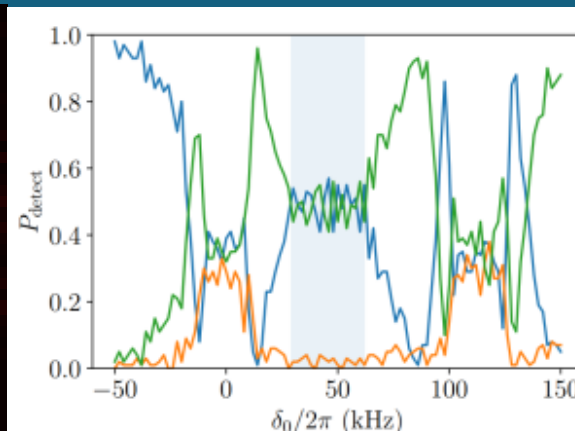
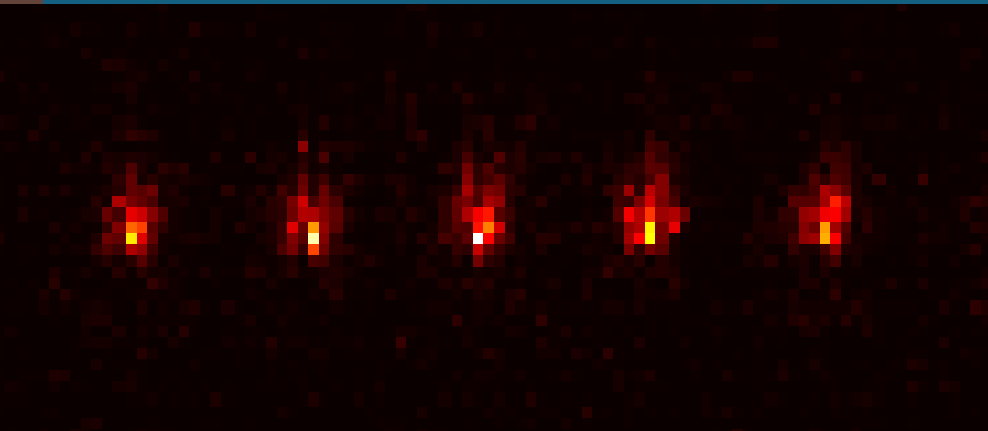
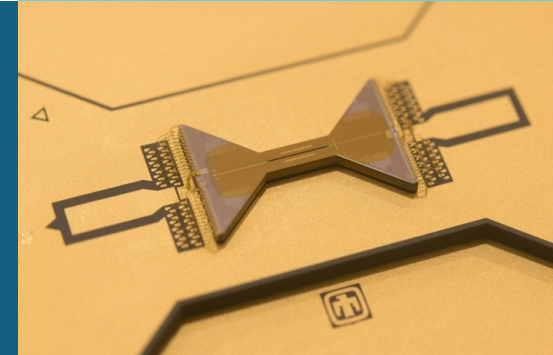


# Demonstration of Mølmer–Sørensen Gates Robust to $\pm 10$ kHz Trap Frequency Error



Presenting: Matthew Chow<sup>1,2,3</sup>

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2. Department of Physics and Astronomy, Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

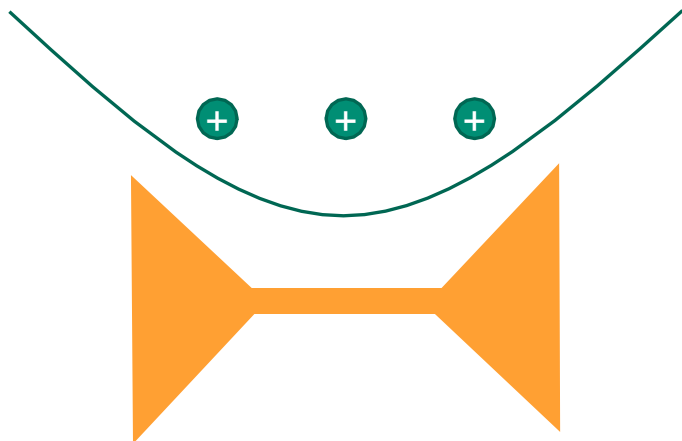
3. Center for Quantum Information and Control, University of New Mexico, Albuquerque, NM

625.02372

SAND #

Overview: We develop and demonstrate an entangling gate on trapped ions that is robust to a dominant noise source.

## Ion RF Paul Trap



Critical challenge: Error mitigation for entangling gates



Design pulses for robust operation

## Schedule:

- Intro to trapped ion QC
- Mølmer-Sørensen Gates
- How our gate design achieves robustness to trap frequency drift.



Trapped ions show great promise for quantum computing...

but the entangling gates suffer from technical noise.

Trapped ions show great promise

- SPAM  $> 0.99999$

[Zukas 2021]

- Single qubit rotations  $> 0.9999$

[Ballance 2016, Gaebler 2016]

- Peak entangling gate  $\geq 0.999$

[Ballance 2016, Gaebler 2016]

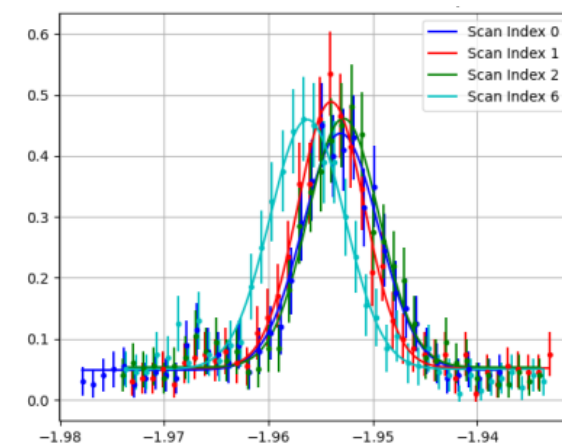
... however

- Motional frequency drifts impact the entangling gate.

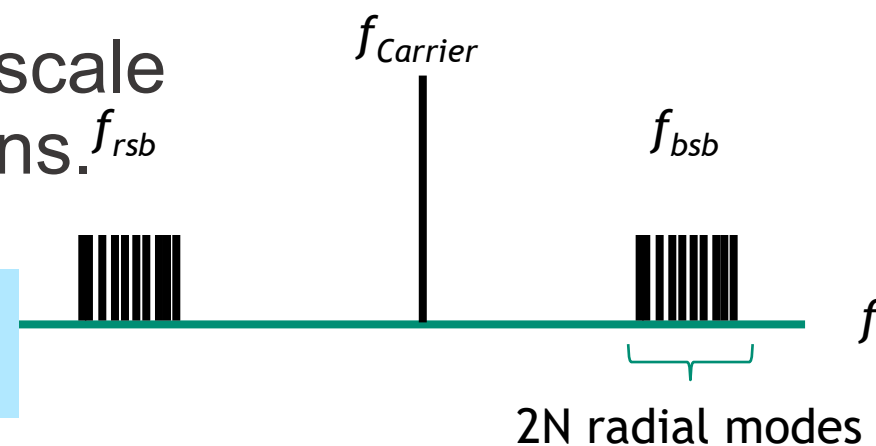
- It's hard to scale to large chains.



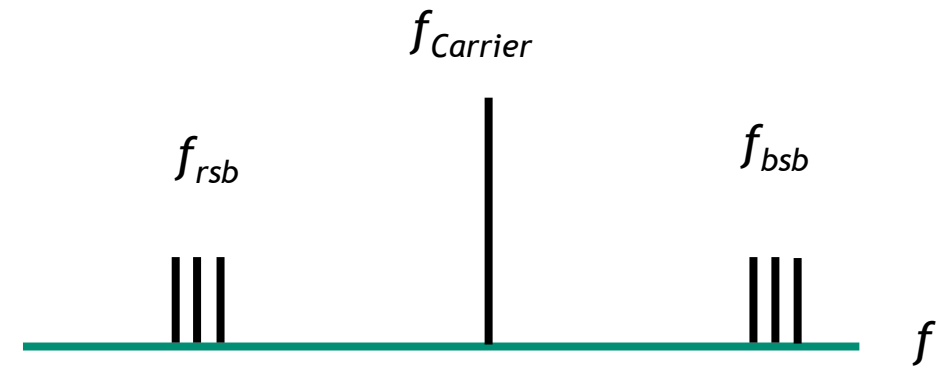
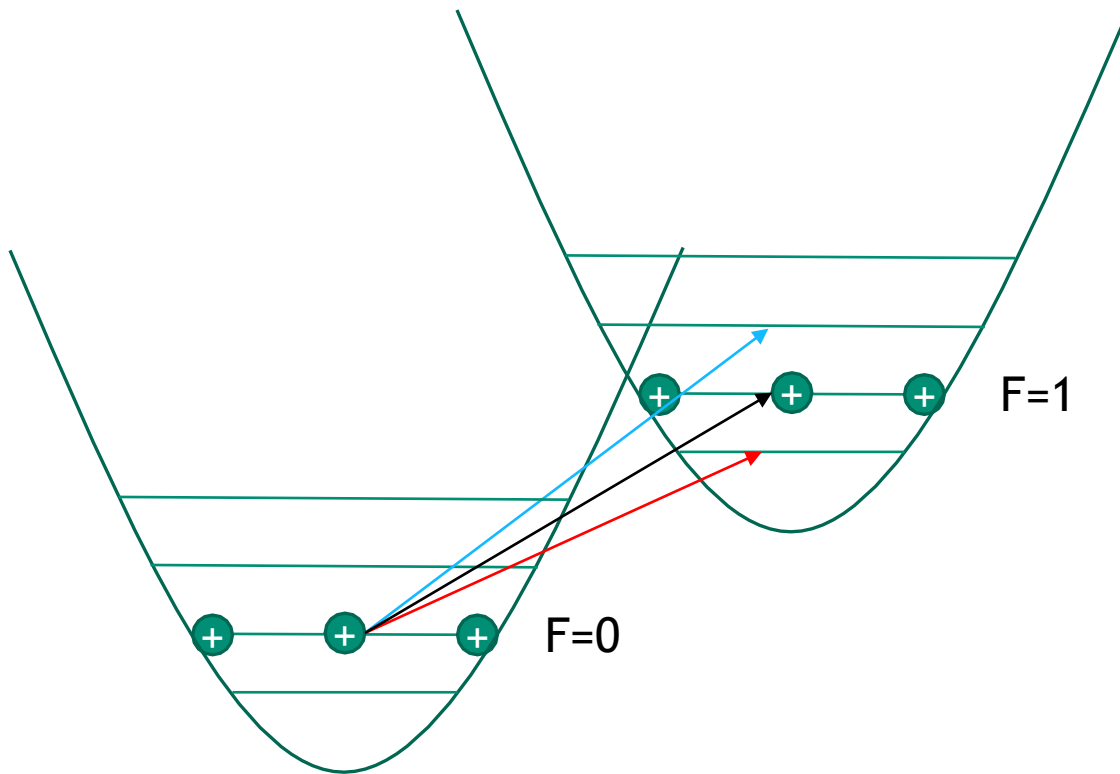
Solution? Develop gates that are robust to trap frequency drift that can be implemented on long chains.



