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Modeling the Effects of Radiation Reaction for Next Generation Pulsed Power Machine

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Next Generation Pulsed Power (NGPP) Machine

- Sandia National Laboratories is designing a Next Generation Pulsed Power machine, which is capable of delivering >60 MA of current to a load.

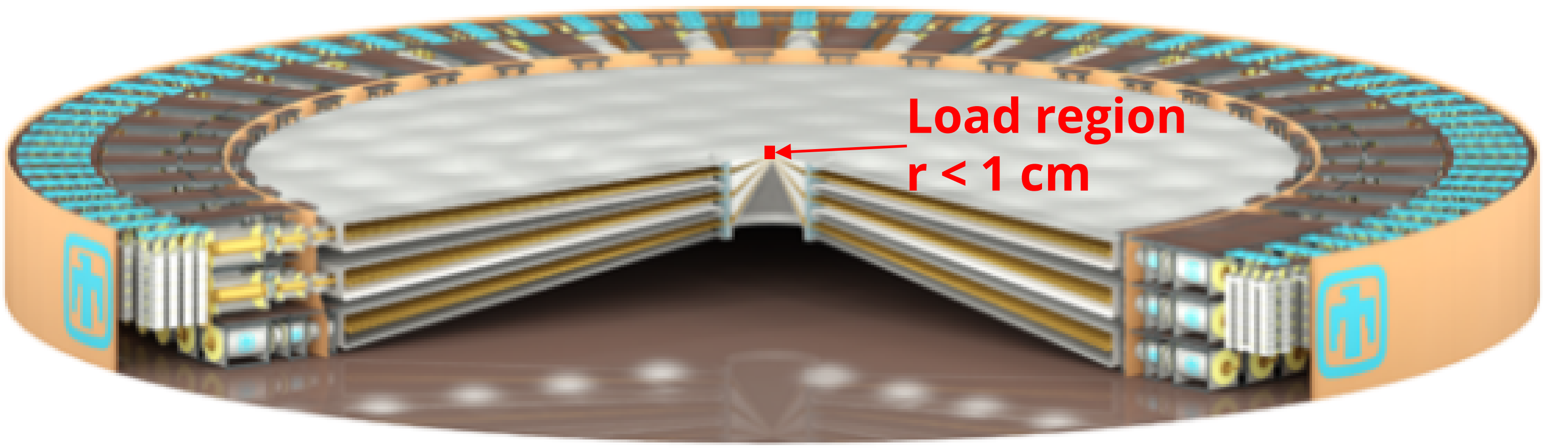
- Present designs are investigating current pulse lengths in the range of 100 ns - 200 ns.

- Strong B-fields cause electrons to emit radiation and lose energy through a radiation reaction force. In this study, we examine how important radiation reaction effects are on power flow electrons outside of the load.

- At a radius of 1 mm: 60 MA -> 12,000 T.

- At a radius of 100 μm: 60 MA -> 120,000 T.

One design being considered for NGPP



What is the radiation reaction force?

- When accelerating charged particles, such as electrons, emit radiation as defined by the Larmor formula, the emission causes a radiation reaction force on the particle.
- From Ford and O'Connell, Phys. Lett. A, 174, pp. 182-184 (1993):

Lorentz Force Law:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

not in existing power flow codes

Lorentz-Abraham-Dirac Equation:

$$\frac{d\mathbf{P}}{dt} = \mathbf{F} + \mathbf{F}_{rad}$$

$$= \mathbf{F} + \tau\left(\gamma \frac{d\mathbf{F}}{dt} - \frac{\gamma^3}{c^2}\left(\frac{d\mathbf{v}}{dt} \times (\mathbf{v} \times \mathbf{F})\right)\right)$$

Radiation Reaction Time-Constant for Electrons:

