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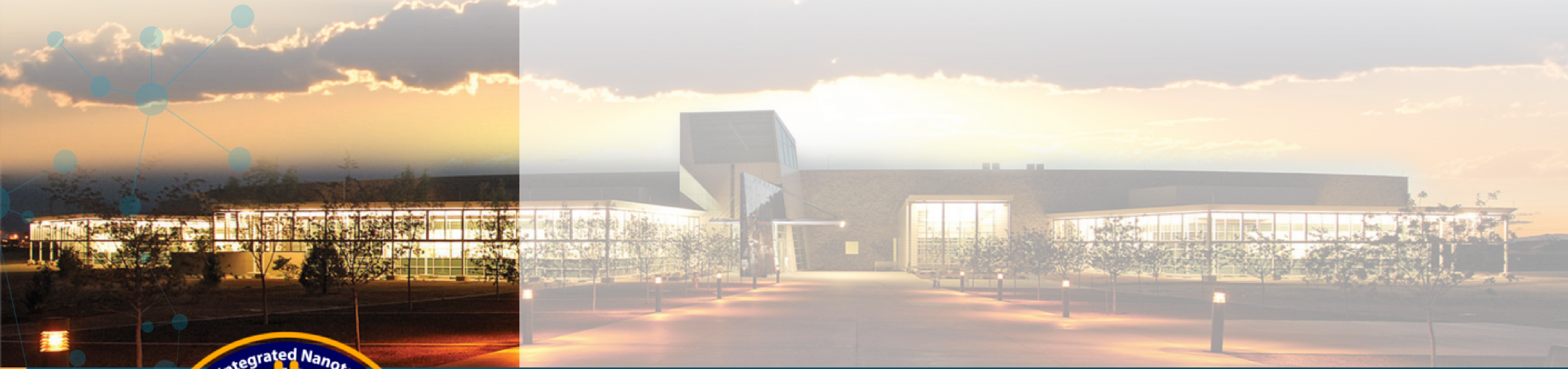


Identifying topology directly from Maxwell's Equations

Alexander Cerjan

International Conference of Quantum, Nonlinear and Nanophotonics

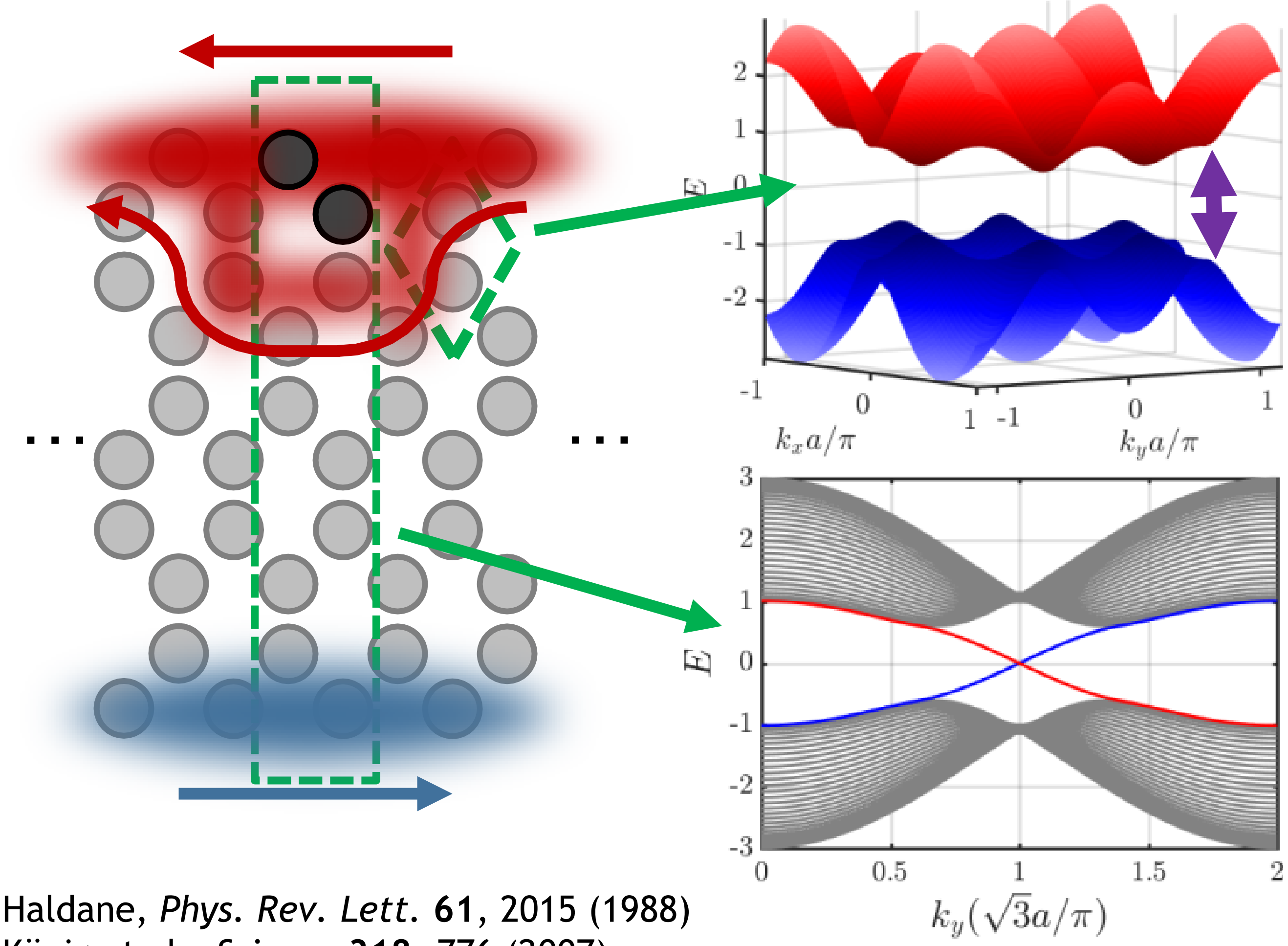
September 7th, 2022

What does a poorly balanced breakfast have to do with photonics?

What is topological photonics?

Chern or Quantum Hall insulators

2D system with *broken* Time Reversal symmetry



Can predict these boundary phenomena from a calculation in the bulk

- Bulk-boundary correspondence

Bulk bandgap → no photons in the bulk

Chern number: (a "topological invariant")

$$C_n = \frac{1}{2\pi i}\int_{BZ}\left(\frac{\partial A_y^n}{\partial k_x} - \frac{\partial A_x^n}{\partial k_y}\right) d^2\mathbf{k}$$

Defects on the edge do not matter!

Still no backscattering!

Berry Connection:

$$\mathbf{A}^n(\mathbf{k}) = i\langle\psi_{n\mathbf{k}}|\nabla_{\mathbf{k}}|\psi_{n\mathbf{k}}\rangle$$