Sideband Frequency (MHz)



# Shared motional modes mediate entangling interactions

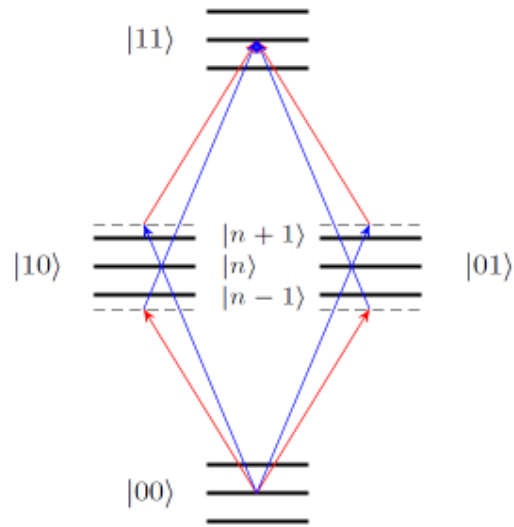


Red sideband:  $n-1$   
Remove a phonon

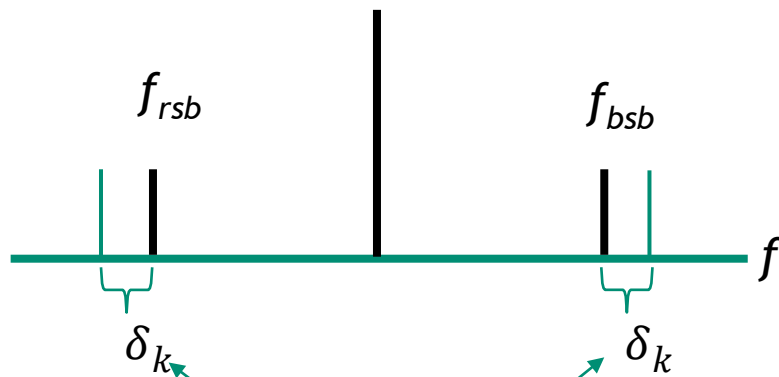
Blue sideband:  $n+1$   
Add a phonon

- Motional sidebands at the trap frequency
- Number of modes  $\propto$  Number of ions
- Coulomb interaction couples ions together
  - Vibrational levels act as “bus” connecting qubits

# The Mølmer-Sørensen (MS) interaction drives spin entanglement and coherent displacement.



$f_{\text{Carrier}}$



Apply 2 Symmetrically Detuned Tones

Interaction Hamiltonian:

$$\hat{H}_I = -\Omega(t) \sum_k \hat{S}_{\phi,k} \hat{a}_k e^{i\delta_k t} + h.c.$$

Bosonic annihilation operator for mode  $k$

Spin operator,

$$\hat{S}_{\phi,k} = (\eta_{1,k}\sigma_{\phi,1} + \eta_{2,k}\sigma_{\phi,2})/2$$

Propagator:

$$\hat{U}(t) = \prod_k e^{-i\beta_k(t)\hat{S}_{\phi,k}^2} \hat{D}(\hat{S}_{\phi,k}\alpha_k(t))$$

Phase space trajectory:  $\alpha_k(t) = i \int_0^t \Omega(t') e^{-i\delta_k t'} dt'$

Governs spin entanglement:  $\beta_k(t) = \frac{i}{2} \int_0^t \frac{d\alpha_k(t')}{dt'} \alpha_k^*(t') - \alpha(t') \frac{d\alpha_k^*}{dt'} dt'$

Gate angle:

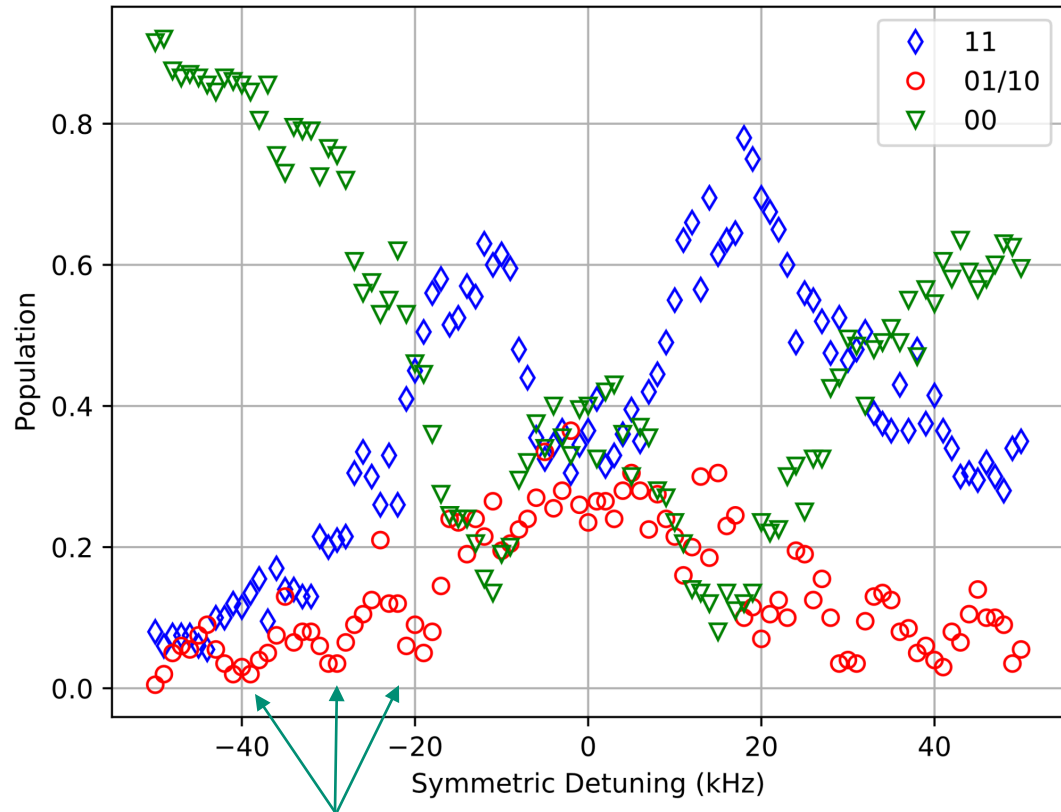
$$\theta(t) = \sum_k \eta_{1,k} \eta_{2,k} \beta_k(t)$$

# The Standard (square, constant frequency) MS gates have issues.

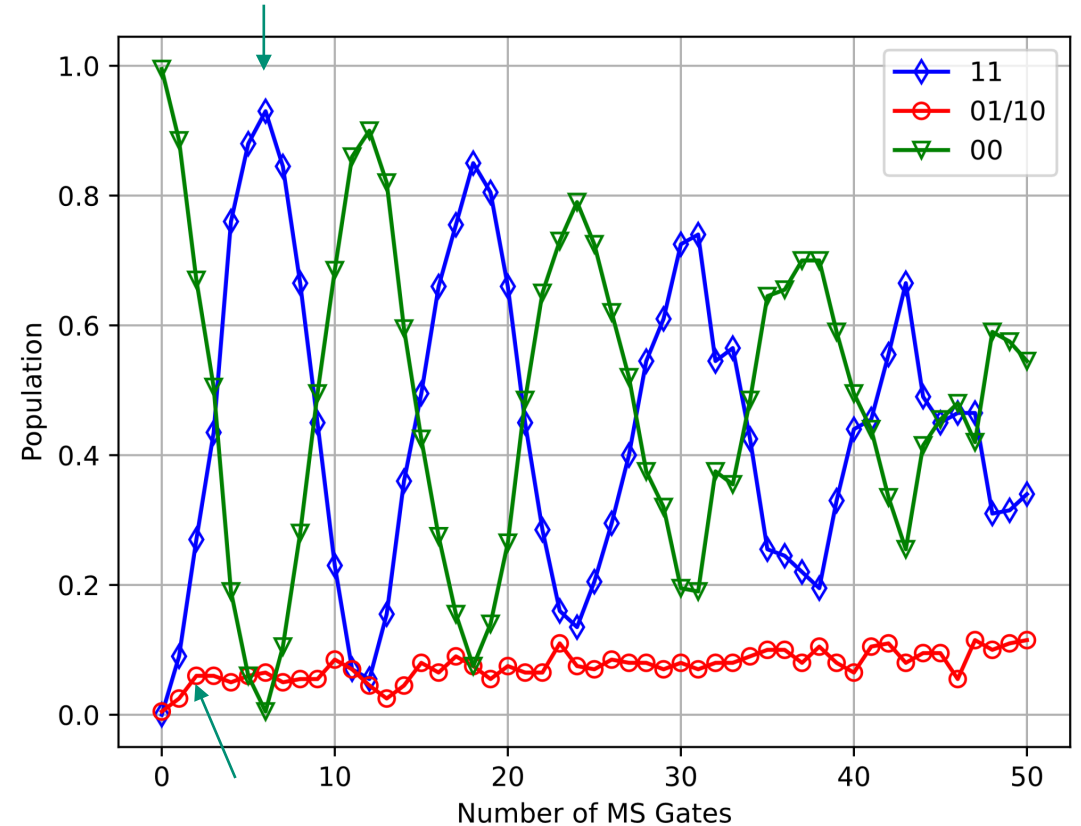


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Ideally, the gate generates flopping between  $|00\rangle$  and  $|11\rangle$  - never populating odd parity states.



- Odd parity population does not go to zero.
- Narrow acceptable detuning range



- *Best Fidelity  $\approx 96.4\%$ , estimated from the max 11 population*
- Odd parity population persistent

# Separating displacement and rotation error clarifies gate failure analysis.



If  $\alpha_k(\tau) \neq \alpha(0)$ ,

then the motional state at the end of the gate is not where it started.

We call this **displacement error**,  $\epsilon_d$ .

Experimental Indicator :  $|01\rangle$  and  $|10\rangle$

If  $\theta(\tau) \neq \pi/2$ ,

then the entangling phase accumulated is not equal to the target value.

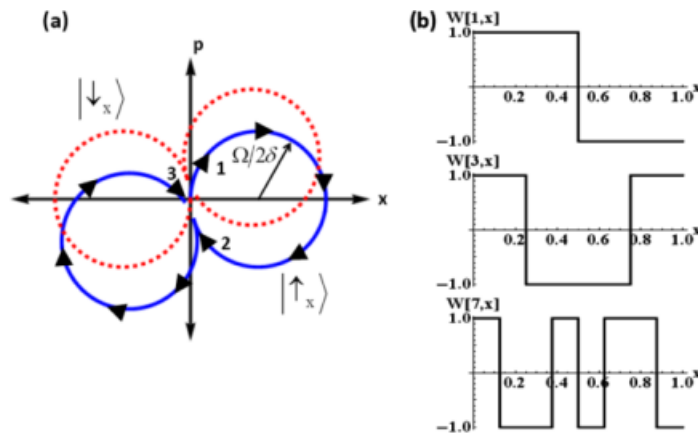
We call this **rotation error**,  $\epsilon_r$ .

Experimental indicator: ratio of  $|00\rangle$  to  $|11\rangle$

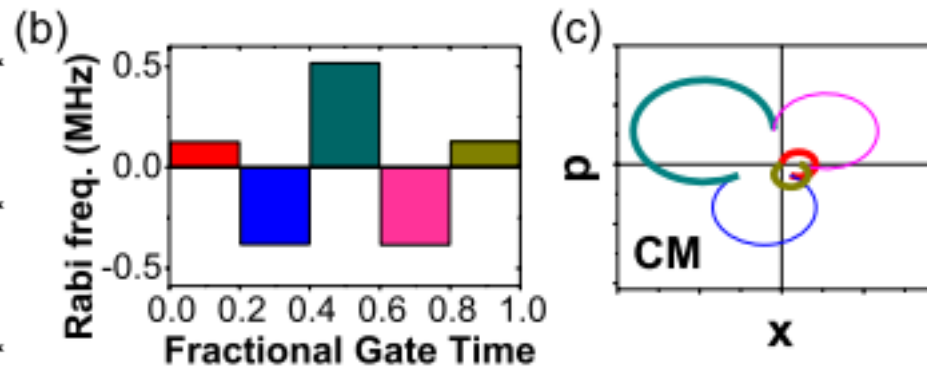
For robust gate, both  $\epsilon_d$  and  $\epsilon_r$  to be small over a broad acceptable range of input parameters.

