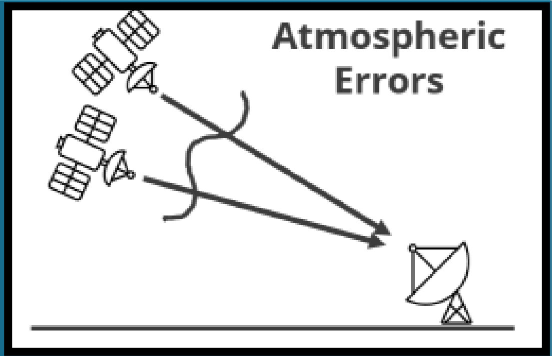
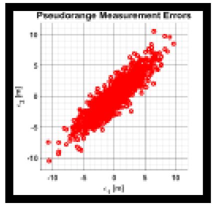
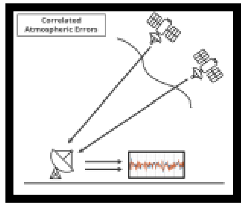
SAND2022-12054C

Sandia National Laboratories

A Monitor for Correlated Kalman Filter Innovations







	$\rho^* = 0.0$		$\rho^* = 0.5$	
	True Negative	False Negative	True Positive	False Positive
Perfect Monitor	1.0	0.0	1.0	0.0
Snapshot Monitor	0.99	0.01	0.01	0.99
Sequence Monitor	0.99	0.01	0.02	0.98

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

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Sandia National Labs

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Tuning a Kalman Filter

When designing and implementing a navigation filter, I often model measurements as

$$\boldsymbol{z_k} = \boldsymbol{h}(\boldsymbol{x_k^*}) + \boldsymbol{w_k}$$
$$\boldsymbol{w_k} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R_k})$$
$$\mathbf{R_k} = \sigma_k^2 \boldsymbol{I}$$

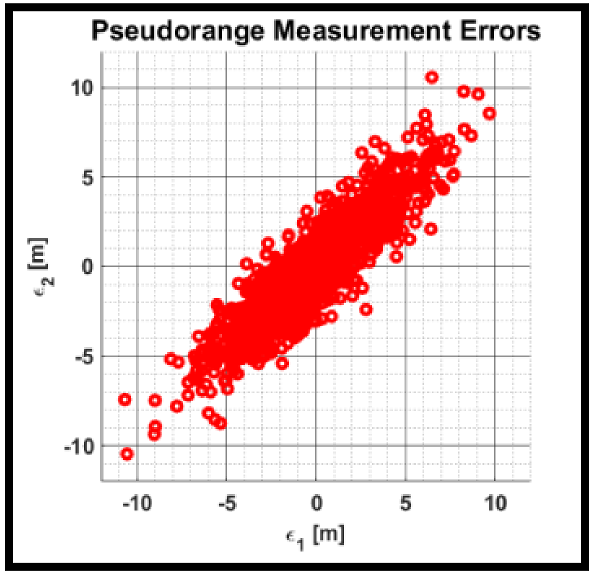
where σ_k^2 may be based on a textbook or prior experience.

Then, as data is collected and processed, I inflate σ_k^2 and underweight the measurements as needed until the Kalman filter is behaving properly.

Claim: A Kalman filter's performance is degraded by the assumption that the measurement noise is independent (diagonal).

