Total Variation Denoising with Slack Variables

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Introduction

- Image denoising is an essential step in numerous fields, finding its application in medical imaging, video processing, pattern recognition, remote sensing, and more
- Introduce a new method that can eliminate the noise in the background of the image while preserving the image signal.
- Data input: the images that have smooth backgrounds (with a light tone gradient) and distinct foreground shapes to emulate the sharp well-localized transitions.

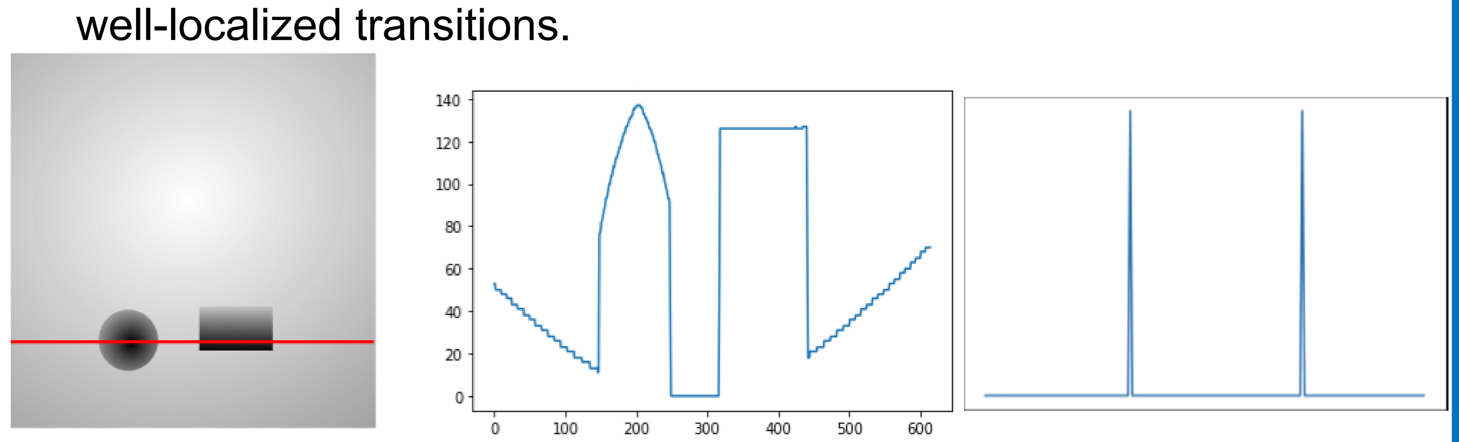

Figure1: From left to right: (1st) The sample image used, the red line marks the row of pixels extracted. (2nd) The extracted row of pixels shown as a signal line graph. (3rd) examples of sharp well-localization

Methodology

Representing the well-localization in the TV Model

$$\underset{\theta_i, s_i \geq 0\ \forall i}{\text{minimize}}\ \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{s} - \boldsymbol{\theta}\|_2^2 + \lambda \sum |\theta_{i+1} - \theta_i| + \nu\|\boldsymbol{s}\|_1,$$

TV Denoising is a common L1-regularized optimization problem:

$$\frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\theta}\|_2^2 + \lambda \sum |\theta_{i+1} - \theta_i|$$

Incorporate the well-localization into the traditional 1D TV Denoising

$$y_i = \theta_i + s_i + \varepsilon_i,$$

Python Implementation and ProxTV Library Utilization

Step 1:
Where k is the iteration index

$$\theta^{k+1} = argmin_\theta \frac{1}{2}\|y - s^k - \theta\|_2^2 + \lambda * TV(y - s^k)$$

Step 2: $s^{k+1} = max(0, softthreshold(y - \theta^{k+1}, \nu))$

Spikes Configuration

- Placing the spikes before, or after, or in the middle of sharp localized transitions.
- Test different spike configurations at and around gradual localized transitions to test their effectiveness