Pioneering work by Brown and Monroe groups have found ways to minimize  $\epsilon_d$ :

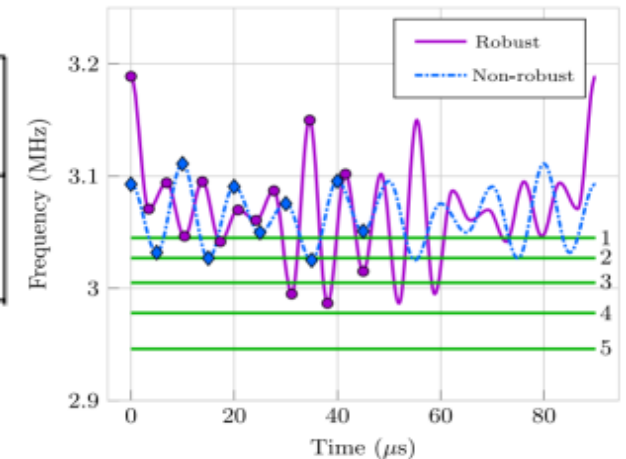
[Hayes et al 2012]



[Choi et al 2014]



[Leung et al 2018]

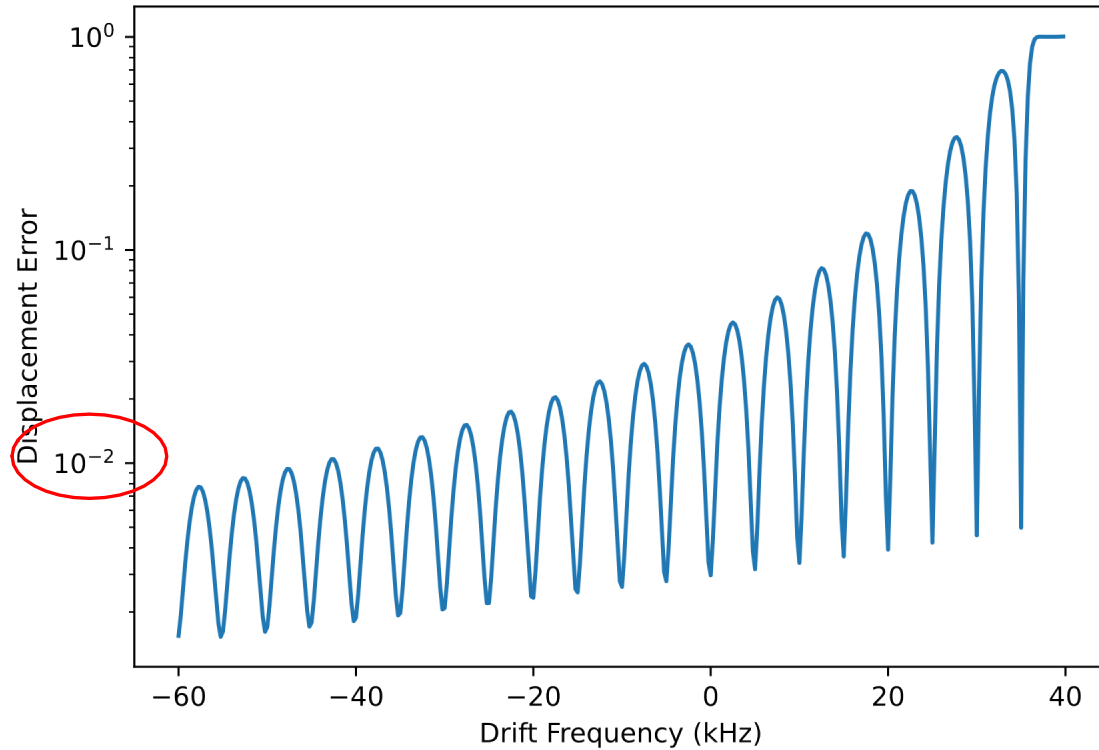


# Gaussian pulse shape is 'naturally' robust to displacement error.

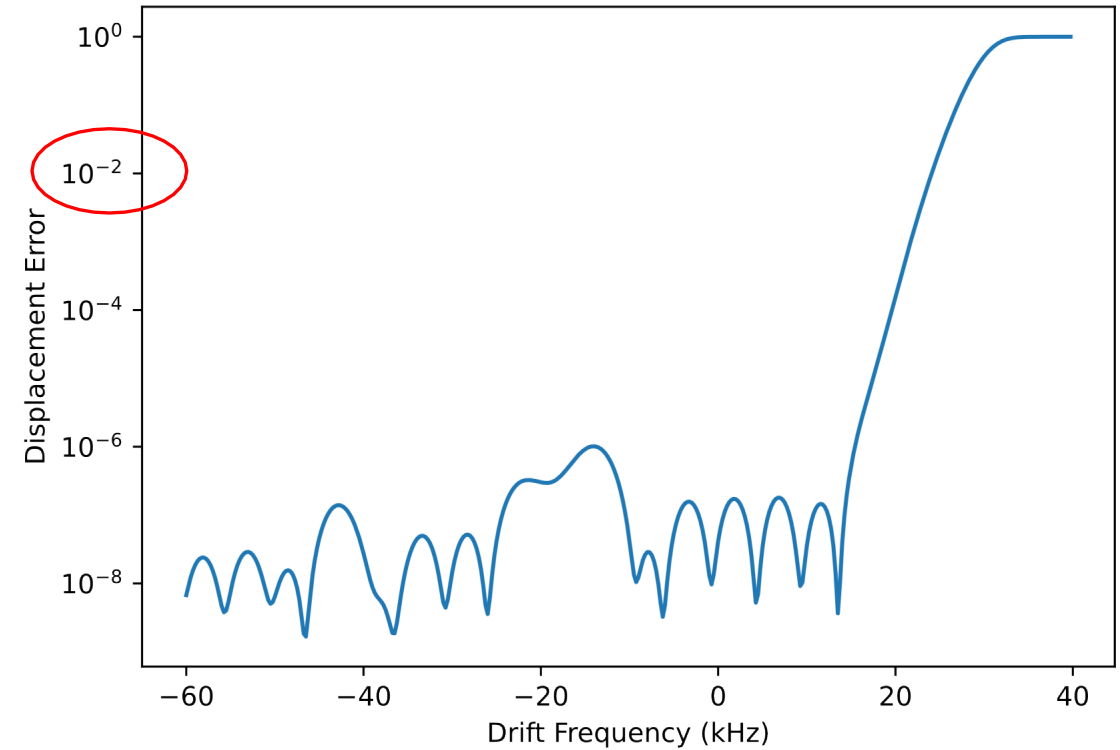


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Square Gate



Gaussian Gate

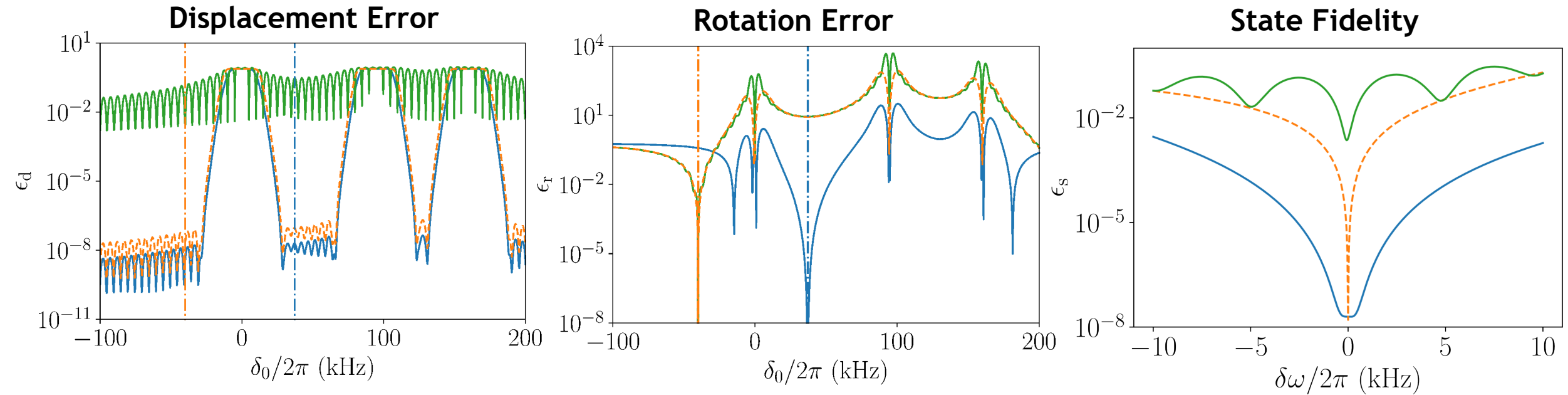


In the Gaussian gate, we still need the right detuning and amplitude to get the right area enclosed, but this shows broadly robust spin-motion disentanglement at the end of a Gaussian gate.

# 9 'Balanced' gaussian gates take care of both rotation and displacement error



Gaussian amplitude modulation and a specific frequency give broad robustness to frequency error



