A MIMO Time Waveform Replication Control Implementation

Ryan Schultz and Steven Carter

Sandia National Laboratories[1]
P.O. Box 5800 – MS0346
Albuquerque, NM, 87185

ABSTRACT
The importance of user-accessible multiple-input/multiple-output (MIMO) control methods has been highlighted in recent years. Several user-created control laws have been integrated into Rattlesnake, an open-source MIMO vibration controller developed at Sandia National Laboratories. Much of the effort to-date has focused on stationary random vibration control. However, there are many field environments which are not well captured by stationary random vibration testing, for example shock, sine or arbitrary waveform environments. This work details a time waveform replication technique that uses frequency domain deconvolution, including a theoretical overview and implementation details. Example usage is demonstrated using a simple structural dynamics system and complicated control waveforms at multiple degrees of freedom.

Keywords: MIMO, vibration testing, control, time waveform replication

1 INTRODUCTION

Sandia National Laboratories has been investigating and using multiple-input/multiple-output (MIMO) vibration testing for many years, however much of that experience has focused on stationary, random vibration environments [1, 2]. Mature MIMO random vibration control methods exist in the literature, and some have been integrated into Sandia's in-house, open-source vibration control software, Rattlesnake [3]. However, there are many service environments that are not well represented as stationary, random vibration environments. For example, shock environments, sine sweep environments, or complex transient environments are typical in many aerospace and other industrial applications. As such, recent efforts have focused on understanding and implementing MIMO transient control capabilities with the goal of implementing them in Rattlesnake and performing vibration tests for complicated, multi-axis or multi-input environments.

Time waveform replication (TWR) is a vibration control technique useful for these kinds of non-stationary signals or even arbitrary waveforms which may include combinations of shocks, sines, random vibrations or any other time-varying content. In TWR control, the objective is to match a control signal time history whereas the objective of random vibration control is to match a control power spectral density (PSD). Like random vibration control, the system output/input relationship is used in an inverse solution to estimate required inputs to best match some desired output. In TWR, the desired output is a linear spectrum representation of a control signal. Extending to multiple inputs and multiple outputs, MIMO TWR again uses an inverse solution of the system output/input relationships but now estimates a set of input linear spectra which best match linear spectra from a set of control signals. The control signals

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can be unique at each output sensor or degree of freedom (DOF) on the system, and similarly the estimated inputs at each input DOF can be unique. In this way, complicated, multi-axis motion can be replicated using MIMO TWR.

TWR can be implemented in the time domain via impulse response function representations of the system output/input relationships, however it is more conveniently implemented in the frequency domain using the familiar FRF matrix system representation [4, 5]. Among other reasons, the frequency domain implementation is used because source estimation in the time domain (i.e. time domain deconvolution) is often computationally burdensome. Frequency domain TWR is sometimes called frequency domain deconvolution because it is effectively deconvolving the output and impulse response to estimate the system inputs. However, in practice it simply involves multiplications of the inverse of the FRF matrix and the Fourier transform of the control signals, the output linear spectra. The inverse Fourier transform is then used to convert the estimated input signal linear spectra to the time domain. These input signals could then be used as shaker drives in a MIMO vibration test.

In many cases the environment of interest is long-duration, well in excess of the frame length for typical frequency resolution. It is often not convenient or possible to estimate an FRF matrix or linear spectra with small enough frequency resolution to cover a record that is many seconds or even minutes long. Instead, the TWR method presented here divides the long-duration signals into shorter-duration segments or frames. These shorter-duration frames can then be used, one at a time, in MIMO TWR control to determine the input signals needed for each frame. Then, these frames are added together to form the long-duration input signals needed for a TWR test. The process used to add the frames together is called constant-overlap-add (COLA). This process adds together overlapping, windowed frames of data [6, 7, 8, 9]. The frame-to-frame overlap and window is chosen to provide a smooth transition of the signals between frames and preserve the signal amplitudes across frame boundaries.

This paper provides an overview of MIMO TWR concepts, theory, and example usage. Section 2 provides a high-level, step-by-step process of the MIMO TWR process being utilized at Sandia and implemented into the Rattlesnake control software. Section 3 shows some of the theory behind MIMO TWR control. Section 4 demonstrates MIMO TWR usage on a simple multiple degree-of-freedom (MDOF) system. Two versions of that system are presented to demonstrate how MIMO TWR can provide nearly perfect control if the test setup is good, and how it may not perfectly control if the test setup is not ideal. To compare results, both time and frequency domain metrics are presented as it is often not sufficiently informative to simply compare control (desired) and achieved output signals.