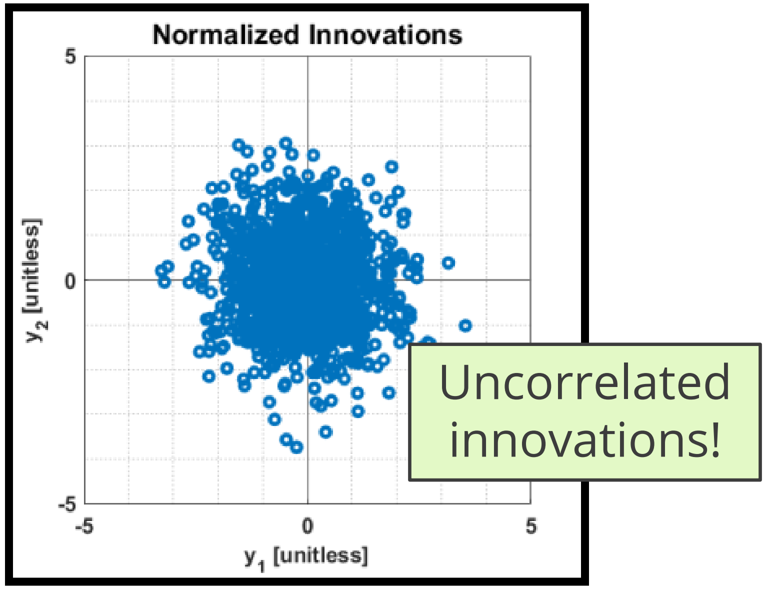
Correlated Measurements to Correlated Innovations

$$z_k = h(x_k^*) + w_k$$



$$\mathbf{R}_k^* = \begin{bmatrix} 9 & 4.5 \\ 4.5 & 9 \end{bmatrix}$$

$$\hat{y}_k = S_k^{-1/2}(z_k - h(\bar{x}_k))$$



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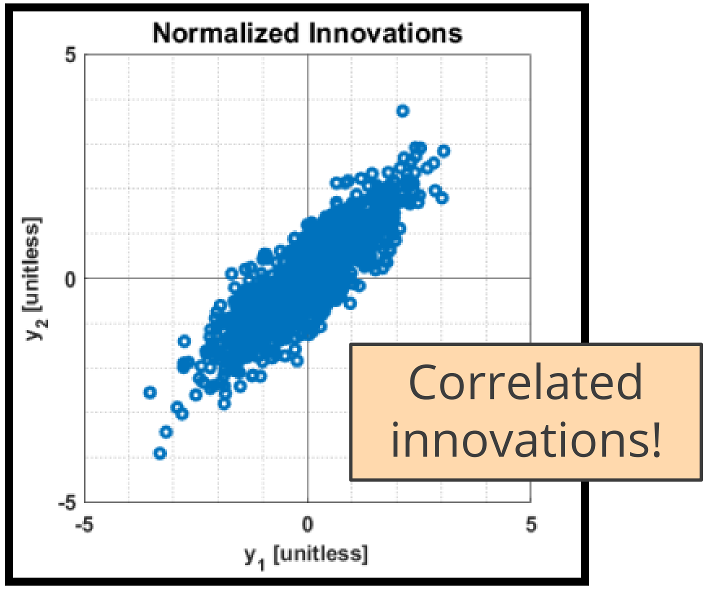


$$R_k = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

Conclusion: When correlations in measurement errors are ignored, a filter’s innovations may themselves be correlated, implying residual information.

Why Should You Care?

Modeling your measurement noise as $\mathbf{R} = \sigma^2 \boldsymbol{I}$ will not break your Kalman filter, but there are situations in which the discarded performance has a meaningful impact.

1) High-performance systems.

Modeling your measurement noise as independent when correlations exist discards potential information that could improve your filter’s accuracy.

Ex: Spacecraft attitude estimators.

2) Correlated small signal detection.

The sudden appearance of correlated measurement innovations may imply changes to your system model assumptions.

Ex: GNSS anti-spoofing.

Measurement correlations may have a meaningful impact in several niche areas.