Edges that are waveguides

Bloch eigenstates

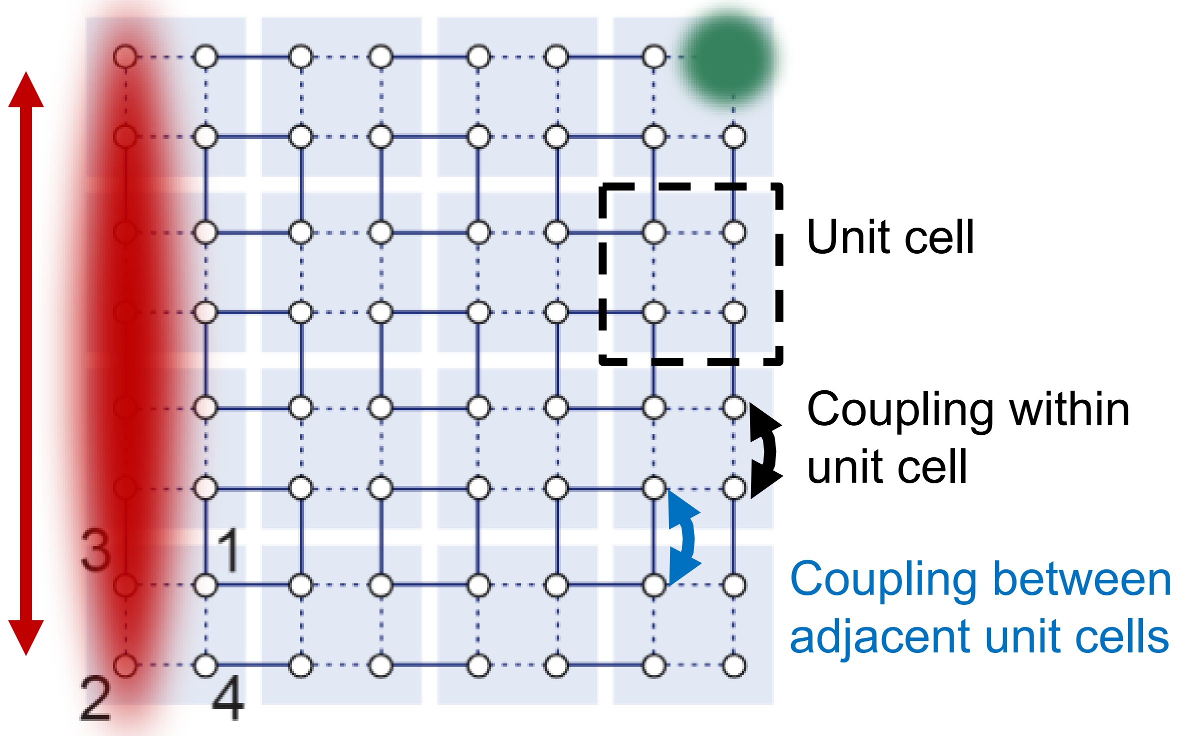
Haldane, *Phys. Rev. Lett.* **61**, 2015 (1988)
König et al., *Science* **318**, 776 (2007)

What is topological photonics?

Topological crystalline insulators

Reciprocal system, requires crystalline symmetry

Can have both edge-, and corner-localized modes



Benalcazar, Bernevig, Hughes, *Science* **357**, 61 (2017)

Predicted using symmetry indicators:
(i.e., a different topological invariant)

$$[\Pi_n] = \#\mathbf{\Pi}_n - \#\mathbf{\Gamma}_n$$

Calculated from terms such as

$$\langle \psi_{n\mathbf{k}} | O | \psi_{n\mathbf{k}} \rangle$$

O – is a crystalline symmetry

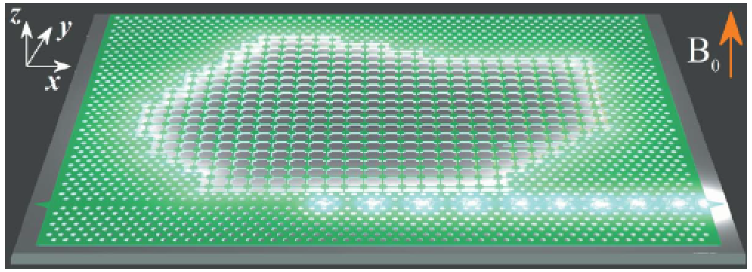
Why make photonics topological?

- No structural fine tuning required
  - If the invariant identifies topology, desired states are guaranteed
- Frequencies of desired states are guaranteed to be in the bandgap
- Systems are robust to fabrication imperfections
  - So long as defects don't close the bandgap.

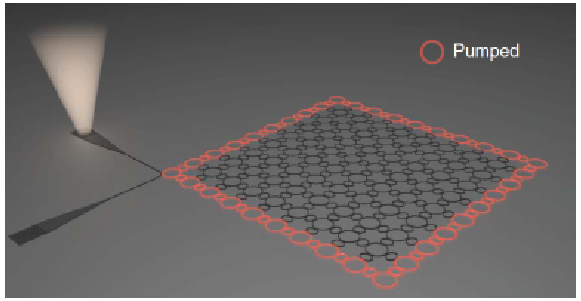
Why make photonics topological?

Topological lasers

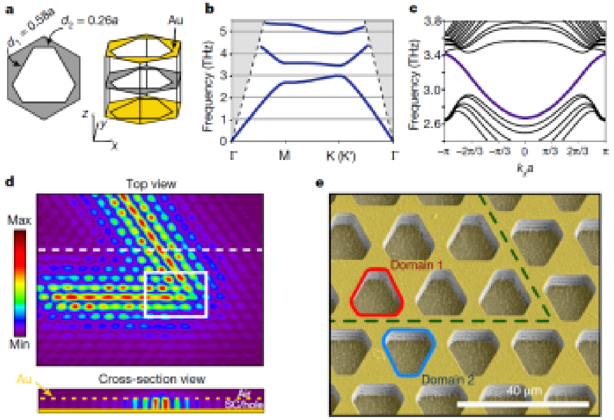
- Robust against disorder
- Efficient phase locking



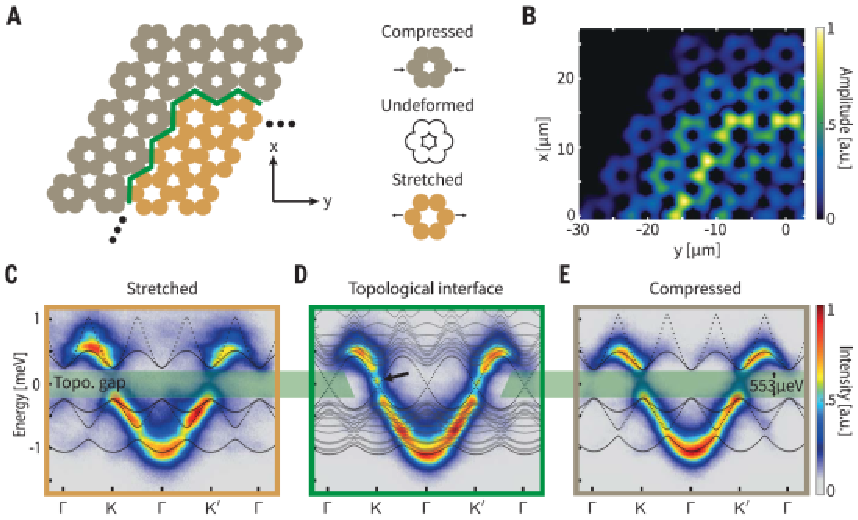
Bahari et al., *Science* **358**, 636 (2017)



Bandres et al., *Science* **359**, 1231 (2018)



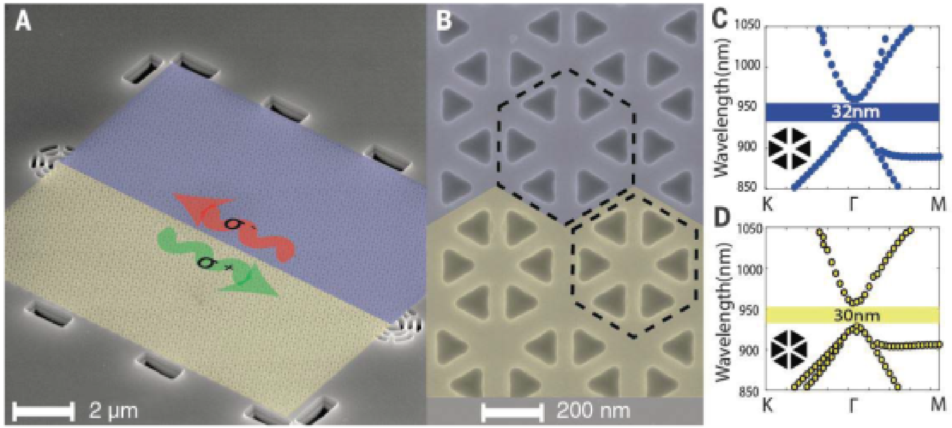
Zeng et al., *Nature* **578**, 246 (2020)



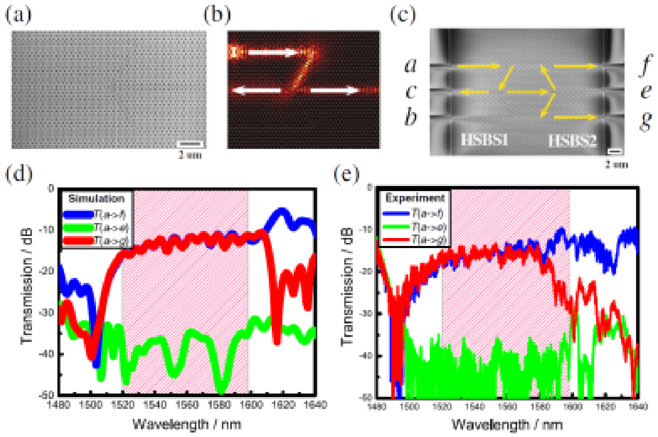
Dikopoltsev et al., *Science* **373**, 1514 (2021)

Why make photonics topological?

