SAND2022-13151C

Sensitivity Analysis in Coupled Radiation Transport Simulations

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Introduction

- Sensitivity coefficients describe the fractional change in a response that is due to perturbations or uncertainties in system parameters.

[illegible]

- Several codes (TSUNAMI, Serpent, MCNP, OpenMC) estimate sensitivity coefficients for critical eigenvalue or reaction rate ratio responses:

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- This new sensitivity methodology was implemented into a 1-D Monte Carlo code and Direct Perturbation calculations were used to confirm its accuracy.

Computing Adjoints using Contributon Theory

$$\square = \square\square\,\Phi$$

$$\langle \Phi^{\dagger} Q \rangle = \langle \lambda\, \Phi^{\dagger} P \Phi \rangle$$

$$Q = Q_s \delta(\tau - \tau_s)$$

$$\Phi^{\dagger}(\tau_s) = \langle \lambda\, \Phi^{\dagger}(r) P(r) \Phi(\tau_s \rightarrow \mathrm{r}) \rangle$$

Generalized Perturbation Theory

- Generalized Perturbation Theory (GPT) calculates sensitivity coefficients for responses that can be expressed as the ratio of reaction rates.

$$S_{R,\Sigma_x} = \frac{\delta R/R}{\delta\Sigma_x/\Sigma_x} \qquad R = \frac{\langle \Sigma_1 \phi \rangle}{\langle \Sigma_2 \phi \rangle}$$

- Calculating generalized sensitivity coefficients requires solving an inhomogeneous, or generalized, adjoint equation:

$$(L^* - \lambda P^*)\Gamma^* = S^*$$

- Applications for GPT sensitivity/uncertainty analysis include:
  - Relative Powers
  - Isotope Conversion Ratios
  - Multigroup Cross Sections

+ C. M. Perfetti, B. T. Rearden, "Development of a Generalized Perturbation Theory Method for Uncertainty and Sensitivity Analysis using Continuous-Energy Monte Carlo Methods," *Nucl. Sci. Eng.*, **182**, *3*, 354–368 (2016).

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Radiation Transport Equations

Neutron Transport

$$(L - \lambda P)\phi \;= 0$$

Photon + Electron Transpor

Photons: $T\,\Phi = S_{total} = S_{pho.} + P\,\psi$

Electrons: $t\,\psi = s_{total} = s_{ele.} + p\,\Phi$

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Responses of Interest

$$R \equiv \frac{\langle \Sigma_1 \phi \rangle}{\langle \Sigma_2 \phi \rangle}$$

$$R \equiv \langle \Sigma_{\mathrm{R},pho.} \Phi \rangle$$

$$R \equiv \langle \Sigma_{\mathrm{R},ele.} \psi \rangle$$

$$R \equiv \langle \Sigma_{\mathrm{R},pho.} \Phi \rangle + \langle \Sigma_{\mathrm{R},ele.} \psi \rangle$$

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Adjoint Radiation Transport Equations

Neutron Transport

$$\left(L^{\dagger} - \lambda P^{\dagger}\right)\Gamma^{\dagger} = S^{\dagger}$$

Photon + Electron Transpor

Photons: $\dagger \quad \dagger = \dagger = \dagger + \dagger \quad \dagger$

Electrons: $\dagger \quad \dagger = \dagger = \dagger + \dagger \quad \dagger$

Adjoint Source Terms

$$S^{\dagger} \equiv \frac{1}{R}\frac{\delta R}{\delta \phi} = \frac{\Sigma_1}{\langle \Sigma_1 \phi \rangle} - \frac{\Sigma_2}{\langle \Sigma_2 \phi \rangle}$$

Photons: $S^{\dagger} \equiv \frac{1}{R}\frac{\delta R}{\delta \Phi} = \Sigma_{\mathrm{R},pho.}$

Electrons: $s^{\dagger} \equiv \frac{1}{R}\frac{\delta R}{\delta \psi} = \Sigma_{\mathrm{R},ele.}$

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Deriving GPT Sensitivity Coefficients for Coupled Transport

Introducing perturbations to the BTEs, multiplying by $\Phi^{\dagger}$ and $\psi^{\dagger}$, and taking the inner product:

Photons: $\langle \Phi^{\dagger} \delta T \, \Phi \rangle + \langle \Phi^{\dagger} T \, \delta\Phi \rangle = \langle \Phi^{\dagger} \delta S \rangle + \langle \Phi^{\dagger} \delta P \, \psi \rangle + \langle \Phi^{\dagger} P \delta\psi \rangle$

Electrons: $\langle \psi^{\dagger} \delta t \, \psi \rangle + \langle \psi^{\dagger} t \, \delta\psi \rangle = \langle \psi^{\dagger} \delta s \rangle + \langle \psi^{\dagger} \delta p \, \Phi \rangle + \langle \psi^{\dagger} p \, \delta\Phi \rangle$

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Multiplying the adjoint BTEs by $\delta\Phi$ and $\delta\psi$, taking the inner product, and applying the Property of the Adjoint:

