

Dissipativity-based Voltage Control in Distribution Grids

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Abstract

We consider the problem of decentralized voltage control using reactive power from distributed energy resources where the reactance matrix of the grid is unknown and potentially time-varying. We present an adaptive controller to set the reactive power at each inverter using a droop curve and analyze the conditions for stability.

Problem Formulation

1. Radial distribution network with n buses; bus 0 has fixed voltage.
2. Voltage magnitudes $v(k)$ can be observed and reactive power injections $q(k)$ can be controlled.
3. **Assumption:** Reactive power injections specified at time step k will reach these values before $k + 1$; the voltages also reach steady state before each time step.
4. Linearized power flow equations at 1 p.u. voltage result in the evolution:

$$\Sigma_l: \begin{cases} q(k+1) = u(k) \\ y(k) = v(k) \end{cases} \quad (1)$$

5. The voltage $v(k)$ and reactive power $q(k)$ satisfy $v(k) = Xq(k) + \bar{v}$, where X is a positive definite matrix that characterizes the reactance of the network and \bar{v} depends on the real power injections and the network parameters and is not controllable.

Motivation

1. Decentralized controllers are simple to implement but can create oscillations.
2. System parameters (e.g. the matrix X) are often unknown and time-varying.

Control Objective: Design $u(k)$ to locally asymptotically stabilize $v(k)$ to a desired set point v^* , $\lim_{k \rightarrow \infty} v(k) \rightarrow v^* = 1$, through a control input $u(k)$ that is (i) a causal function of the output $y(0), \dots, y(k)$, (ii) a local controller, i.e., each input $u_i(k)$ depends only on the local voltages at bus i , and (iii) satisfies the inverter's limits to inject or absorb reactive power: $u(k) \in [q_{\min}, q_{\max}]$.

Methodology

1. **Dissipativity:** Using a scattering transformation technique, we show that the distribution grid described by (1) is dissipative [3].
2. **Control Design:** We propose a new droop-like controller and prove the asymptotic stability using the above dissipativity property.
3. **Extremum Seeking Controller (ESC):** We use ESC [1] to estimate the desired reactive power setpoint and prove the stability of the overall system.

Step 1: Proving Dissipativity

1. Let the set of feasible operating points be $\mathcal{C} = \{(q, v) \in \mathbb{R}^n \times \mathbb{R}^n | v = Xq + \bar{v}\}$. Let $(q^*, v^*) \in \mathcal{C}$ be the desired operating point corresponding to the voltage set point v^* and the reactive power $v^* = Xq^* + \bar{v}$, with $q^* \in [q_{\min}, q_{\max}]$.
2. Denote the incremental quantities $\Delta q(k) = q(k) - q^*$, $\Delta u(k) = u(k) - u^*$, and $\Delta v(k) = v(k) - v^*$, $\nu(k) = \Delta v(k) + X\Delta u(k)$, and $\omega(k) = -\Delta v(k) + X\Delta u(k)$.

Dissipativity

Proposition: The linearized system Σ_l is passive with respect to the input $\nu(k)$ and output $\omega(k)$ irrespective of how $u(k)$ is designed.

Step 2: Controller Design

We propose the droop-like controller:

$$u(k) = \begin{cases} q_{\max} & v(k) < v_l \\ u^* - \bar{K}(v(k) - v^*) & v_l \leq v(k) \leq v_h \\ q_{\min} & v(k) > v_h, \end{cases} \quad (2)$$

Stability

Theorem: Consider Σ_l with the controller (2). Let \bar{K} be a diagonal matrix with

$$K := (I + X\bar{K})^{-1}(X\bar{K} - I) < 0 \quad (3)$$

in the sense that $K + K^T$ is negative-definite. Then, the closed loop system is asymptotically stabilized to the desired operating point (q^*, v^*) .

Step 3: Extremum Seeking Controller (ESC)

1. The proposed controller requires the knowledge of the set point u^* , which, in turn, requires the value of X
2. We show that a controller of the form:

$$u(k) = \hat{u}^*(k) - \bar{K}(v(k) - v^*), \quad (4)$$

where $\hat{u}^*(k)$ is the current estimate of the desired reactive power u^* ($= q^*$) stabilizes the system asymptotically.