Snapshot & Sequence Monitors

$$\hat{y}_1 = \begin{bmatrix}-0.66\\-0.28\end{bmatrix}, \quad \hat{y}_2 = \begin{bmatrix}-0.13\\-0.18\end{bmatrix}, \quad \hat{y}_3 = \begin{bmatrix}-0.97\\+0.17\end{bmatrix}, \quad \dots, \quad \hat{y}_N = \begin{bmatrix}+0.74\\+0.48\end{bmatrix}$$

Innovations Snapshot Monitor

$$\Lambda^* = \hat{y}_{i,j}$$

$$\Lambda^* \sim \mathcal{N}(0,1)$$

$$\Lambda^* \underset{H_0}{\overset{H_A}{\gtrless}} \Lambda_0$$

Detects large instantaneous faults

(e.g. clock failures)

Innovations Sequence Monitor

$$\Lambda^* = \sum_{i=1}^{N} \sum_{j=1}^{p} \hat{y}_{i,j}^2$$

$$\Lambda^* \sim \chi_{Np}^2$$

$$\Lambda^* \underset{H_0}{\overset{H_A}{\gtrless}} \Lambda_0$$

Detects consistent small errors (biases)

(e.g. multipath)

Claim: Neither of these monitors are effective at detecting *correlated* innovations.

Epoch-Correlated Innovations – Monte Carlo

Random samples were drawn from a zero-mean 2D multivariate Gaussian distribution with covariance:

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

Innovation Snapshot and Sequence monitors were run 100,000 times with an allowable false negative ratio of 0.01.

Sequence monitors accumulated 100 samples before testing.

	$\rho^* = 0.0$		$\rho^* = 0.5$	
	True Negative	False Negative	True Positive	False Positive
Perfect Monitor	1.0	0.0	1.0	0.0
Snapshot Monitor	0.99	0.01	0.01	0.99
Sequence Monitor	0.99	0.01	0.02	0.98

The Innovation Snapshot and Sequence monitors are ineffective at detecting correlated model errors.

Solution: Innovations Sphericity Monitor

Suppose a Kalman Filter produced p innovations for every measurement epoch $1..N$.

Let $\boldsymbol{B}$ be the scaled sample covariance of the innovations over the horizon:

$$\boldsymbol{B} = \sum_{i=1}^{N} (\hat{y}_i - \bar{y})(\hat{y}_i - \bar{y})^T$$

Then, as $N/p \gg 1$, the test statistic:

$$\boxed{\Lambda^* = -Np(1 - \ln(N)) - N\ln(\det(\boldsymbol{B})) + \mathrm{tr}(\boldsymbol{B})}$$

approaches a chi-squared distribution with $p(p+1)/2$ degrees of freedom under the hypothesis that the innovations are uncorrelated.

Epoch-Correlated Innovations – Monte Carlo Revisited

The Monte Carlo simulation was re-run with the addition of the Sphericity Monitor.

Random samples were drawn from a zero-mean 2D multivariate Gaussian distribution with covariance:

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

All monitors were run 100,000 times with an allowable false alarm ratio of 0.01.

Sequence and Sphericity monitors accumulated 100 samples before testing.

	$\rho^* = 0.0$		$\rho^* = 0.5$	
	True Negative	False Negative	True Positive	False Positive
Perfect Monitor	1.0	0.0	1.0	0.0
Snapshot Monitor	0.99	0.01	0.01	0.99
Sequence Monitor	0.99	0.01	0.02	0.98
Sphericity Monitor	0.99	0.01	0.99	0.01

The Sphericity Monitor effectively detected modeling errors due to the correlated innovations!

