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2022 Sandia FORCEE Summer Research Symposium

Estimating Seismic Source Time Functions using Time Domain Inversion with Uncertainty Quantification

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2022 FORCEE Symposium

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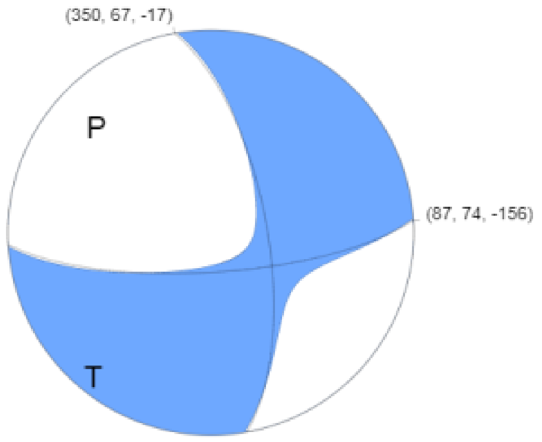
The Problem



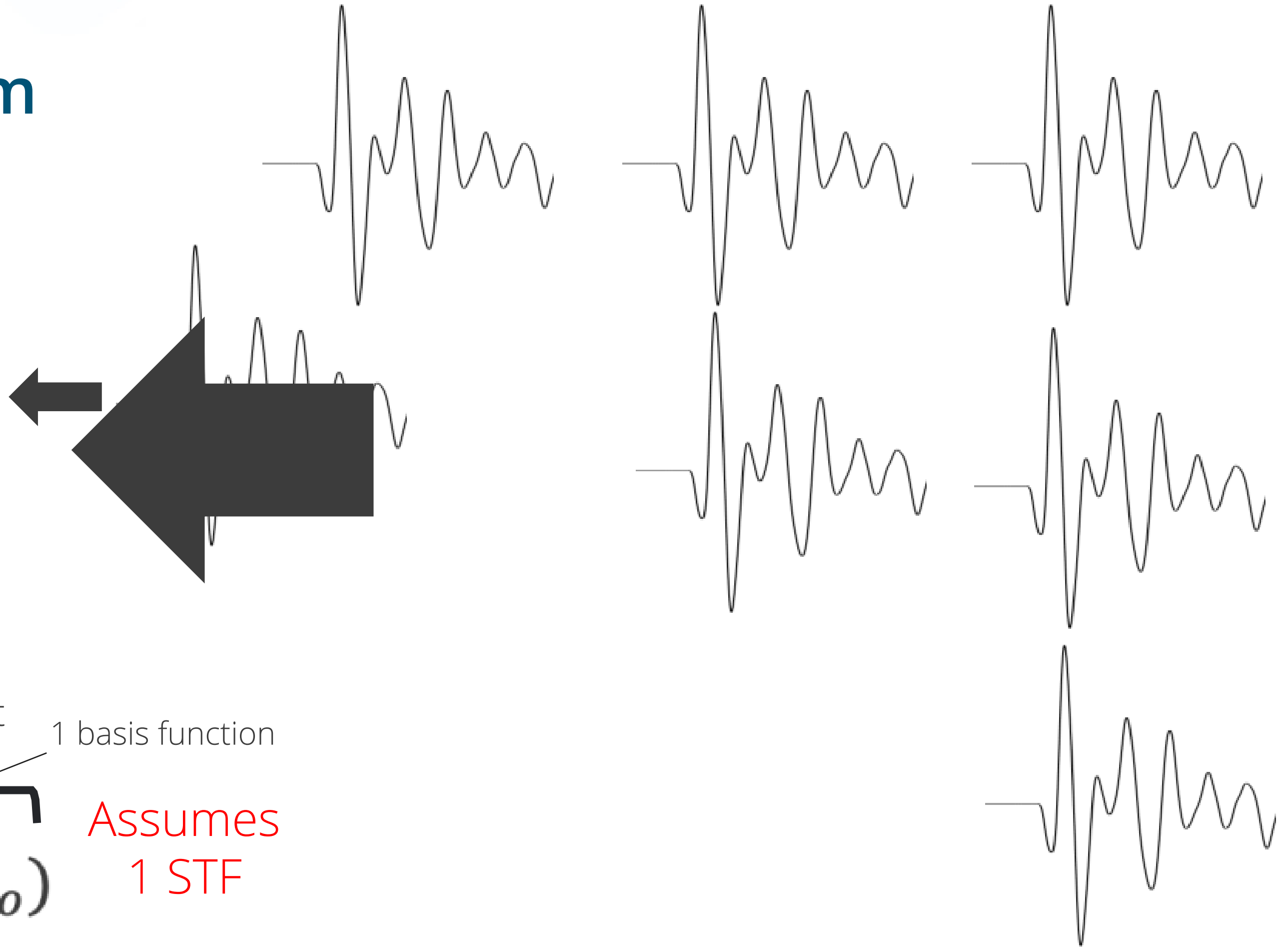
- Problem Itself
  - Compute 6 independent Source-Time Functions
  - In the time domain
  - Fully quantify uncertainty
- Importance
  - Better estimate data and model limitations
  - Understand what we don’t know, as much as what we do
  - Understand limitations when using seismic sources for practical applications
    - event discrimination (earthquake vs explosion)