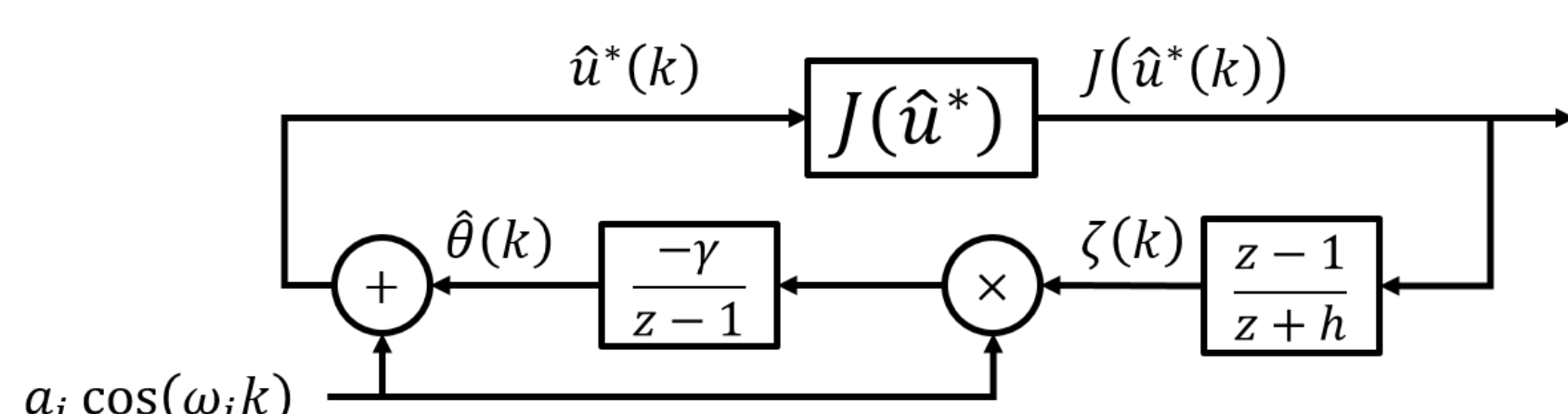


Figure 1. Proposed algorithm for identifying $\hat{u}^*(k)$ with $J(\hat{u}^*(k)) = \|Xu(k) + \bar{v} - v^*\|_2^2$.

Stability with Time-varying System Parameters

1. A time-varying X may not always satisfy (3). Thus, the stability proof above (and in similar works in the literature) will be violated.

Stability with Time-Varying X

Suppose $X \in \{X_0, \dots, X_n\}$, with X_0 satisfying (3). Let $K_i := (I + X_i\bar{K})^{-1}(X_i\bar{K} - I)$, λ_i be the smallest eigenvalue of the matrix K_i , and $\lambda_{\min} = \min_{i=1, \dots, n} \lambda_i$. Then, a sufficient condition for stability is to ensure that for every block of N steps, the nominal matrix X_0 is active at least m times such that $-m\lambda_0 - (N - m)\lambda_{\min} < 0$.