2D Simulation

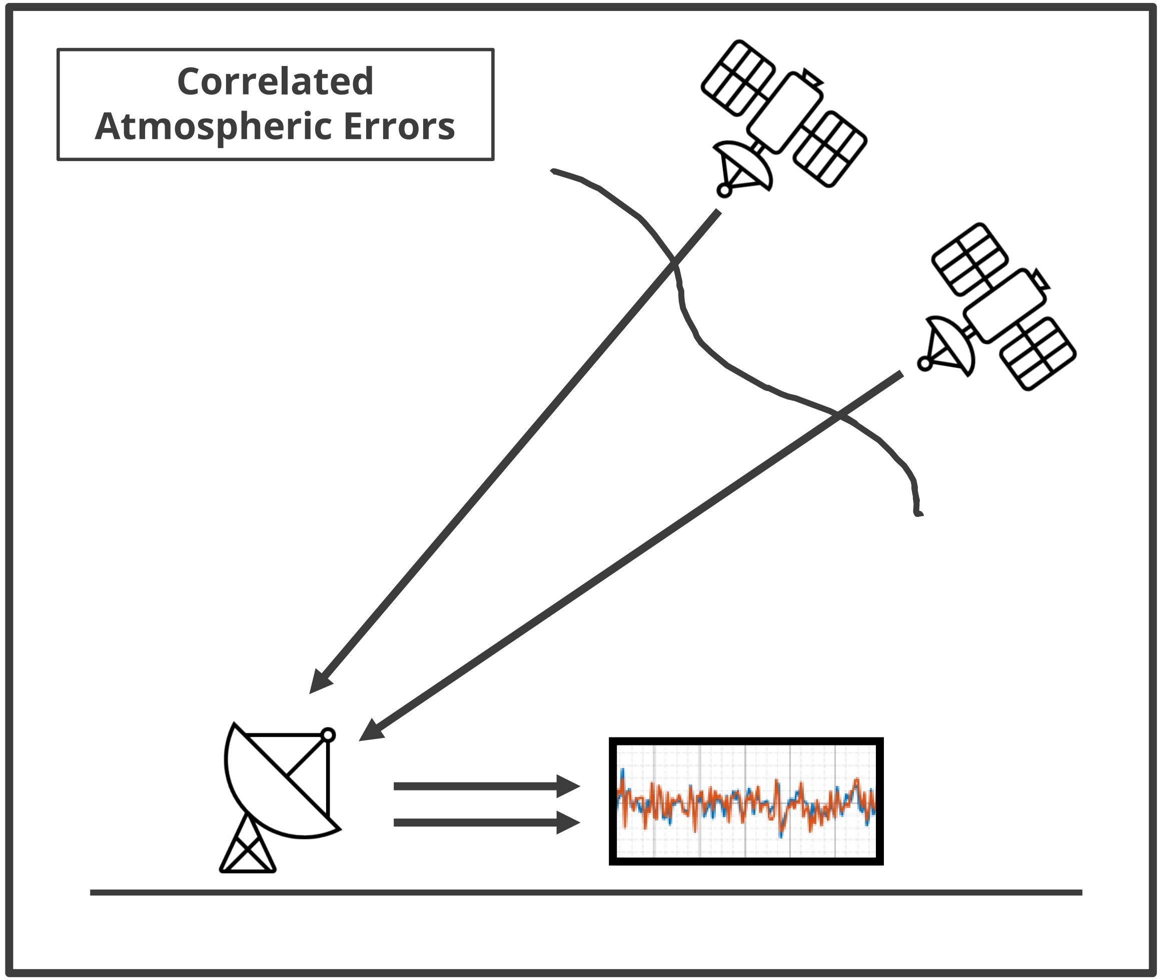
Consider a simple 2D example wherein a receiver estimates its position from a set of two range measurements (ignoring clock biases).

A navigation engineer models the range measurements as the geometric range plus additive independent Gaussian noise:

$$z^{(i)} = \|\boldsymbol{r}\| + \epsilon^{(i)}$$

$$E\left[\epsilon^{(i)}\epsilon^{(j)}\right] = \begin{cases} 0, & i \neq j \\ \sigma^2, & i = j \end{cases}$$