2 TIME WAVEFORM REPLICATION PROCESS

Time waveform replication using frequency domain deconvolution can be summarized in a few steps:

1. Measure the system FRFs
2. Break up the control signals into sequential, overlapping frames which span the total duration
3. Zero pad the frame of control signals
4. For each frame, convert the control signals to the frequency domain with the Fourier transform
5. Solve the frequency-domain deconvolution problem to estimate the inputs to best match each frame's control linear spectra given the system FRFs
6. Convert the input linear spectra to time histories with the inverse Fourier transform
7. Use the COLA process to add the input signals for all frames together to give input signals which span the entire duration

While this is a straightforward process, there are several details in the implementation and usage which can be critical to the TWR control performance. For example, the system FRFs must be properly estimated and at sufficient frequency resolution to capture important features. The FRFs and impulse responses should be carefully inspected to understand factors such as conditioning and singular value distribution, high- and low-frequency FRF accuracy, and impulse response causality. Additionally, the control signals should be filtered to fit well within the frequency range of interest (i.e. where the FRFs are nicely estimated) and the signals should be high-pass filtered to avoid very low frequency content which will be difficult to match due to the windowing and zero-padding needed for each frame. The frame length should be determined based on signal frequency content, signal duration, and FRF frequency resolution.

Interpolation may be needed to get the FRF frequency resolution to align with the needed frame length. Prior to taking the Fourier transform in Step 4 above, the frame must be zero padded to avoid the convolution wrap-around error that may occur [9]. An additional improvement can be made if the signals are partially extended beyond the frame and smoothly tapered toward zero in the zero-padded sections. Another important consideration is the sample rate of the control signals and the test. The sample rate must be sufficiently high to resolve the waveforms, and typically this should be higher than just twice the maximum frequency of interest. Resampling may be needed to increase the sample rate prior to a TWR test. There are other factors which may need to be considered such as noise on the control signals and regularization in the inverse solution.

3 THEORY

This section provides a brief overview of frequency-domain MIMO TWR theory. For a linear system with inputs x and outputs y, the output/input relationships for the i-th input and j-th output can be represented in the time domain as a convolution of the input and impulse response:

$$y_j(t) = h_{y_j x_i}(t) * x_i(t) \tag{1}$$

where $h_{y_j x_i}(t)$ is the impulse response matrix for that input-output pair at time t and the symbol $*$ denotes the convolution operator.

The system output/input relationship can also be represented in the frequency domain for a set of inputs and outputs as

$$Y(\omega) = H_{yx}(\omega) X(\omega) \tag{2}$$

where $X(\omega)$ is a vector of the input linear spectra, $Y(\omega)$ is a vector of the output linear spectra and $H_{yx}(\omega)$ is the system frequency response function (FRF) matrix at a frequency ω. Multiplication in the frequency domain is more convenient that convolution in the time domain so frequency domain analysis is used here. Although no longer applying convolution directly in the time domain, zero padding the input time history is still necessary to avoid circular convolution wrap around error.

Determining inputs to best match some control response signals can be achieved in the frequency domain by

$$X(\omega) = H_{yx}^{+}(\omega) Y(\omega) \tag{3}$$

where $H_{yx}^{+}(\omega)$ is the pseudo-inverse of the FRF matrix.

A Fourier transform converts the time domain signals to frequency domain linear spectra. Similarly, an inverse Fourier transform converts the frequency domain linear spectra to time domain signals. This can be done for an entire time record or for segments (frames) of the record. For long-duration TWR testing, the entire record is split into sequential, overlapping frames that can be added together to recreate the entire record as

$$x(t) = \sum_{i=0}^{n} w(t) x_i(t) \tag{4}$$

where the signal for a given frame is $x_i(t)$, n is the number of frames and the entire signal is $x(t)$. $w(t)$ is a window function which smoothly tapers each frame's signal to zero. The window shape and the overlap frame-to-frame are chosen to ensure a constant amplitude over the entire record (i.e. the summation of overlapped windows is equal to one over the entire record). More details on COLA can be found in [8].