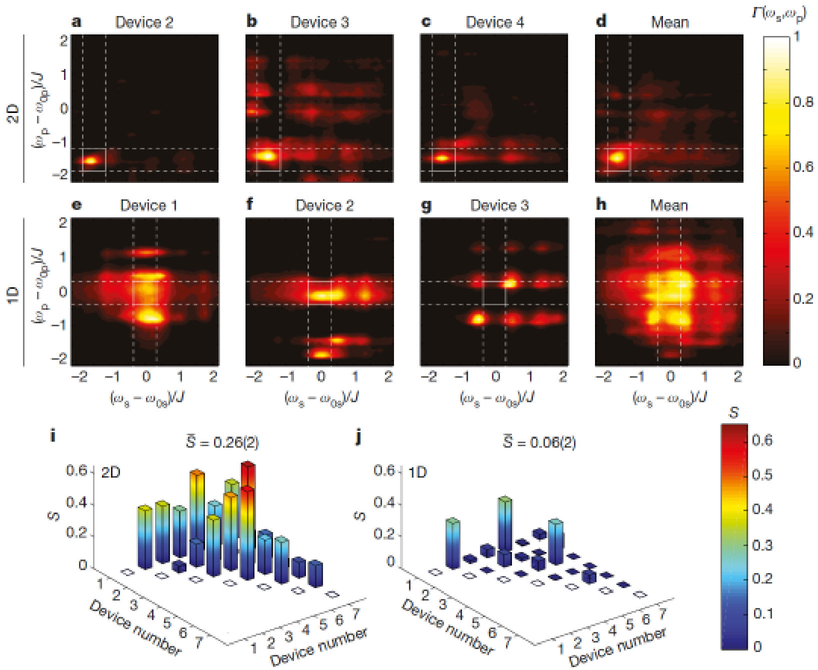
Routing of quantum information



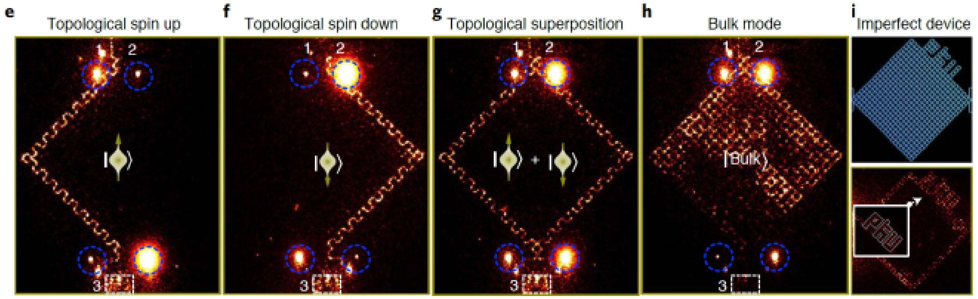
Barik et al., *Science* **359**, 666 (2018)



Chen et al., *Phys. Rev. Lett.* **126**, 230503 (2021)



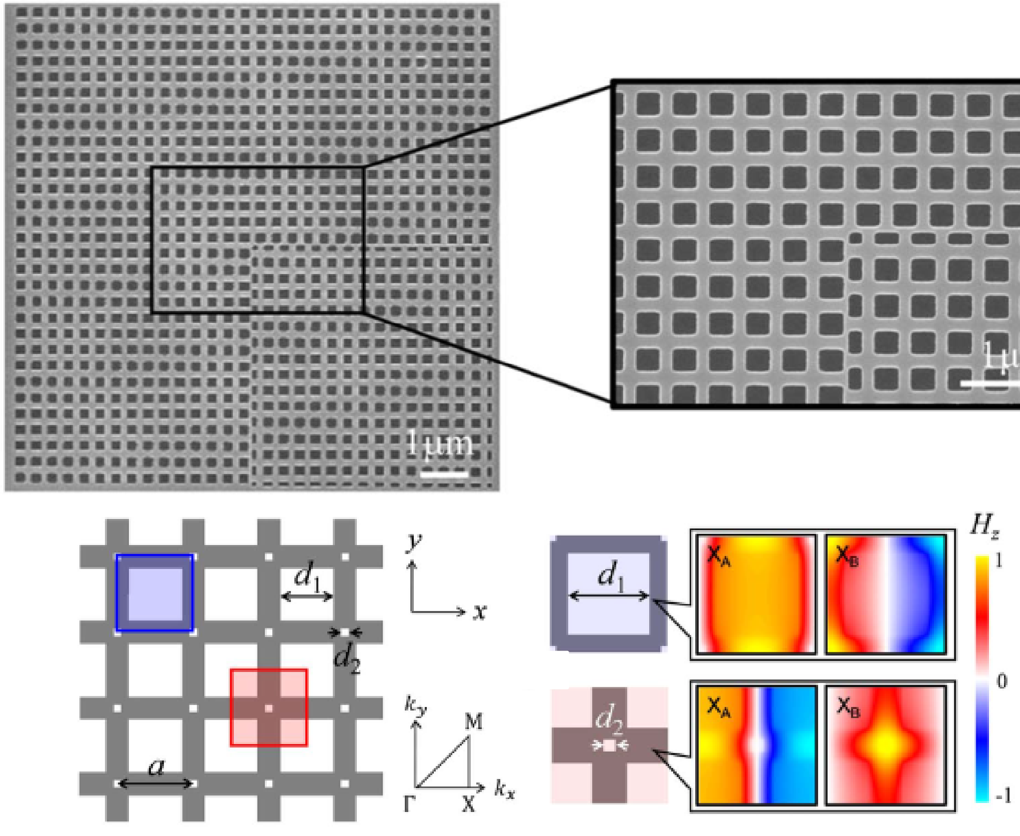
Mittal et al., *Nature* **561**, 502 (2018)



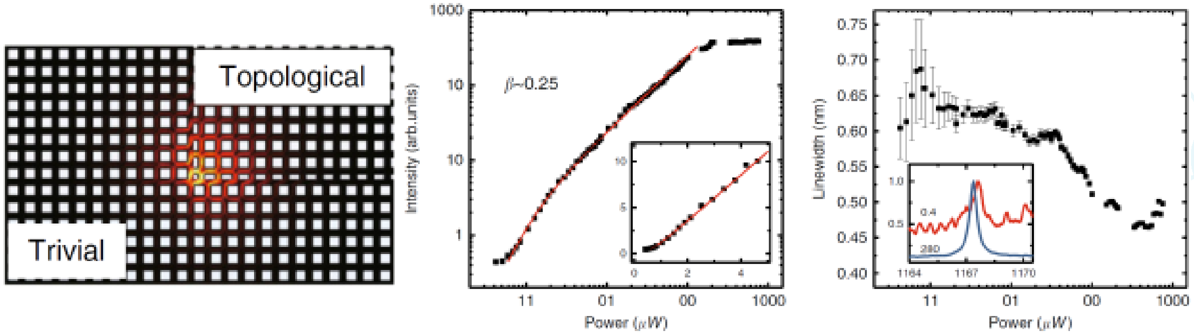
Dai et al., *Nat. Photonics* **16**, 248 (2022)

Why make photonics topological?