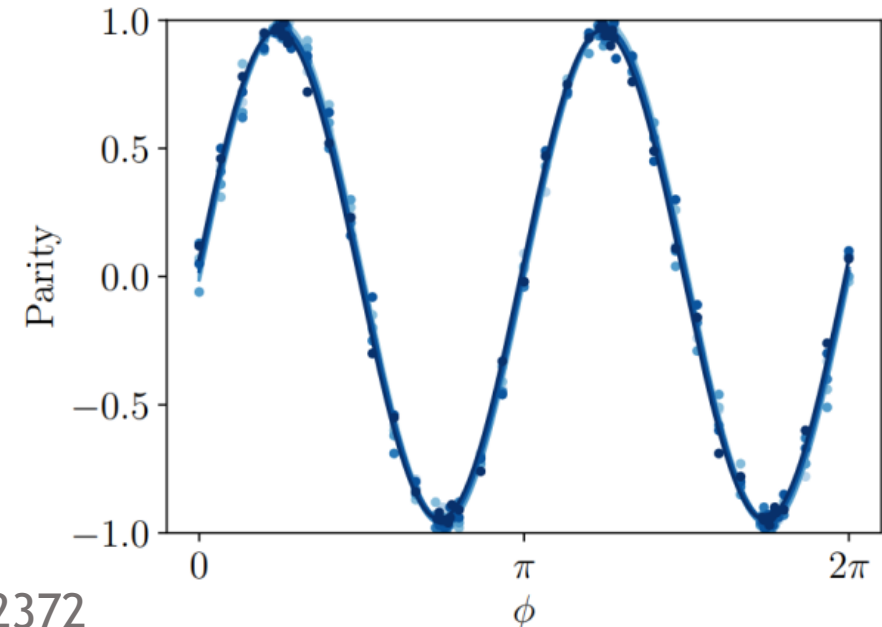
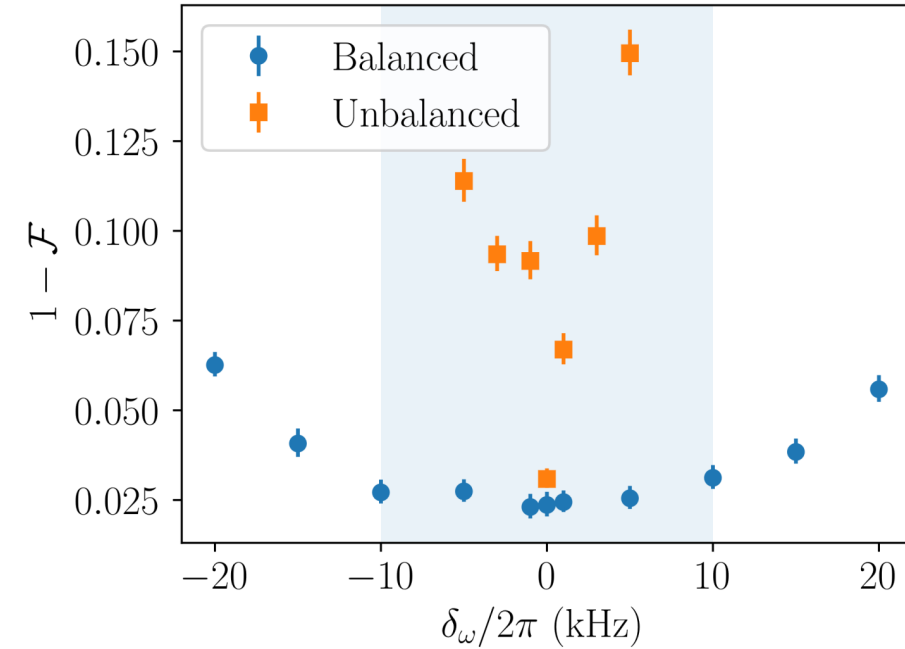
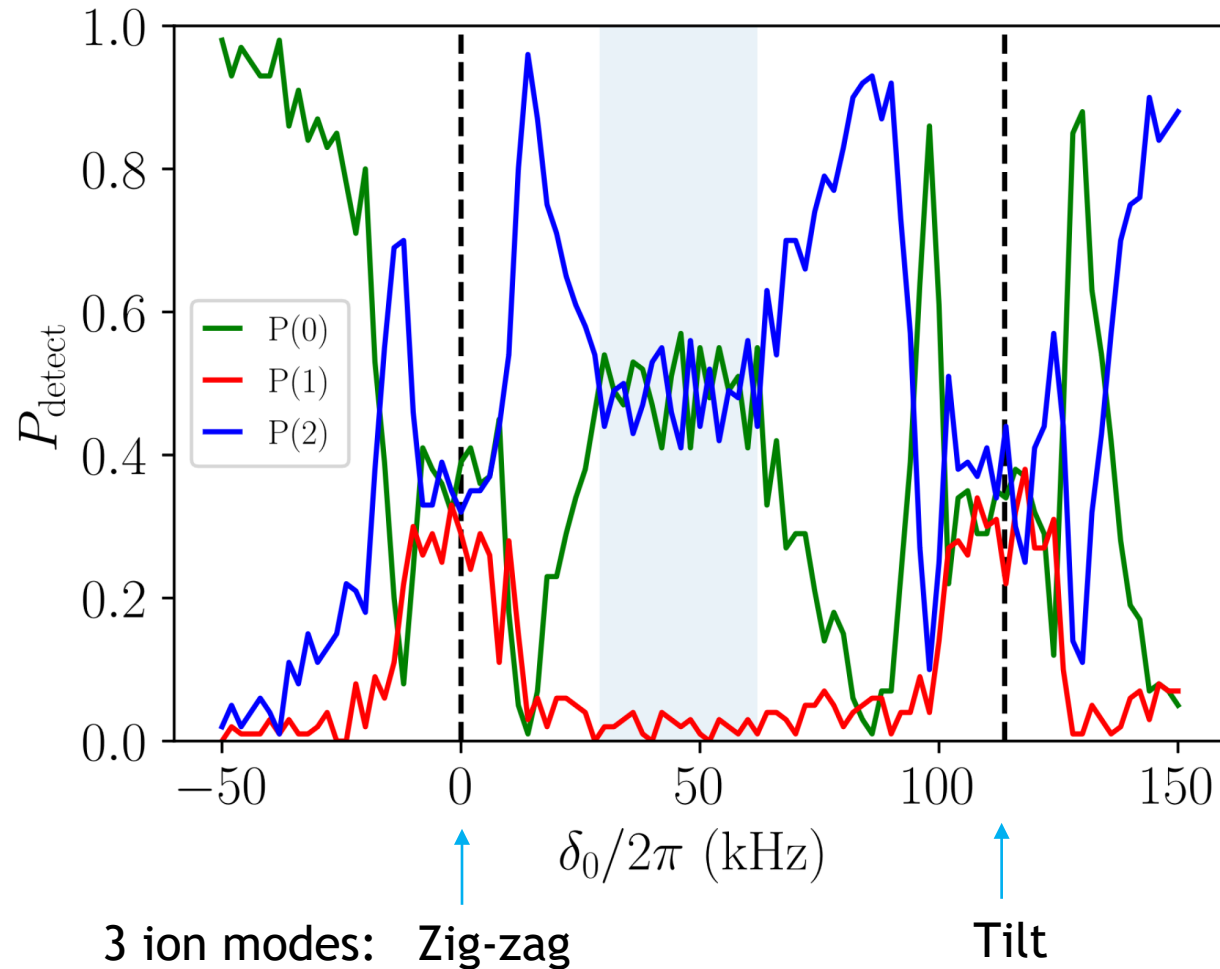
Pulse shape: **Balanced Gaussian**, **Gaussian**, **Square**

Gates are simple to implement: no need to optimize tons of pulse-shape parameters

# Experiment shows balanced Gaussian is robust to $\pm 10$ kHz trap frequency drift

Peak fidelity = 97.7(4)%

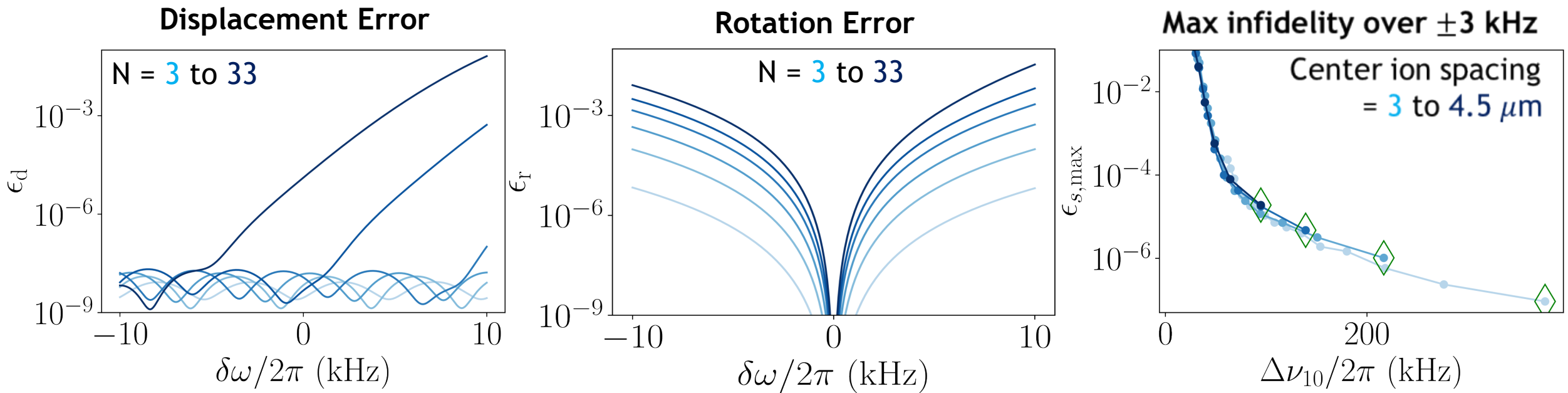
Drop in fidelity over  $\pm 10$  kHz < 1%



# We find good prospects for scaling to more ions.



- Numerical simulations for chains of up to 33 ions
- Fixed center ion separation: results shown are for  $3 \mu\text{m}$



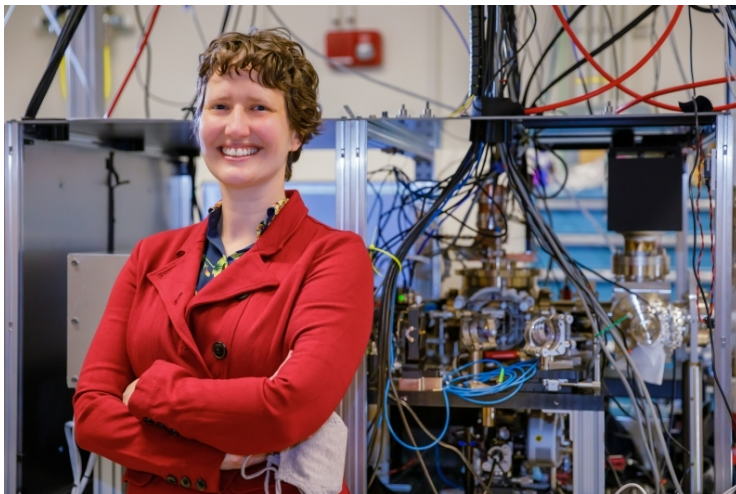
- Sensitivity (right plot) depends almost entirely on the splitting of nearest two modes

# QSCOUT Team

Email: [qscout@sandia.gov](mailto:qscout@sandia.gov) to be added to mailing list

Website: [qscout.sandia.gov](http://qscout.sandia.gov)

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Corrie Sadler  
Becky Loviza  
John Rembetski  
Eric Ou  
Matt Delaney