Photons: $$\langle \Phi^\dagger T\, \delta\Phi \rangle = \langle \delta\Phi\, S^\dagger \rangle + \langle \psi^\dagger p\, \delta\Phi \rangle$$

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$$
\begin{aligned}
\frac{\delta R^{indirect}}{R} &= \left\langle \frac{1}{R} \frac{\partial R}{\partial \Phi} \delta\Phi \right\rangle + \left\langle \frac{1}{R} \frac{\partial R}{\partial \psi} \delta\psi \right\rangle \\
&= \langle \delta\Phi\, S^{\dagger} \rangle + \langle \delta\psi\, s^{\dagger} \rangle \\
&= \langle \Phi^{\dagger} \delta S \rangle - \langle \Phi^{\dagger} \delta T\, \Phi \rangle + \langle \Phi^{\dagger} P \delta\psi \rangle \\
&\quad + \langle \psi^{\dagger} \delta s \rangle - \langle \psi^{\dagger} \delta t\, \psi \rangle + \langle \psi^{\dagger} \delta p\, \Phi \rangle
\end{aligned}
$$

Deriving GPT Sensitivity Coefficients for Coupled Transport

$$\frac{\delta R^{indirect}}{R} = \left\langle \frac{1}{R}\frac{\partial R}{\partial \Phi}\delta\Phi \right\rangle + \left\langle \frac{1}{R}\frac{\partial R}{\partial \psi}\delta\psi \right\rangle$$
$$= \langle \delta\Phi\, S^{\dagger} \rangle + \langle \delta\psi\, s^{\dagger} \rangle$$
$$= \langle \Phi^{\dagger}\delta S \rangle - \langle \Phi^{\dagger}\delta T\, \Phi \rangle + \langle \Phi^{\dagger} P \delta\psi \rangle$$
$$+ \langle \psi^{\dagger}\delta s \rangle - \langle \psi^{\dagger}\delta t\, \psi \rangle + \langle \psi^{\dagger}\delta p\, \Phi \rangle$$

Through Contributon Theory:

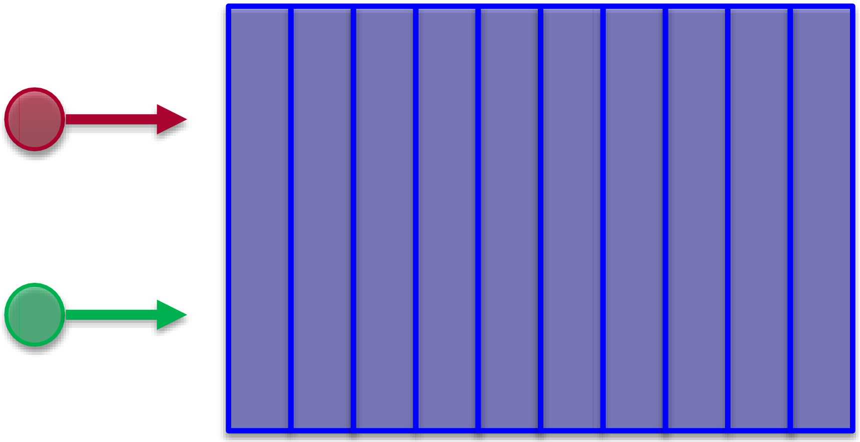
$$\Phi^{\dagger}(\tau) = \langle \Phi(\tau \to r)\, S^{\dagger}(r) \rangle + \langle \psi^{\dagger}(r)\, P(r)\, \Phi(\tau \to r) \rangle$$
$$\psi^{\dagger}(\tau) = \langle \psi(\tau \to r)\, s^{\dagger}(r) \rangle + \langle \Phi^{\dagger}(r)\, p(r) \psi(\tau \to r) \rangle$$

Proof of Principle

- This methodology was implemented and tested in a 1-D Monte Carlo Code.
  - 3 energy groups.
  - Physics set to emulate true physics.
  - Photons were assumed to only produce secondary electrons, and vice versa.
- Response set to the overall photon and electron absorption rates:

$$R_1 \equiv \langle \Sigma_{R,pho.} \Phi \rangle \qquad R_2 \equiv \langle \Sigma_{R,ele.} \psi \rangle$$

- Responses and sensitivities were tallied across 10 equal width regions.
- Reference sensitivities obtained through Direct Perturbation.