Case Study: IEEE 13-bus test feeder system

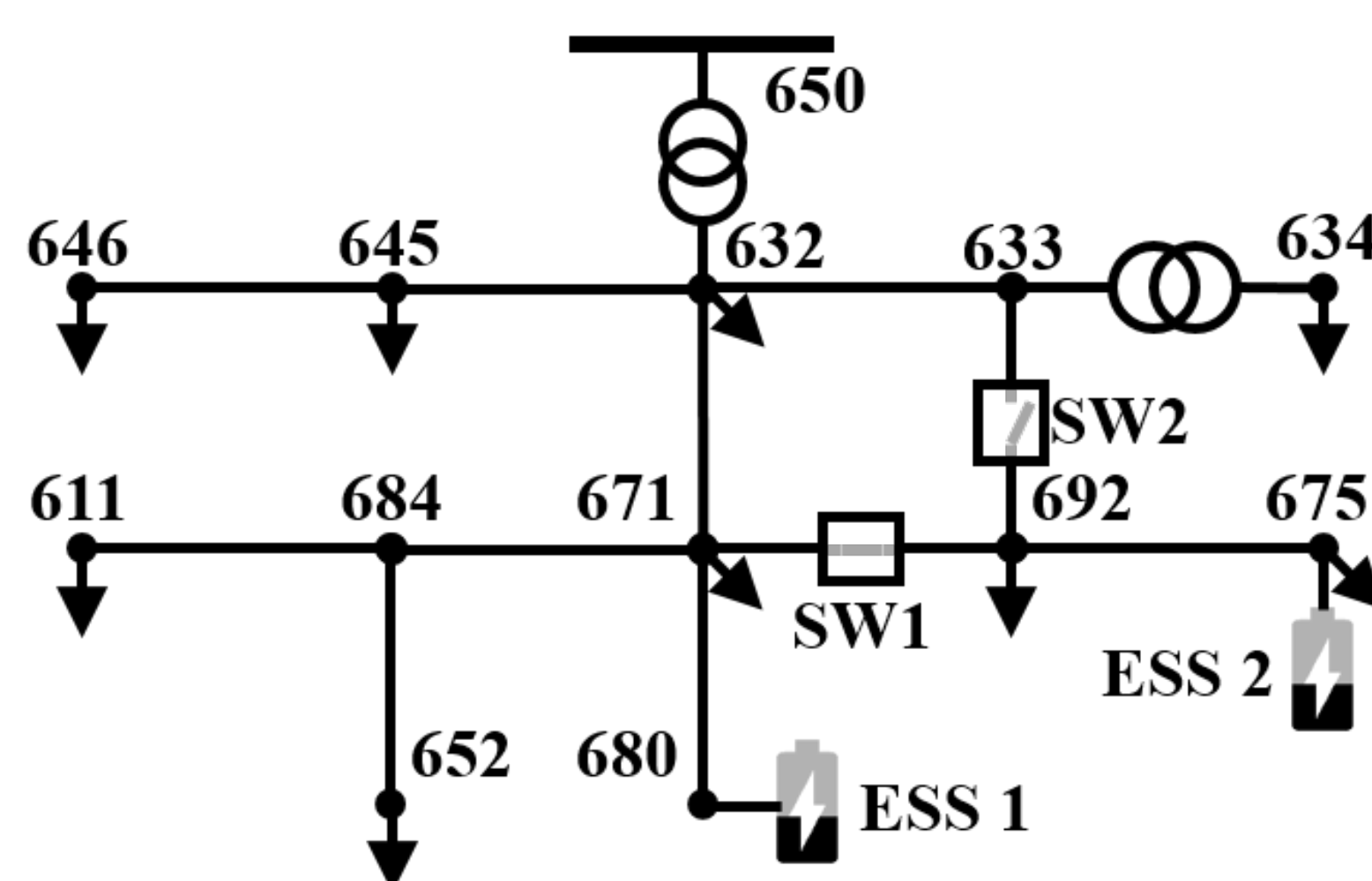


Figure 2. Modified test feeder with two 3-phase 600 kW energy storage systems (ESSs) and switch SW2.

OpenDSS simulation with time step of 1s. SW1 opens and SW2 closes at 12h.