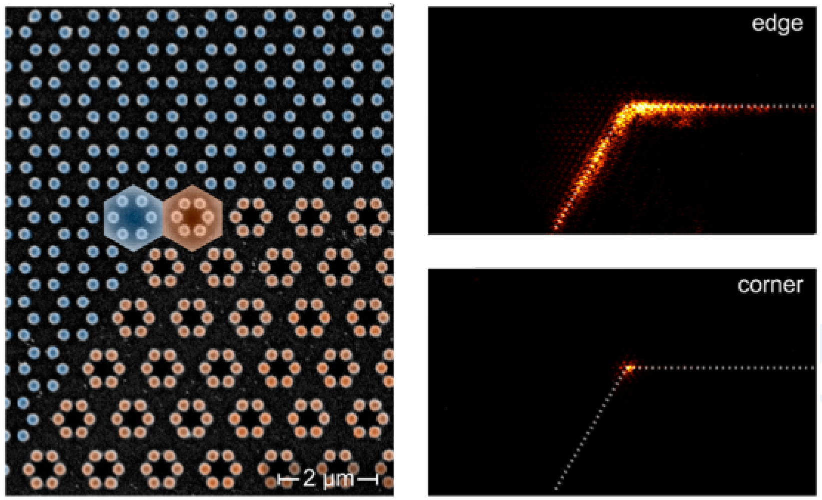
Creating cavities for light-matter interaction



Ota et al., *Optica* **6**, 786 (2019)



Zhang et al., *Light Sci. Appl.* **9**, 109 (2020)



Kruk et al., *Nano Lett.* **21**, 4592 (2021)

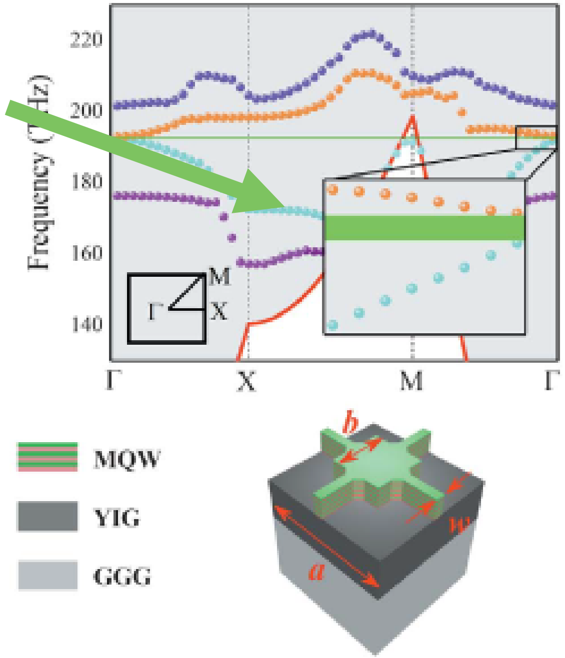
So what's the problem?

We'd like nanophotonic Chern insulators

- Non-reciprocal edge states

But… it's hard to break time-reversal symmetry

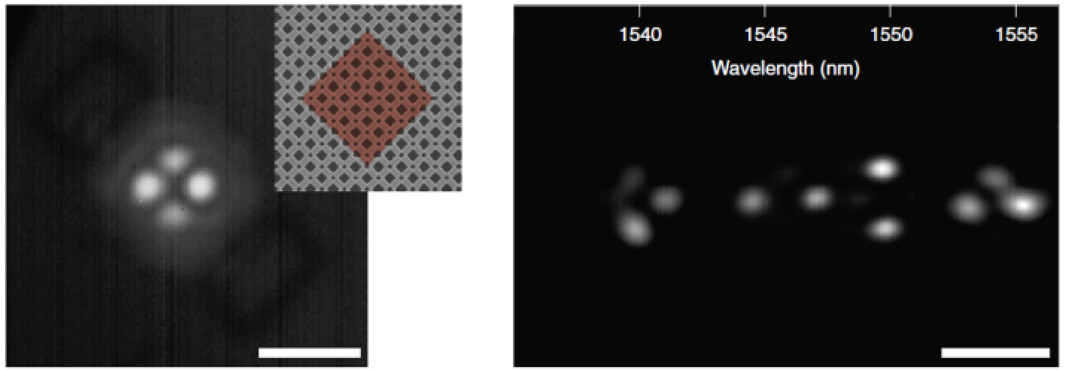
Small complete bandgap (42 pm)



Bahari et al., *Science* **358**, 636 (2017)

We don't have a theory that handles finite systems

How close can two topological cavities be, while maintaining protection?

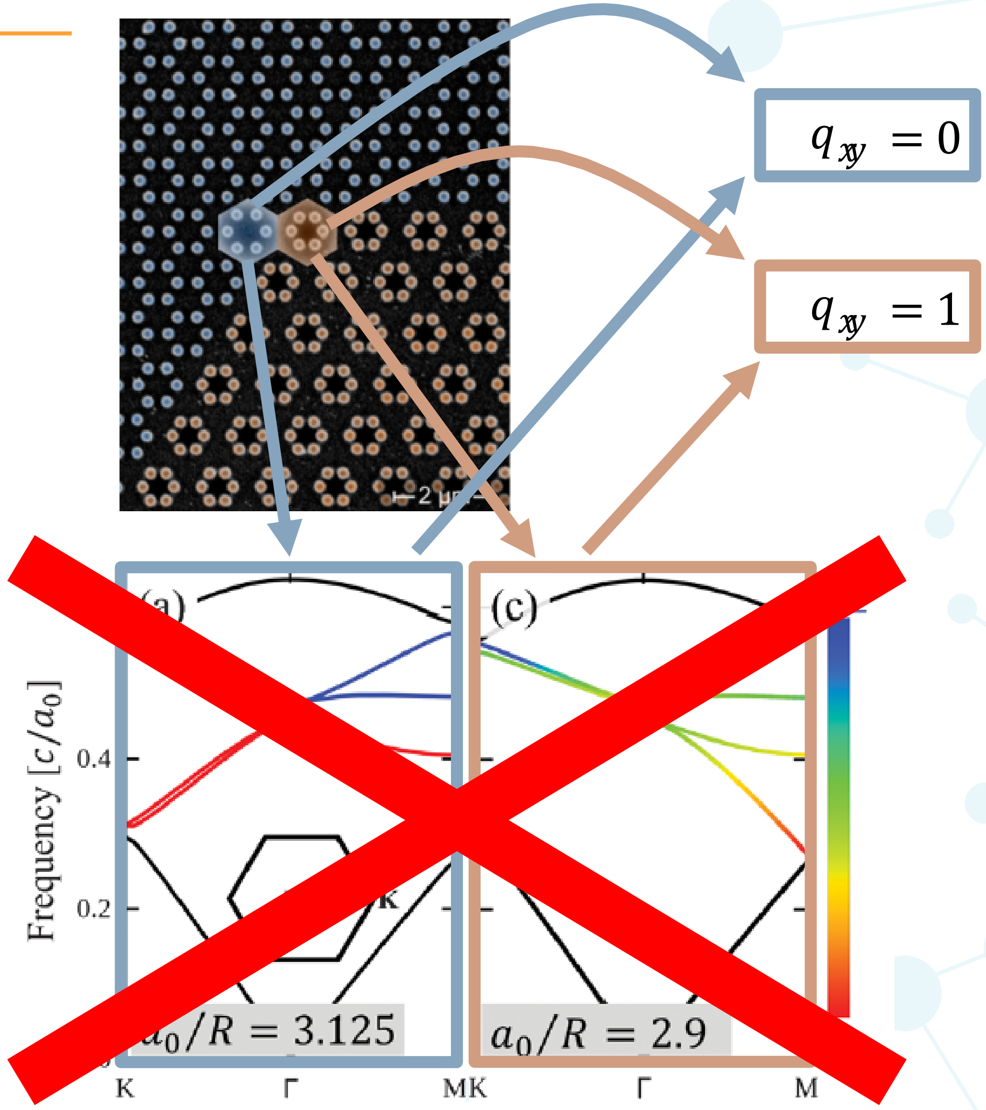


Kim et al., *Nat. Commun.* **11**, 5758 (2020)

So what's the problem?

