



# WEC Array Optimization with Multi-Resonance and Phase Control of Electrical Power Take-Off

Madelyn G. Veurink<sup>1</sup>, Wayne W. Weaver<sup>1</sup>, David G. Wilson<sup>2</sup>,  
Rush D. Robinett III<sup>1</sup>, Ronald C. Matthews<sup>2</sup>

<sup>1</sup> Michigan Technological University, Houghton, Michigan, USA,

<sup>2</sup> Sandia National Labs, Albuquerque, New Mexico, USA

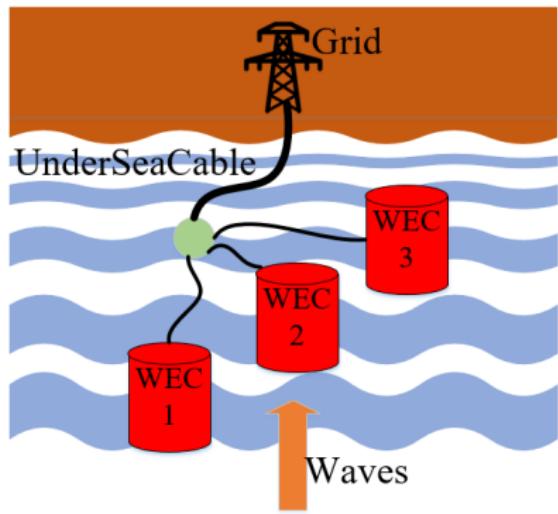
IFAC Symposium on Mechnatronic Systems  
September 6-9, 2022

# Introduction

- Wave Energy Converters (WECs) can be used to extract energy from the world's oceans.
- The electric machine on each buoy must be controlled with a multi-resonance controller to optimize the power output of each WEC.
- The electrical energy from each of the electric machines oscillates with the wave period
- Shifting the physical placement of the buoys will create a phase shift in the electrical signals

# Main Components of a WEC

- Buoys
- Mechanical Drive-trains
- Electric Machines
- Power Electronics
- Energy Storage
- Under-sea Cable
- Grid connection

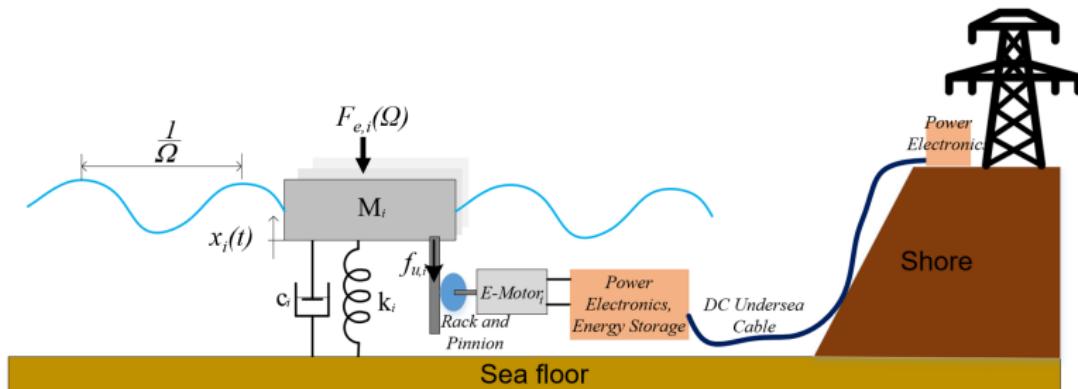


# Mechanical Model of the WEC Array

- The mechanical systems of the buoys are modelled as mass-spring-damper systems:

$$m_i \ddot{x}_i + c_i \dot{x}_i + kx_i = f_{e,i} + f_{u,i} \quad (1)$$

- Power Electronics convert mechanical energy from E-Motor to dc
- Transmit dc power to the shore - Shore side inverter couples to local transmission grid



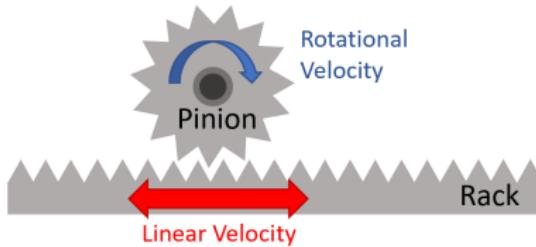
# Electrical Drives of a WEC Array

The control force  $f_{u,i}$  is replaced with the linear force on a PMDC machine

$$f_{u,i} = \frac{\tau}{r} = \frac{i_{a,i} K_m}{r} \quad (2)$$

The linear motion of the buoys is translated to a rotational motion that turns the electric machines.