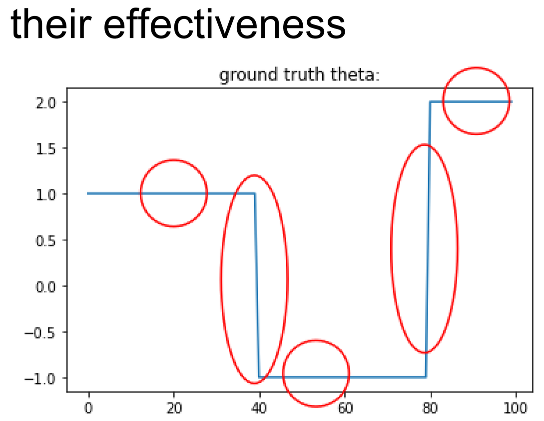

Figure 2: Possible calcification spikes placement positions in a simple synthetic data set (before noise is applied)

Analysis and Results

Optimal Spikes Placement

- For **sharp well-localized transitions**, the spike should be placed at the exact location of where the transition takes place.
- For **a more gradual transition**, the spike should be placed at the local maxima
- Other well-localization configurations such as placing spikes at the local maxima of sharp localized transitions, or at local minima just before transitions yielded consistently worse results.

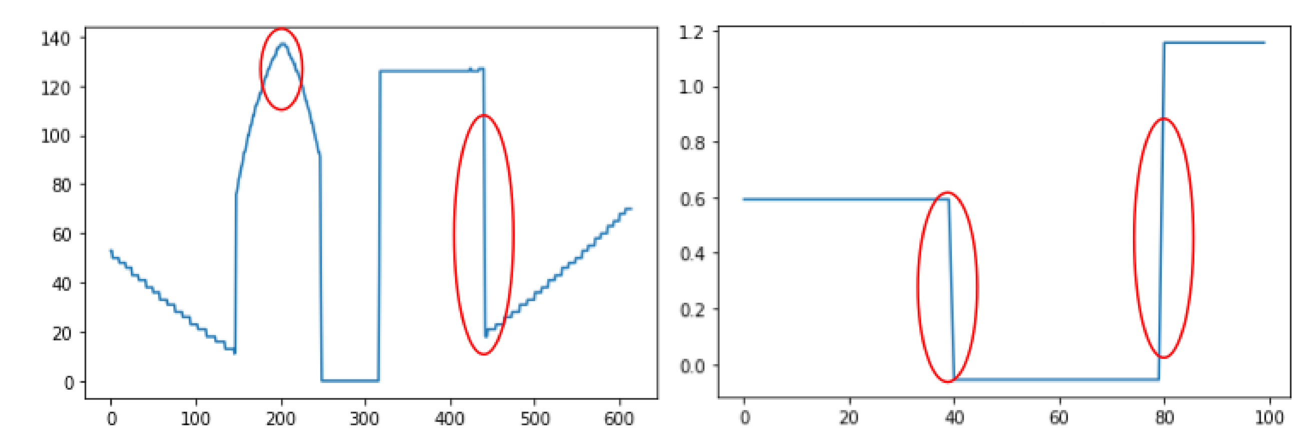

Figure 3: Optimal spike placement locations on two sample inputs (before noise is introduced)