Is it possible to define topology without Bloch eigenstates or band structures?

- For electromagnetic fields, calculating $|\psi_{n\mathbf{k}}\rangle$ and $\nabla_{\mathbf{k}}|\psi_{n\mathbf{k}}\rangle$ can be challenging



Kruk et al., *Nano Lett.* **21**, 4592 (2021)
Wu and Hu *Phys. Rev. Lett.* **114**, 223901 (2015)

Outline

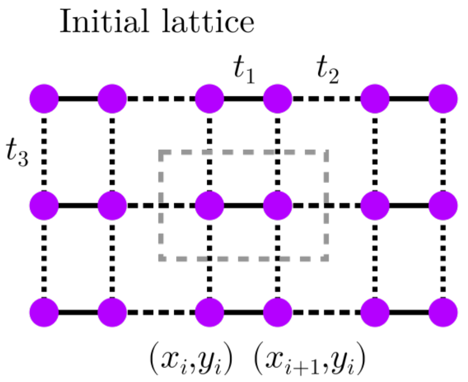
- An operator-based approach to topological photonics
  - Uses a framework called the “spectral localizer”
- Topology without a band gap
- Universal approach to topological crystalline photonics
- Realization in acoustic metamaterial

Topology as "Wannierizability"

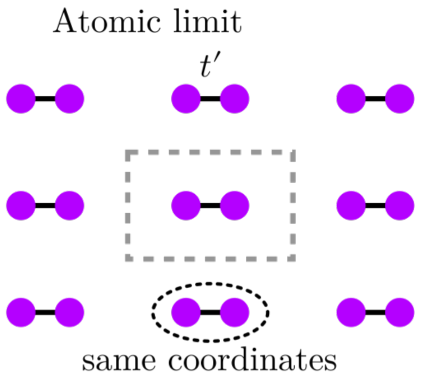
Instead of an invariant, "Can the system be permuted to an *atomic limit*?"

- Band gap stays open
- Symmetries are preserved

Can a lattice



be permuted to



without violating?

If yes
- ➢ Trivial

If no
- ➢ Topological

Can address this from band theory of original lattice

- ➢ Topological quantum chemistry

Determine if a complete Wannier basis exists.

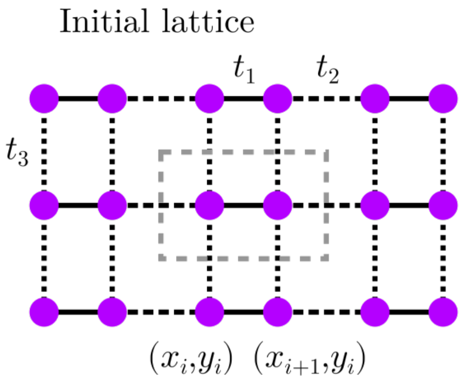
Kitaev, *AIP Conference Proceedings* **1134**, 22 (2009)
Brouder et al., *Phys. Rev. Lett.* **98**, 046402 (2007)
Soluyanov and Vanderbilt, *Phys. Rev. B* **83**, 035108 (2011)
Taherinejad et al., *Phys. Rev. B* **89**, 115102 (2014)
Bradlyn et al., *Nature* **547**, 298 (2017)
Po et al., *Nat. Commun.* **8**, 50 (2017)

Topology as “Wannierizability”

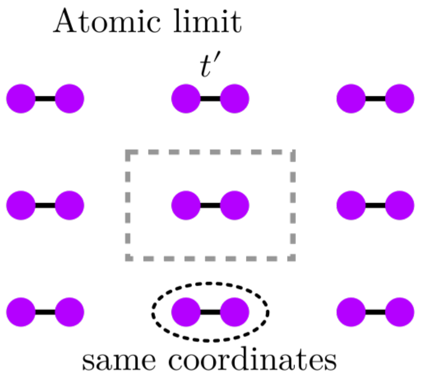
Instead of an invariant, “Can the system be permuted to an *atomic limit*?”

- Band gap stays open
- Symmetries are preserved

Can a lattice



be permuted to



without violating?

If yes
➢ Trivial

If no
➢ Topological

$$H = \begin{bmatrix} \ddots & -t_2 & & -t_3 & & \\ -t_2 & \varepsilon & -t_1 & & -t_3 & \\ & -t_1 & \varepsilon & -t_2 & & -t_3 \\ -t_3 & & -t_2 & \varepsilon & -t_1 & \\ & -t_3 & & -t_1 & \varepsilon & -t_2 \\ & & -t_3 & & -t_2 & \ddots \end{bmatrix}$$

$$H_a = \begin{bmatrix} \ddots & & & & & \\ & \varepsilon' & -t' & & & \\ & -t' & \varepsilon' & & & \\ & & & \varepsilon' & -t' & \\ & & & -t' & \varepsilon' & \\ & & & & & \ddots \end{bmatrix}$$

Can the operators

be permuted to

without violating similar restrictions?

$[H, X] \neq 0$

$$X = \begin{bmatrix} \ddots & & & & & \\ & x_{i-1} & & & & \\ & & x_i & & & \\ & & & x_{i+1} & & \\ & & & & x_{i+2} & \\ & & & & & \ddots \end{bmatrix}$$

$[H_a, X_a] = 0$

$$X_a = \begin{bmatrix} \ddots & & & & & \\ & x'_i & & & & \\ & & x'_i & & & \\ & & & x'_{i+1} & & \\ & & & & x'_{i+1} & \\ & & & & & \ddots \end{bmatrix}$$

Topology from operators

Instead of an invariant, "Can the system be permuted to an *atomic limit*?"

"Can the system's operators be permuted to be *commuting*?"

- Diagnose using recent advances in the mathematics of C^*-algebras

Construct the *spectral localizer* using a non-trivial Clifford representation

$$L_{(x_1,\ldots,x_d,E)}(X_1,\ldots,X_d,H) = \sum_{j=1}^{d} (X_j - x_j I) \otimes \Gamma_j + (H - EI) \otimes \Gamma_{d+1}$$

Different topological invariants are given by this composite operator's properties

Example: local Chern number $C_L(x,y,E) = \frac{1}{2}\,\mathrm{sig}[L_{(x,y,E)}(X,Y,H)]$

As is the system's "local gap" (i.e., something like a spatially resolved band gap)

$$\mu^C(x_1,\ldots,x_d,E) = \sigma_{\min}[L_{(x_1,\ldots,x_d,E)}(X_1,\ldots,X_d,H)]$$ (smallest eigenvalue of $L_{(x_1,\ldots,x_d,E)}$)

Reformulating Maxwell's equations

Linear, local media, allow for dispersion

$$\nabla \times \mathbf{E}(\mathbf{x}) = i\omega\bar{\mu}(\mathbf{x},\omega)\mathbf{H}(\mathbf{x}),$$
$$\nabla \times \mathbf{H}(\mathbf{x}) = -i\omega\bar{\varepsilon}(\mathbf{x},\omega)\mathbf{E}(\mathbf{x}),$$
$$\nabla \cdot [\bar{\varepsilon}(\mathbf{x},\omega)\mathbf{E}(\mathbf{x})] = 0,$$
$$\nabla \cdot [\bar{\mu}(\mathbf{x},\omega)\mathbf{H}(\mathbf{x})] = 0.$$

For non-zero frequencies, can recast as:

$$\left[\begin{pmatrix} 0 & -i\nabla\times \\ i\nabla\times & 0 \end{pmatrix} - \omega \begin{pmatrix} \bar{\mu}(\mathbf{x}) & 0 \\ 0 & \bar{\varepsilon}(\mathbf{x}) \end{pmatrix}\right] \begin{pmatrix} \mathbf{H}(\mathbf{x}) \\ \mathbf{E}(\mathbf{x}) \end{pmatrix} = 0,$$