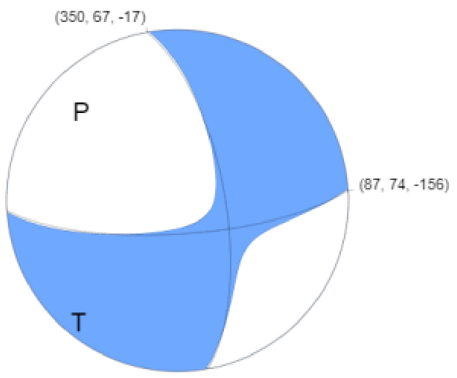
The Problem


Recorded data
Earth
Green's function
Source
Source-Time Function

The problem

$$M_{nj} = \begin{matrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{matrix}$$





STF, function of t

1 coefficient

1 basis function

$$M_{nj}(t) = M_{nj}\emptyset(t - t_o)$$

Assumes 1 STF

STF, function of t

N coefficients

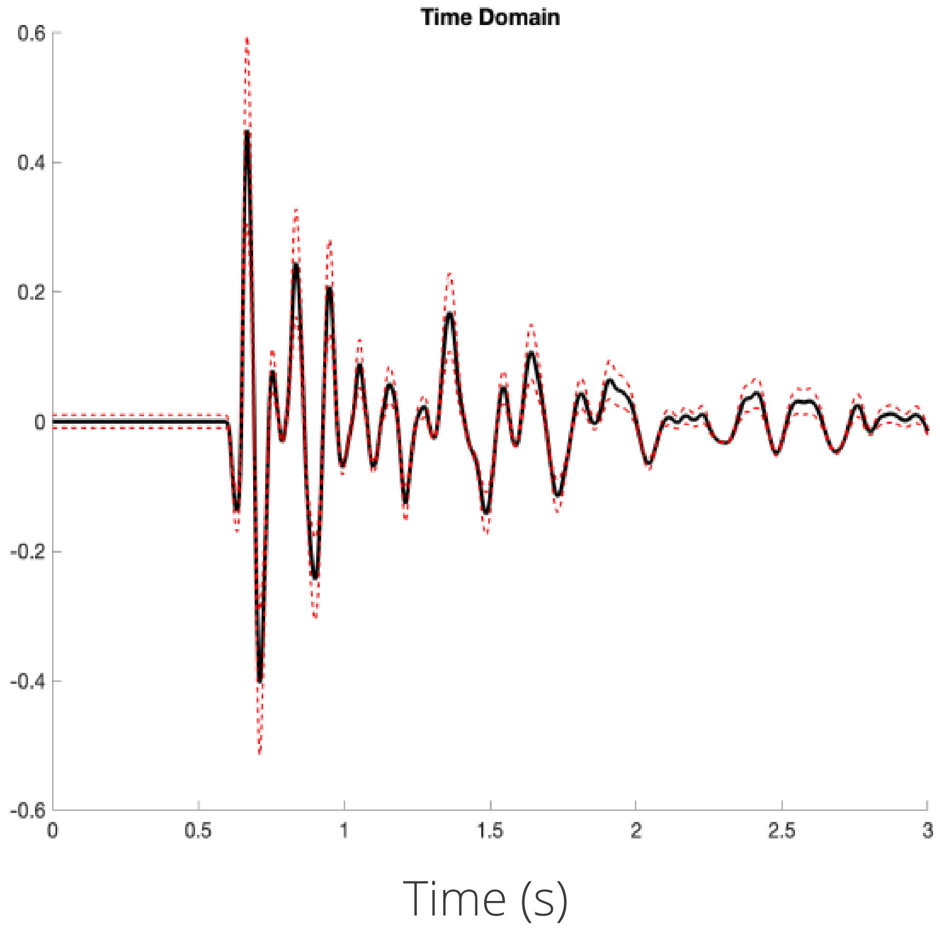
N basis functions

$$M_{nj}(t) = \sum_{k=1}^{N} s_{knj}\emptyset_k(t - t_o)$$

Assumes 6 different STFs, built with N basis functions and N coefficients