$$v_i = \dot{x}_i = r w_{m,i} \quad (3)$$



# Electrical Model of the WEC Array

The electric machine on each of the buoys is

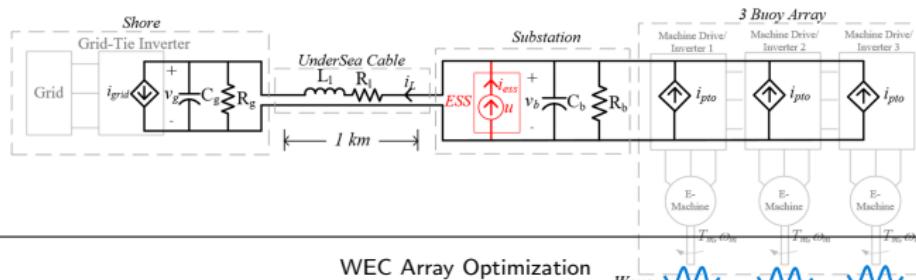
$$\dot{i}_{a,i} = \frac{1}{L_a} (v_{a,i} - i_{a,i} R_a - \frac{K_m v_i}{r}) \quad (4)$$

The current injected into the electrical bus from each of the DC electric machines is

$$i_{pto,i} = \frac{P_{pto,i}}{v_b} = \frac{v_{a,i} i_{a,i}}{v_b} \quad (5)$$

The sum of the currents is

$$i_{ptosum} = \sum_{i=1}^N i_{pto,i} \quad (6)$$



# Array Connection to the Onshore Grid

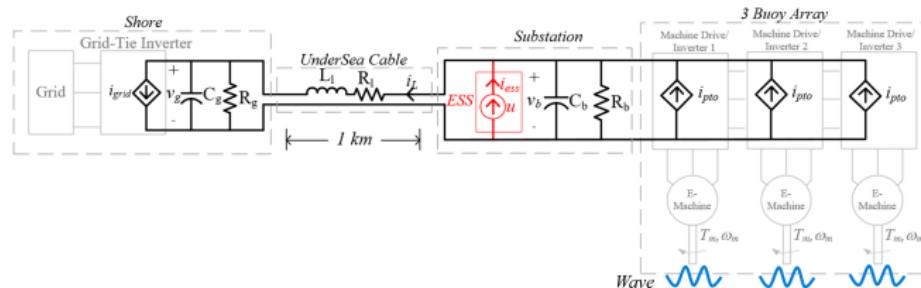
The electrical bus, the line to shore, and the grid can be modelled by the following equations.

$$\dot{v}_b = \frac{1}{C_b} (i_{ptosum} - \frac{v_b}{R_b} - u - i_L) \quad (7)$$

$$\dot{i}_L = \frac{1}{L_L} (v_b - i_L R_L - v_g) \quad (8)$$

$$\dot{v}_g = \frac{1}{C_g} (i_L - i_{grid} - \frac{v_g}{R_g}) \quad (9)$$

$u$  is the current injected to the bus by the ideal energy storage system (ESS).



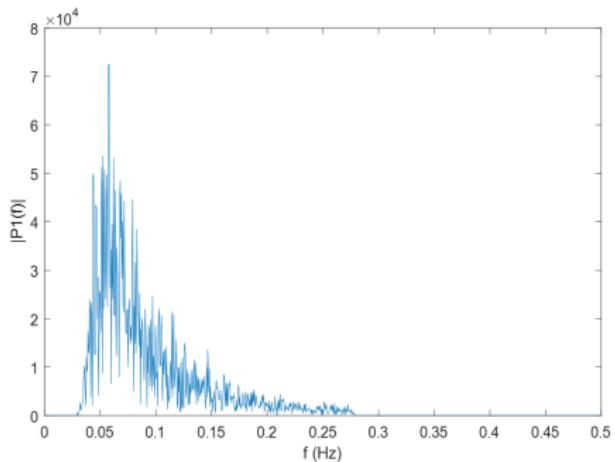
# Excitation Force on the WEC Array

- Ocean waves contain many frequencies.
- One model of a sea state is the *Bretschneider Spectrum*.
- The multi-frequency excitation force is the sum of multiple frequency components.

$$f_e = \sum_{n=1}^N A_n \sin(w_n t + \phi_n) \quad (10)$$

- To maximize power output of the Array the buoys must resonate with the peak frequencies of the waves.