The divergence equations can be recovered using

$\nabla \cdot \nabla \times \mathbf{F}(\mathbf{x}) = 0$ for any vector field $\mathbf{F}(\mathbf{x})$

This yields a "self-consistent" generalized eigenvalue equation:

$$\boxed{W\boldsymbol{\psi}(\mathbf{x}) = \omega M(\mathbf{x},\omega)\boldsymbol{\psi}(\mathbf{x})}$$

In which: $\boldsymbol{\psi}(\mathbf{x}) = (\mathbf{H}(\mathbf{x}), \mathbf{E}(\mathbf{x}))^\top,$

$$W = \begin{pmatrix} 0 & -i\nabla\times \\ i\nabla\times & 0 \end{pmatrix},$$

$$M(\mathbf{x},\omega) = \begin{pmatrix} \bar{\mu}(\mathbf{x},\omega) & 0 \\ 0 & \bar{\varepsilon}(\mathbf{x},\omega) \end{pmatrix},$$

And finally an ordinary eigenvalue equation:

$$\boxed{H_{\text{eff}}(\omega)\boldsymbol{\phi}(\mathbf{x}) = \omega\boldsymbol{\phi}(\mathbf{x})}$$

$$H_{\text{eff}}(\omega) = M^{-1/2}(\mathbf{x},\omega)WM^{-1/2}(\mathbf{x},\omega)$$

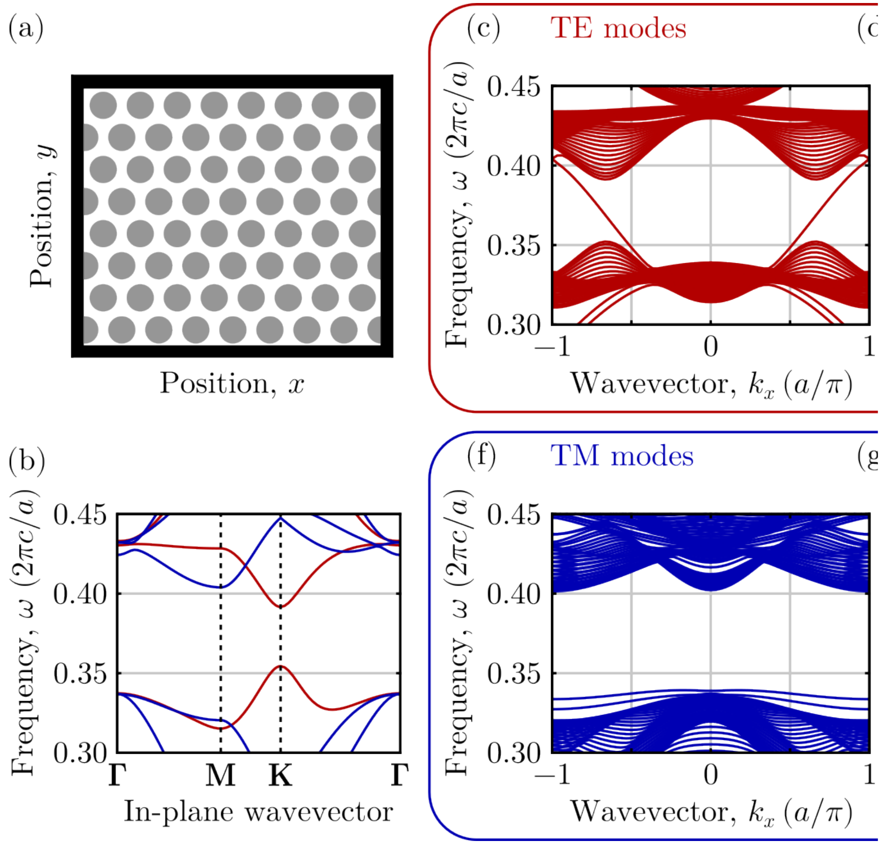
$$\boldsymbol{\phi}(\mathbf{x}) = M^{1/2}(\mathbf{x},\omega)\boldsymbol{\psi}(\mathbf{x})$$

The Haldane and Raghu photonic Chern insulator

$$H_{\text{eff}}(\omega) = M^{-1/2}(\mathbf{x},\omega) W M^{-1/2}(\mathbf{x},\omega)$$

$$L_{(x,y,\omega)}(H,X,Y) = \begin{bmatrix} H - \omega I & (X - xI) - i(Y - yI) \\ (X - xI) + i(Y - yI) & -(H - \omega I) \end{bmatrix}$$

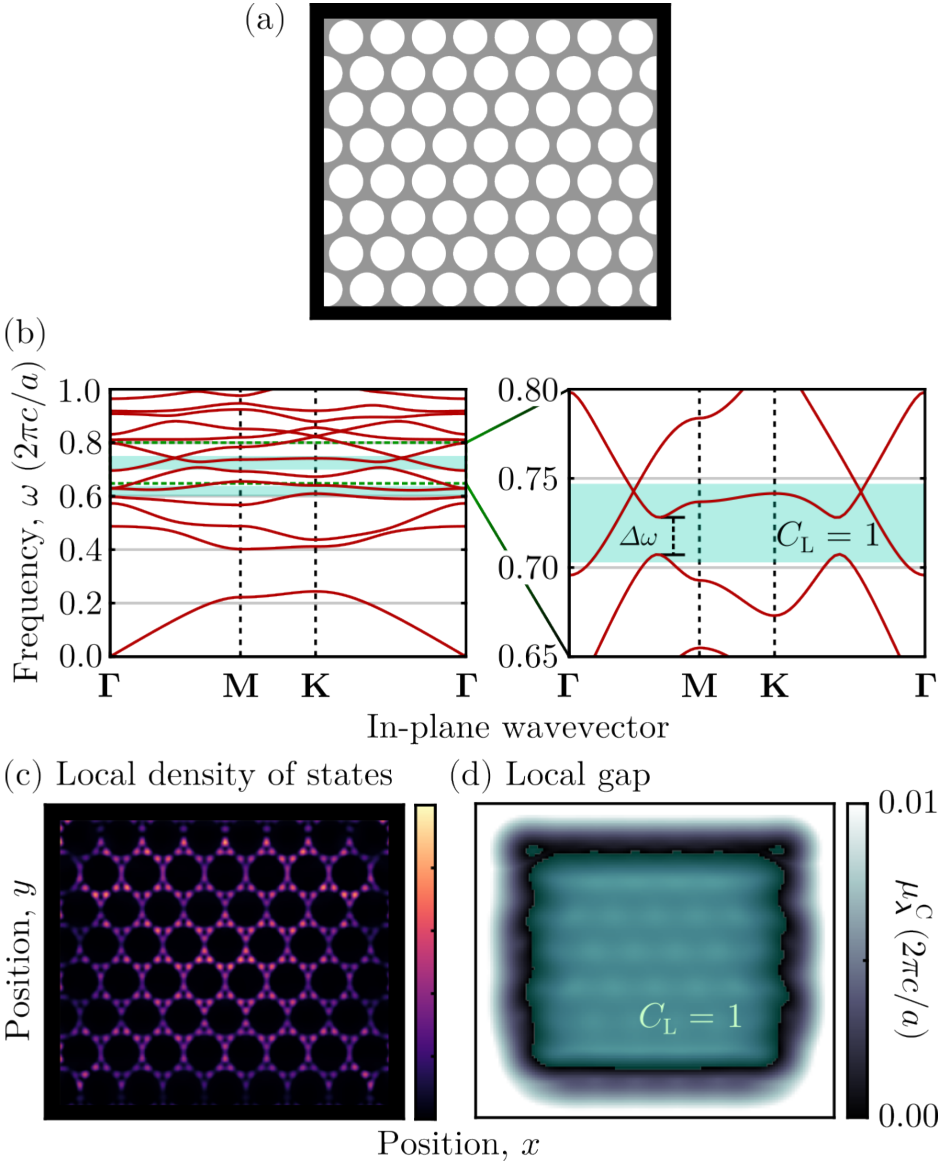
2D photonic crystal of dielectric pillars in gyro-electric air



Haldane and Raghu, *Phys. Rev. Lett.* **100**, 013904 (2008)
Raghu and Haldane, *Phys. Rev. A* **78**, 033834 (2008)

AC and Loring, in submission.