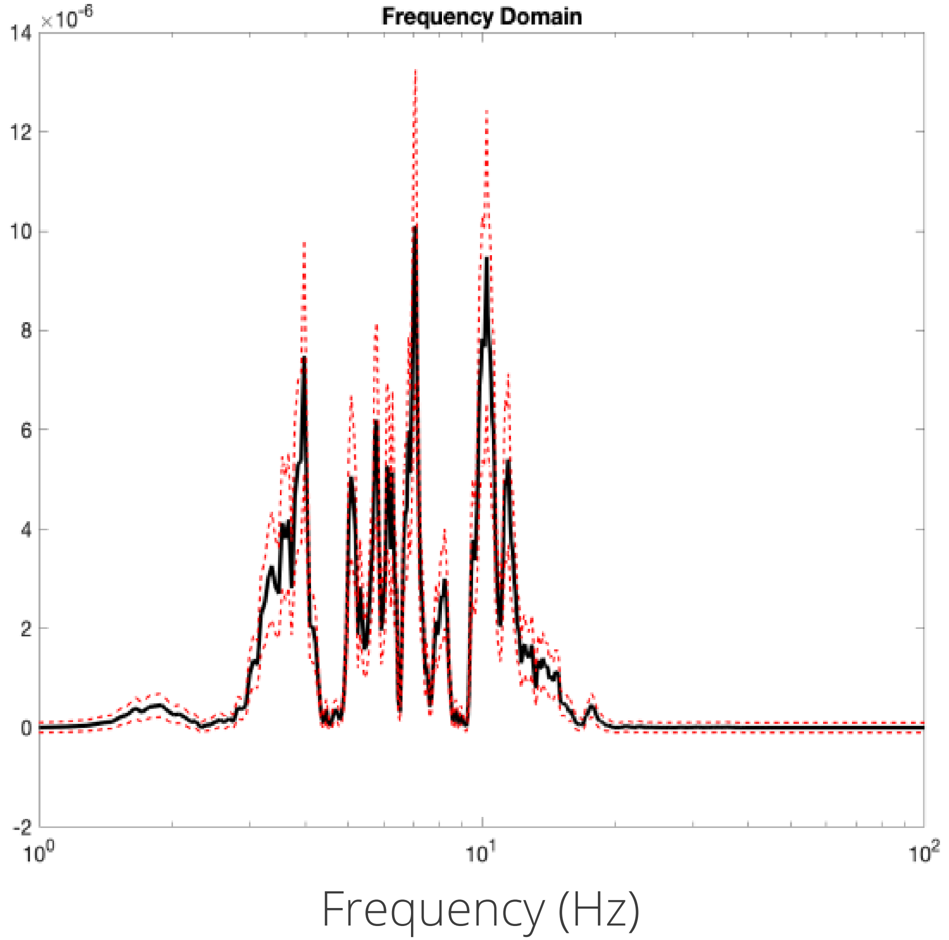
Prior Work

Time and frequency domain – without full uncertainty quantification



Fourier Transform

Inverse Fourier Transform



Methods – Least Squares

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \\ \vdots \\ u_{2n-1} \end{bmatrix} = \begin{bmatrix} G_1 & 0 & 0 & 0 \\ G_2 & G_1 & 0 & 0 \\ \vdots & G_2 & G_1 & 0 \\ G_n & \vdots & G_2 & G_1 \\ 0 & G_n & \vdots & G_2 \\ 0 & 0 & G_n & \vdots \\ 0 & 0 & 0 & G_n \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}$$