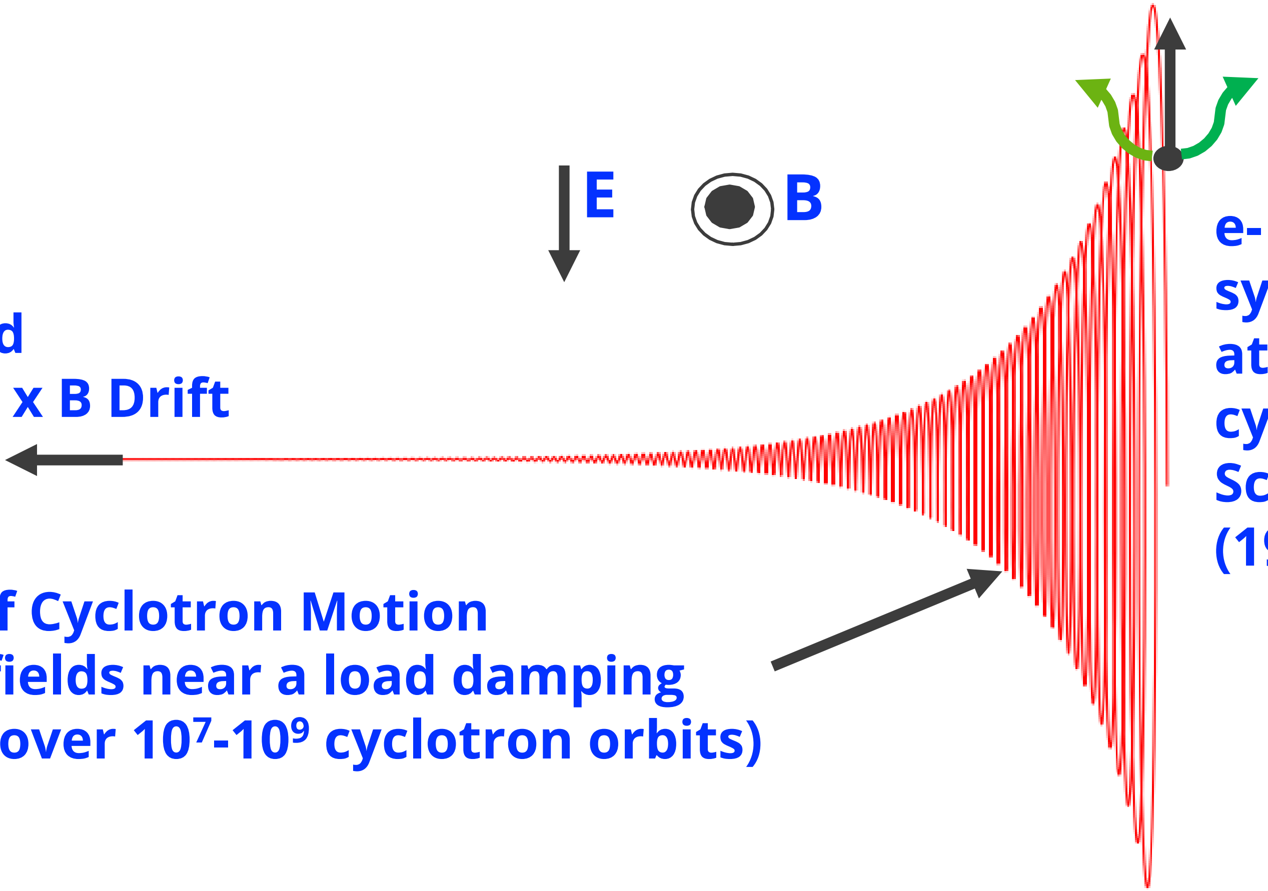
$$\tau = \frac{q^2}{6\pi\epsilon_0 m c^3} = 6.3 \times 10^{-24} s$$

Radiation Reaction in Crossed Uniform E and B Fields

Lorentz-Abraham-Dirac Equation:
In E x B Drift Frame ($P'_{||} = 0$)

$$\frac{d\mathbf{P}'_\perp}{dt'} = -e\mathbf{v}'_\perp \times \mathbf{B}' - \frac{\gamma'\mathbf{P}'_\perp}{\tau_{damp}}$$

$$\tau_{damp} = \frac{m^2}{e^2 B'^2 \tau} \qquad B' = B\sqrt{1 - \frac{E^2}{B^2c^2}} \simeq B$$