The Bretschneider Spectrum



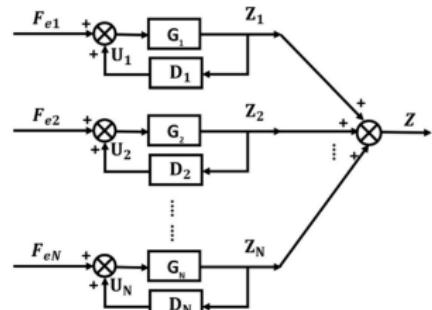
# Multi-Resonance Control

- The multi-frequency excitation force can be decomposed into its individual components using a Discrete Fourier Transform.
- PDC3 requires that a PD controller be designed for each of the main frequencies.

$$k_{p,i} = w_i^2 m_i - k. \quad (11)$$

$$k_{d,i} = c_i. \quad (12)$$

- The individual channels are then summed to create the full control force.



# Buoy Shifting and Electrical phasing

- For a shifted WEC in an array the output power is

$$p_i(t) = \frac{1}{2}(\cos(2\omega_n t - 2(i-1)\phi) + 1) \quad (13)$$

- The power from N WECs is

$$P_{array} = \sum_{i=1}^N p_i(t) = \frac{1}{2}(\csc(\phi)\sin(N\phi)\cos(2\omega_n t + \phi(1-N))) + N. \quad (14)$$

- The total sum of the power is constant if

$$\csc(\phi)\sin(N\phi) = 0 \text{ when } \phi \in \left\{ \frac{\pi}{N}, \frac{2\pi}{N} \right\}. \quad (15)$$

# Example System

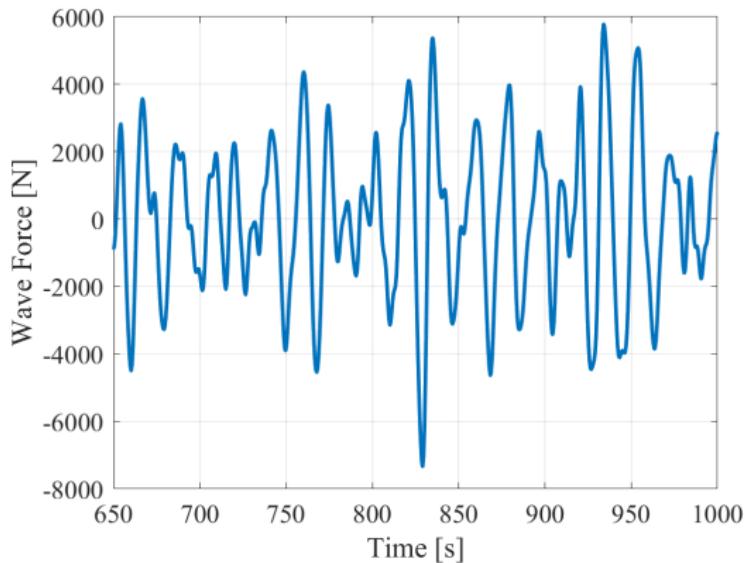
- 3 WECs
- Bretschneider Wave Force
- DC machines on the buoys connect to DC bus

## Model Parameters

Parameter	Description	Value
$m$	Buoy Masses	268 kg
$c$	Damper Coefficients	1226 N/ $\frac{m}{s}$
$k$	Spring Coefficients	1194 $\frac{N}{m}$
$r$	Rack and Pinion Gear Radii	0.025 m
$K_m$	Torque Constants	2 $\frac{Nm}{A}$
$L_a$	Armature Inductances	$1e^{-3}$ H
$R_a$	Armature Resistances	$1e^{-3}$ Ω
$C_b$	Bus Capacitance	$2e^{-6}$ F
$R_b$	Bus Parasitic Resistance	$80e^{-9}$ Ω
$R_L$	Undersea Cable Resistance	2.5 Ω
$L_L$	Undersea Cable Inductance	$95.6e^{-6}$ L
$R_g$	Grid Inverter Resistance	1000 Ω
$C_g$	Grid Inverter Capacitance	$2e^{-6}$ F