arXiv : 2210.02372



## Users



Tameem Albash  
Elizabeth Crosson  
Namitha Pradeep

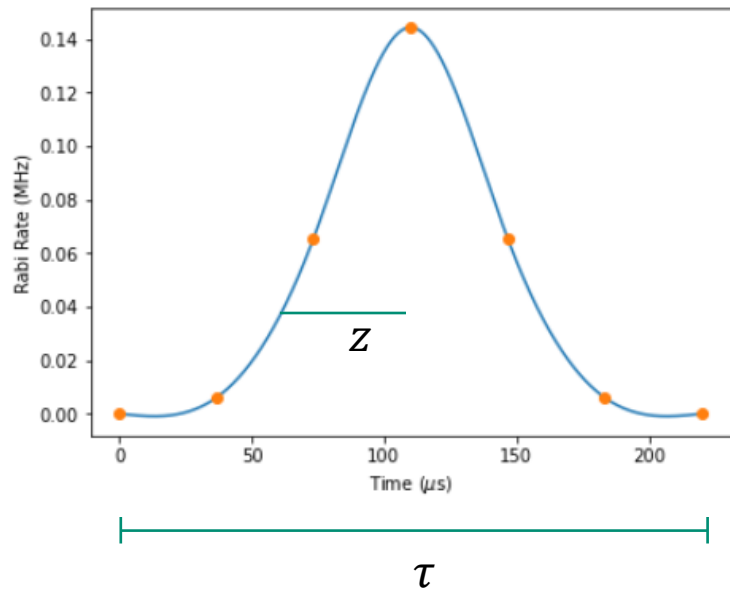
Berkeley  
UNIVERSITY OF CALIFORNIA



# Extra Slides Beyond

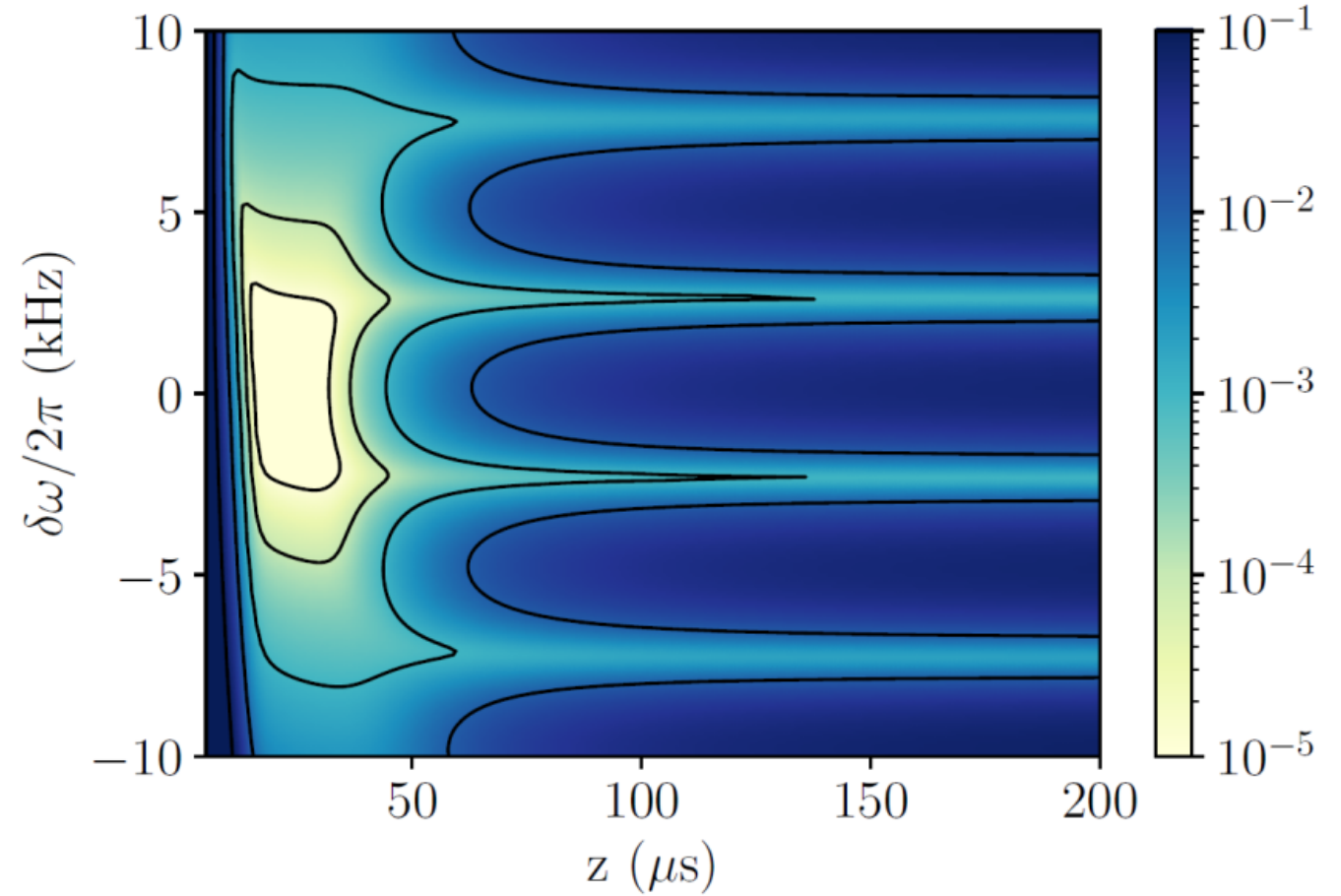


# Gaussian pulse parameters



$z$  : time-domain standard deviation

$\tau$  : gate time, truncation window

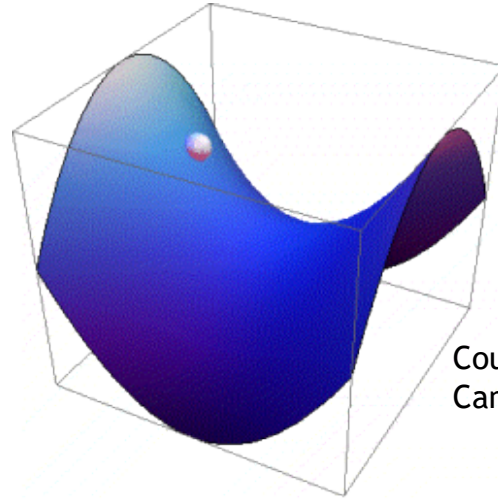


# RF Traps for Ions



Trapping requirement: A restoring force when displaced from trap center  
(in any direction)

Cannot use  
t  
Field lines  
start/en

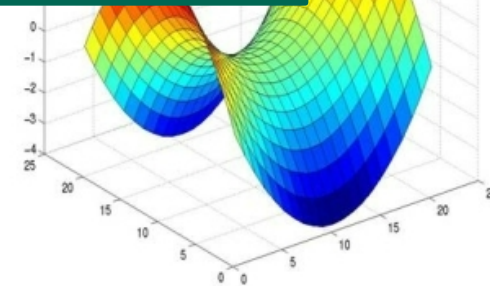
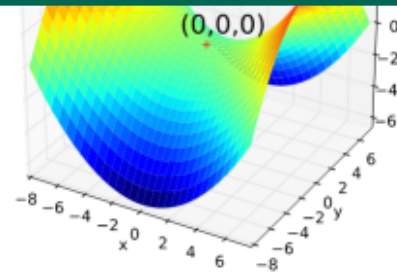


Courtesy of Wes Campbell

ut” and  
rections



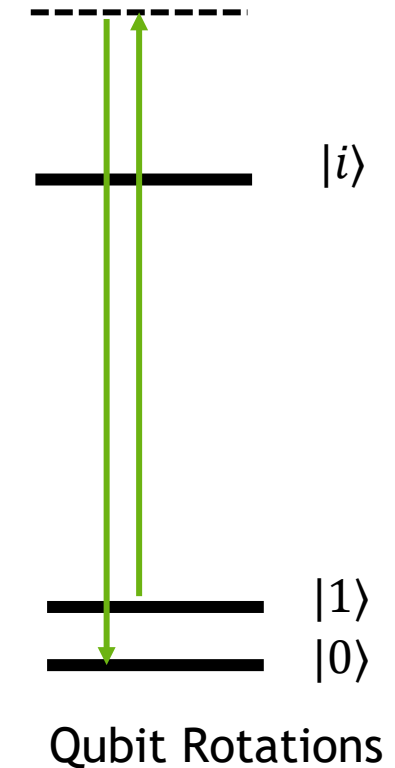
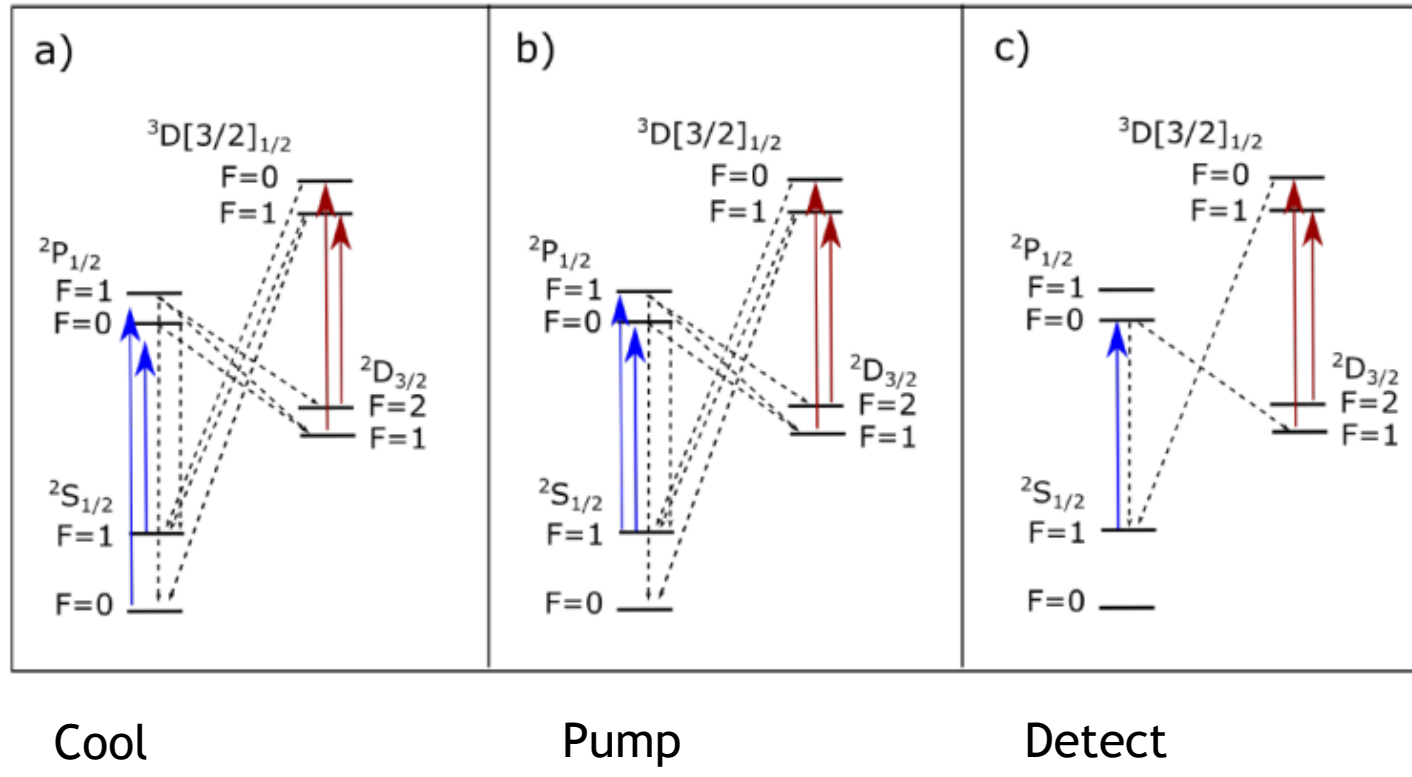
Before ion escapes, field reverses!



# Crash Course in Trapped Ion Quantum Computing

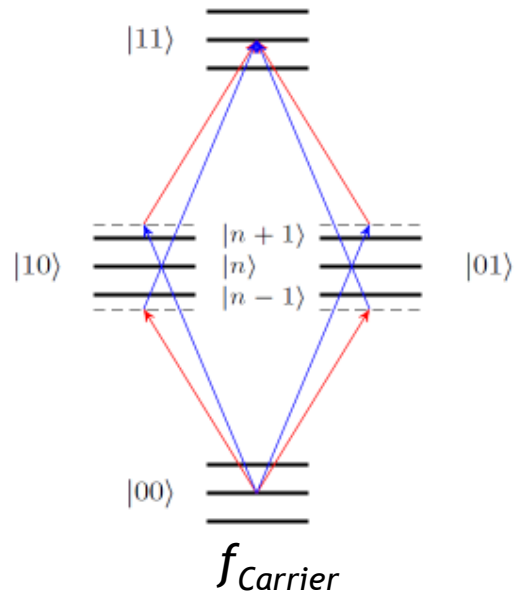


$^{171}\text{Yb}^+$



- Sidebands on cooling laser (370nm) allow incoherent control processes
- Pulsed laser (355nm) Raman transitions for qubit rotations