Undamped
Uniform E x B Drift

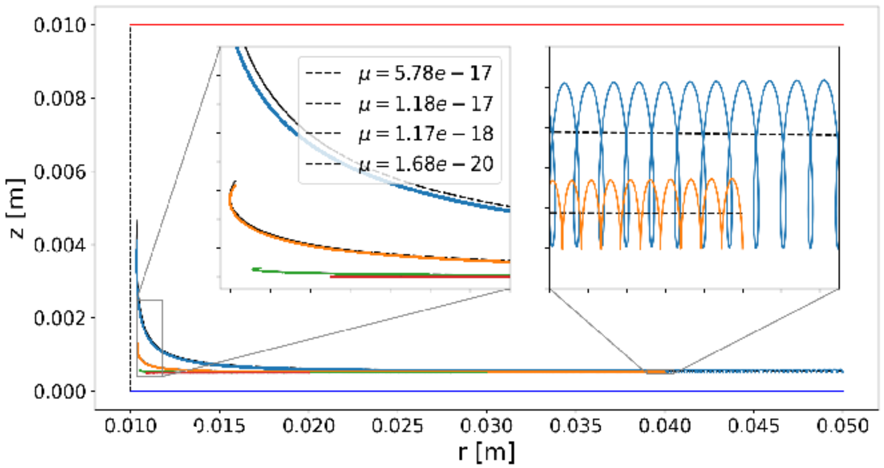
e- emitting synchrotron radiation at harmonics of the cyclotron frequency: Schwinger, Phys. Rev. (1949)

Damping of Cyclotron Motion
(for NGPP fields near a load damping may occur over 10^7-10^9 cyclotron orbits)

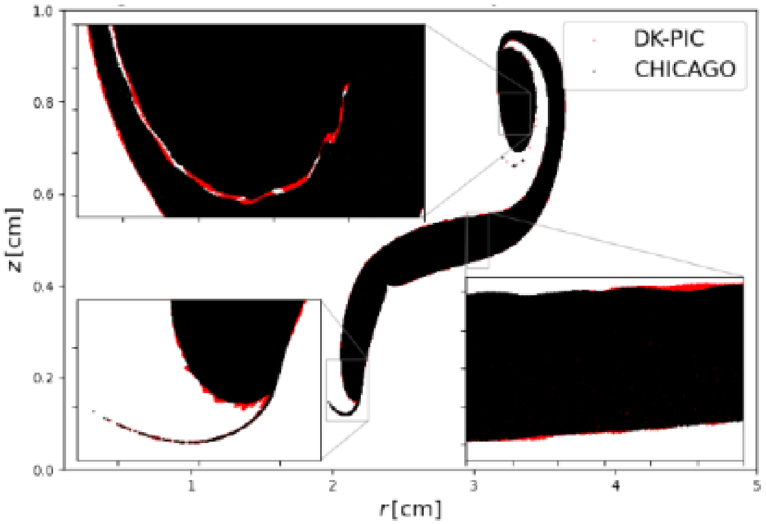
Drift – Kinetic Modeling Approach for Radial MITLs

$$\mathbf{v}_{\rm gc} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu}{q\gamma}\frac{\mathbf{B}\times\nabla B}{B^2} \qquad \mu = \frac{p^2_{r,\rm osc} + p^2_{z,\rm osc}}{2mB}$$

Guiding Center Drift Motion and Magnetic Moment



Hess and Evstatiev, IEEE Trans. Plasma Sci. (2021) (vacuum fields with magnetic moment conserved)



Evstatiev and Hess, TP11.00092 (vacuum/space-charge fields with magnetic moment conserved)