Results

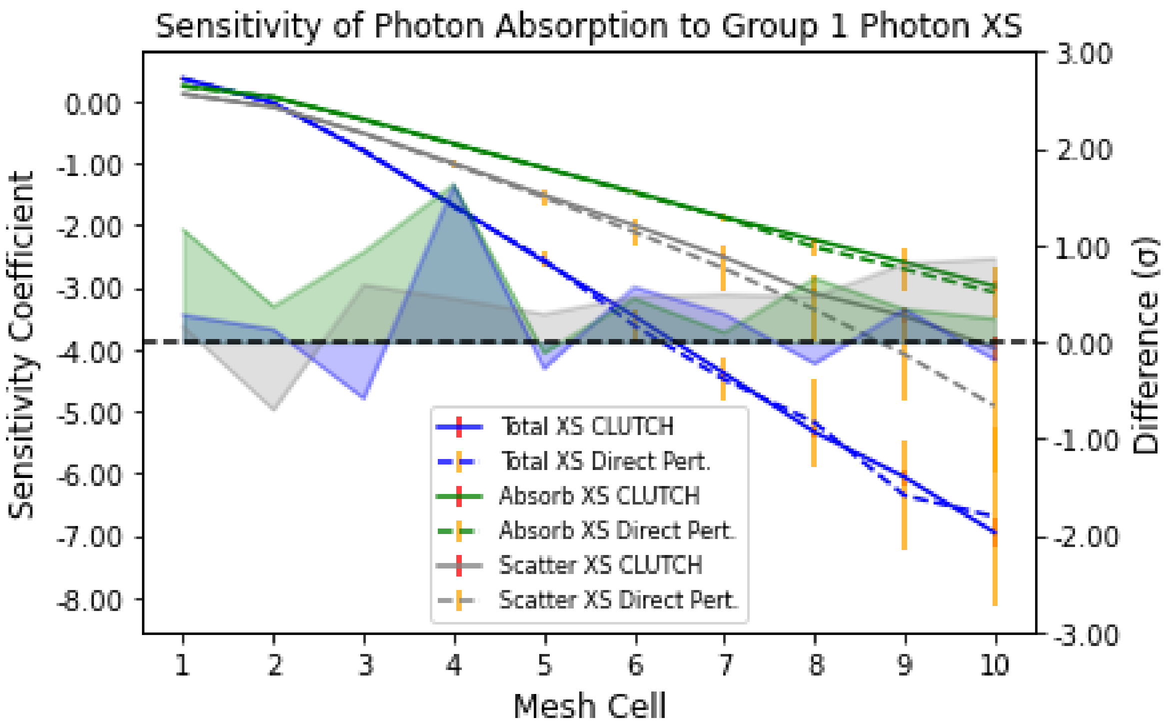
Sensitivity of Photon Absorption to Photon Total Cross Sections

Response in Cell:	Calculated Sensitivity	Direct Pert. Sensitivity	Diff.
Photon Absorption Response Sensitivity			
1	0.368 ± 0.001	0.365 ± 0.010	0.28 σ
2	-0.023 ± 0.003	-0.025 ± 0.013	0.13 σ
3	-0.802 ± 0.006	-0.783 ± 0.033	-0.58 σ
4	-1.674 ± 0.010	-1.700 ± 0.012	1.62 σ
5	-2.583 ± 0.018	-2.549 ± 0.126	-0.27 σ
6	-3.459 ± 0.030	-3.611 ± 0.264	0.57 σ
7	-4.384 ± 0.048	-4.484 ± 0.335	0.29 σ
8	-5.332 ± 0.079	-5.176 ± 0.709	-0.22 σ
9	-6.055 ± 0.131	-6.353 ± 0.865	0.34 σ
10	-6.952 ± 0.226	-6.694 ± 1.439	-0.18 σ

Sensitivity of Electron Absorption to Photon Total Cross Sections

Response in Cell:	Calculated Sensitivity	Direct Pert. Sensitivity	Diff.
Electron Absorption Response Sensitivity			
1	0.214 ± 0.001	0.211 ± 0.022	0.14 σ
2	0.002 ± 0.005	-0.043 ± 0.024	1.88 σ
3	-0.719 ± 0.010	-0.671 ± 0.053	-0.90 σ
4	-1.612 ± 0.018	-1.669 ± 0.070	0.80 σ
5	-2.544 ± 0.031	-2.338 ± 0.035	-4.42 σ
6	-3.451 ± 0.049	-3.472 ± 0.119	0.16 σ
7	-4.187 ± 0.079	-3.897 ± 0.397	-0.72 σ
8	-5.676 ± 0.136	-4.806 ± 1.113	-0.78 σ
9	-5.923 ± 0.197	-6.728 ± 1.206	0.66 σ
10	-6.924 ± 0.370	-8.258 ± 2.000	0.66 σ