$\mathbf{u}$ $\mathbf{m}$

A

$$\begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \vdots \\ \boldsymbol{u}_{N-1} \\ \boldsymbol{u}_N \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1\,M-1} & A_{1M} \\ A_{21} & A_{22} & \cdots & A_{2\,M-1} & A_{2M} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{N-1\,1} & A_{N-1\,2} & \cdots & A_{N-1\,M-1} & A_{N-1\,M} \\ A_{N1} & A_{N2} & \cdots & A_{N\,M-1} & A_{NM} \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \\ \vdots \\ \boldsymbol{m}_{M-1} \\ \boldsymbol{m}_M \end{bmatrix}$$

Forward problem:

$\mathbf{u} = \mathbf{A}\mathbf{m}$

data — Forward operator — Model/stf

Inverse problem:

$$\mathbf{m} = (\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}\mathrm{A}^{\mathrm{T}}\mathbf{u}$$

Regularized, smoothed Inverse problem:

$$\mathbf{m} = (\mathrm{A}^{\mathrm{T}}\mathrm{A} + \alpha\mathrm{I} + \beta\mathrm{L}^{\mathrm{T}}\mathrm{L})^{-1}\mathrm{A}^{\mathrm{T}}\mathbf{u}$$

Regularizing term — Smoothing term

Methods – Least Squares

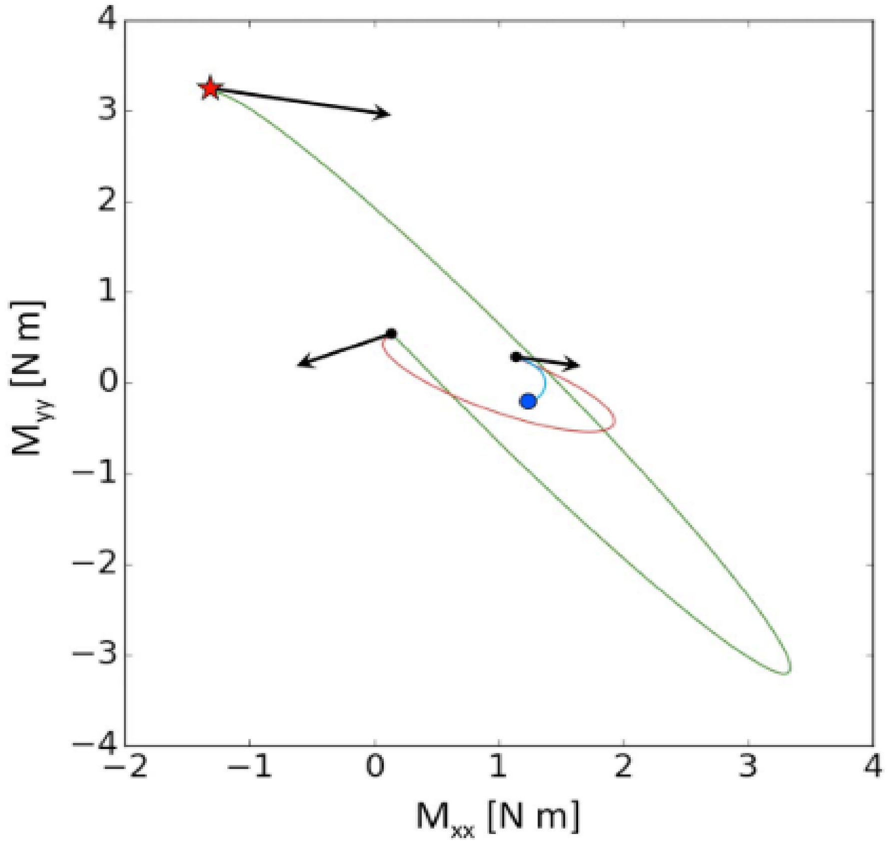
Least squares estimation

Add random and coherent noise

Uncertainty/Error:
Iterative (100-1000) times

Uncertainty/Error:
Hamiltonian Monte Carlo

Methods – Hamiltonian Monte Carlo (HMC)



Example model space trajectory of two model parameters, based on kinetic and potential energy