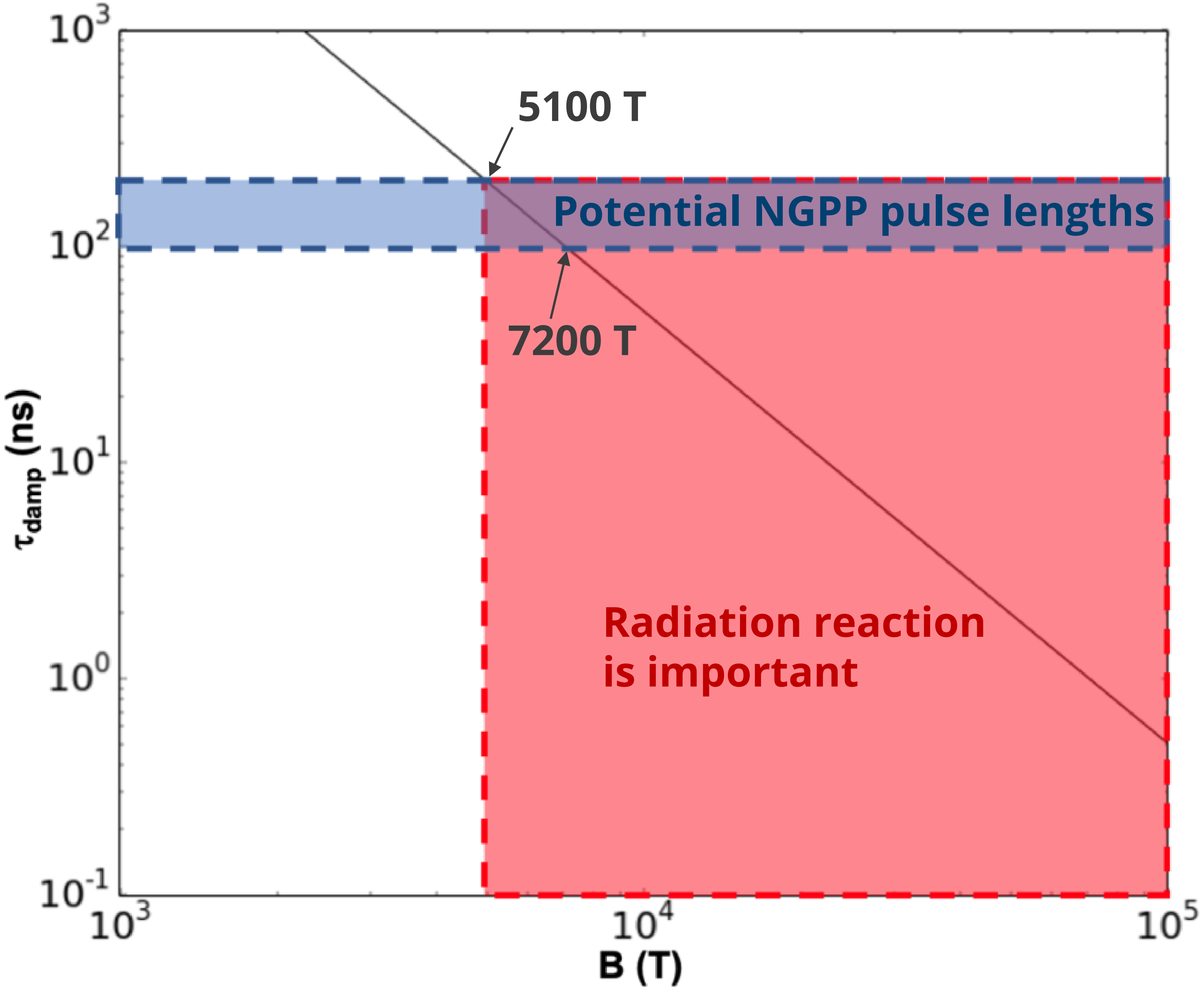
$$\mathbf{v}_{gc} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu(t)}{q\gamma}\frac{\mathbf{B}\times\nabla B}{B^2} \qquad \frac{d\mu}{dt}\Big|_{rad} = -\frac{2\mu}{\tau_{damp}}\sqrt{1+2\mu B/mc^2}$$

Present work* (vacuum fields with time-dependent magnetic moment due to radiation reaction)

*Hess and Evstatiev, SAND Report (2021) https://www.osti.gov/servlets/purl/1814239

When does radiation reaction become important for NGPP?