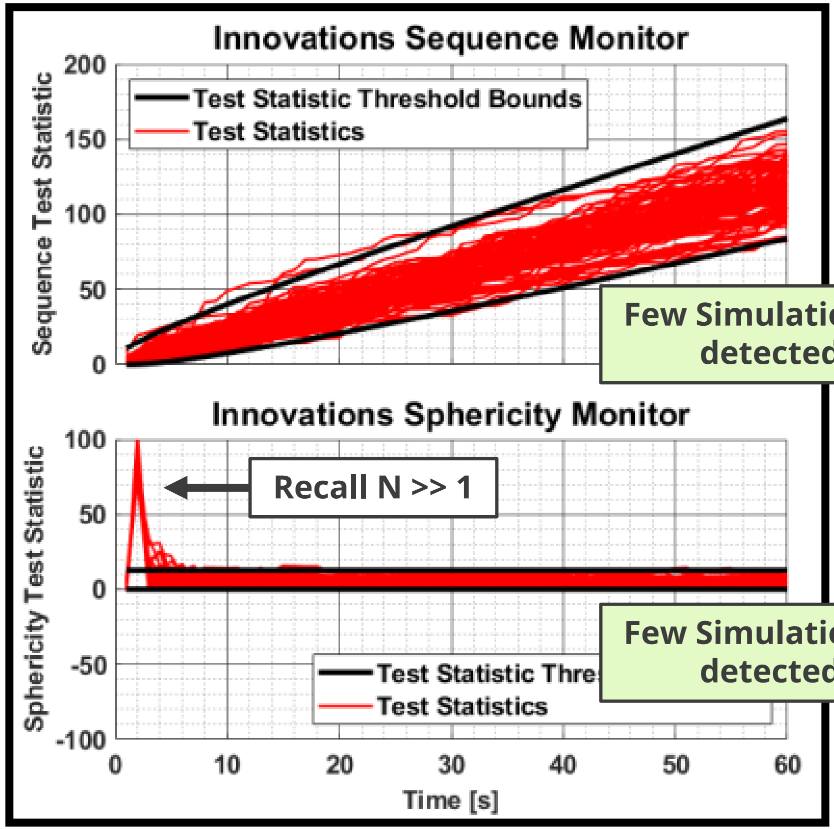
Unbeknownst to the engineer, atmospheric effects cause correlated errors in the range measurements.



2D Example: Monte Carlo

True Negative Scenario

$(\rho^* = 0.9, \rho = 0.9)$

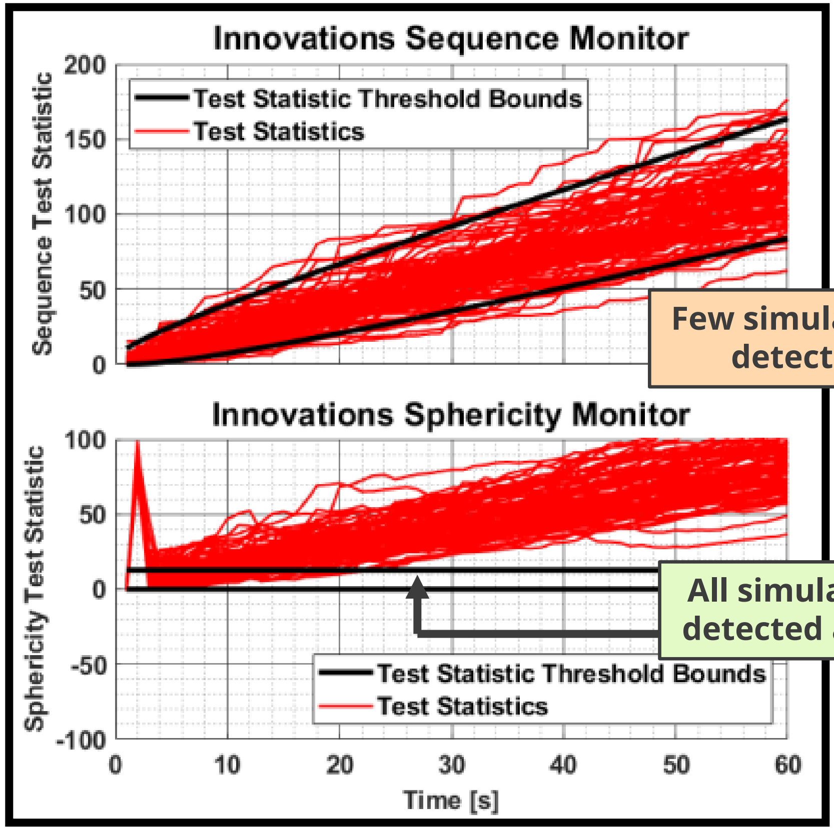


Few Simulations incorrectly detected an error.