Fichtner and Simute, 2018

$$u_i(\mathbf{x}, t) = \sum_{j=1}^{6} M_j(t) * G_{ij}(\mathbf{x}, t)$$

Calculated displacement — Moment tensor component — Green's function set

Forward Problem: computing displacement

Potential Energy:

$$U(\mathbf{q}) = \frac{1}{2T} \sum_{r=1}^{N_r} \sum_{n=1}^{3} \int_0^T \sigma_d^{-2} \left[u_n(\mathbf{x}_r, t; \mathbf{q}) - u_n^{\text{obs}}(\mathbf{x}_r, t)\right]^2 dt$$

1 scalar

Integrate over time, and sum over each component

Calculated displacement — Observed displacement

Synthetic vs data misfit

Kinetic Energy:

$$K(\mathbf{p}) = \frac{1}{2} \sum_{i,j=1}^{n} p_i M_{ij}^{-1} p_j \,,$$

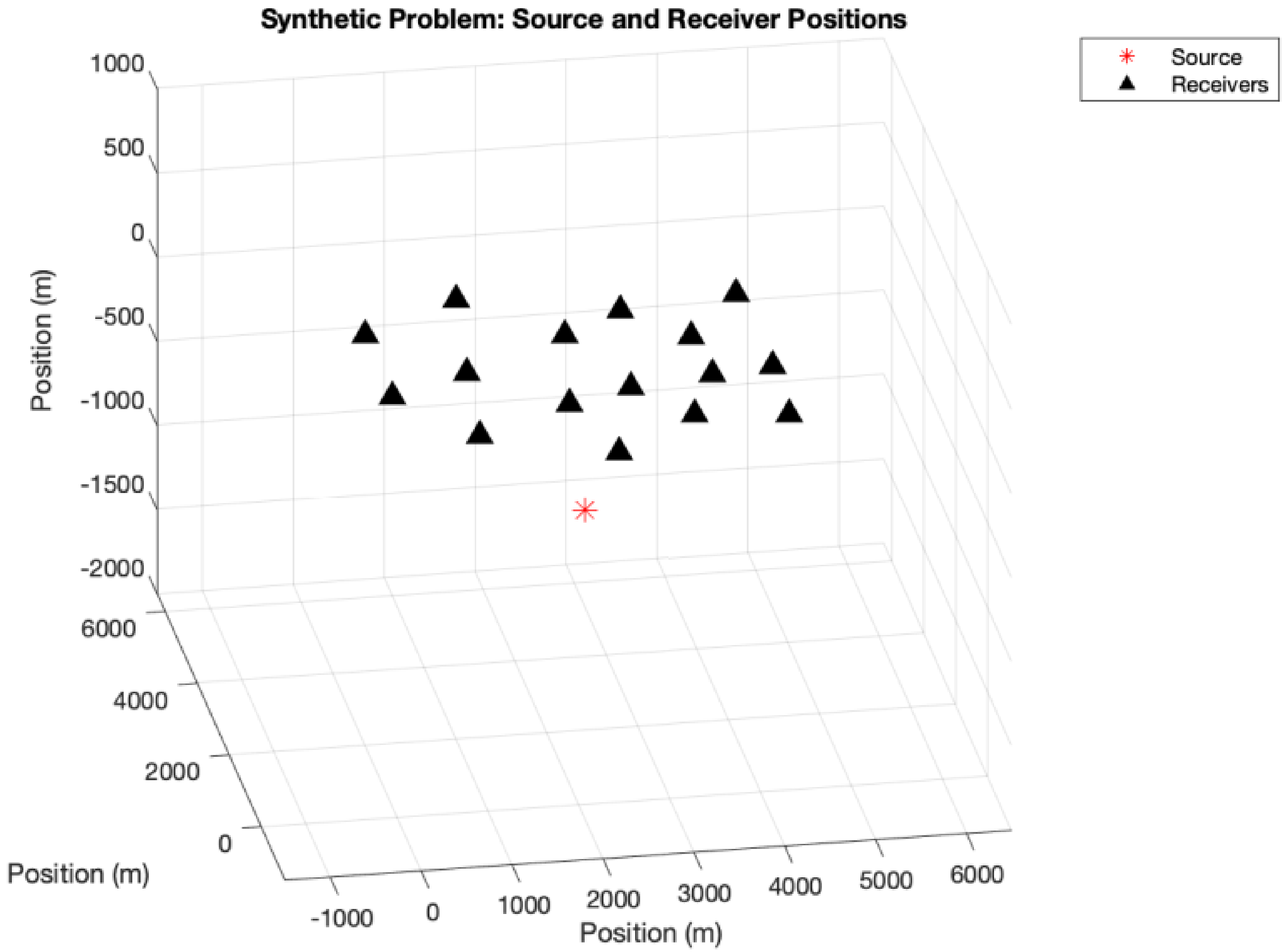
Mass matrix

momentum

Total Energy:

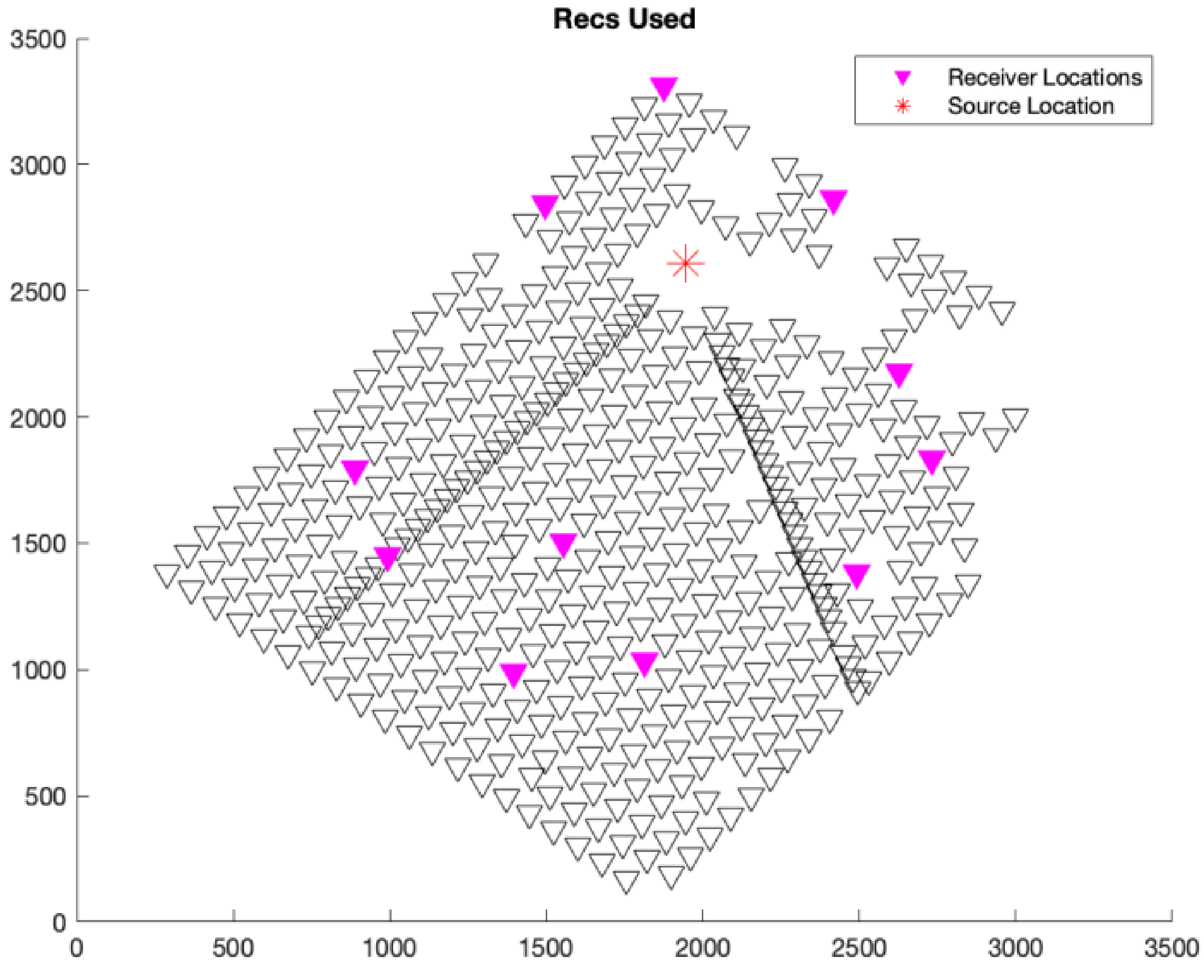
H = K + U

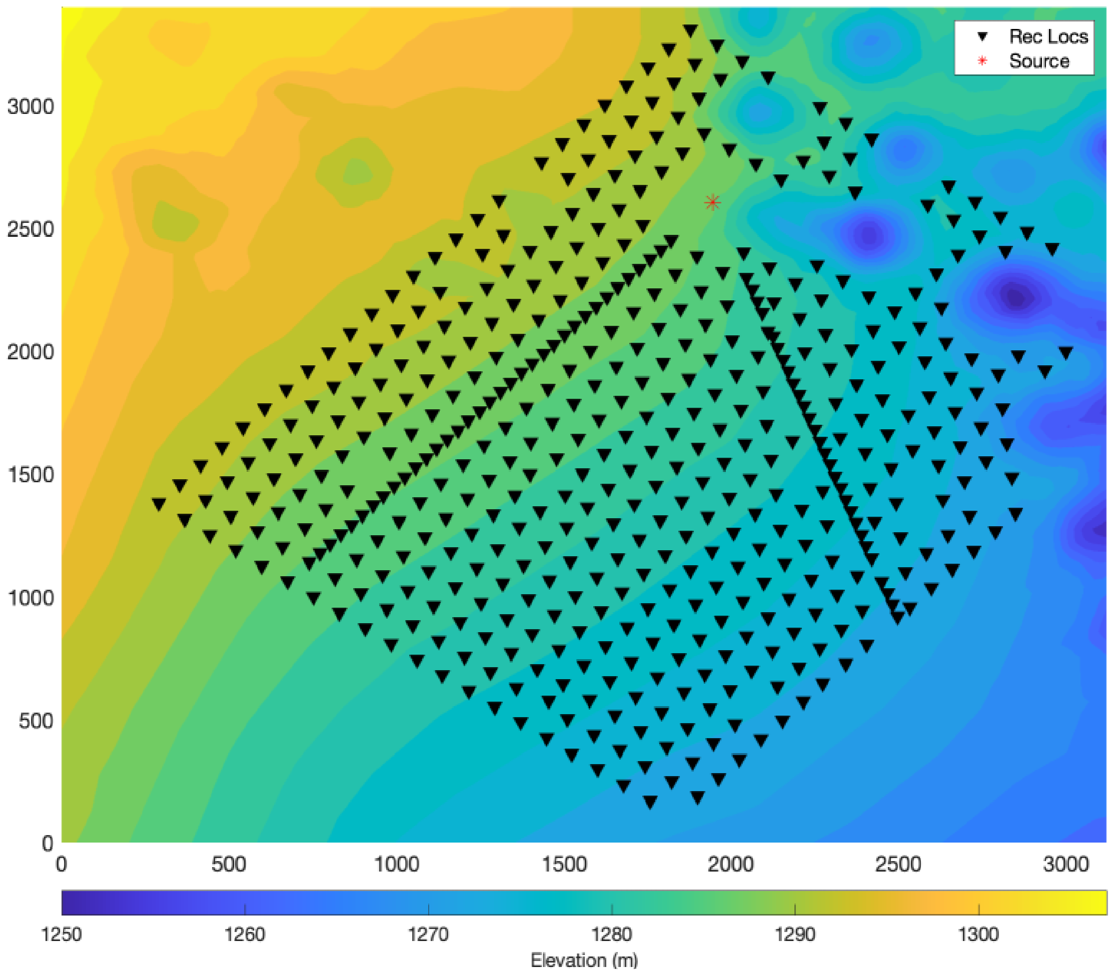
Data - Synthetic



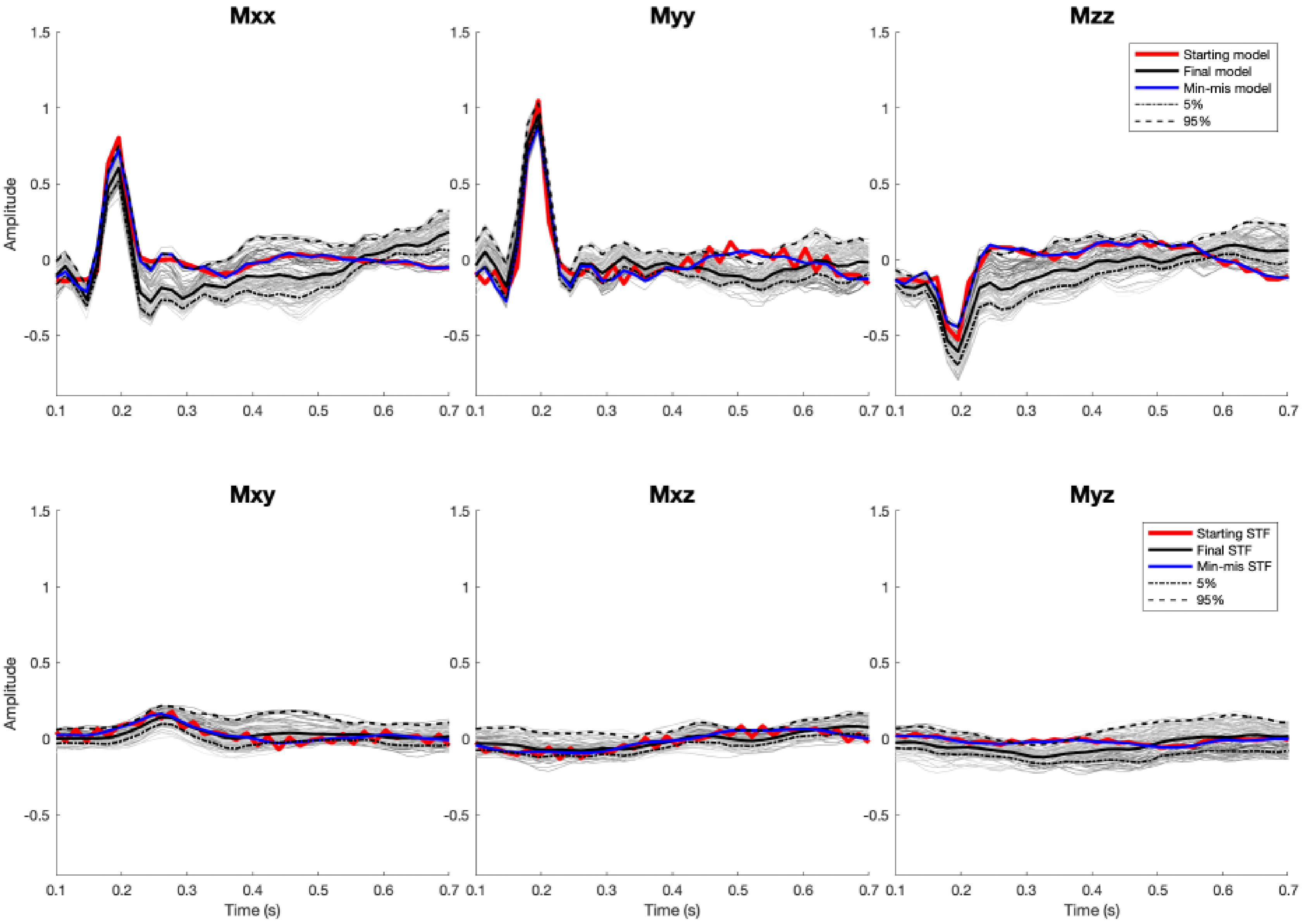
- All receivers are at 0m depth (Z = 0)
- Source is at 800m depth (Z = -800 m)
- Sources are scattered in various horizontal directions

Data – Real: Source Physics Experiment Dry Alluvium Geology





Results – Synthetic Data



Results – Real Data

Work in Progress – Coming Soon!

Impact of these data to the Problem Statement

- Methodology works on synthetic data

- HMC achieves the same uncertainty quantification goal as iterative linear inversion
  - Faster
  - Makes fewer assumptions

- Real data will demonstrate proof of concept

Conclusions

- New, robust time domain method for uncertainty quantification
- As effective as iterative methods with stochastic perturbations
- Method works on synthetic data
- Will demonstrate concept on real data

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Thank you for your attention!

Questions? Email me at dewells@sandia.gov

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