4 MODEL-BASED EXAMPLE

To provide an example of how this TWR process works, a model-based example is provided. Here, a simple dynamic system is first subjected to some non-stationary loads to produce a set of control signals. Next, the TWR control methodology is used to derive new inputs which can best match those control signals and then those inputs are applied to the system to predict the response. The results are compared using plots of the control and predicted waveforms, the time response assurance criteria (TRAC), running root mean square (RMS) response, and the time-evolving PSD via spectrograms.

4.1 MODEL DESCRIPTION

The example free-free, multi-DOF model is shown in Figure 1. The model is comprised of six point masses connected with linear springs and has motion is in one direction only. Each mass is 1 kg. The spring stiffness between DOF 1-2, 2-3, 4-5, and 5-6 is 5×10^5 N/m and the spring stiffness between DOF 1-4, 2-5, and 3-6 is 1×10^6 N/m. There are three outputs on DOFs 1, 2, and 3 and two inputs on DOFs 4 and 6. Modes are computed from the model mass and stiffness matrices. There are five elastic modes that range between 112 and 297 Hz in addition to one rigid body mode. All six modes are used to synthesize the acceleration/force frequency response functions (FRFs) between the three outputs and two inputs, evaluated in 1 Hz steps up to 4096 Hz.

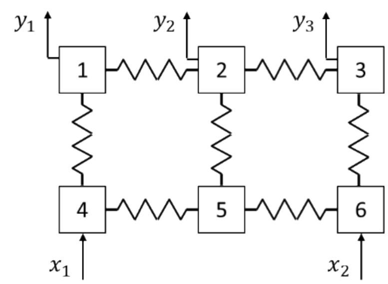

Figure 1: MDOF model with three outputs and two inputs

4.2 EXAMPLE TRANSIENT INPUTS AND SYSTEM RESPONSE

Two different, non-stationary force inputs are created to excite the system and the acceleration response is measured at the three outputs. The input signals are comprised of two band-limited burst random events which start and end at different times in a two-second record with an 8192 Hz sample rate (twice the max FRF frequency). The first burst random event has frequency content between 50 and 200 Hz and the second has frequency content between 200 and 500 Hz. Band-limiting is accomplished by applying a finite impulse response filter to pure random noise.

Modal time integration is used to simulate the system's response to these two inputs. To minimize error due to the time integration process, time integration is performed at 16x the nominal sample rate (131072 Hz). The resulting acceleration response at the three output DOFs are the control signals that will be used in time waveform replication. If the time waveform replication process is successful, the original input signals should be recovered. Figure 2 shows the acceleration response signals for the three output DOFs and Figure 3 shows the time histories for the two input DOFs.

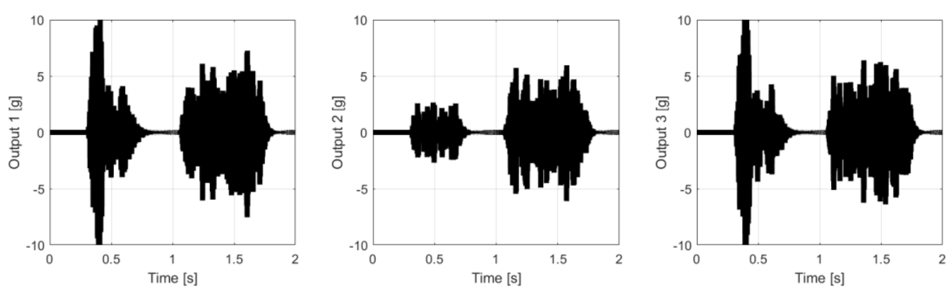

Figure 2: Control time histories at the three output DOFs

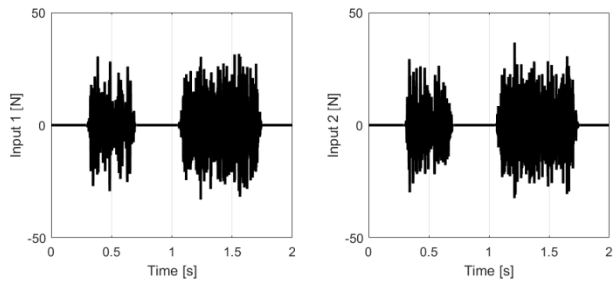

Figure 3: Input time histories at the two input DOFs

4.3 TWR CONTROL RESULTS

The control signals are fed into the time waveform replication algorithm along with the system FRF matrix pseudo-inverse to compute the estimated input signals needed to best match the output at each time. The pseudo-inverse of the FRF matrix is computed using a singular value decomposition at each frequency line. In this case, no regularization was utilized though that is typically necessary to minimize the effects of noise on the estimated inputs.