# Applied Wave Force

- Developed from a Bretschneider Spectrum



Chosen Controller  
Frequencies for the Three  
PDC3 Channels

Parameter	Value
$f_{Channel1}$	0.053 Hz
$f_{Channel2}$	0.0505 Hz
$f_{Channel3}$	0.0655 Hz
$f_{Channel4}$	0.0485 Hz

# Force Shifting on the WECs

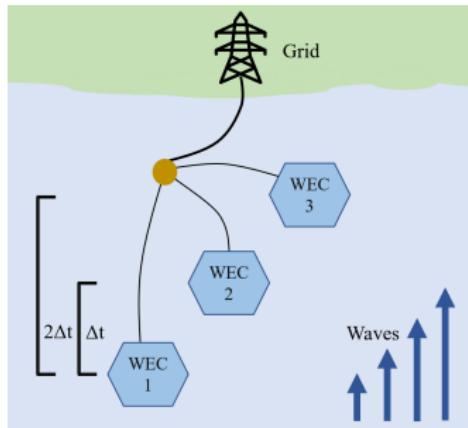
- The WECs are shifted spatially in the water.
- This spatial shift is represented by a time shift in the excitation forces.

$$f_{e,1}(t) = \hat{f}_e(t) \quad (16)$$

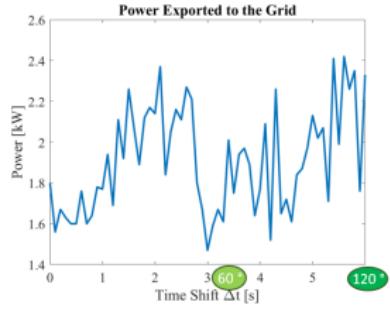
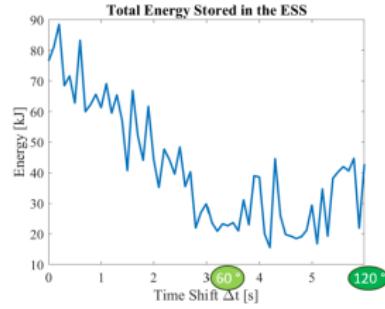
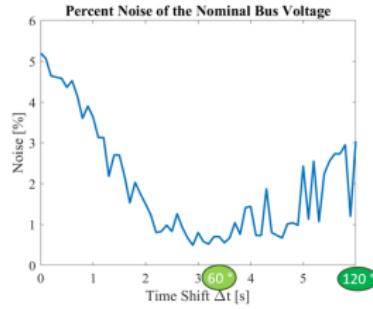
$$f_{e,2}(t) = \hat{f}_e(t - \Delta t) \quad (17)$$

$$f_{e,3}(t) = \hat{f}_e(t - 2\Delta t) \quad (18)$$

- $\Delta t$  is a design parameter used to determine the optimal spacing of the WECs.



# Optimizing the Time Shift



# Time Shifting Results

Table: Results Varying Time Shifts on the 3 WECs

$\Delta t$ [s]	Pk-Pk Voltage Noise [%]	ESS Energy [kJ]	Grid Power [kW]
0	5.20	76.5	1.8
1	3.63	61.3	1.77
2	1.50	44.5	2.14
3	0.80	29.8	1.47
4	1.44	38.6	1.77
5	2.43	29.4	2.13
6	3.04	42.8	2.33

60°

120°

Best Cases

# Conclusions

- Changing the physical spacing of the buoys creates an electrical phase shift.
- Phase shifting the electrical signals reduces bus voltage ripple, ESS size, and maximizes the grid power.
- For three buoys the optimal phase shift occurs at  $60^\circ$  or  $120^\circ$ .
- Due to additional frequency components in the wave spectrum the optimal time shift is slightly less than 3 seconds or  $60^\circ$ .

# Acknowledgments

- This study was funded by the Laboratory Directed Research & Development (LDRD) program at Sandia National Laboratories. Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.
- This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

*Special thanks to Dr. Ray Byrne at Sandia, for his technical review and programmatic leadership for this LDRD project.*

Thank You for Your  
Attention!

Questions?