# The Mølmer-Sørensen (MS) gate is derived from a spin-dependent force.



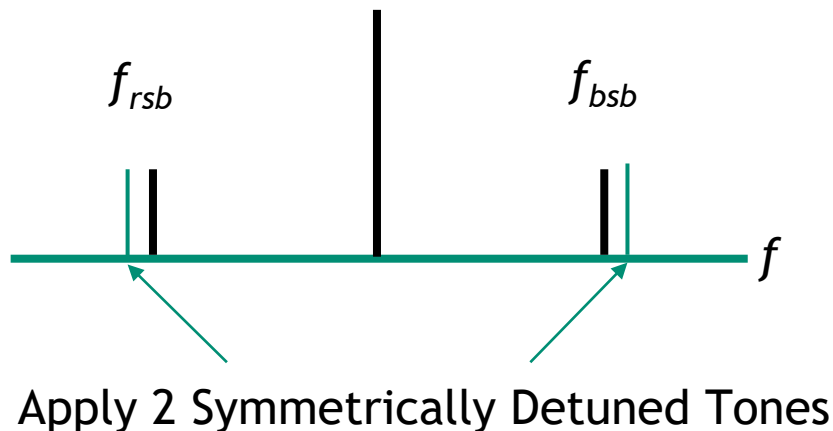
Spin dependent force<sup>1</sup>:

$$\hat{H}_I = \frac{\hbar\eta\Omega}{2} (\hat{\sigma}_+ \hat{a} e^{-i(\delta t - \phi_r)} + \hat{\sigma}_- \hat{a}^\dagger e^{i(\delta t - \phi_r)}) + \frac{\hbar\eta\Omega}{2} (\hat{\sigma}_+ \hat{a}^\dagger e^{i(\delta t + \phi_b)} + \hat{\sigma}_- \hat{a} e^{-i(\delta t + \phi_b)})$$

$$\hat{H}_I = \frac{\hbar\eta\Omega}{2} (\hat{a} e^{-i(\delta t + \phi_M)} + \hat{a}^\dagger e^{i(\delta t + \phi_M)}) \hat{\sigma} \cdot \phi_s$$

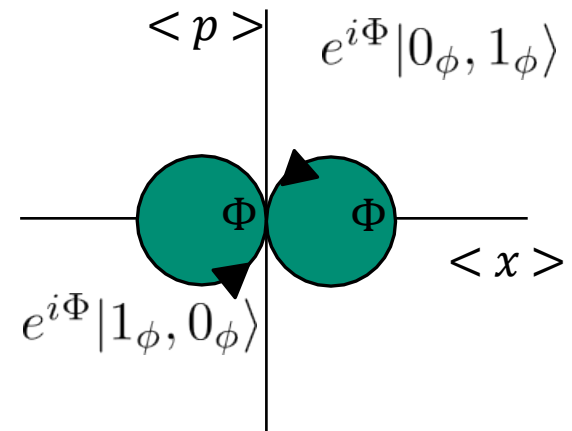
Jaynes-Cummings +  
Anti Jaynes-Cummings

Rearrange into Spin  
and Motion Phases



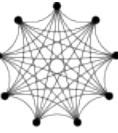
Drive loops in phase space, pick up geometric phase.

- Spin and motional parts of the wavefunction are disentangled when phase loops close.
- Fully Entangling Gate encloses a phase space area of  $\frac{\pi}{2}$



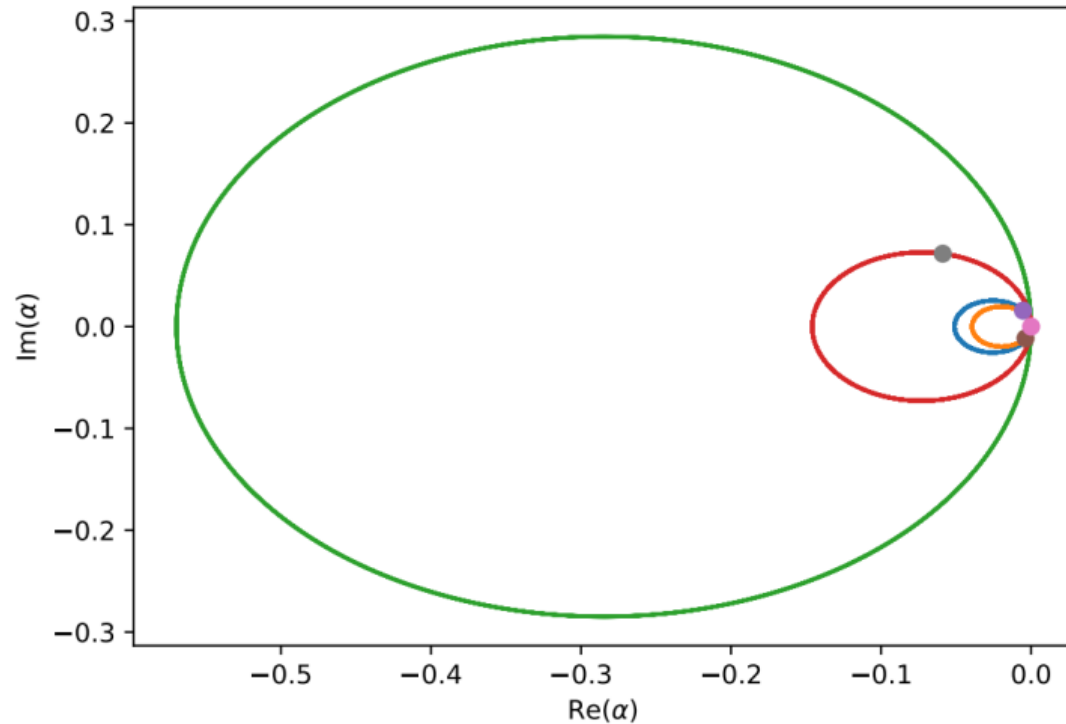
<sup>1</sup> Lee, Patricia J., "Quantum Information Processing with Two Trapped Cadmium Ions", Univ of Michigan, 2006

# Gaussian MS Gates Show Better Loop Closure than Square MS

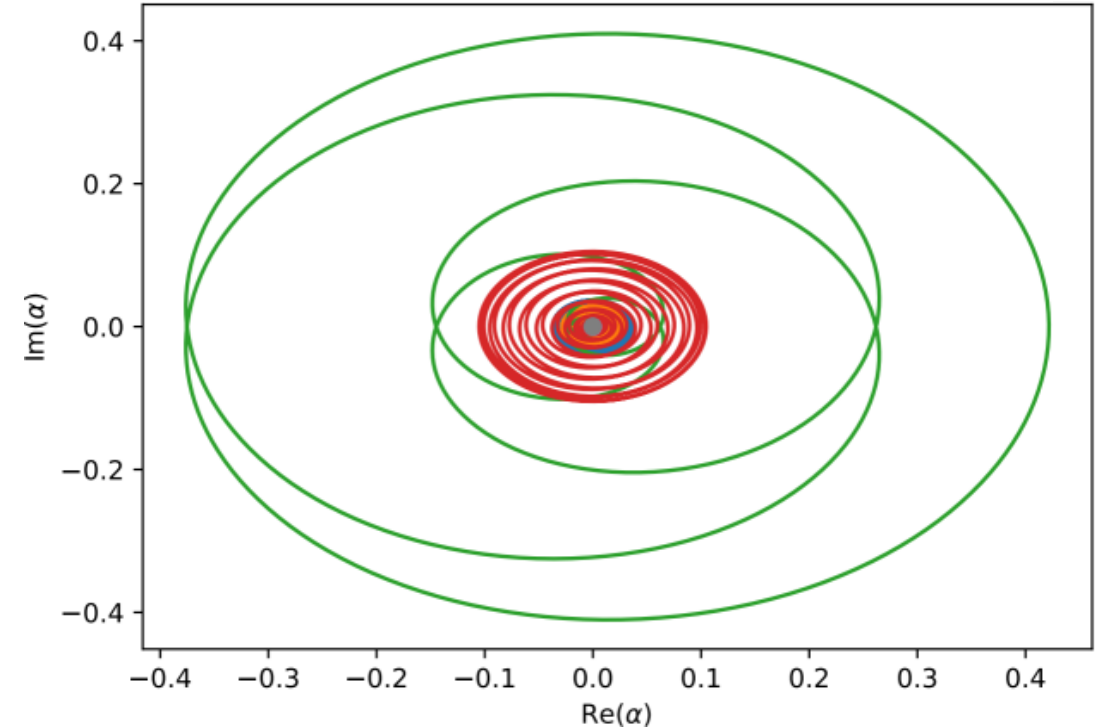


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## Square Gate



## Gaussian Gate



### Gate parameters (for both):

- Detuning is -40 kHz below the lowest frequency mode in a 2 ion chain.
- 200  $\mu s$  duration

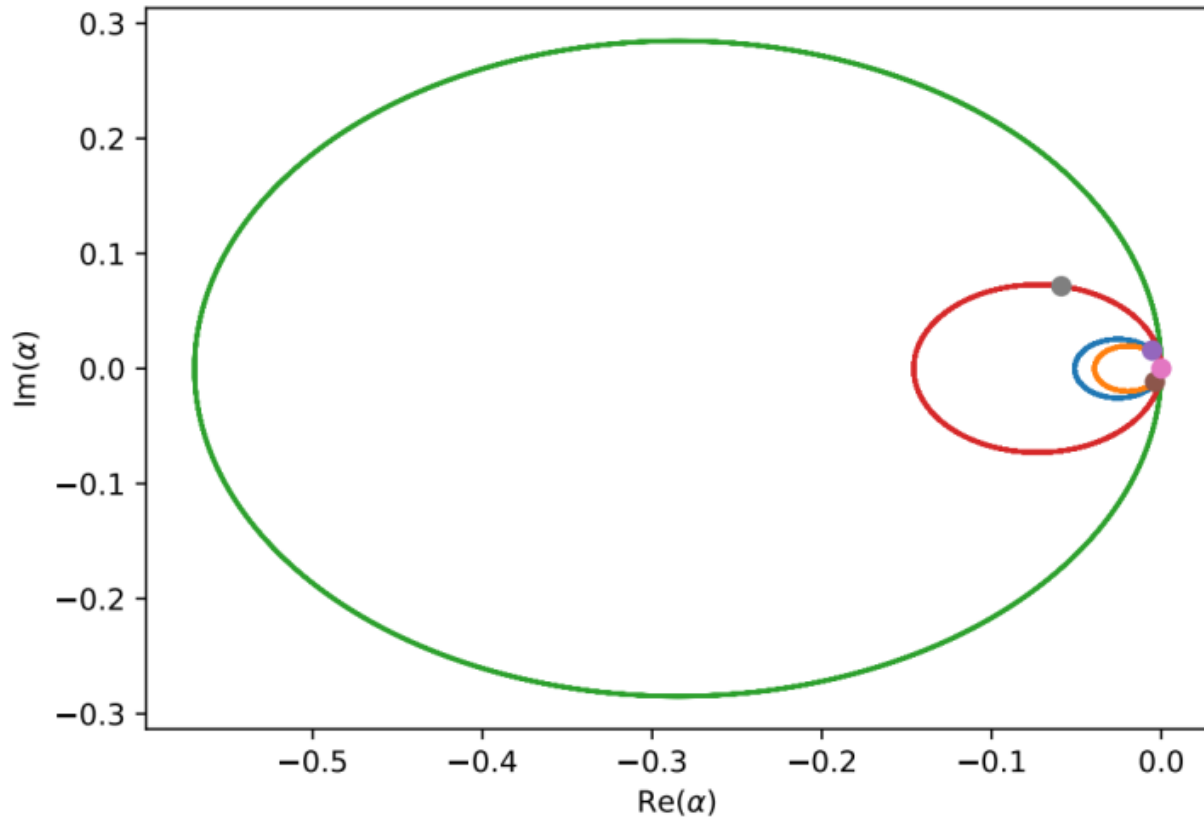
### Notable differences:

- All modes end close to the starting point for Gaussian gate.
- Time averaged displacement close to zero for the Gaussian gate.

# Intuition for Phase Space Trajectories (PSTs)



PST for Square MS Gate



## How pulse parameters come into play:

- Detuning controls the angular velocity and radius in phase space.
  - Smaller detuning  $\rightarrow$  Bigger, slower circles
- Rabi rate controls only radius.
- Phase can change the direction of rotation.