In this example, the environment duration is long, in excess of the block length determined by the FRF matrix frequency resolution (i.e. $T = 1/df$). Thus, the entire record is broken up into five overlapping frames. Each frame is 0.5 seconds long and is overlapped with the adjacent frames by 0.125 seconds. The amount of overlap frame-to-frame could be changed with negligible effect on the results. Figure 4 shows the five overlapping frames (in blue, red, green, purple, cyan) and how the estimated input signal (in black) is composed of a summation of estimates for each frame. The Tukey window which was used to taper each set of signals to zero at the ends of the frames is shown by the gray dotted lines (window amplitude is scaled to be visible in this plot).

Next, the estimated input signals are input to the model. Again, modal time integration is used with the same oversampling as in the original simulation. The resulting response can then be compared with the original, control responses to assess how well time waveform replication could control this MDOF system.

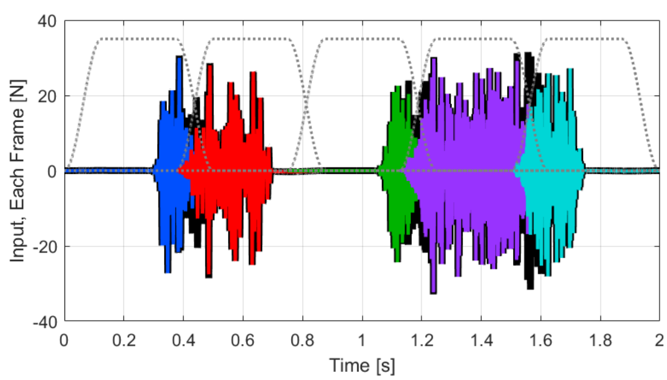

Figure 4: Estimated input time histories for each of five overlapping frames which sum to the total input signal

4.3.1 TIME HISTORY COMPARISONS

Time histories can be compared by simply plotting the specification and predicted signals for each DOF, along with the error signal, $e(t) = y_{predicted}(t) - y_{control}(t)$, as shown in Figure 5. In this case, the true inputs are known so the same comparison can be done for the input time histories as well, Figure 6. Here, both the inputs and outputs match the original (specification) time histories very well, with only minor errors. The error signal is near zero for all DOFs over the entire record. Computing a running root mean square (RMS) can provide a simple view of the DOF energy versus time as shown in Figure 7. This shows how each DOF's energy is accurately replicated by TWR control. A time response assurance criterion (TRAC) provides a scalar metric of the overall match of a pair of time histories (i.e. specification versus prediction for a given DOF) [10]. Here, the TRAC for each of the output DOFs is 1.0 and the TRAC for each of the input DOFs is 0.99, indicating a near perfect match of the time histories. Note that the TRAC will be sensitive to any phase shift or frequency errors so this metric should be used with caution or at least in conjunction with other metrics.

Moreover, it is important to note that comparisons in the time domain can, in general, be misleading and may be of little use to the practitioner. This is because they are essentially showing the errors at every frequency line for every moment in time. As such, errors that are limited to a specific frequency band (while the remaining frequency lines are considered accurate) could lead to significant amplitude or phase errors in the time domain. While it is important to understand this time domain error, it is unlikely that the practitioner would notice that it is due to a specific frequency band, limiting the actionable information in the comparison.

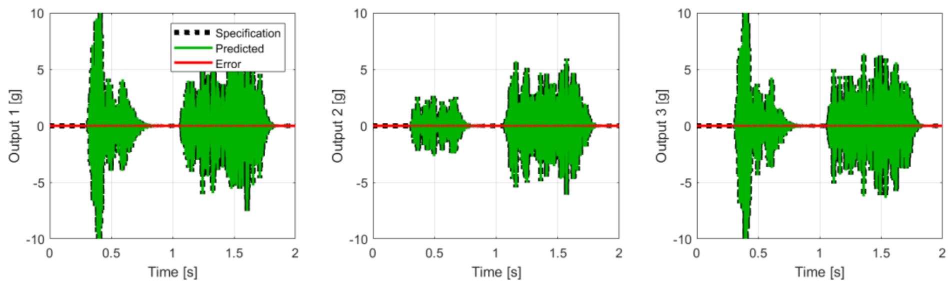

Figure 5: Output time histories comparing the original (specified) versus predicted

