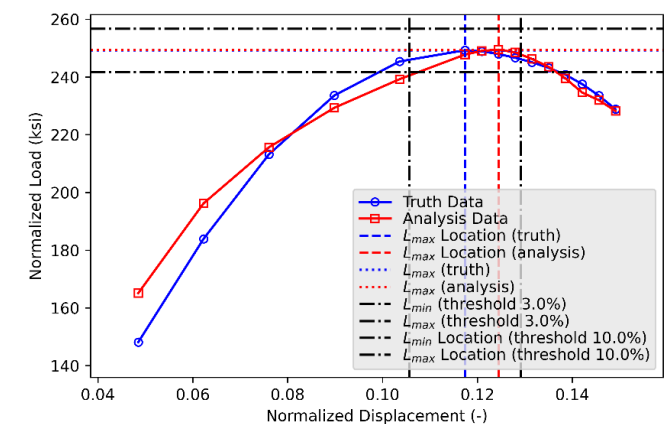
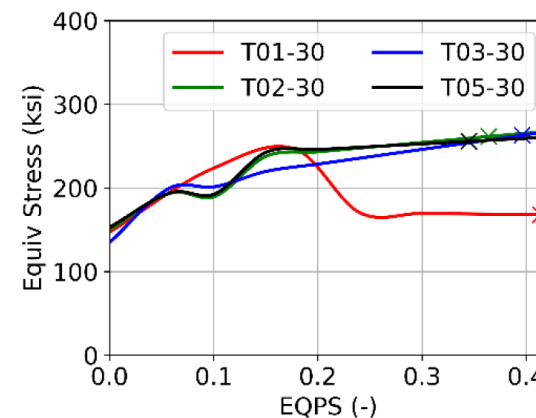
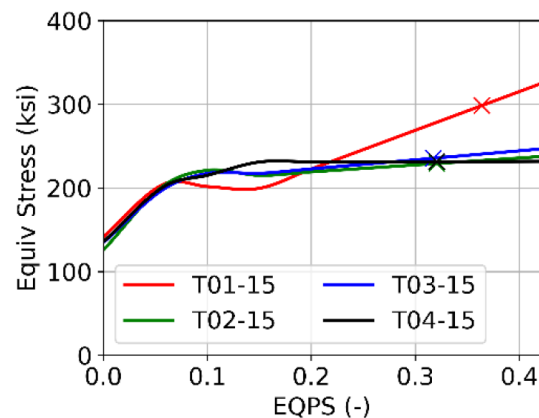
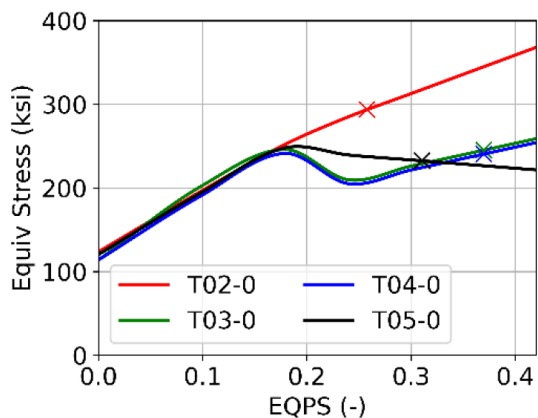




# Calibration Results

- Calibrated models for  $0^\circ$ ,  $15^\circ$ , and  $30^\circ$  loading angles.
- Each loading angle had 4 sets of test data.
- 12 distinct calibrated models!
- Observable variability between most calibrations

Parameter \ Test	T02-0	T03-0	...	T05-30
$\sigma_y$ (ksi)	124	114		153
$k_2$ (ksi)	196	202		195
$k_3$ (ksi)	253	246		192
$k_4$ (ksi)	285	210		243
$k_5$ (ksi)	313	226		246
$f_{EQPS}$ (-)	0.26	0.37		0.34