Gapless photonic Chern materials



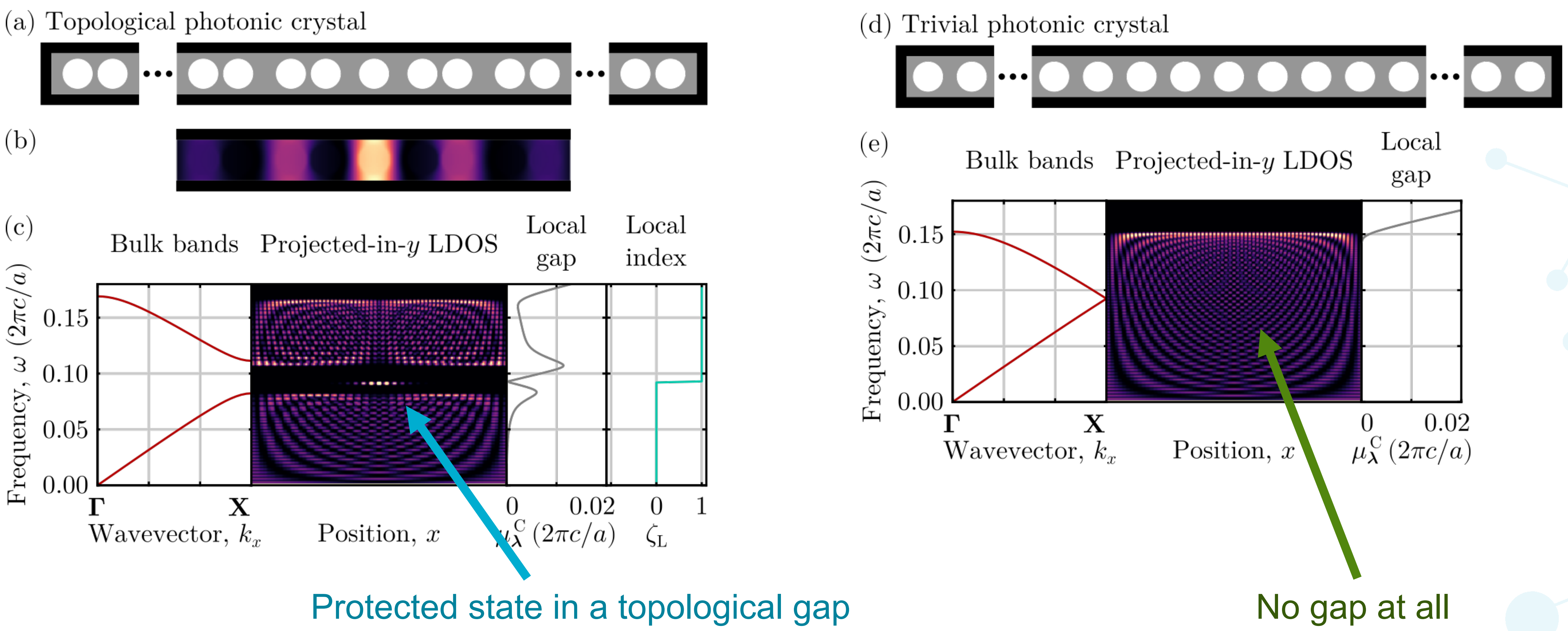
Completely gapless in the frequency range of interest

Even so, can identify topology

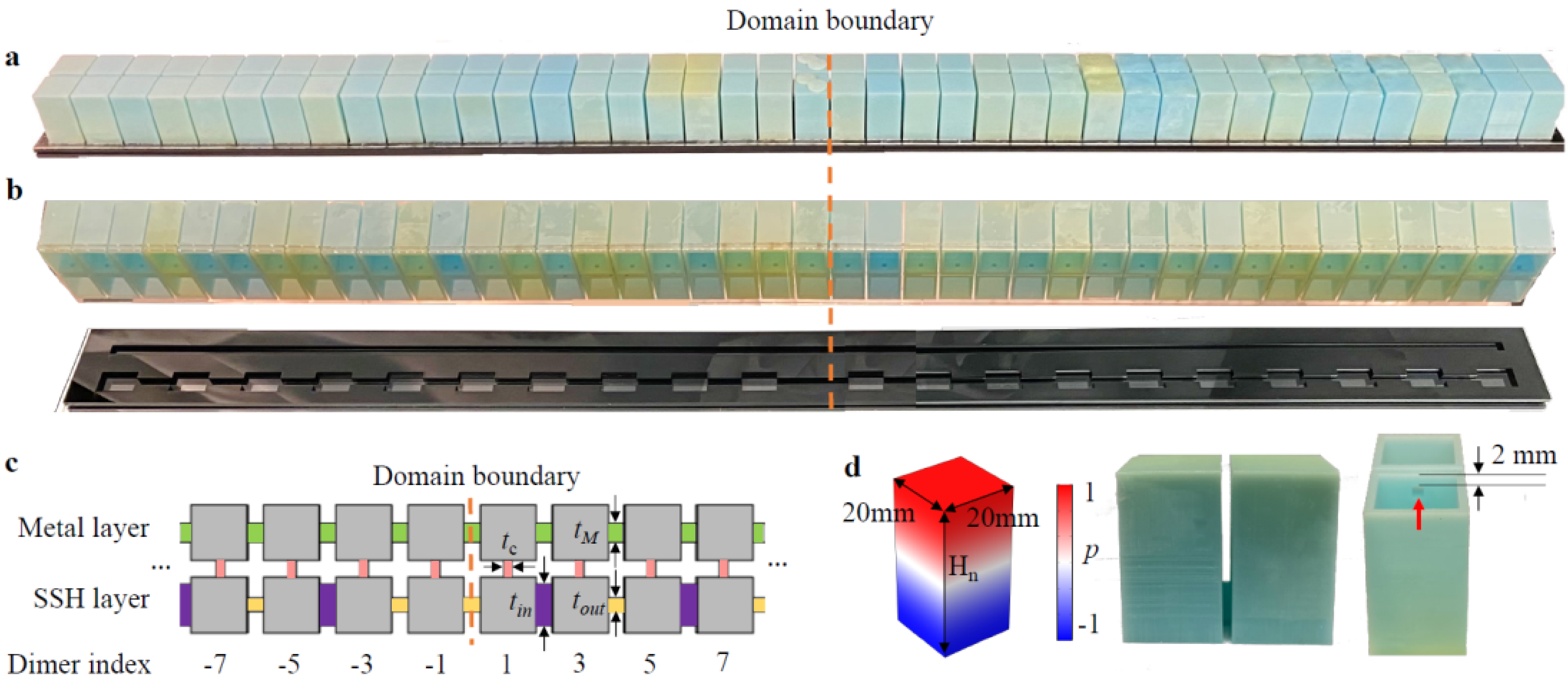
And see bulk-boundary correspondence

New designs for non-linear topological photonics?