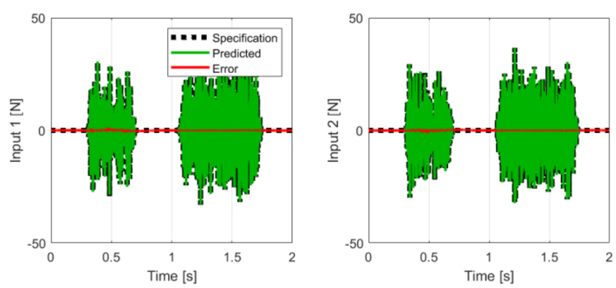

Figure 6: Input time histories comparing the original (specified) versus predicted

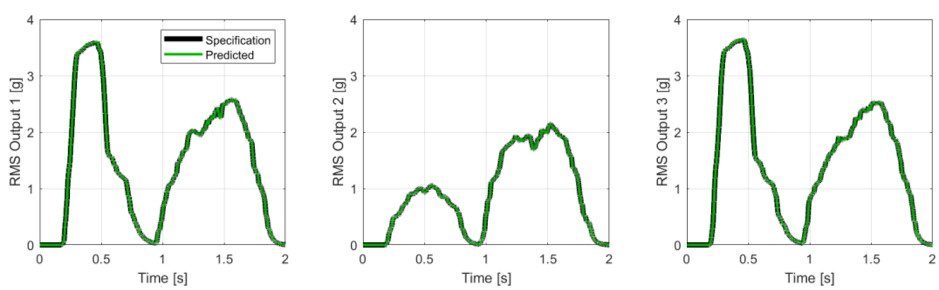

Figure 7: Running RMS of the output time histories comparing the original (specified) versus predicted

4.3.2 SPECTROGRAM COMPARISONS

In addition to the time history comparisons shown above, it is informative to compare TWR results in terms of time-evolving frequency content. This can be done by computing a spectrogram, which is the signal power spectral density (PSD) as a function of both time and frequency. Figure 8 shows the control and predicted response spectrograms for each of the three output DOFs, note that the Z-axis (i.e. color) is set to show the dominant frequency content. Qualitatively, the spectrograms match very closely, indicating good control at each DOF. To quantify the spectrogram comparison, an error spectrogram can be computed by taking the ratio of the predicted and specified spectrograms and converting to decibels (dB). In this way, a single plot can show the TWR performance in terms of under- or -over test for both time and frequency. Figure 9 shows this dB error spectrogram for each output DOF. As the TWR control worked well, the errors are low for all times and frequencies. To show some error in these plots, the limits of the plot are just +/- 0.15 dB.

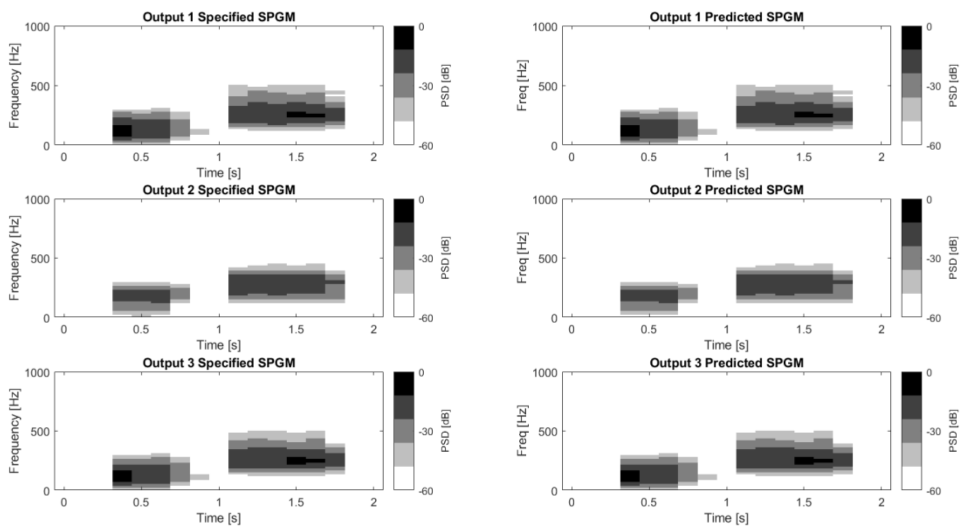

Figure 8: Spectrograms to compare the original outputs (left) and predicted outputs (right)