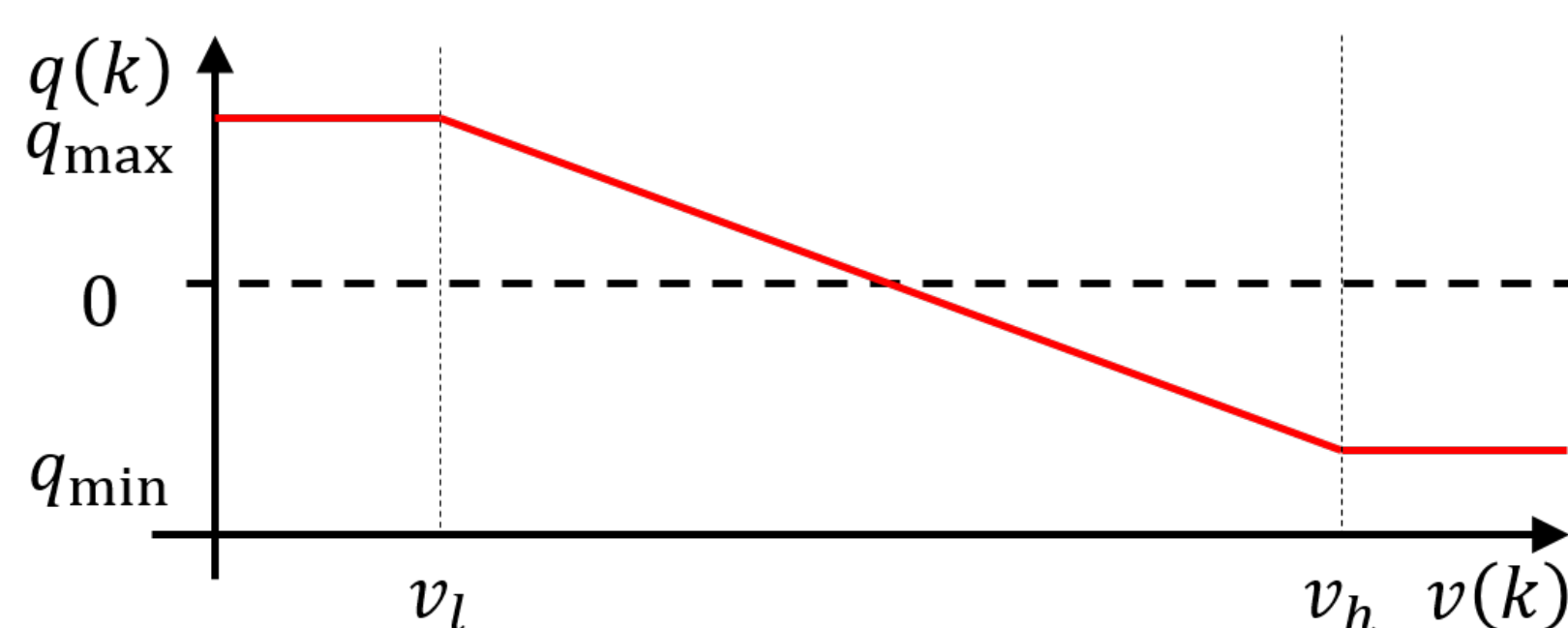


Figure 3. Voltage-reactive power control settings chosen within DER A range of standard IEEE 1547-2018.