Results: Sensitivity of Photon Absorption to Group 1 Photon XS



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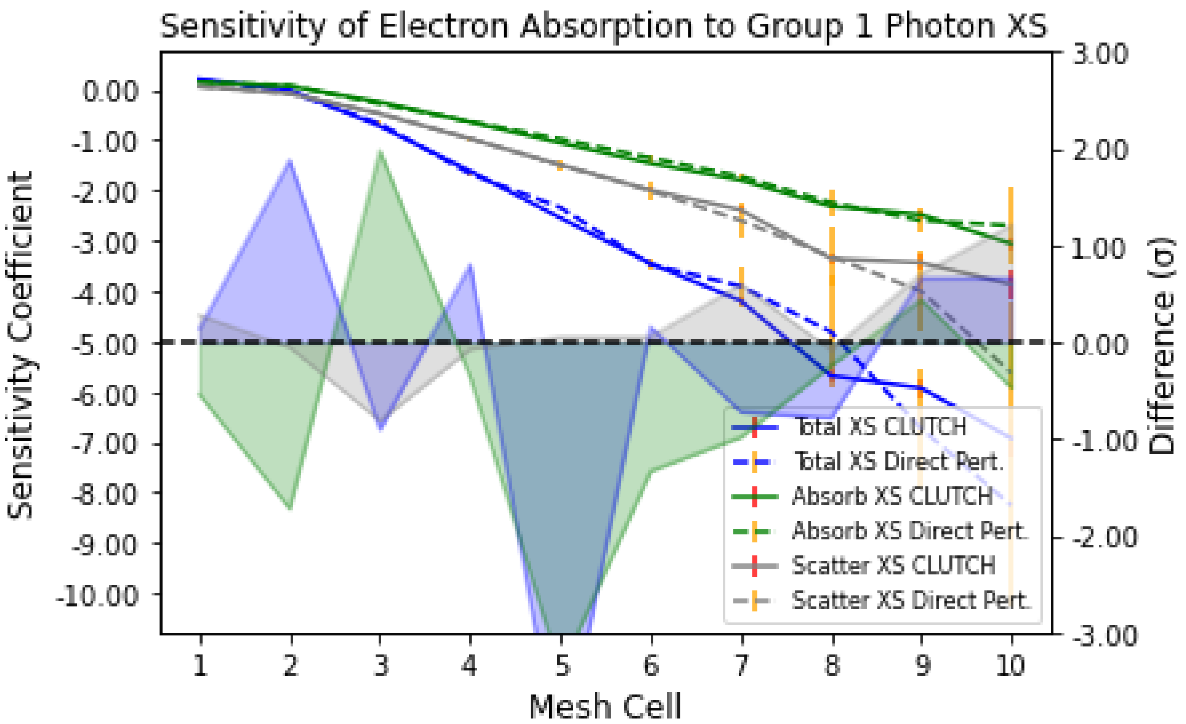
Results: Sensitivity of *Electron* Absorption to Group 1 Photon XS



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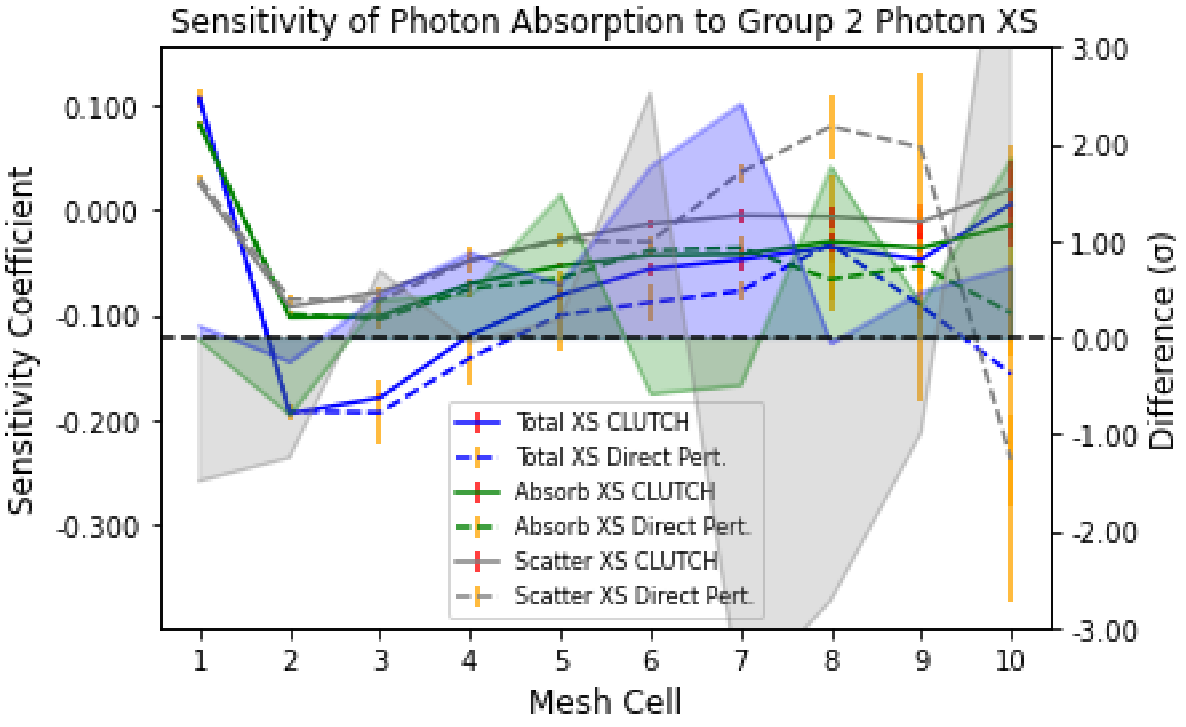
Results: Sensitivity of Photon Absorption to Group 2 Photon XS