Few Simulations incorrectly detected an error.

True Positive Scenario

$(\rho^* = 0.9, \rho = 0.0)$



Few simulations correctly detected an error.

All simulations correctly detected an error by 26s.

The Sphericity Monitor effectively detects a correlation modeling error while the Sequence Monitor does not.

Conclusions

- Unmodeled correlated measurement errors may degrade Kalman filter performance.
- Snapshot and Sequence monitors are ineffective at detecting correlated innovations.
- The proposed Sphericity monitor effectively detects correlated innovations.

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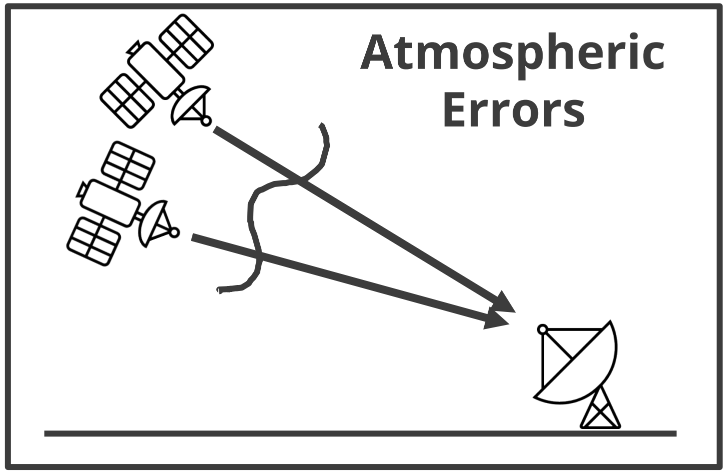
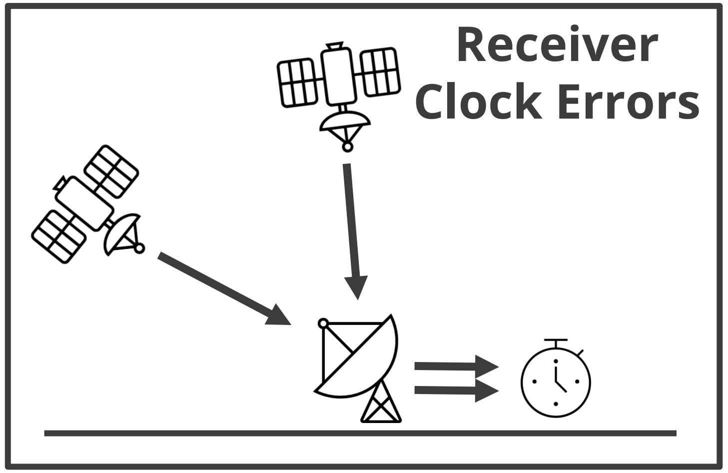
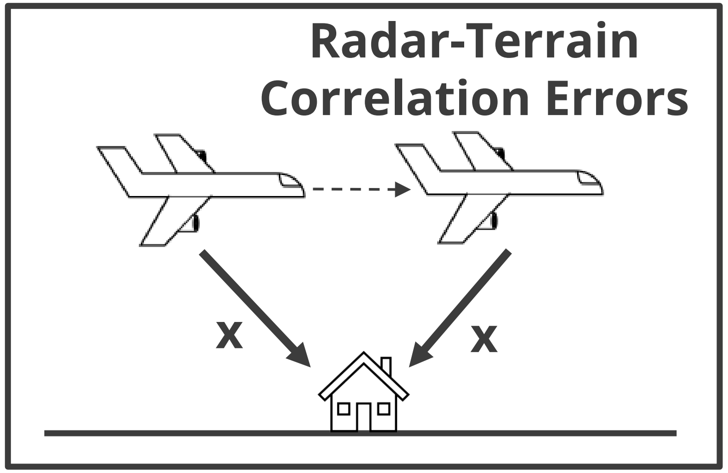
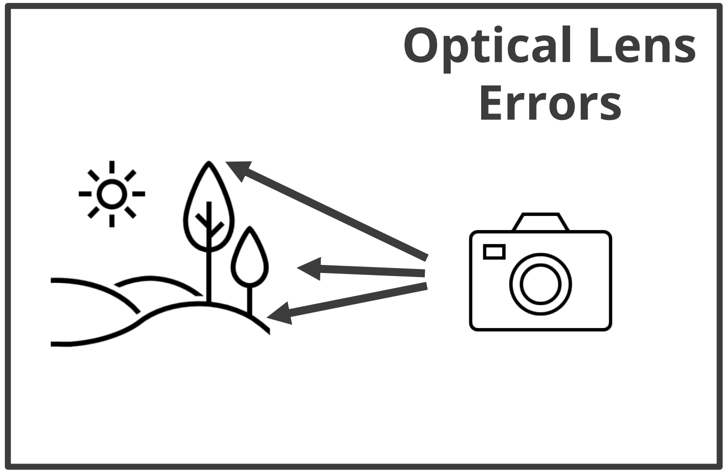
Backup Slides

