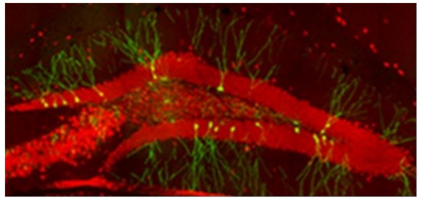
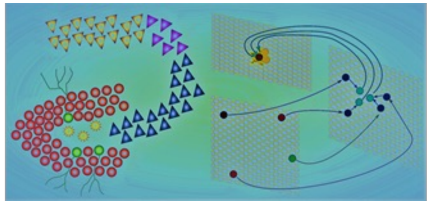
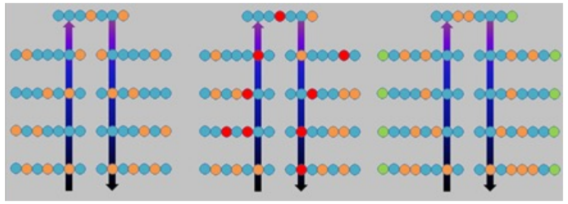
SAND2022-9993C

Sandia National Laboratories

Distributed Localization with Grid-based Representations on Digital Elevation Models







PRESENTED BY

Felix Wang - felwang@sandia.gov

Corinne Teeter, Sarah Luca, Srideep Musuvathy, Brad Aimone

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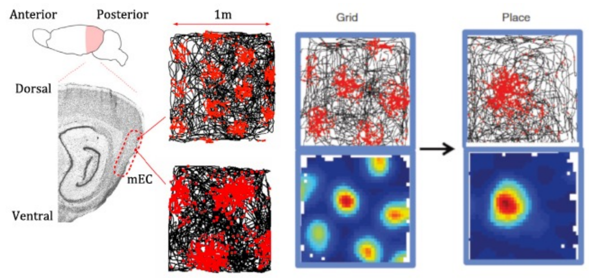
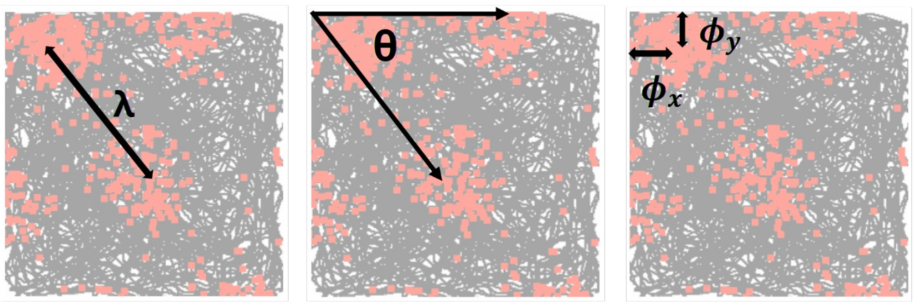
Introduction and Motivation

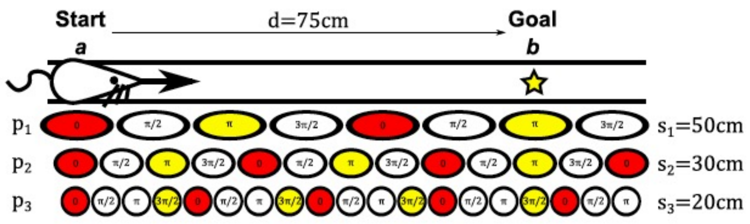
- Robust sensor-aided localization
  - We take a neuro-inspired model of distributed grid-based computation and apply it in the context of navigation-based datasets (e.g. digital elevation models)
  - Applications to intelligent navigation from sensor inputs in challenging environments and/or with resource constraints (e.g. terrain relative navigation, simultaneous localization and mapping)
- Neural inspiration from grid cells
  - Hippocampal representation of space using grid cells (in addition to place cells)
  - Characterized by a periodic, hexagonal tiling with different spatial scales, orientations, and offsets
  - Intersection of multiple grid modules can be decoded yield unique locations



Grid cell activations of the rat hippocampus collected over square arena [*Moser et al. Place Cells, Grid Cells, and Memory*]