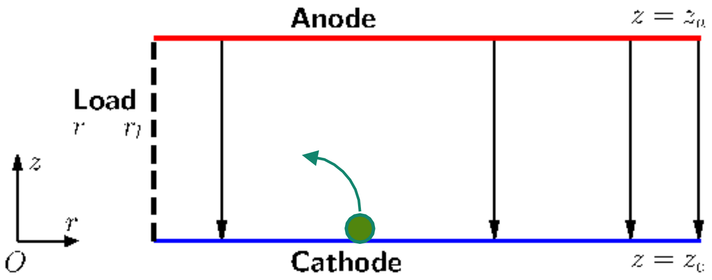
- One criterion for radiation reaction effects to become important on NGPP is the damping time becomes equal to (or less than) the NGPP pulse length.
- For a pulse length of 100 ns, radiation reaction becomes important B > 7200 T.
- For a pulse length of 200 ns, radiation reaction becomes important B > 5100 T.



Electron Dynamics in the Radial MITL Modeled with Drift Kinetics

Radial MITL Field Equations:

$$E = -\frac{\mu_0 i}{2p}\ln\left(\frac{r}{r_l}\right) e_z \qquad B = -\frac{\mu_0 I(t) e_f}{2pr} \qquad I(t) = I_{peak}\sin^2\left(\frac{pt}{2t_{pulse}}\right)$$



Radial MITL with fixed load driven by time-dependent current / electron emitted at cathode

Drift Equations:

$$\mathbf{v}_{gc} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu(t)}{q\gamma}\frac{\mathbf{B}\times\nabla B}{B^2} \qquad \mu|_{t_0} = \frac{mE^2}{2B^3}$$

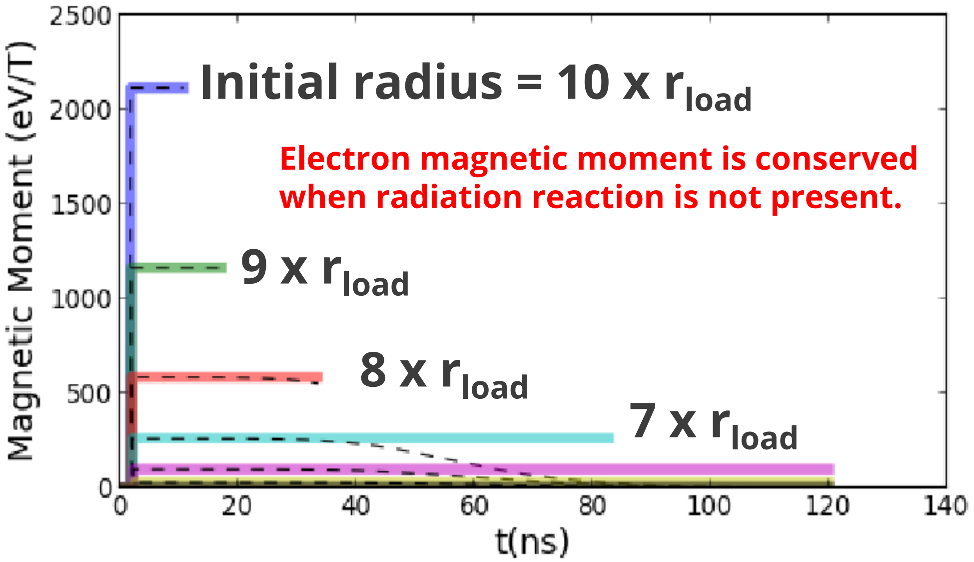
Magnetic Moment / Energy Equations:

