Sandia National Laboratories

When is better state preparation worthwhile?

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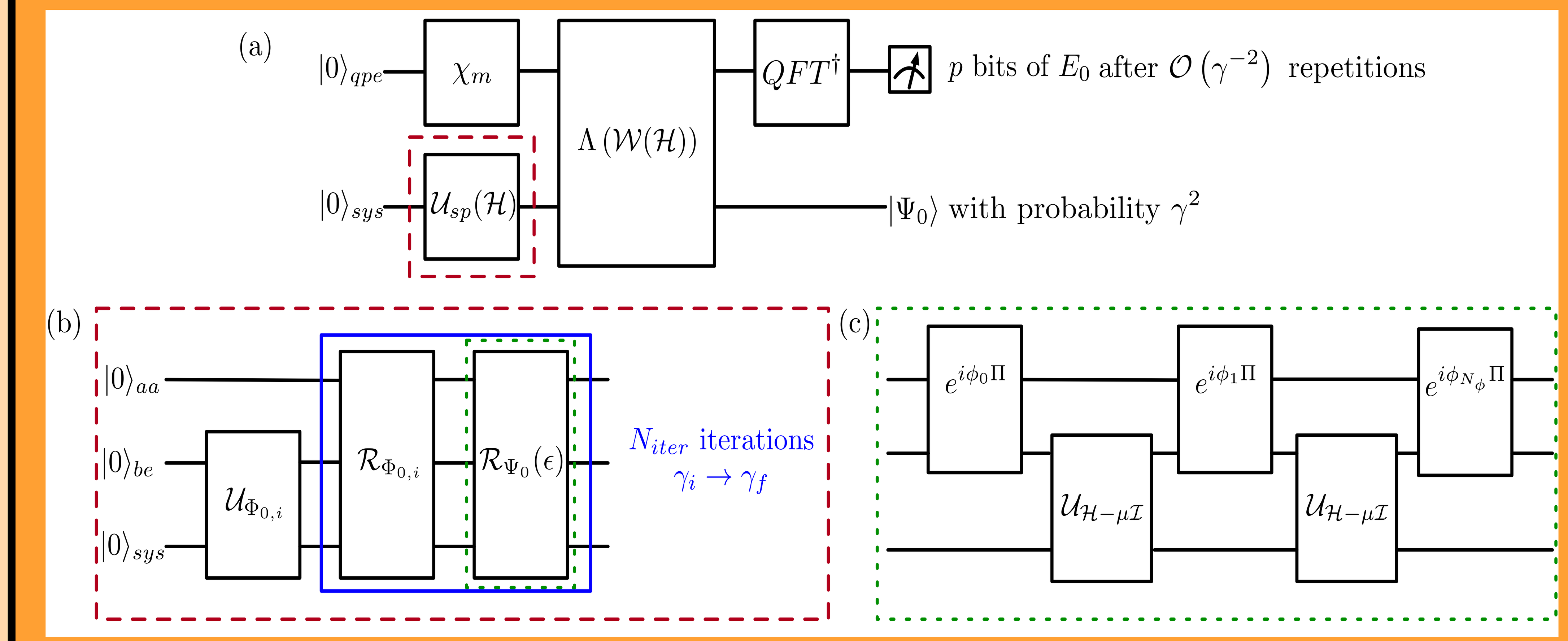
State Preparation — 1

- Want to prepare an approximation to the ground state Ψ_0 of (local-) Hamiltonian $\mathcal{H}$
- Start with a guess: $\Phi_{0,i}$; $\gamma_i = |\langle \Phi_{0,i} | \Psi_0 \rangle|$
- Improve the guess (e.g. Adiabatic, Filter-based): $\Phi_{0,f} = \mathcal{U}_{SP}(\mathcal{H})\Phi_{0,i}$; $\gamma_f = |\langle \Phi_{0,f} | \Psi_0 \rangle|$

State preparation is expensive. **When improving from $\Phi_{0,i}$ to $\Phi_{0,f}$ is worth it?**

Context: phase estimation with state preparation (amplitude amplification) — 2



a) Quantum phase estimation benefits from large overlap [1]

b) Use amplitude amplification (AA) state preparation proposed by Lin and Tong [2]

c) Approximate reflector $\mathcal{R}_{\Psi_0}(\epsilon)$ implemented with quantum signal processing [2]

How do we quantify benefit?
How does error in $\mathcal{R}_{\Psi_0}(\epsilon)$ propagate/affect benefit?