## Figures of merit:

- Loop closure of each mode:  $\alpha_k(0) = \alpha_k(t_{gate})$ ?
- Area enclosed: Is the gate angle right?

# Displacements During the MS Gate



Interaction Hamiltonian:

$$\hat{H}_I = \frac{\hbar\eta\Omega}{2} (\hat{a}e^{-i(\delta t + \phi_M)} + \hat{a}^\dagger e^{+i(\delta t + \phi_M)}) \hat{\sigma} \cdot \phi_s$$

State dependent drive force gives coherent displacement by  $\alpha_k$  on mode  $k$  [1]:

$$\hat{D}(\hat{\alpha}_k) = \exp(\hat{\alpha}_k a_k^\dagger - \hat{\alpha}_k^\dagger a_k)$$

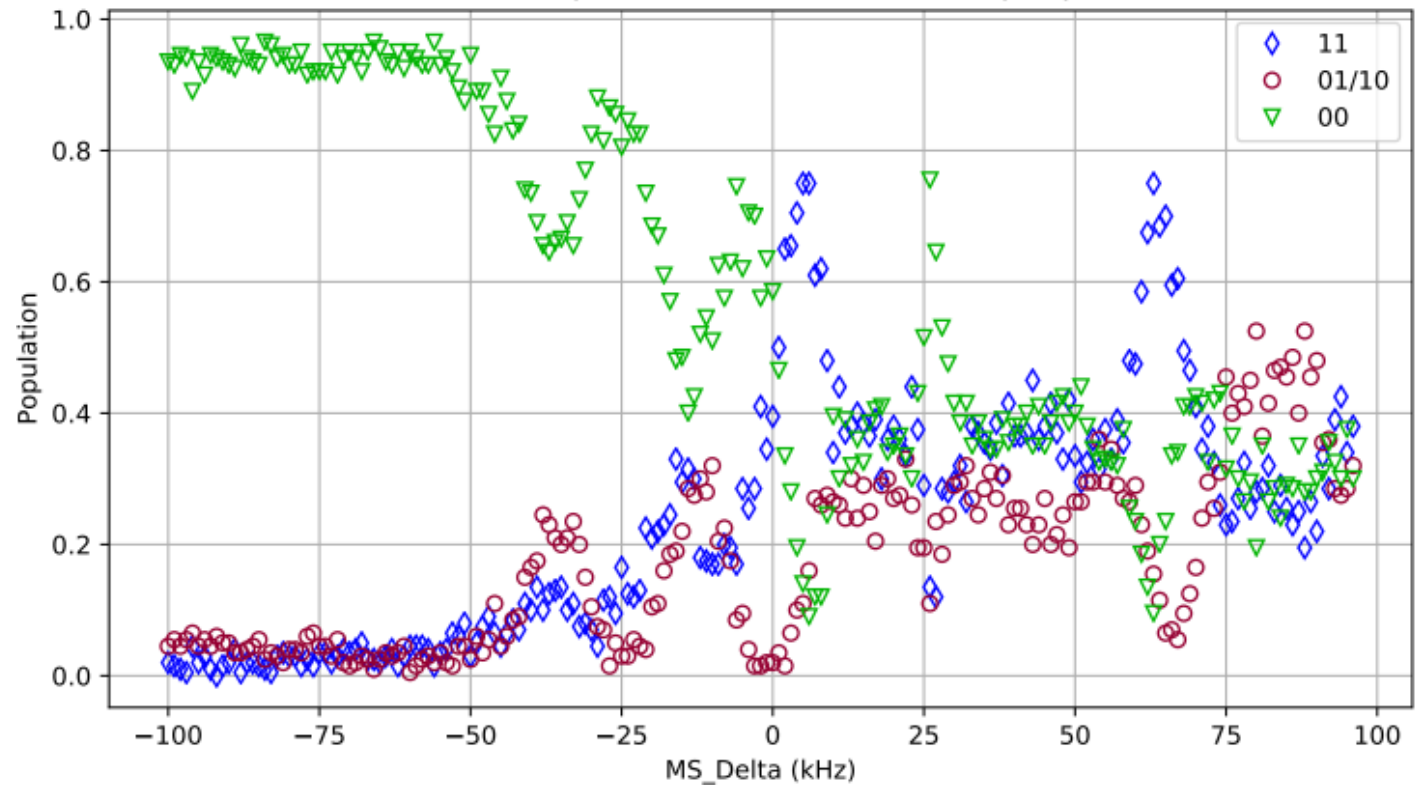
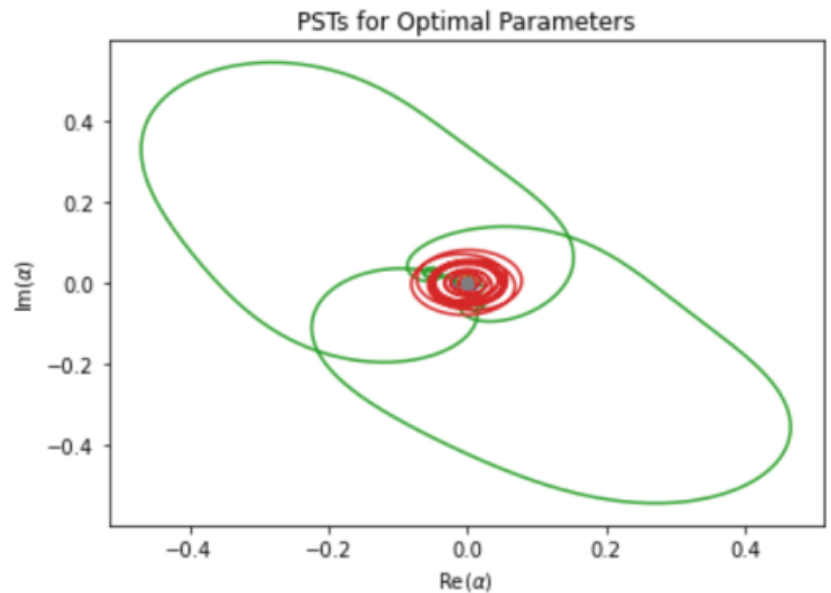
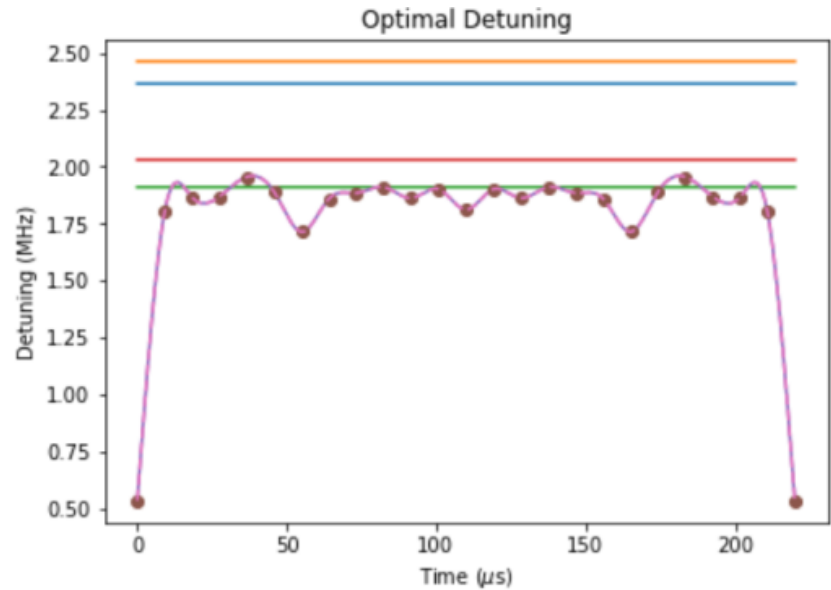
In the simplest case (+1 eigenstate of  $\hat{\sigma}_\phi$ , no laser phase difference) [1]:

$$\alpha_k = \frac{\Omega}{2} (\eta_{i,k} + \eta_{j,k}) \int_0^t e^{i\theta_k(t')} dt' \text{ where } \theta_k(t') = \int_0^{t'} \delta_k(t'') dt'' \text{ is the accumulated phase}$$

So, for a constant detuning and Rabi rate, we get circular trajectories in phase space, with closure condition:  $t_{gate} = 2\pi m / \delta_k$

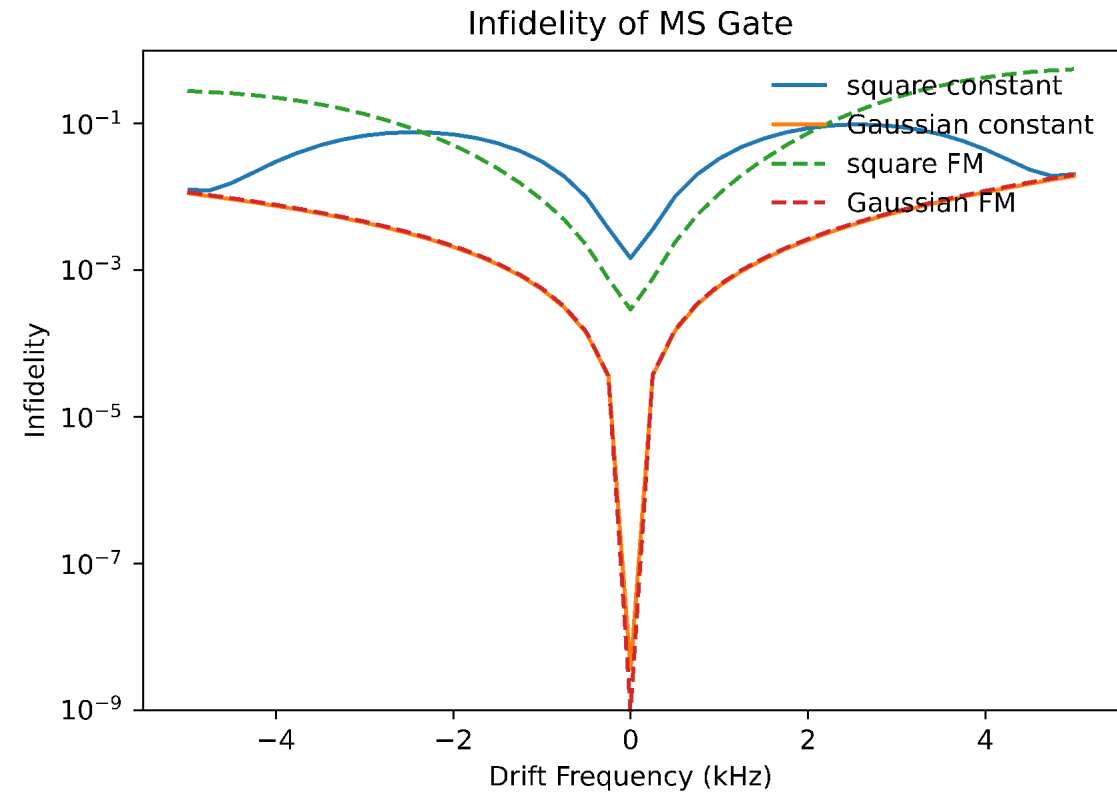
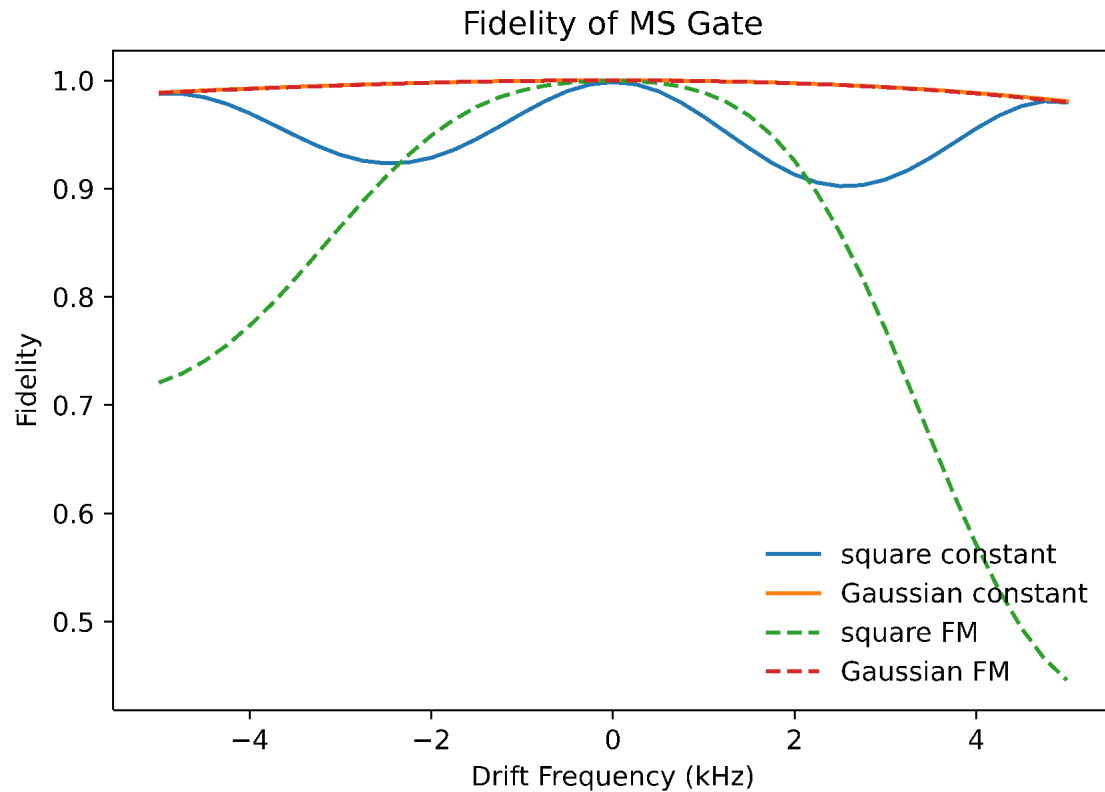
Closed loops are required for spin-motion disentanglement at the end of the gate.