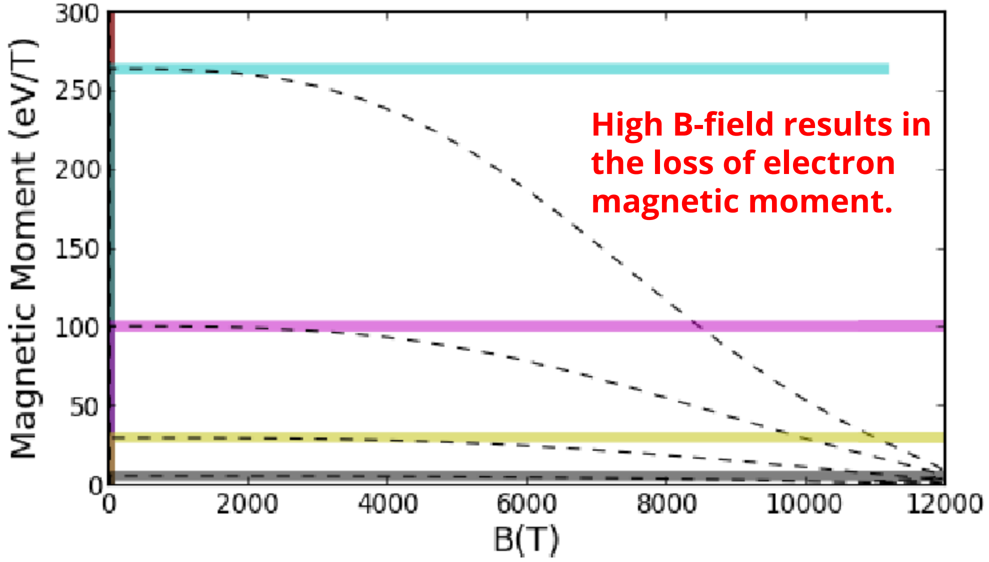
$$p_{cyc} = \sqrt{2\mu m B} \qquad K_{cyc} = (\sqrt{1 + 2\mu B/mc^2} - 1)mc^2$$

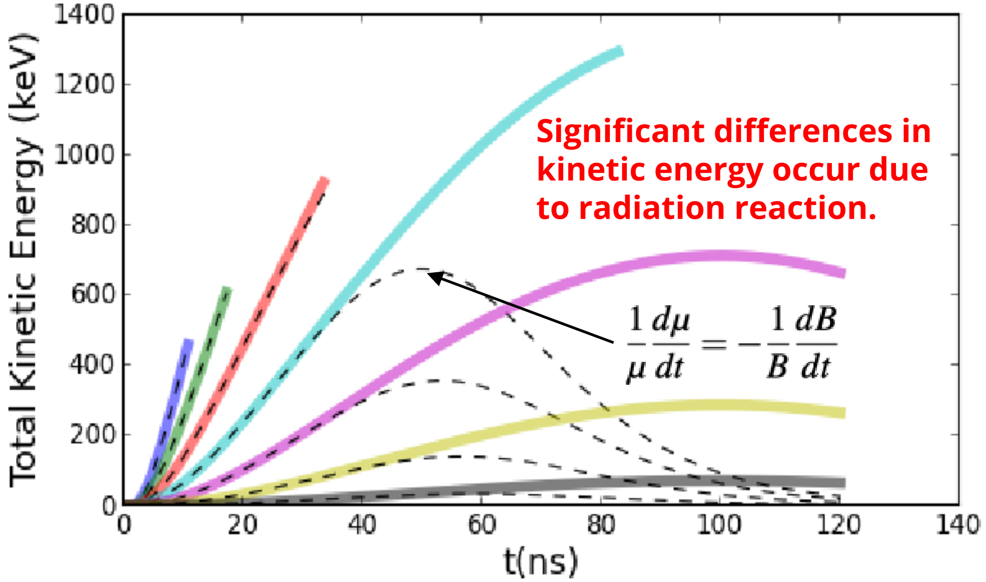
$$\frac{d\mu}{dt}\Big|_{rad} = \frac{2\mu}{p_{cyc}}\frac{dp_{cyc}}{dt}\Big|_{rad} = -\frac{2\mu}{\tau_{damp}}\sqrt{1 + 2\mu B/mc^2}$$