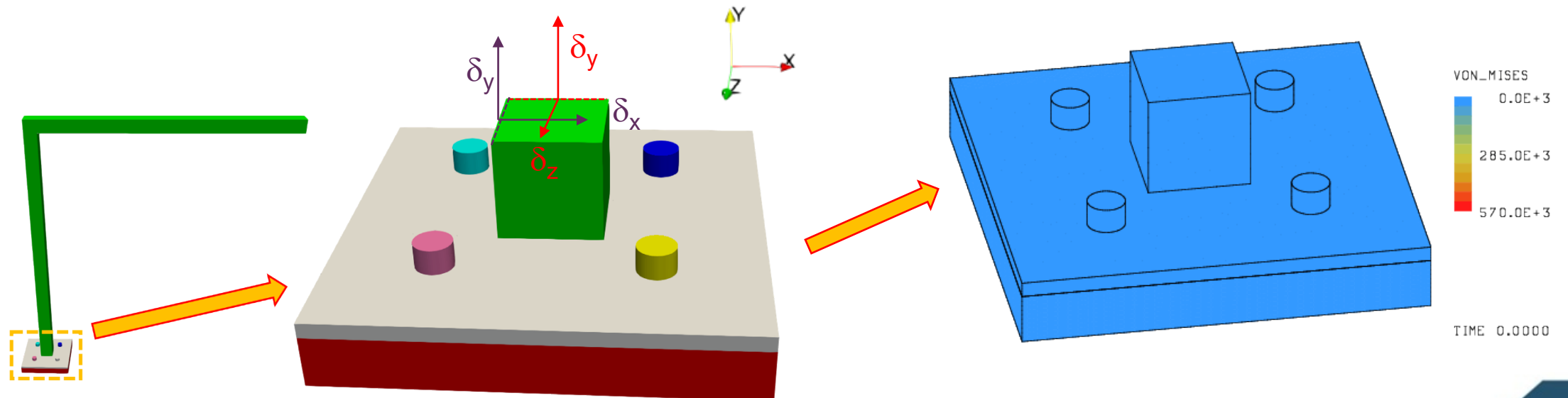
## Exemplar: Street Light

- Street light has four fasteners connected to base.
- Created submodel for ease of simulation.
- Simplified loading conditions:
  - Prescribed displacements in all three coordinate axis directions
- Quantity of Interest (QoI)
  - Load carrying capacity of structure (total force magnitude).



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<https://www.shutterstock.com/image-photo/structural-detail-lighting-post-steel-base-388752364>







# Exemplar: Discrete Sampling Study

- Compute peak load using a random, discrete sampling strategy.
- **UQ Strategy [4]:**
  - Sample from model library using “once and only once” principle.
  - 1) treat all calibrations as one “family”
    - Note: violates IID (Independent and Identically Distributed) assumption.
  - 2) Treat each angle “family” separately.
  - Use small sampling strategies to determine upper and lower bounds on load carrying capacity.
  - “tolerance interval equivalent normal” approach[5].
    - “the 1-D sparse sample UQ methods are targeted by design to give **high reliability of confidence** of providing **conservative estimates** of various statistics...”  
--Vicente Romero, SNL
  - **90% Coverage, 90% Confidence**

Each column represents a fastener

Bolt (0°) -- Example Trial			
1	2	3	4
T03-0	T05-0	T02-0	T05-0
T02-0	T03-0	T04-0	T03-0
T05-0	T02-0	T03-0	T04-0
T04-0	T04-0	T05-0	T02-0

Bolt (15°) -- Example Trial			
1	2	3	4
T01-15	T03-15	T01-15	T04-15
T02-15	T02-15	T02-15	T01-15
T03-15	T01-15	T04-15	T02-15
T04-15	T04-15	T03-15	T03-15

Bolt (30°) -- Example Trial			
1	2	3	4
T05-30	T02-30	T02-30	T03-30
T01-30	T03-30	T03-30	T02-30
T03-30	T01-30	T01-30	T01-30
T02-30	T05-30	T05-30	T05-30

One Family, 12 Samples -- Example

1	2	3	4
T02-15	T02-0	T03-30	T03-15
T02-30	T03-15	T05-30	T03-0
T04-0	T02-15	T03-15	T05-0
T03-0	T04-15	T02-30	T04-15
T05-30	T01-30	T02-0	T03-30
T03-15	T05-30	T02-15	T02-15
T05-0	T03-30	T03-0	T01-15
T01-15	T03-0	T04-0	T01-30
T02-0	T01-15	T01-15	T04-0
T01-30	T05-0	T04-15	T02-0
T04-15	T04-0	T05-0	T05-30
T03-30	T02-30	T01-30	T02-30