Scenarios with potentially correlated measurement noise









Any two measurements which are influenced by the same system component may be afflicted by correlated errors.

The Property-Monitor Triad

$$\hat{y}_k \sim \mathcal{N}(0, I)$$

Innovations Property		Innovations Monitor
Unit Variance	→	Snapshot Monitor
Zero Mean	→	Sequence Monitor
Uncorrelated	→	Sphericity Monitor

The Innovations Sphericity Monitor completes a satisfying triad of property-monitor relationships.

Snapshot, Sequence, & Sphericity Monitors

$$\hat{\boldsymbol{y}}_1 = \begin{bmatrix} -0.66 \\ -0.28 \end{bmatrix}, \quad \hat{\boldsymbol{y}}_2 = \begin{bmatrix} -0.13 \\ -0.18 \end{bmatrix}, \quad \hat{\boldsymbol{y}}_3 = \begin{bmatrix} -0.97 \\ +0.17 \end{bmatrix}, \quad \dots, \quad \hat{\boldsymbol{y}}_N = \begin{bmatrix} +0.74 \\ +0.48 \end{bmatrix}$$

Snapshot Monitor	Sequence Monitor	Sphericity Monitor
		$\boldsymbol{B} = \sum_{i=1}^{N} (\hat{y}_i - \bar{y})(\hat{y}_i - \bar{y})^T$
$\Lambda^* = \hat{y}_{i,j}$	$\Lambda^* = \sum_{i=1}^{N} \sum_{j=1}^{p} \hat{y}_{i,j}^2$	$\Lambda^* = -Np(1 - \ln(N)) - N \ln(\det(\boldsymbol{B})) + \mathrm{tr}(\boldsymbol{B})$
$\Lambda^* \sim \mathcal{N}(0,1)$ $\Lambda^* \underset{H_0}{\overset{H_A}{\gtrless}} \Lambda_0$	$\Lambda^* \sim \chi^2_{Np}$ $\Lambda^* \underset{H_0}{\overset{H_A}{\gtrless}} \Lambda_0$	$\Lambda^* \sim \chi^2_{p(p+1)/2}$ $\Lambda^* \underset{H_0}{\overset{H_A}{\gtrless}} \Lambda_0$