Optimal Denoising Results

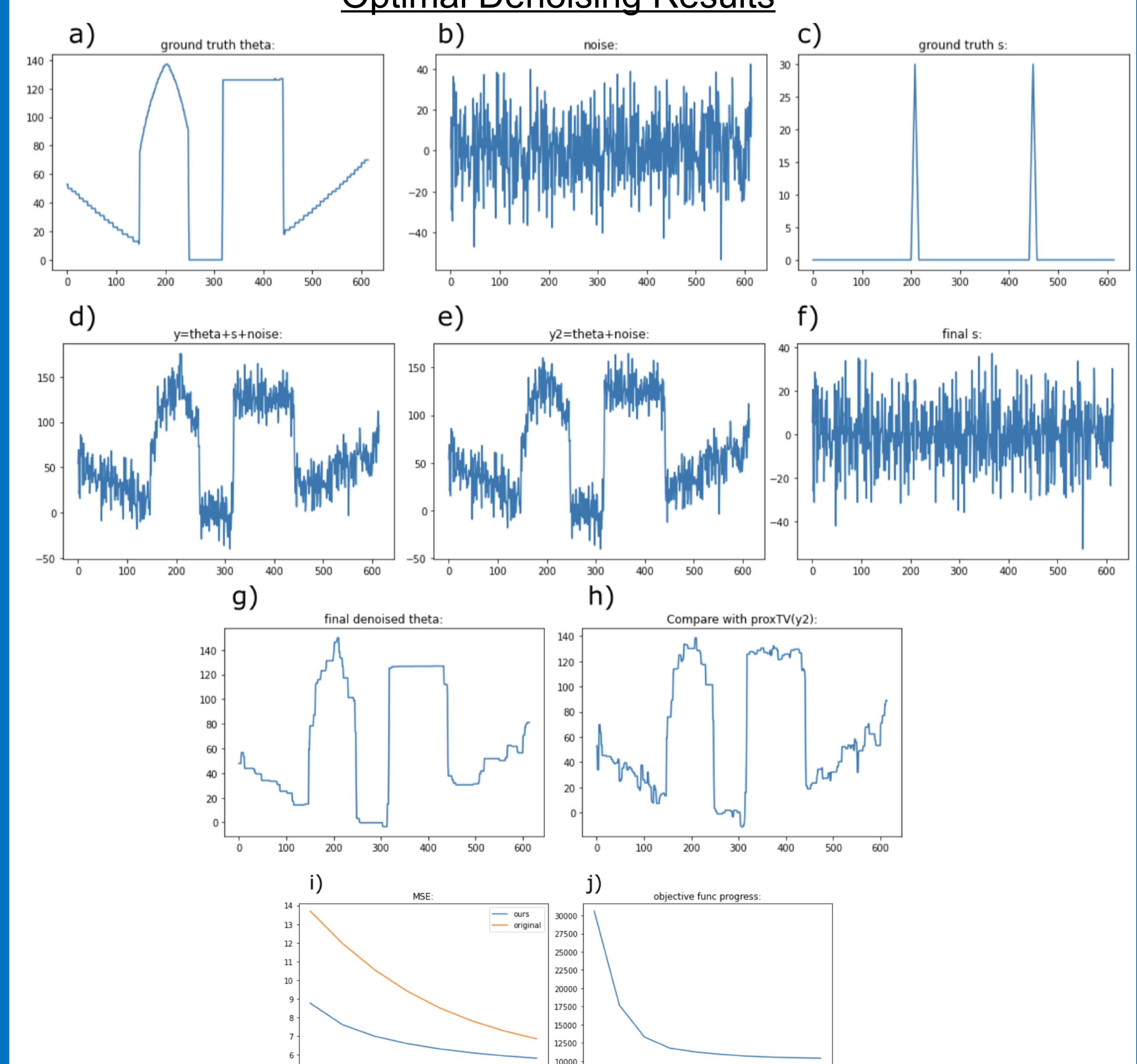

Figure 4 : Most optimal denoising results of our model. The optimal parameters used are as follows: sigma = 15(noise amplitude), random seed = 8 (for numpy.random), λ=6, v = 1, spike height = 30, spike width = 15

Sub-optimal Denoising Results

- The test results is on the same sample image, but with half of the noise amplitude
- Similarly effective at preserving the features of the original image data
- A sufficiently small noise range, the original TV denoise approach converges to a more optimal denoising result than our method.
- With additional parameter optimization (such as adjusting the spike's width and height), we might be able to improve our model's performance in this case.

Discussion

- Spikes placed outside of the optimal locations in "Optimal Spikes Placement" slide had consistently produced sub-optimal results, which further affirms our proposed spike placement strategy.
- The soft threshold parameter ***v***, we observed better performance at ***v*** values close to 0, as ***v*** increases, the performance of our model decreases, and the visual similarity between our model's result with the original TV denoise method's result increases.
- The MSE metric does not always correlate with better visual comparison results

Limitations and Future Work

- Better performance compared to the original TV denoising method at larger noise levels -> optimal the computation cost
- The well-localization vector ***s*** require the manual process -> not scalable for larger and higher dimension data -> Automated solution for constructing the well-localization vector
- The method was not effective at lower noise levels -> improve our model's performance through extensive parameter optimization.
- The same method can be applied to more complex real-world problems, including efficient reconstruction of 3D tomographic images, and the denoising of other types of non-image signal data.

References

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