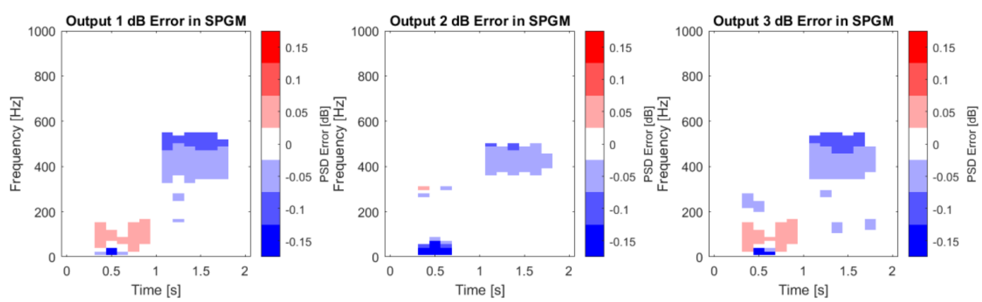

Figure 9: dB error in the spectrograms for each of the three output DOFs

4.4 MODIFIED SYSTEM RESULTS

To better show how these metrics look when results are not perfect, a modified system is created by slightly changing the spring stiffness between the masses. This mimics what could be seen in actual tests, where the specification is derived from a unit in the field and then applied to a TWR test of a different unit in the laboratory test. This change in stiffness results in a slight change in mode frequencies as shown in Table 1. TWR is then applied using the FRFs from this new system, along with the control waveforms from the original system. The change in system dynamics makes it more difficult to perfectly match the control response as can be seen in the comparison plots in the following sections.

Table 1: Mode frequencies of the original and modified systems

Mode #	Original System Mode Frequency [Hz]	Modified System Mode Frequency [Hz]
1	0.2	0.2
2	113	106
3	195	185
4	225	216
5	252	249
6	298	281

4.4.1 TIME HISTORY COMPARISONS

As seen in Figure 10 and 11, the modified system cannot perfectly match the specified response. There is some error in the time histories of each DOF and the running RMS shows an under-test for DOF 1 and 2 and a slight over-test for DOF 3. The TRAC metric also reflects the error with values of 0.96, 0.89, and 0.97 for DOF 1, 2, and 3, respectively.

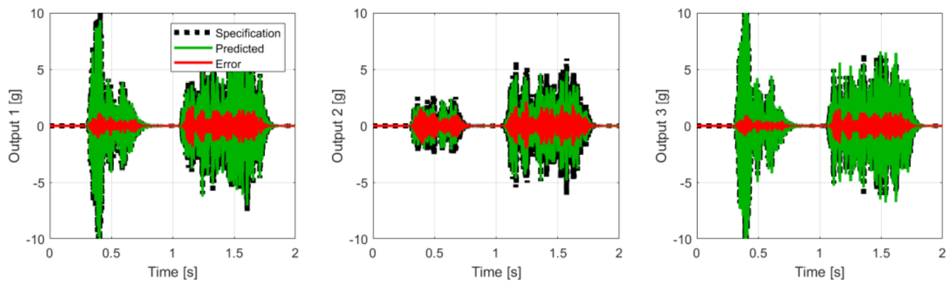

Figure 10: Output time histories comparing the original (specified) versus predicted for the modified system

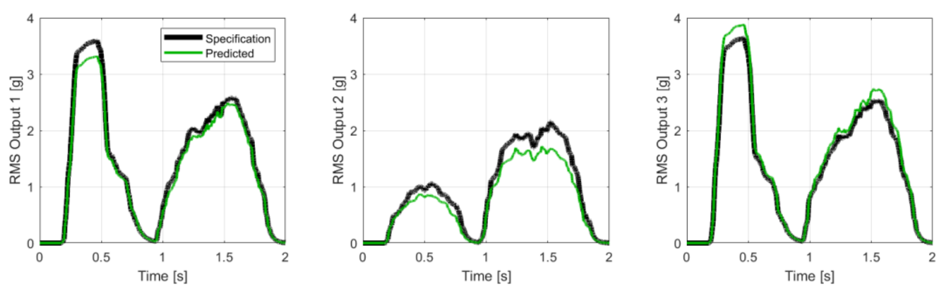

Figure 11: Running RMS of the output time histories comparing the original (specified) versus predicted for the modified system

4.4.2 SPECTROGRAM COMPARISONS

The specified and predicted spectrograms in Figure 12 indicate that the general distribution of energy versus time and frequency is achieved, with approximately the right frequency content occurring at the right time. However, there is some difference in the PSD levels. This can be seen more closely in the dB error spectrograms in Figure 13 where there are indications of a slight over-test (indicated in red) or slight under-test (indicated in blue) for some points in time for certain frequency ranges. For example, DOF 2 shows a general under-test between 250 and 500 Hz in the 1 to 2 second time range. This means that DOF 2 is not being excited properly during the second burst random excitation.