# FM and Constant Frequency Gaussians Are Comparable



- See essentially the same odd parity population near optimized detuning.
- Area enclosed still has roughly the same sensitivity (slope of 11/00 crossing) as constant frequency case.

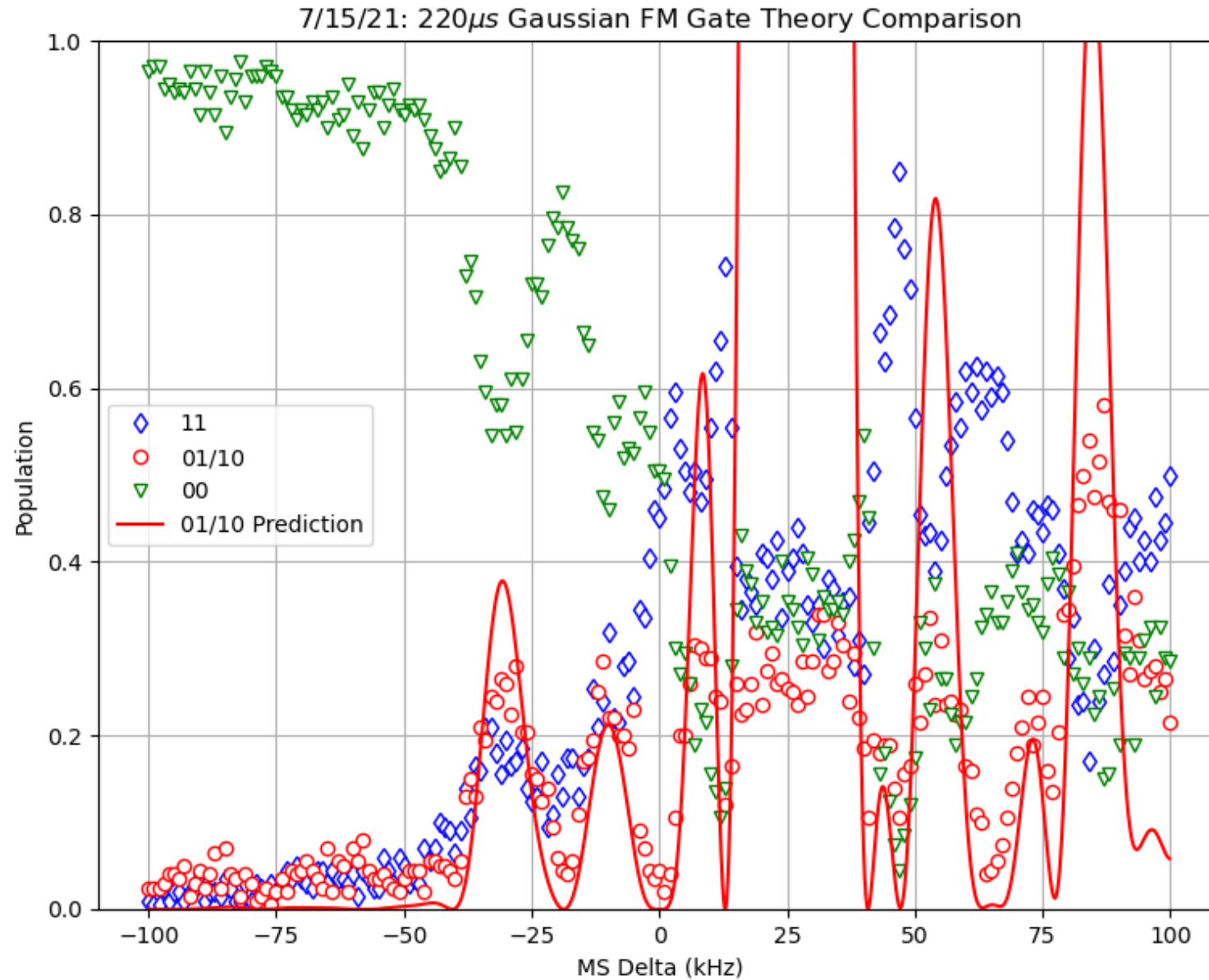
# Gaussian Pulse Shape Dominates Gate Performance (with Current Cost Functions)



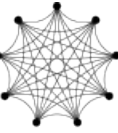
## Next steps:

- Further development of gate angle cost function
- Batch optimization
- Transfer function optimization
- Larger chains

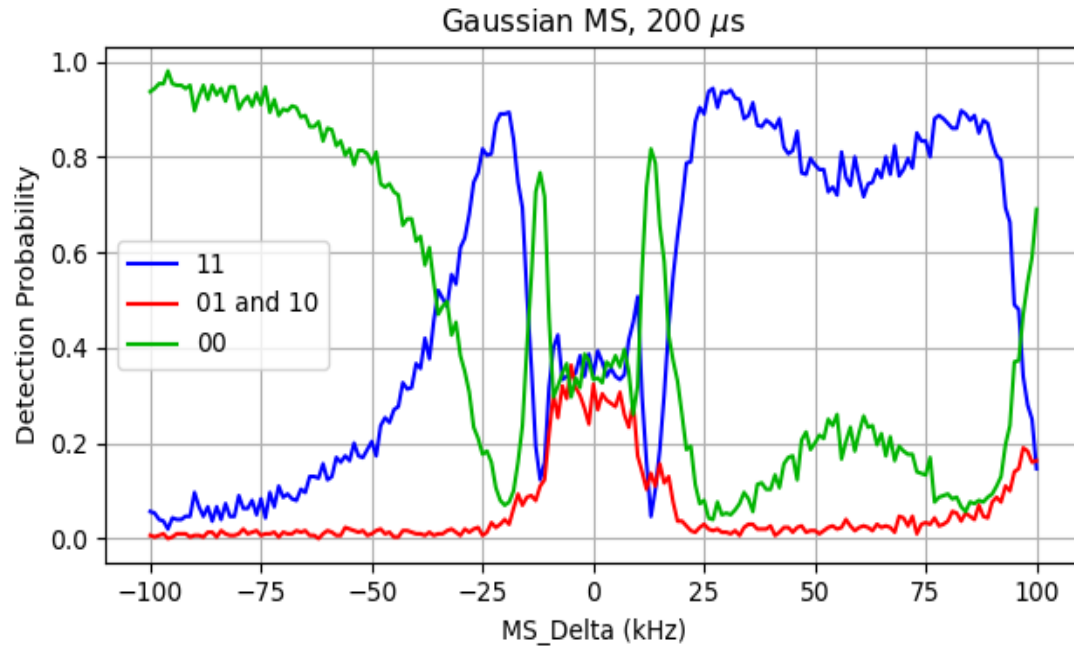
# Optimizer/Data Comparison For FM Gaussian



# Constant Frequency Gaussian MS Gates Work Well



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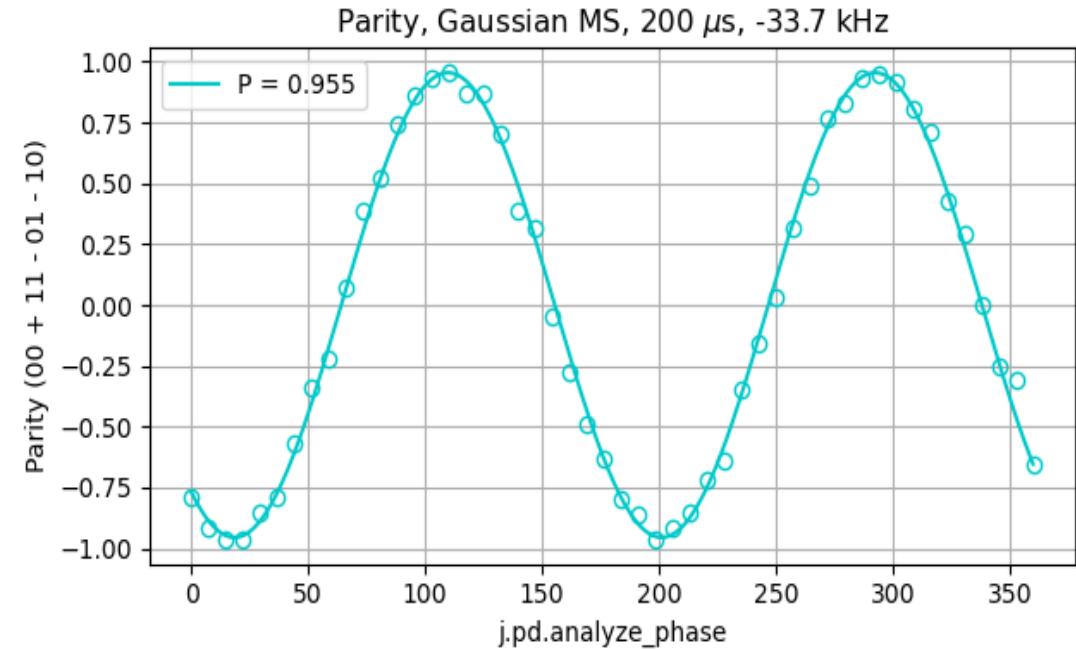


## Mølmer-Sørensen Entangling Gate

- $|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

## Implementing with Octet

- Gaussian pulse shaping drastically improved our two-qubit gate performance



**Entanglement Fidelity  $\sim 0.98$**

- Narrow acceptable detuning and amplitude range
- Area enclosed sensitive to small changes in trap RF