Instability: Oscillations with standard droop-controller

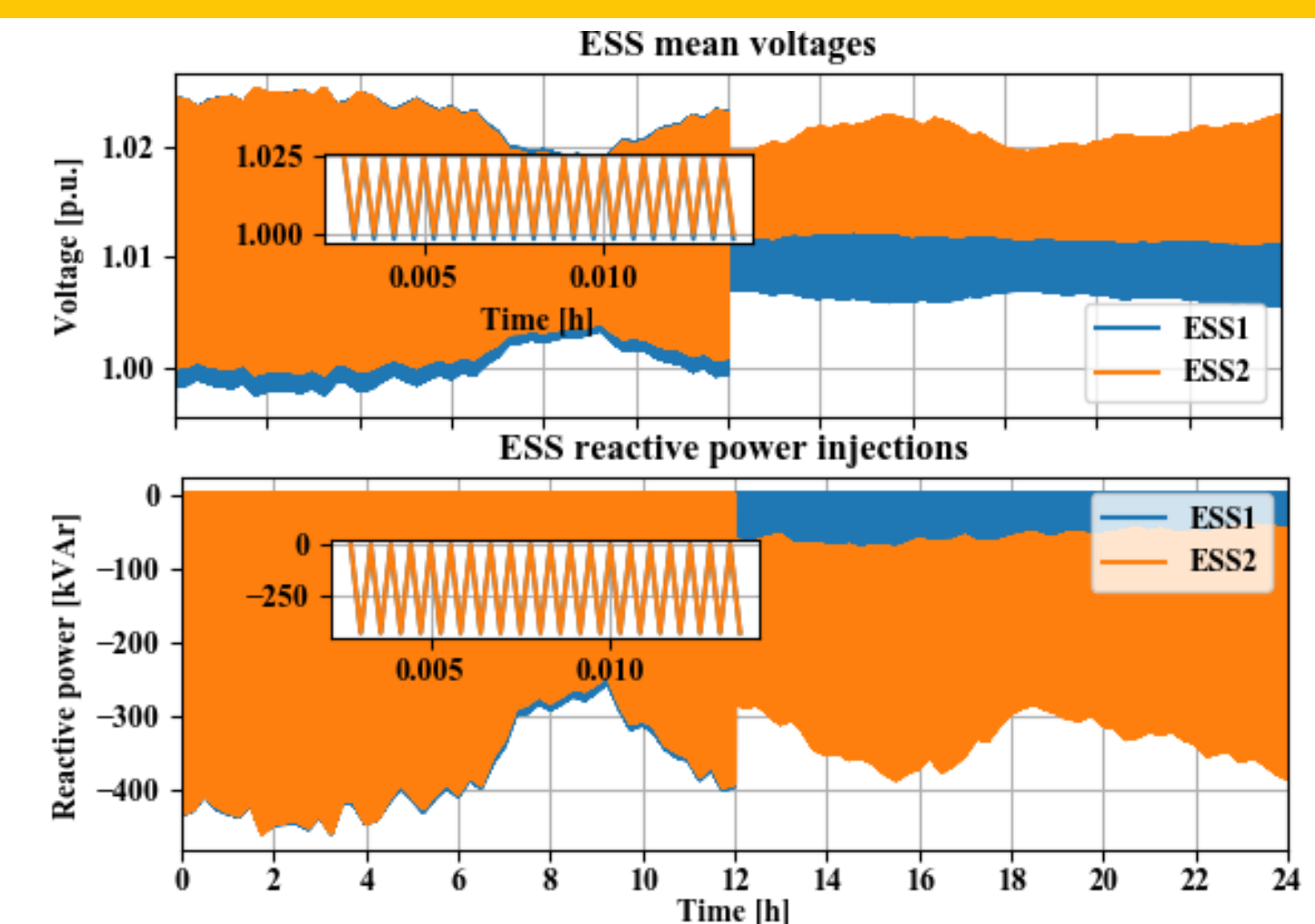


Figure 4. The volt/Var droop controllers saturate at every time step causing oscillations.

1. Similarly to [2], an oscillation in the output of the volt/Var droop controller is found.
2. The controllers saturate at every time step causing oscillations in power flows and voltages.

Results of Proposed Controller

1. Controller (2) designed using $\bar{K} = \text{diag}\{10000, 1000\}$, which resulted in $K < 0$.
2. The parameters were chosen as $a_i = 0.1$, $\omega_i = \pi/2$, $\gamma_i = 0.027$, $h = 0.99$.