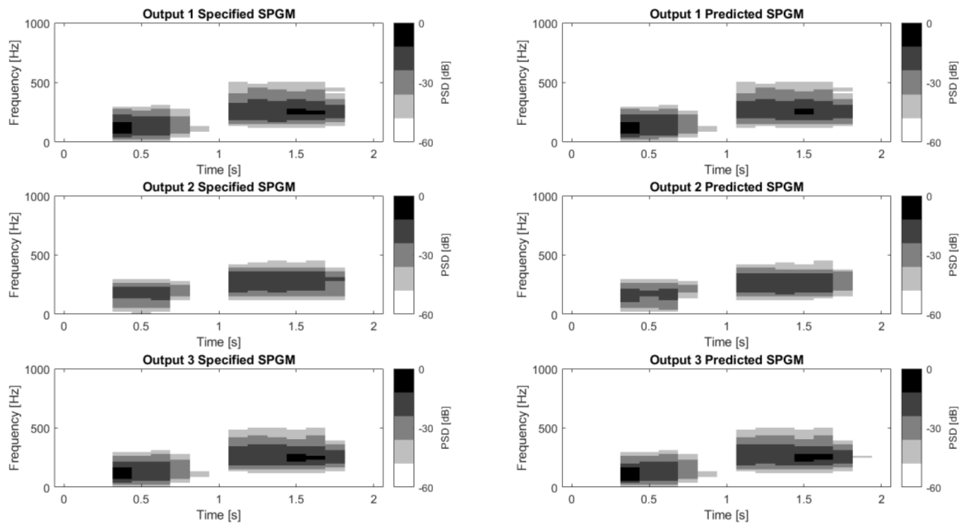

Figure 12: Spectrograms to compare the original outputs (left) and predicted outputs (right) for the modified system

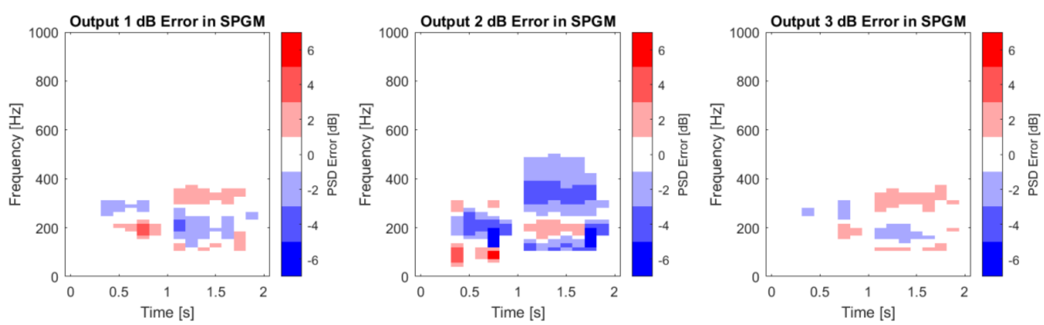

Figure 13: dB error in the spectrograms for each of the three output DOFs for the modified system

5 CONCLUSION

MIMO vibration testing can accurately mimic complicated, multi-axis service environment response. Strong research and development efforts in recent years indicate the industry understands this and is moving toward MIMO vibration testing. To help enable research in MIMO control, Sandia National Laboratories has developed an open-source MIMO vibration control software called Rattlesnake. To increase the capabilities in Rattlesnake, Sandia researchers are investigating TWR control methods. This paper presents a frequency domain deconvolution approach to MIMO TWR which will soon be integrated into Rattlesnake and utilized in multi-axis transient vibration testing. Demonstration of this MIMO TWR control method on an example dynamic system shows that near perfect control is possible when the problem is well posed. To demonstrate how control suffers when the problem is poorly posed, the example system was modified and then used in MIMO TWR predictions. Due to the time-varying, broadband content in these kinds of tests, there is likely no one metric that is best so various comparison metrics were used to assess results, including running RMS, spectrograms, and the TRAC.

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