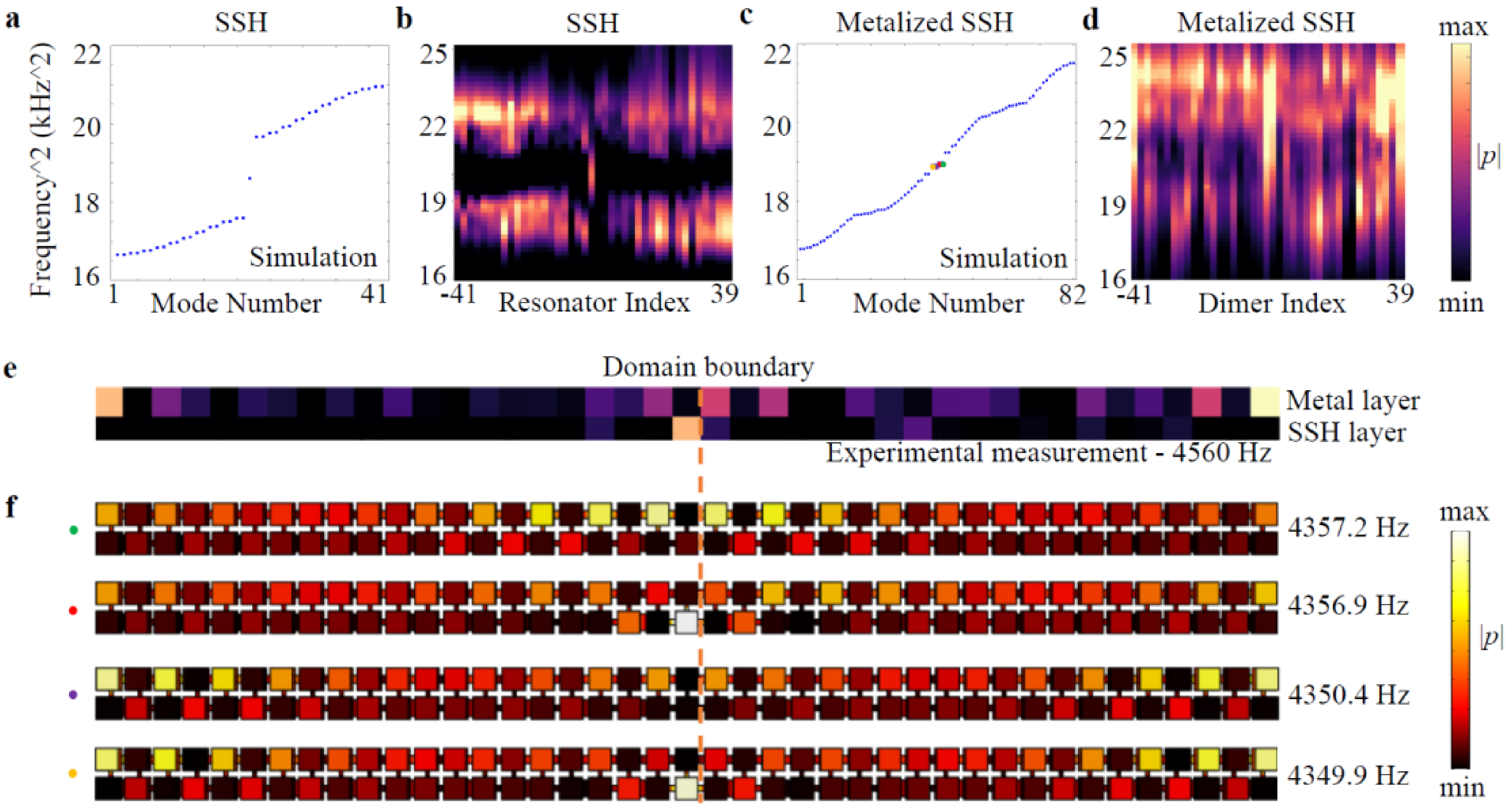
Topological crystalline photonic structures



Gapless topological acoustic metamaterial



Gapless topological acoustic metamaterial



Direct observation of K-theory

New experimental protocol:

➢ Localizer ***as*** Hamiltonian

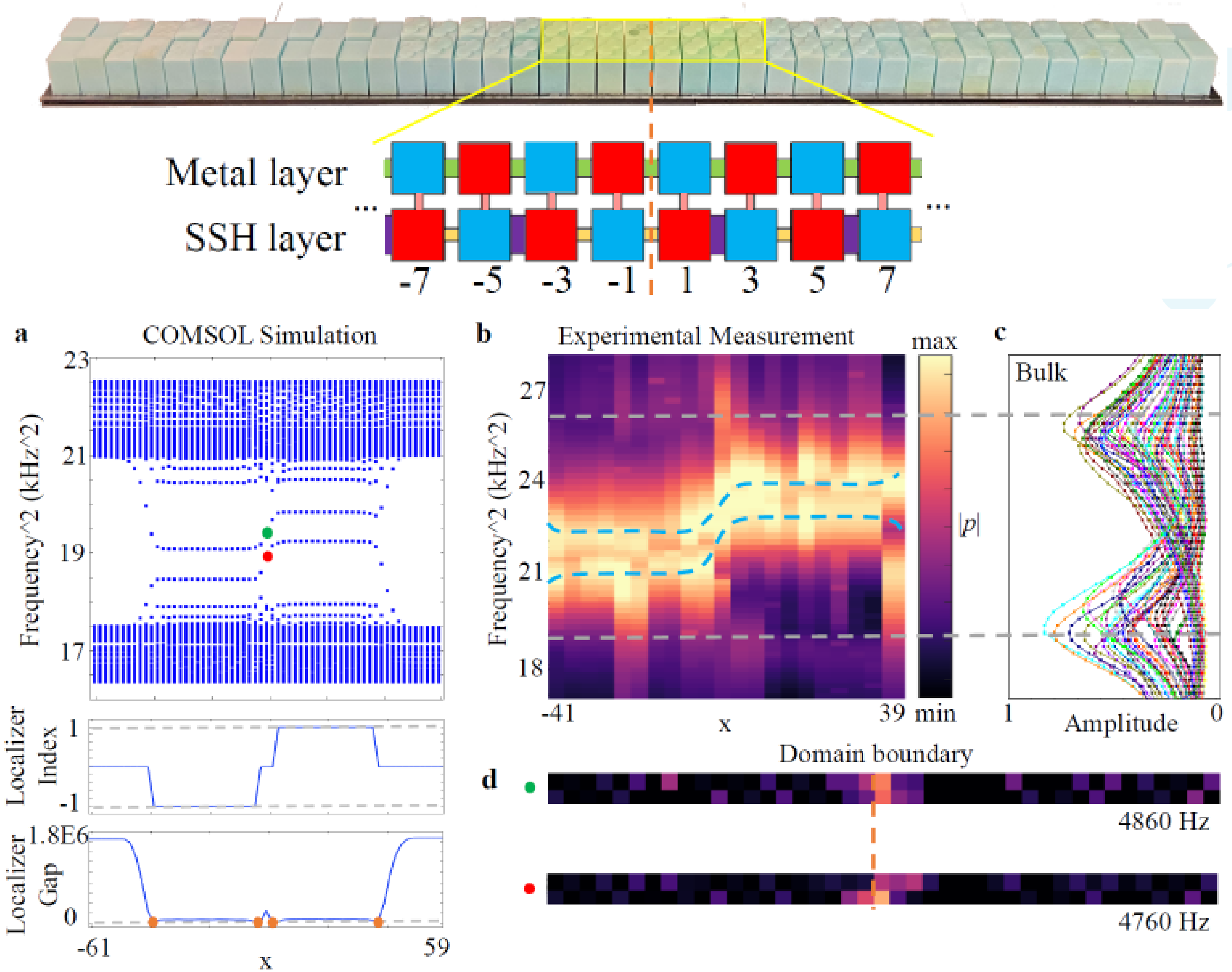
$$\tilde{H}_x = \tilde{L}_{(x,0)}(X, H) = \kappa(X - xI)\Pi + H.$$

Makes topology directly observable.

$$\nu(x) = \tfrac{1}{2}\mathrm{sig}\left(\tilde{L}_{(x,0)}(X, H)\right) \in \mathbb{Z},$$

Slight alteration for physical systems.

$$\tilde{H}_x = \kappa\left[\tanh\left(\frac{X - xI}{\alpha}\right)\right]\Pi + H.$$



Conclusion

- Benefits of topological photonics
  - States with guaranteed properties
  - Robust against disorder
- Topology can be diagnosed using an operator-based framework
  - No band structures or Bloch eigenstates required
- Yields topology in gapless materials (not possible with band theories)
- Rigorous, general framework for finite system effects
- New experimental protocol – Spectral localizer as Hamiltonian

Acknowledgements

Terry A. Loring
University of New Mexico