Spatial Scale | Orientation | Offset

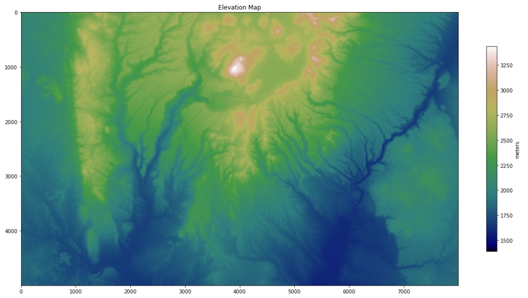


Intersection of multiple grid modules encodes locations as phases [*Bush et al. Using Grid Cells for Navigation*]

Grid Cell Activations Over a Map

- Overlaying grid cell activations onto digital elevation models (DEMs) provides a grid-based representation of locations

Sample DEM map (area around Albuquerque)



$$k_1 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{\pi}{12}) + \sin(\theta + \frac{\pi}{12}) \right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{\pi}{12}) - \sin(\theta + \frac{\pi}{12}) \right) \times (y - \varphi_y) \right)$$

$$k_2 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{5\pi}{12}) + \sin(\theta + \frac{5\pi}{12}) \right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{5\pi}{12}) - \sin(\theta + \frac{5\pi}{12}) \right) \times (y - \varphi_y) \right)$$

$$k_3 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{3\pi}{4}) + \sin(\theta + \frac{3\pi}{4}) \right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{3\pi}{4}) - \sin(\theta + \frac{3\pi}{4}) \right) \times (y - \varphi_y) \right)$$

$$G = \frac{2}{3}\left(\frac{k_1 + k_2 + k_3}{3} + .5 \right)$$

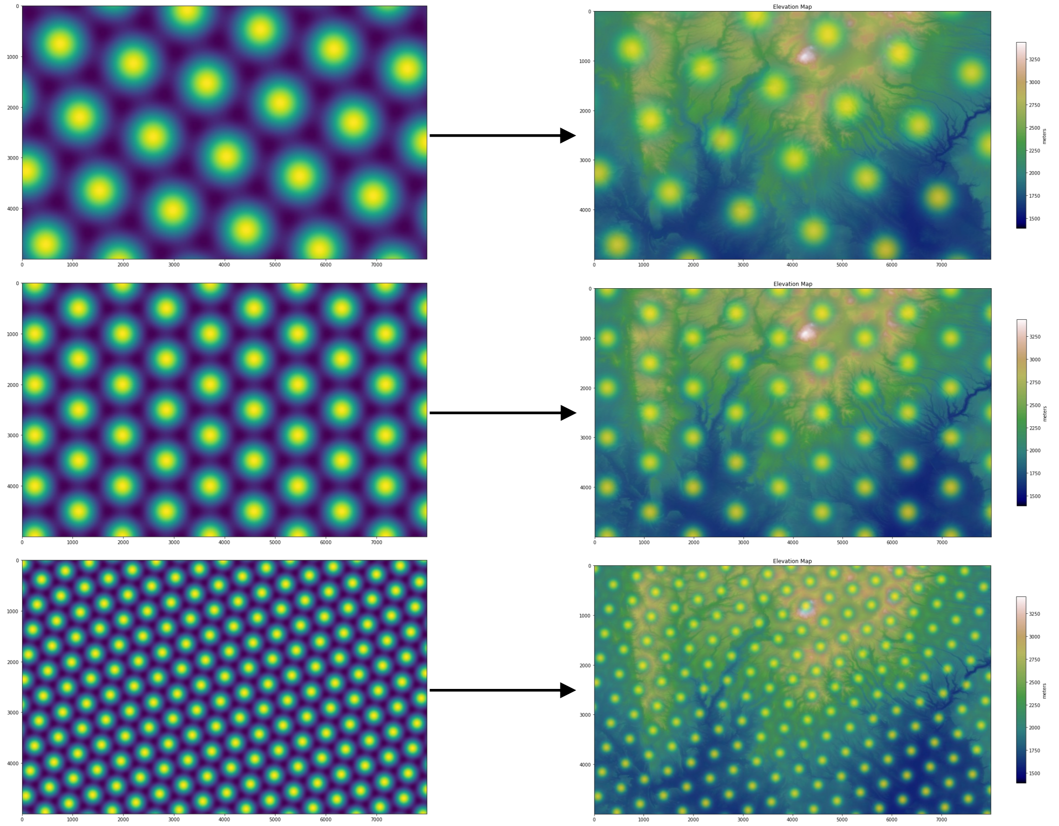
Equations for grid cell activations [*Solstad et al. From grid cells to place cells: a mathematical model*]

Sample grid cells of distinct periods, orientations, and offsets overlaid on the same elevation map. Centroids correspond to locations with high activation.