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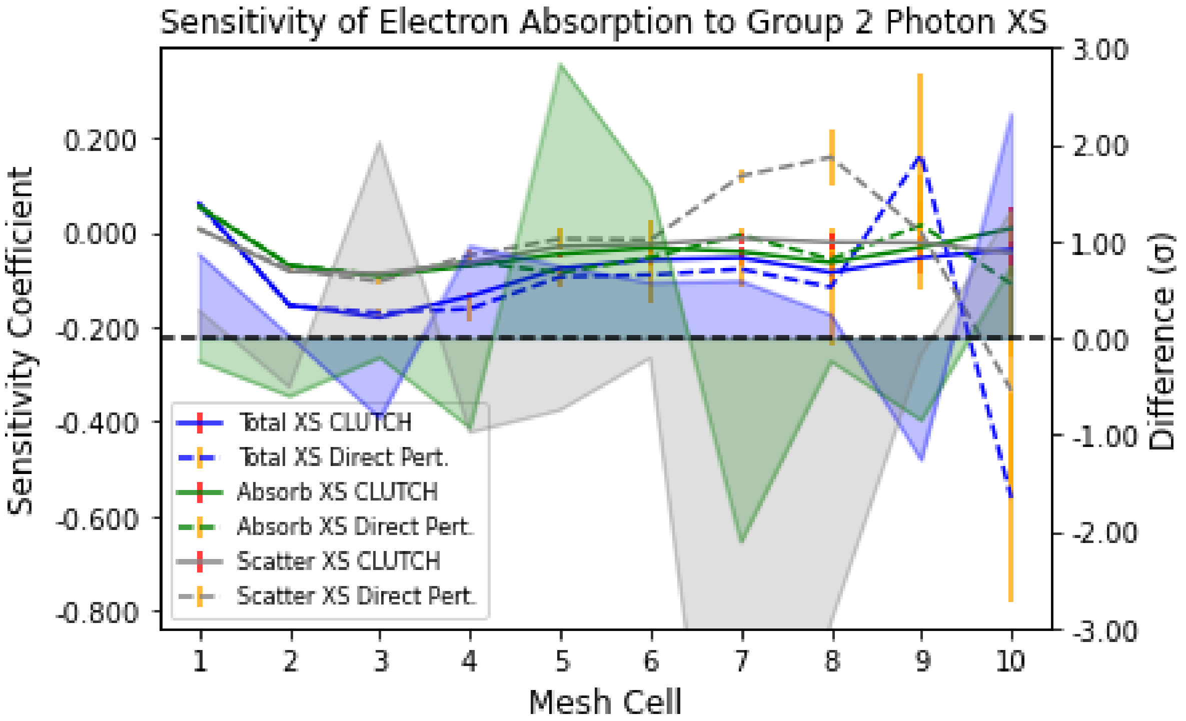

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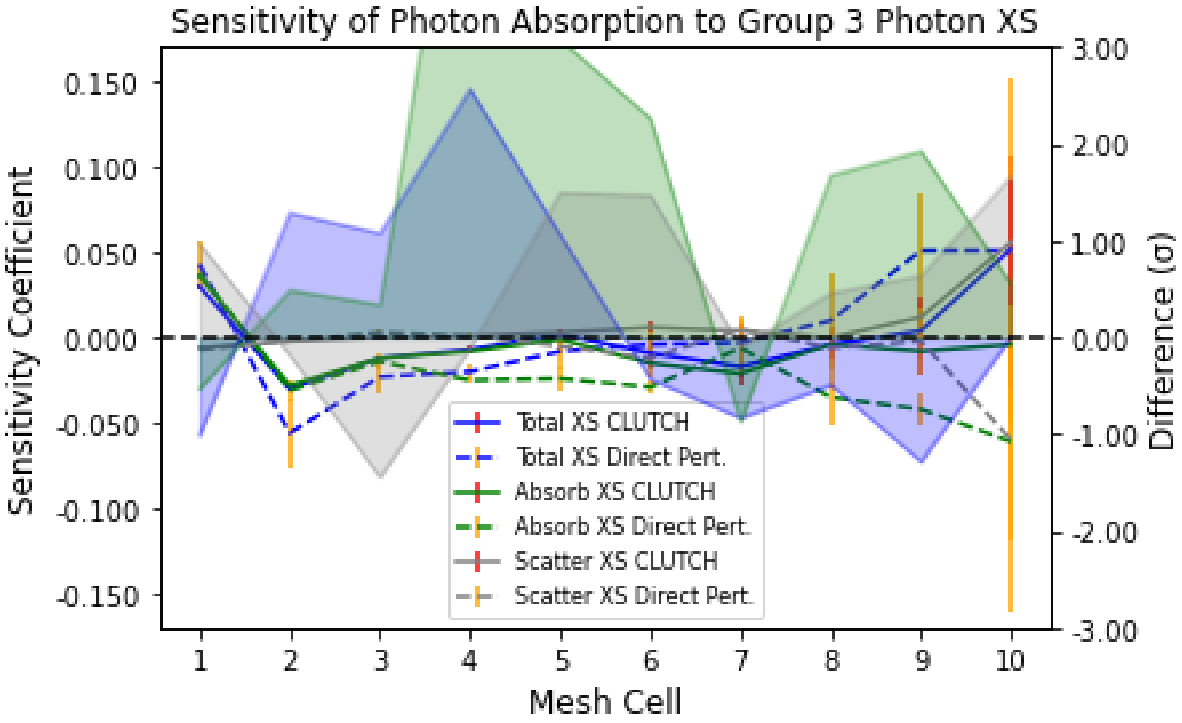
Results: Sensitivity of Photon Absorption to Group 3 Photon XS