Quantification: T counts, Improvement ι — 3

- We compute full T counts for circuit Sec. 2, proxy for fault-tolerant runtimes, with and without state prep

To quantify benefit we **define the improvement**

$$\iota = \frac{\gamma_i^{-2}\left(T_{\Phi_{0,i}} + T_{QPE}(\Delta E)\right)}{\gamma_f^{-2}\left(T_{\Phi_{0,i}} + T_{QPE}(\Delta E) + T_{AA}(\gamma_i, \gamma_f)\right)}$$

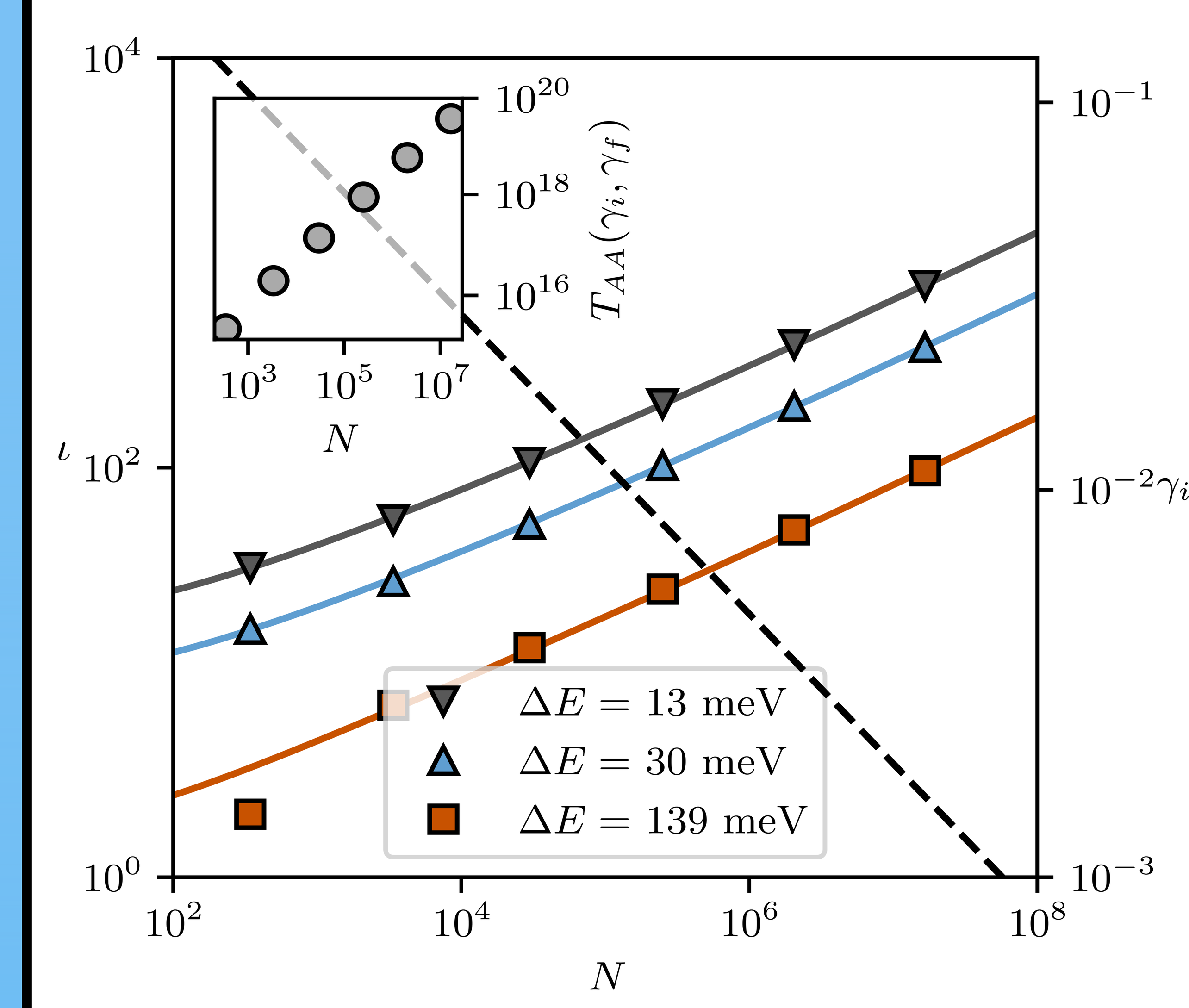
- Asymptotic near-quadratic, $\iota \sim 1/(\gamma_i \log \gamma_i^{-2})$
- *We prove a bound on ϵ in $\mathcal{R}_{\Psi_0}(\epsilon)$ so overlap γ_f is guaranteed:* $\epsilon \leq (1 - \gamma_f^2/\gamma_{max}^2)/8N_{iter}^2$

Computed T counts and Improvements — 4

- Improvement as a function of plane wave basis set size N, QPE accuracy ΔE; inset: T counts for state preparation
- Points are computed values, solid lines are fits to asymptotic form in Sec. 3.
- Dashed descending line is starting point γ_i

Multiple orders of magnitude improvement for large plane wave set with high accuracy: "quantum advantage" regime!

β —Aluminas: $Na_4Al_{22}O_{35}$



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[1] Babbush *et al.* PRX **8,** 041015 (2018)
[2] L. Lin and Y. Tong, Quantum **4,** 372 (2020)

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