$\lambda = 1500\text{px}$, $\theta = \frac{\pi}{4}$, $\phi = (\pi, \pi)$

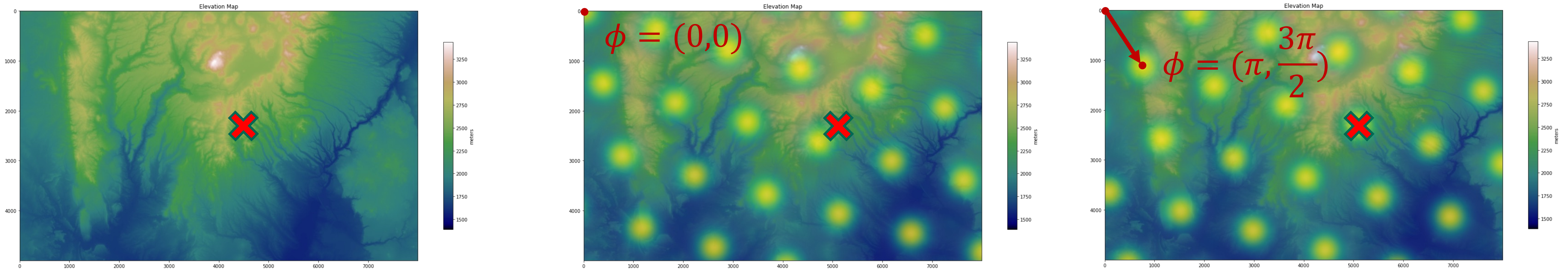
$\lambda = 1000\text{px}$, $\theta = \frac{\pi}{6}$, $\phi = \left(\frac{\pi}{2}, 0\right)$

$\lambda = 500\text{px}$, $\theta = \frac{\pi}{9}$, $\phi = \left(0, \frac{\pi}{4}\right)$

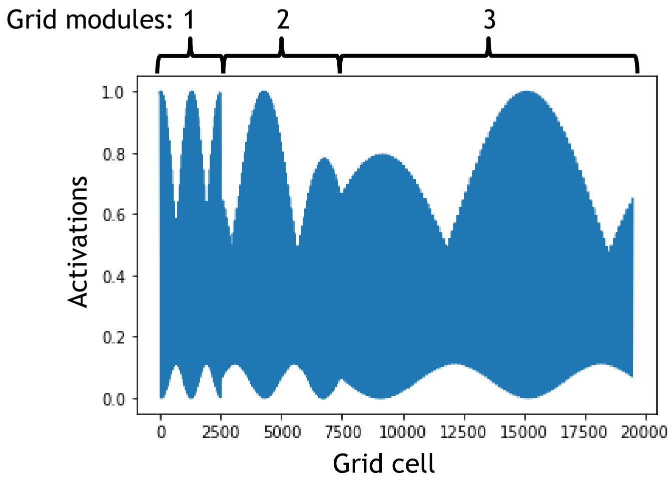


Representing Locations Using Grid Modules

- Refinement of grid-based representation from individual grid cell activations to grid module phase codes enables greater representation and more tractable computation



Grid modules defined by shared period and orientation, whereas their "phase" determines their offset w.r.t. a reference point (e.g. $\phi = (0,0)$)