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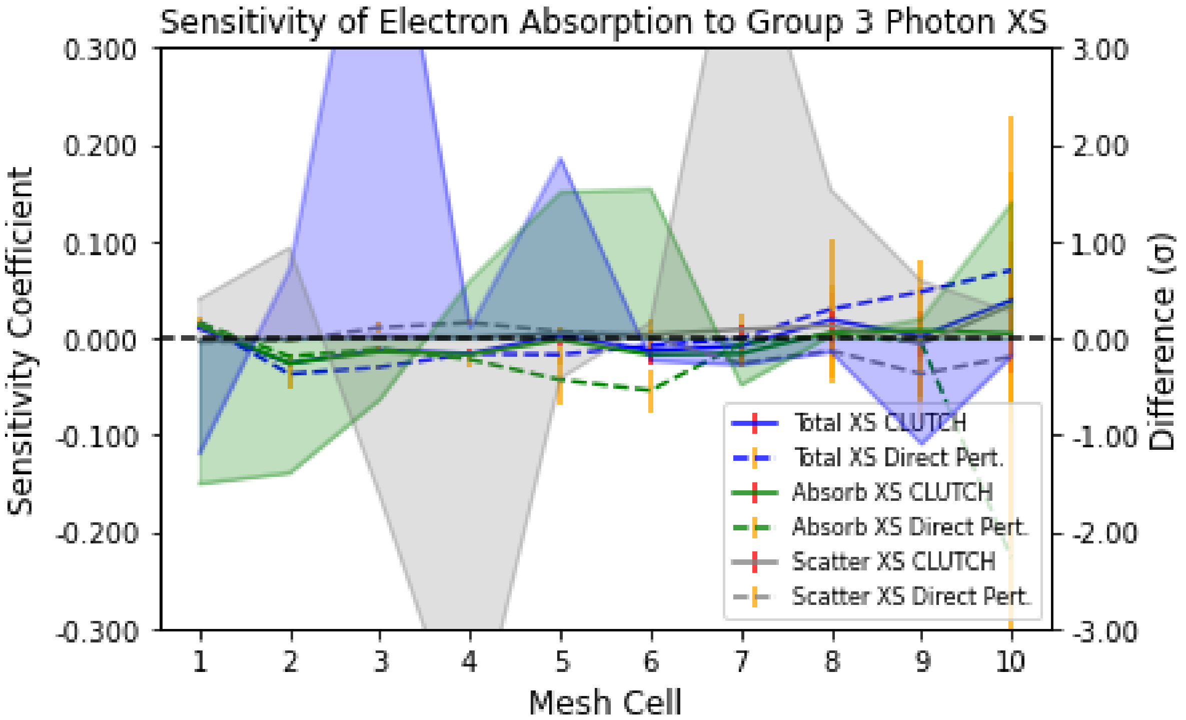

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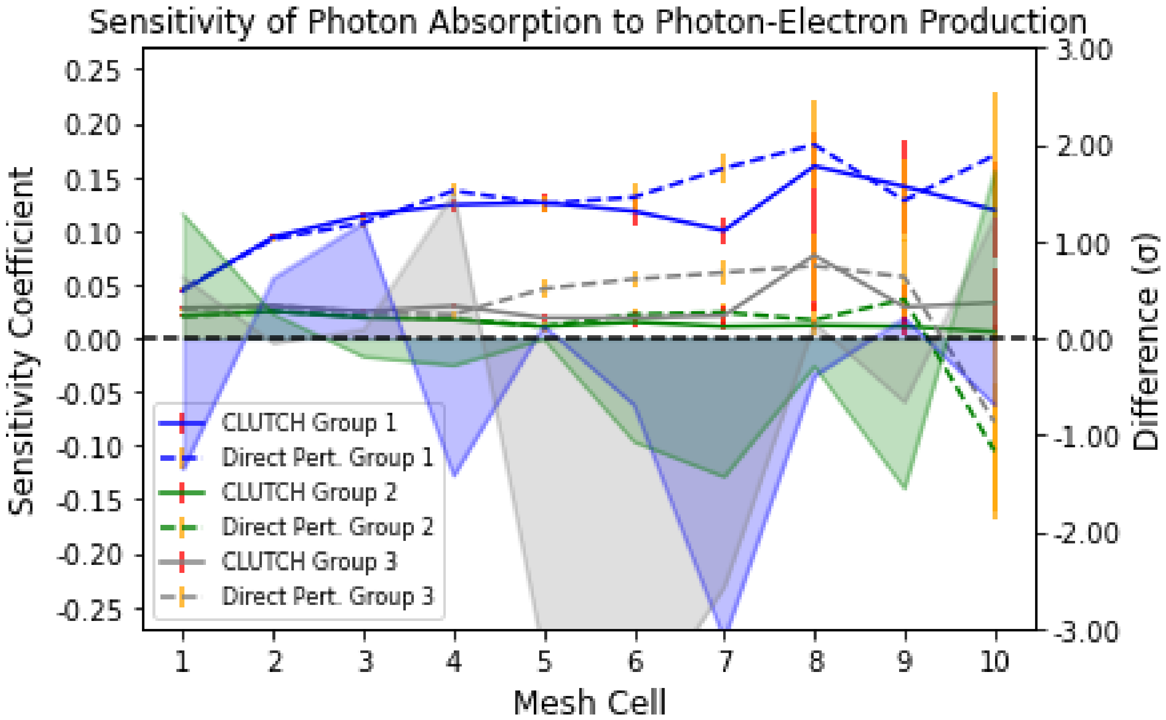
Results: Sensitivity of Photon Absorption to Photon-Electron Secondary Particle Production