One Family, 6 Samples -- Example

1	2	3	4
T02-15	T02-0	T03-30	T03-15
T02-30	T03-15	T05-30	T03-0
T04-0	T02-15	T03-15	T05-0
T03-0	T04-15	T02-30	T04-15
T05-30	T01-30	T02-0	T03-30
T03-15	T05-30	T02-15	T02-15

One Family, 5 Samples --Example

1	2	3	4
T02-15	T02-0	T03-30	T03-15
T02-30	T03-15	T05-30	T03-0
T04-0	T02-15	T03-15	T05-0
T03-0	T04-15	T02-30	T04-15
T05-30	T01-30	T02-0	T03-30

Sampling restricted to angle families

Sampling all angles at once

[4] Romero, V., “Arguments for the Generality and Effectiveness of “Discrete Direct” Model Calibration and Uncertainty Propagation vs. Other Calibration-UQ Approaches”, AIAA Non-Deterministic Approaches Conference, SciTech 2022, Jan. 3-7, San Diego, CA AIAA 2022-2107 ( SAND2021-14785 C).

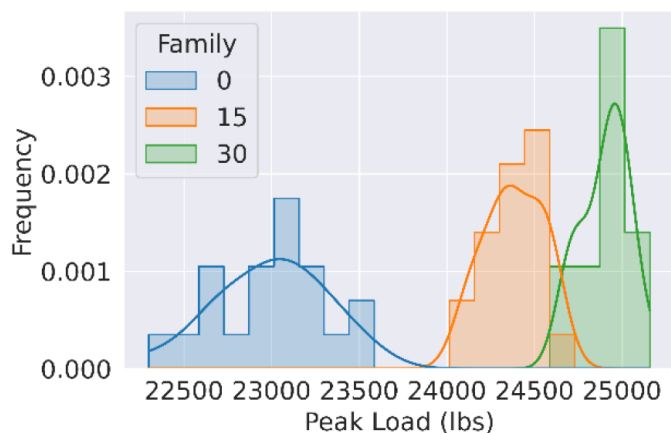
[5] Jekel, C., Romero, V., “Conservative and Efficient Tail Probability Estimation from Sparse Sample Data”, SAND2020-7572J.



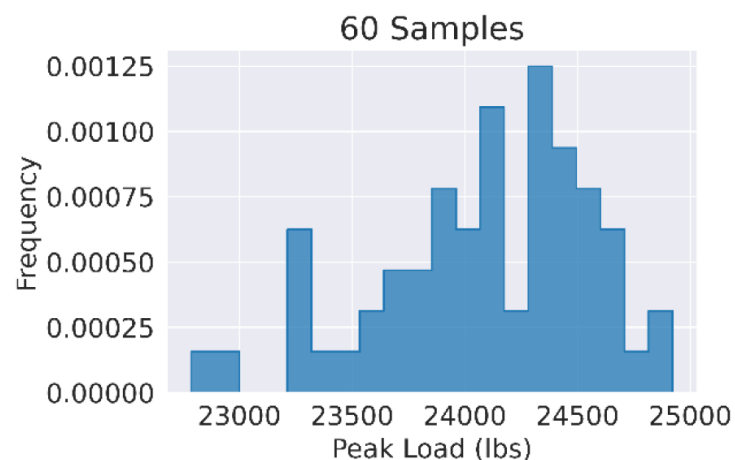
# Exemplar: Uncertainty Propagation – Results and Conclusions

- **Angle Families Histogram:** Reveals that calibrations from different angles behave slightly differently.
  - What's the loading in the exemplar of interest?
- **Mixed Family Histogram:** Appears to fall in between angled results.
  - Note: Violates IID (Independent and Identically Distributed) Assumption
- **TIEN Bound Prediction:** Seem to conservatively predict bounds with small number of samples.
  - Mixed distributions don't change much with sample size.
  - Hopefully obtaining a realistic and conservative prediction of bounds!
- **What's the right answer?**