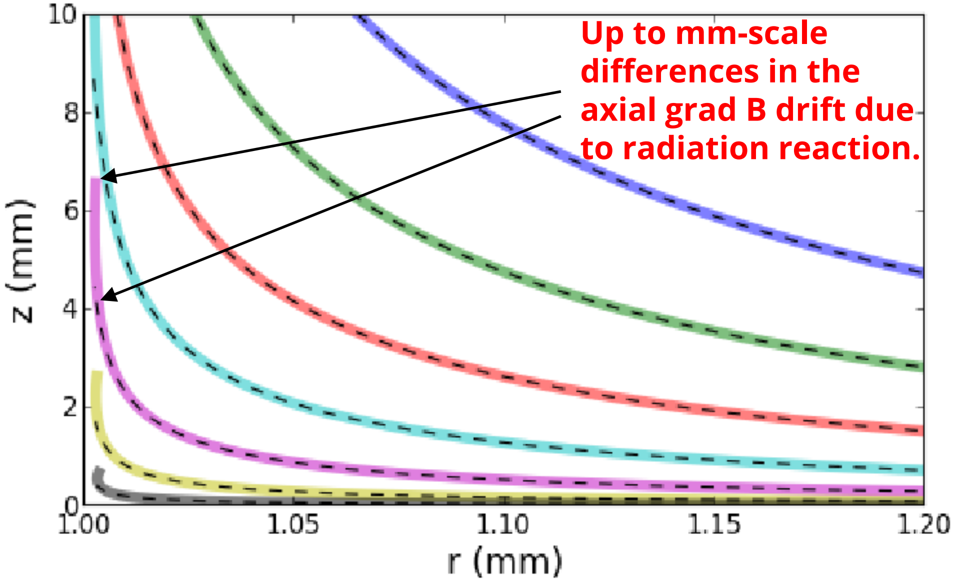
$$K_{tot} = K_{cyc} + \frac{1}{2}mv_{gc}^2$$

Results for Radial MITL with I_{peak} = 60 MA, t_{pulse} = 100 ns, r_{load} = 1 mm (solid no radiation reaction , dashed with radiation reaction)







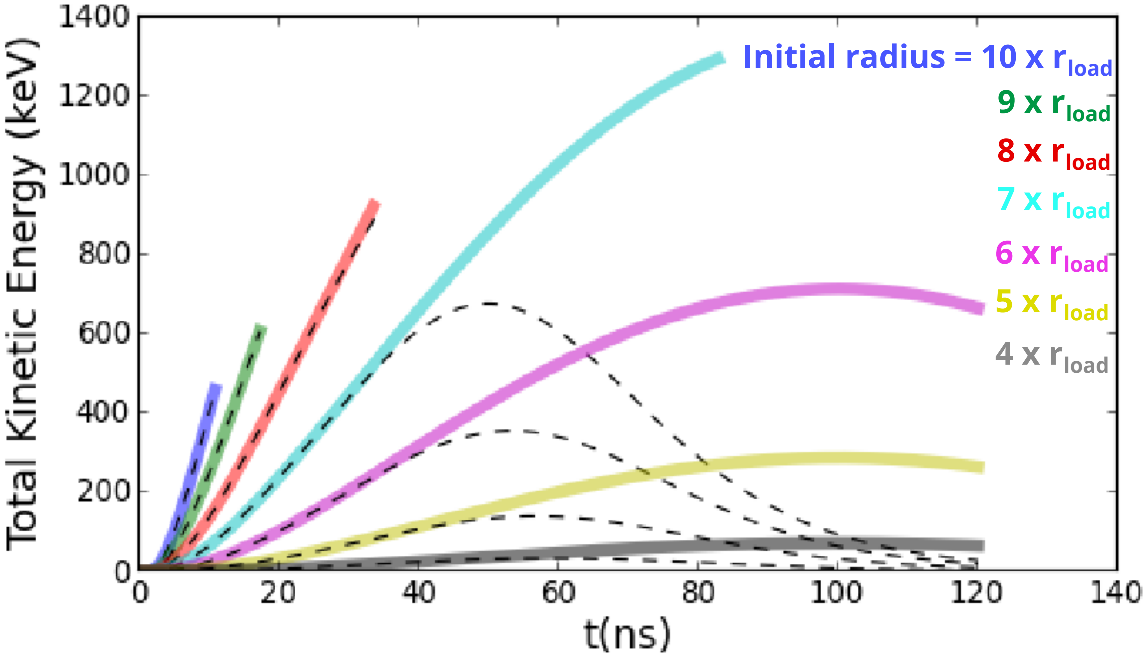


Decreasing the Load Radius Enhances Kinetic Energy Reduction Due to Radiation Reaction