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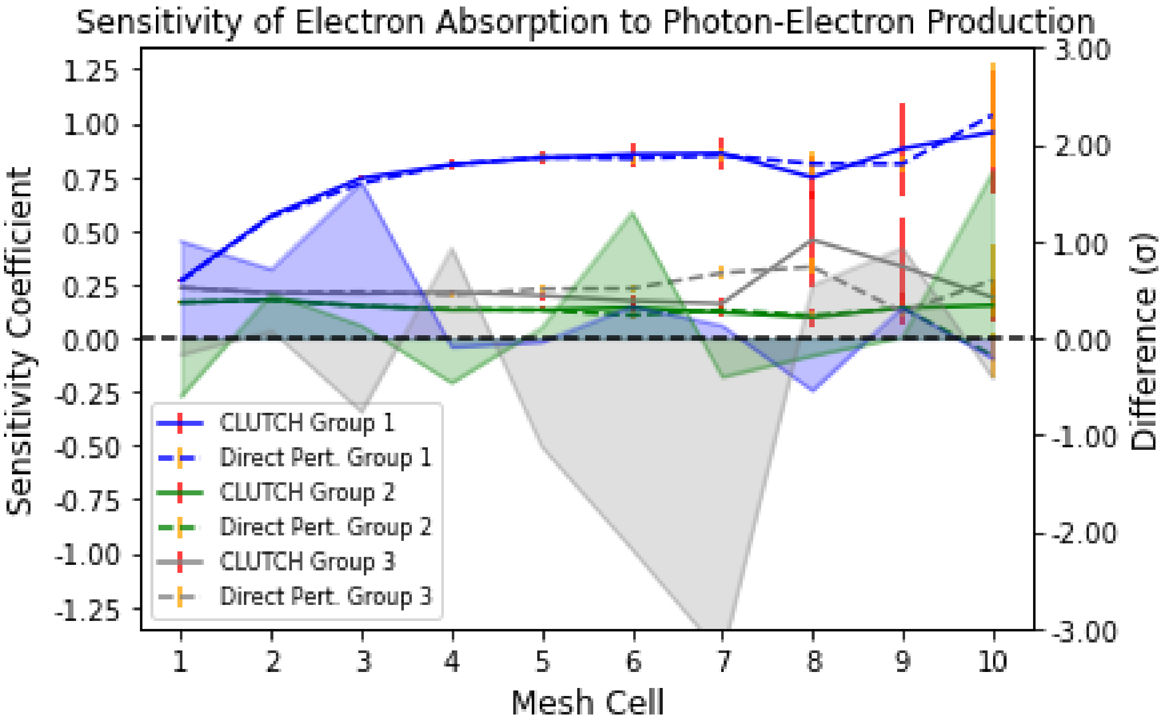