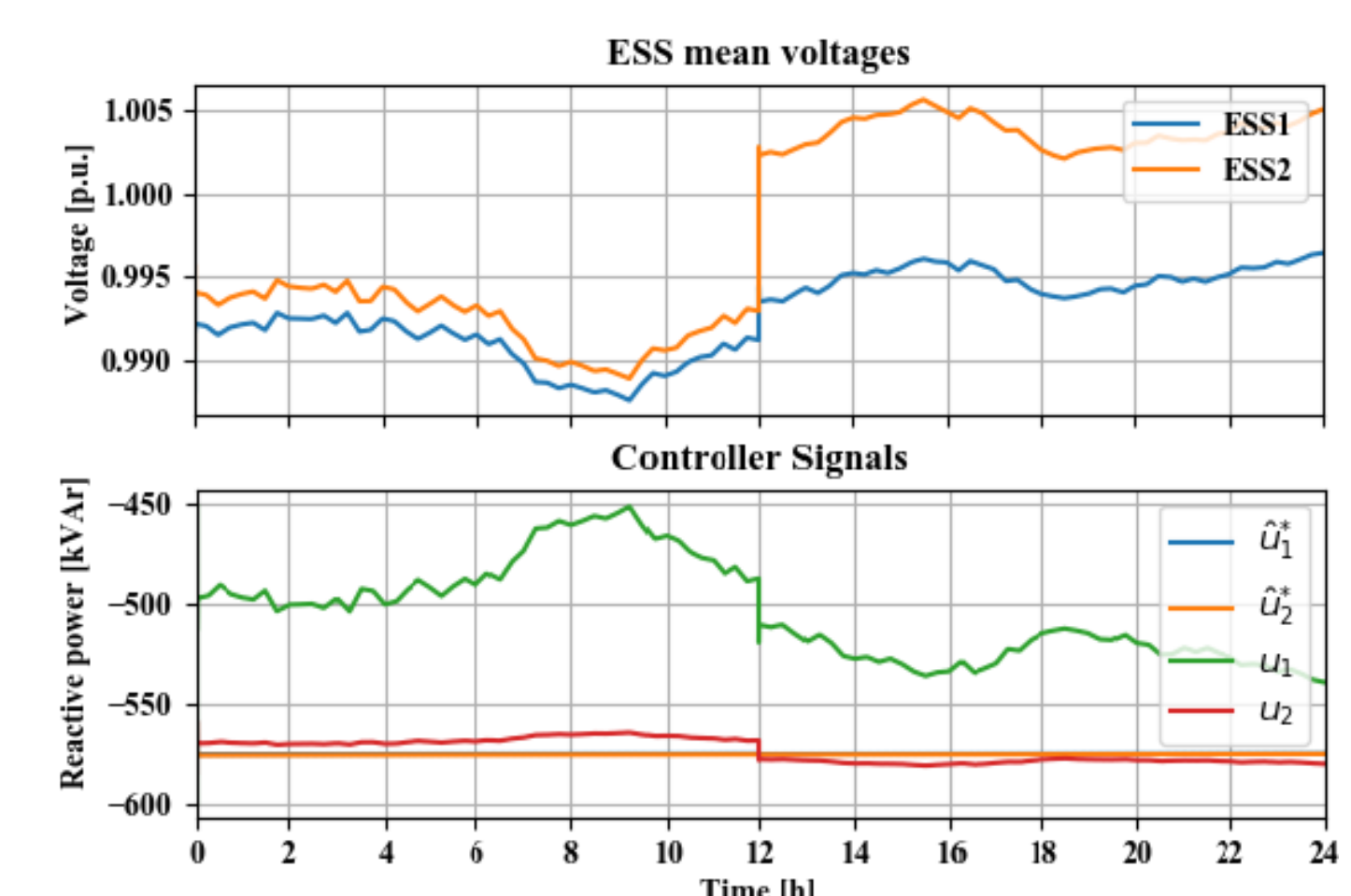


Figure 5. ESS voltages and power injections for $\bar{K} = \text{diag}\{10000, 1000\}$, $a_i = 0.1$, $\omega_i = \pi/2$, $\gamma_i = 0.027$, $h = 0.99$.

Conclusions and Future work

1. Unlike traditional droop-based approaches, the proposed dissipativity-based adaptive controller was able to provide voltage support without introducing oscillations into the distribution system when control gains were set according to the developed criterion. The effectiveness of the controller is currently limited by the saturation of the actuators, which could only be improved with higher ESS power capacity.
2. Future work includes extending this approach to distributed control of DERs. Identifying adversarial agents and how to mitigate their effect.

References

- [1] Joon-Young Choi, Miroslav Krstic, Kartik B Ariyur, and Jin Soo Lee. Extremum seeking control for discrete-time systems. *IEEE Trans. Automat. Control*, 47(2):318–323, 2002.
- [2] Andre Egli, Stavros Karagiannopoulos, Saverio Bolognani, and Gabriela Hug. Stability analysis and design of local control schemes in active distribution grids. *IEEE Trans. Power Syst.*, 2020.
- [3] Hassan K Khalil. *Nonlinear systems third edition*. Patience Hall, 2002.

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