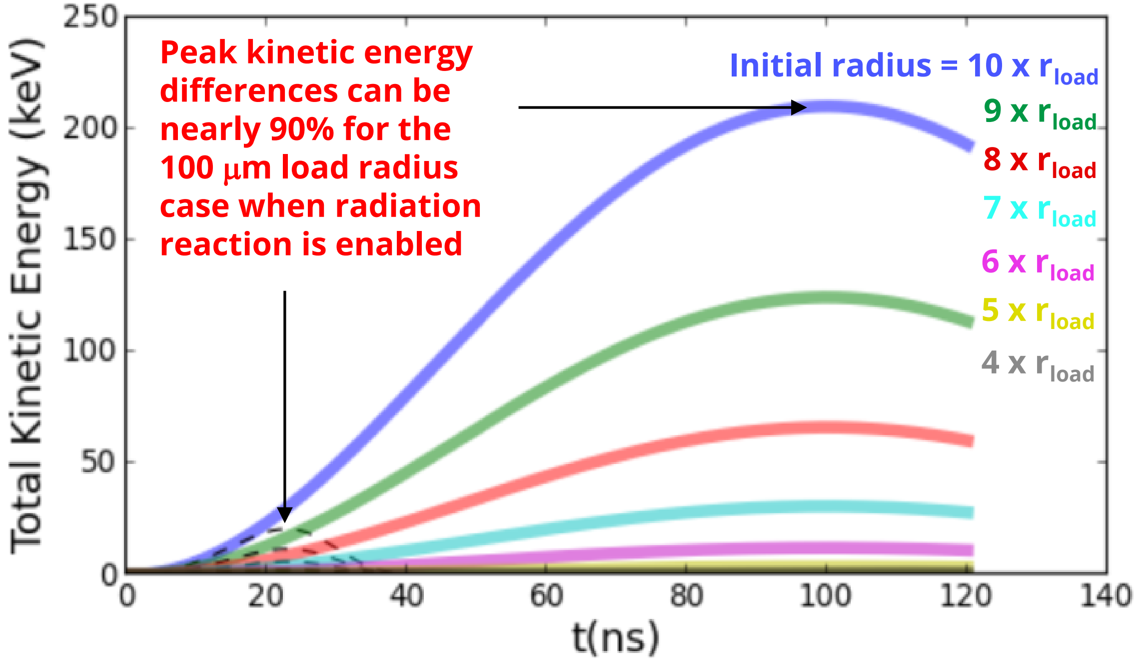
- By decreasing the load radius to 100 μm (and keeping other MITL parameters fixed), the B-field increases by a factor of 10x near the load compared to the previous load radius = 1 mm case.

- The damping time ∝ $1/B^2$ decreases by 100x causing enhanced radiation reaction effects.

Load Radius of 1 mm



Load Radius of 100 μm



Summary

- We simulated single particle electron motion and kinetic energy loss due to radiation reaction in a high-current radial MITL near a load for NGPP design parameters using a guiding center drift approach.
    - The key piece of physics is the introduction of a time-dependent magnetic moment which is damped due to radiation reaction.

- Electrons in B-fields >10,000 T (which will occur in the MITL near a load ≤ 1 mm in 60+ MA drivers) can experience significant cycloidal kinetic energy loss/prevention of kinetic energy gain due to radiation reaction.

- Radiation reaction also affects axial grad B drift motion in the radial MITL by damping the electron’s magnetic moment.

- Future work will include coupling radiation reaction physics with space-charge fields for modeling electron dynamics in radial MITLs.