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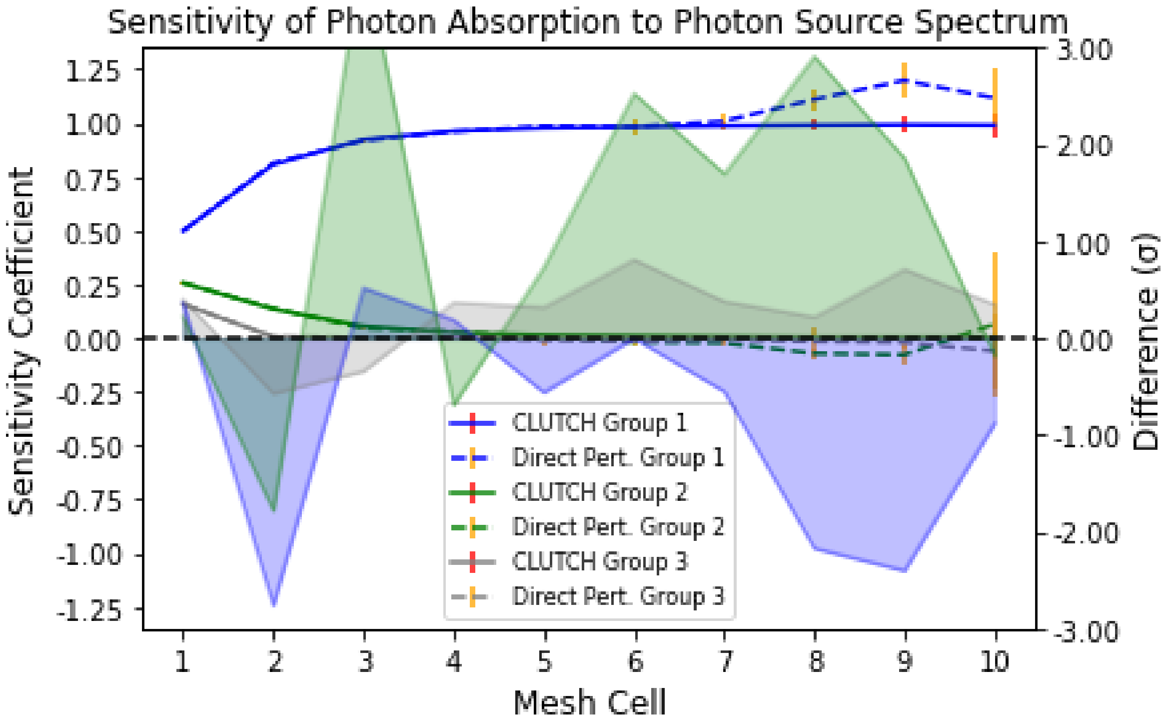
Results: Sensitivity of Photon Absorption to the Incident Photon Source Spectrum



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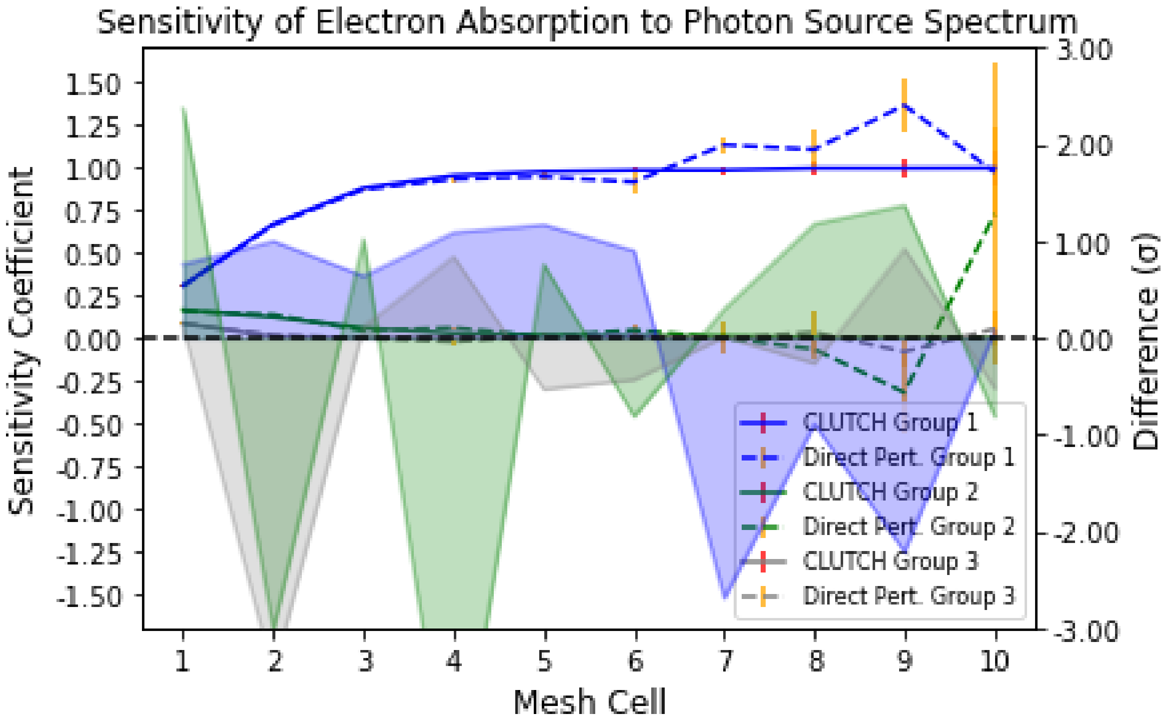

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Conclusions

- This work has extended the CLUTCH method to estimate the sensitivity of responses in coupled radiation transport simulations.
- This methodology was demonstrated in a 1-D Monte Carlo code and its accuracy was confirmed through Direct Perturbation reference calculations.
- Ongoing work will extend this methodology to the ITS radiation transport code.

Questions?

Please contact:

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