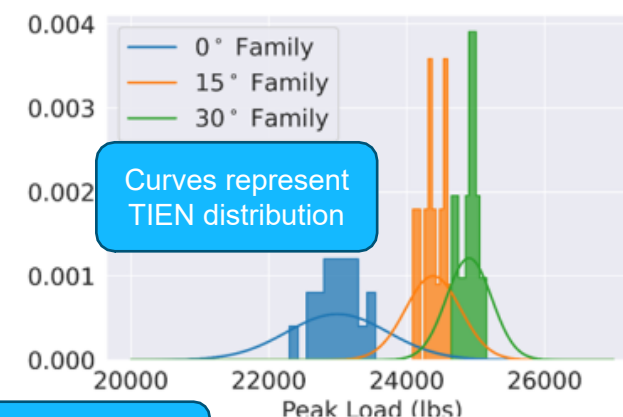
Credibility evidence to support decision-making!



Angle Family Histograms



Mixed Family Histogram



Using average of mean,  $\sigma_{EN}$ , from five trials

Prediction of Bounds

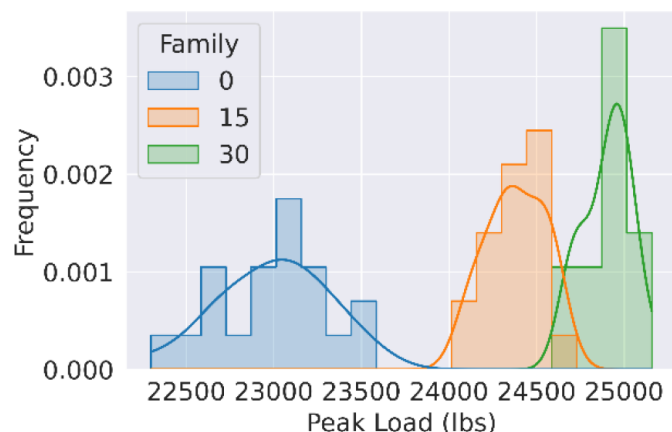
Strategy	5%	95%
<b>0° Family</b> (4 Samples)	21781.75	24206.14
<b>15° Family</b> (4 Samples)	23722.07	25043.21
<b>30° Family</b> (4 Samples)	24359.02	25447.29
<b>Mixed</b> , 12 Samples	22981.37	25203.05
<b>Mixed</b> , 6 Samples	22952.29	25123.25
<b>Mixed</b> , 5 Samples	22860.55	25194.87



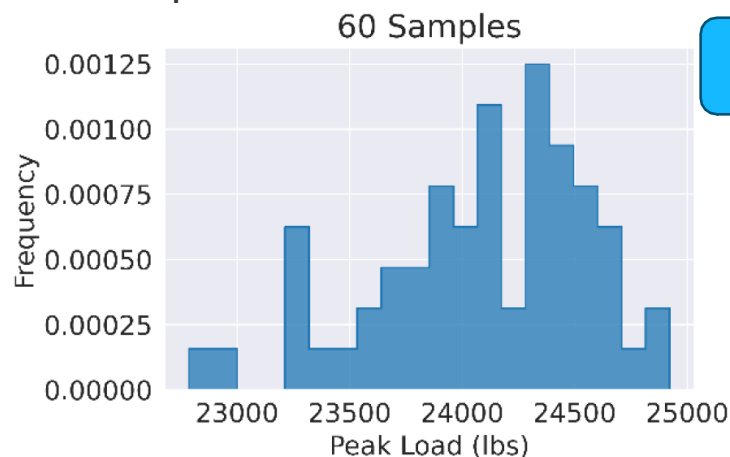
## Future Work

- Implement more realistic loading conditions on street light.
- Complete calibrations of all test data.
- Quantify uncertainty in calibration results:
  - Calculate coefficient of variation of spline knots to estimate uncertainty, upper bound values, then scale up with confidence intervals (due to small sample size).
  - Consider different groups of data (single angle, multiple angle, etc.).
- Alternative calibration strategy:
  - Build surrogate model for reach test angle.
  - Implement hierarchical statistical model.
  - Use Bayesian methods to calibrate parameter distributions in light of sample-to-sample variability.

**Thank you!**



Angle Family  
Histograms



Mixed Family Histogram

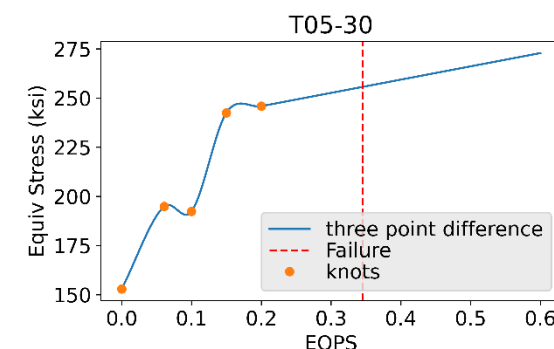
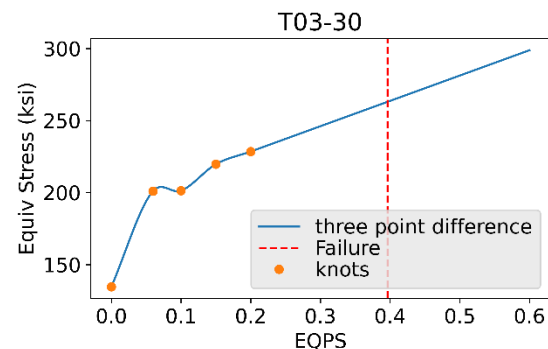
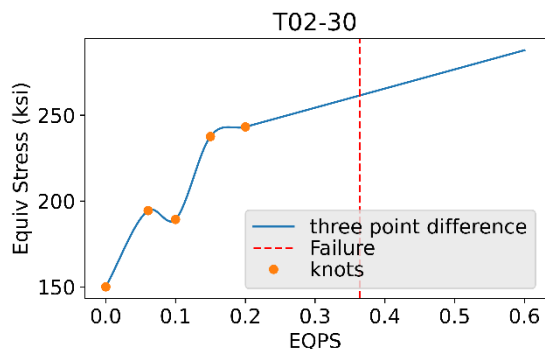
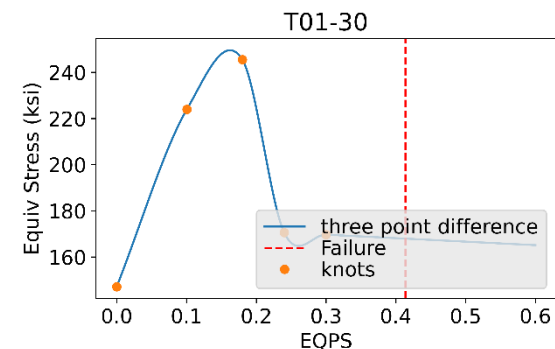
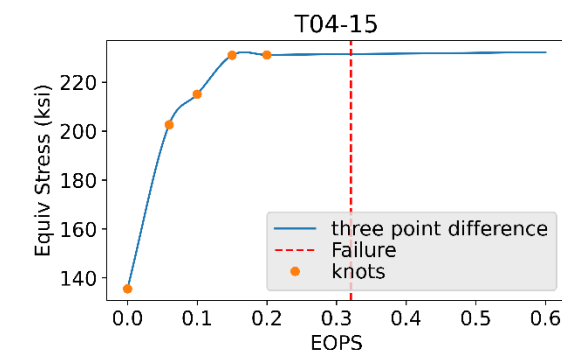
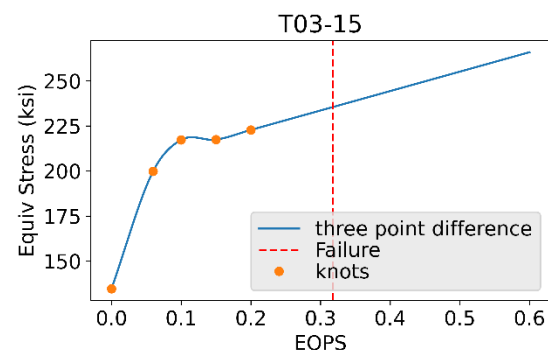
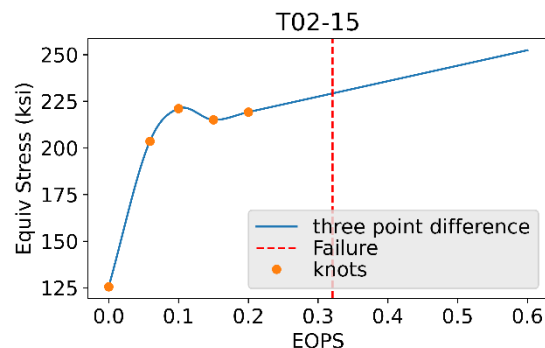
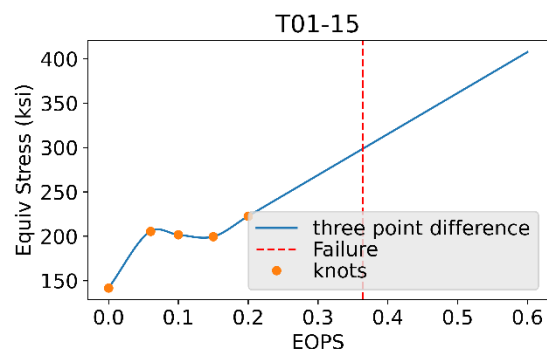
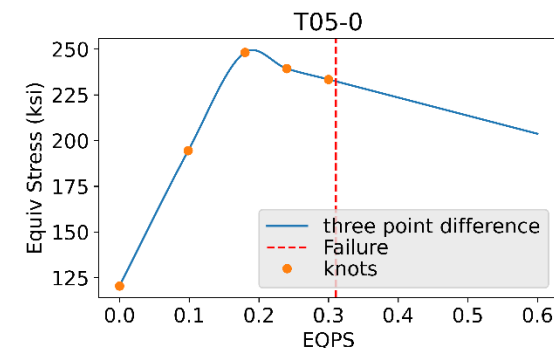
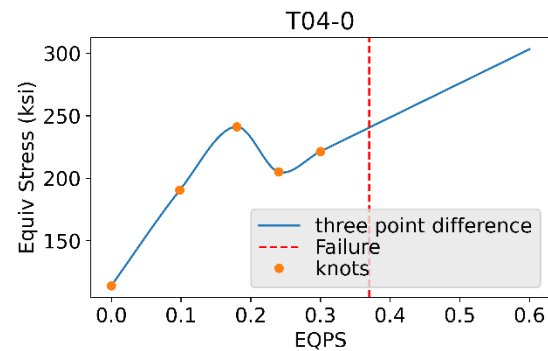
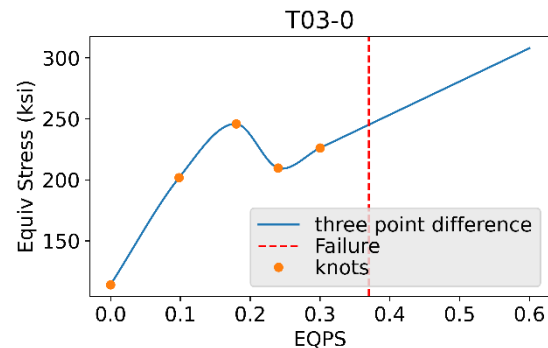
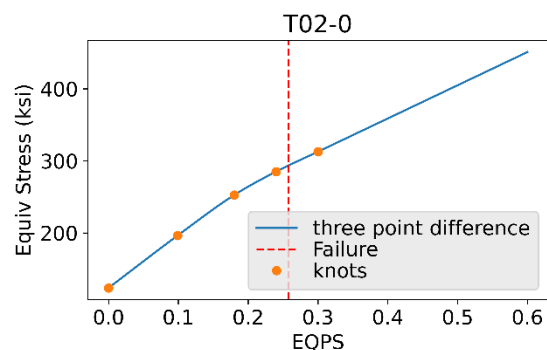
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From TIEN Distributions



# CHS – Hardening Curves





# Calibration Workflow: Dakota

- Optimization: Single-Objective Genetic Algorithm (SOGA)

SOGA Theory<sup>b</sup>:

1. Initialize the population
2. Evaluate the population (calculate the values of the objective function and constraints for each population member)
3. Loop until converged, or stopping criteria reached
  - Perform crossover
  - Perform mutation
  - Evaluate the new population
  - Assess the fitness of each member in the population
  - Replace the population with members selected to continue in the next generation
  - Test for convergence