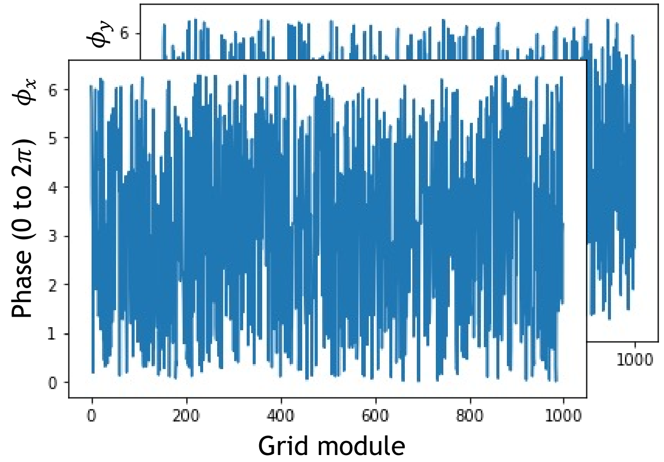
Representation of a location (x,y) using:

- Grid cell activations (left)

$$(x, y) \rightarrow \{a^1_{0,0}, a^1_{\Delta\phi_x,0}, a^1_{\Delta\phi_x,\Delta\phi_y}, \ldots, a^m_{\mathrm{k}\Delta\phi_x,\mathrm{k}\Delta y}\}$$

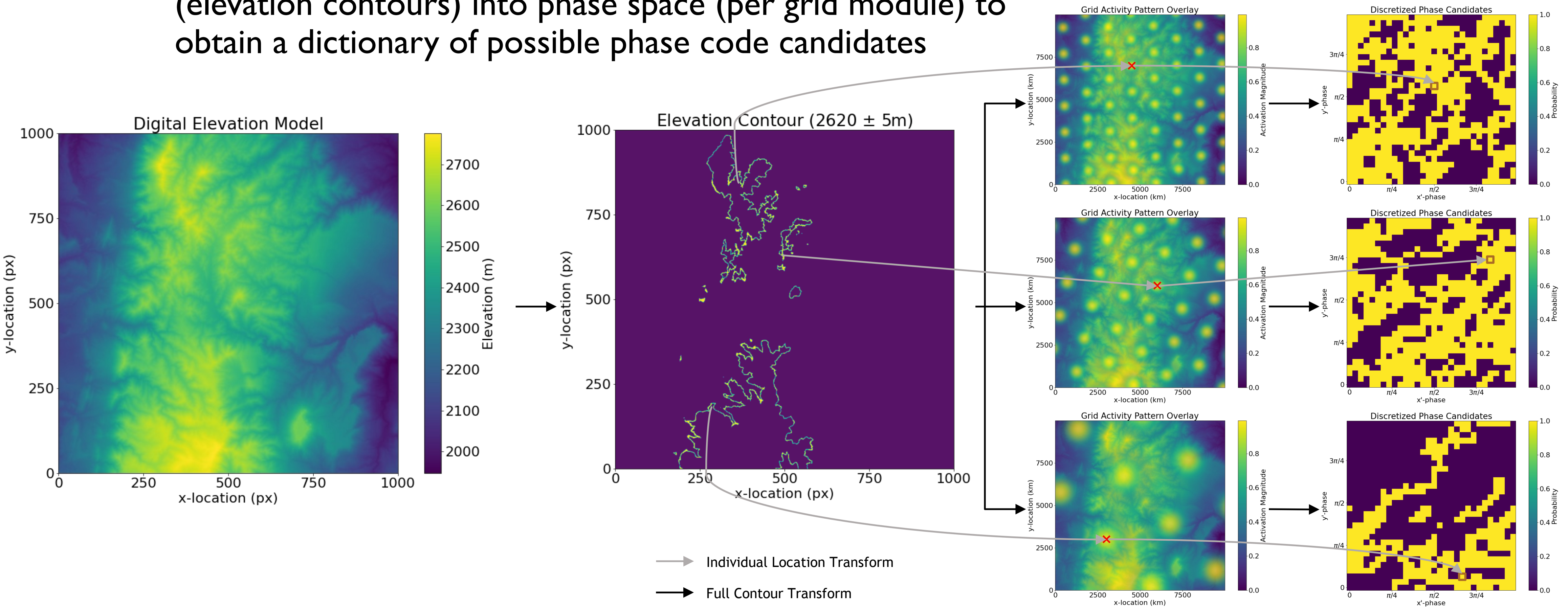
- Grid module phases (right)

$$(x, y) \rightarrow \{\phi^1_x, \phi^1_y, \phi^2_x \ldots, \phi^m_y\}$$



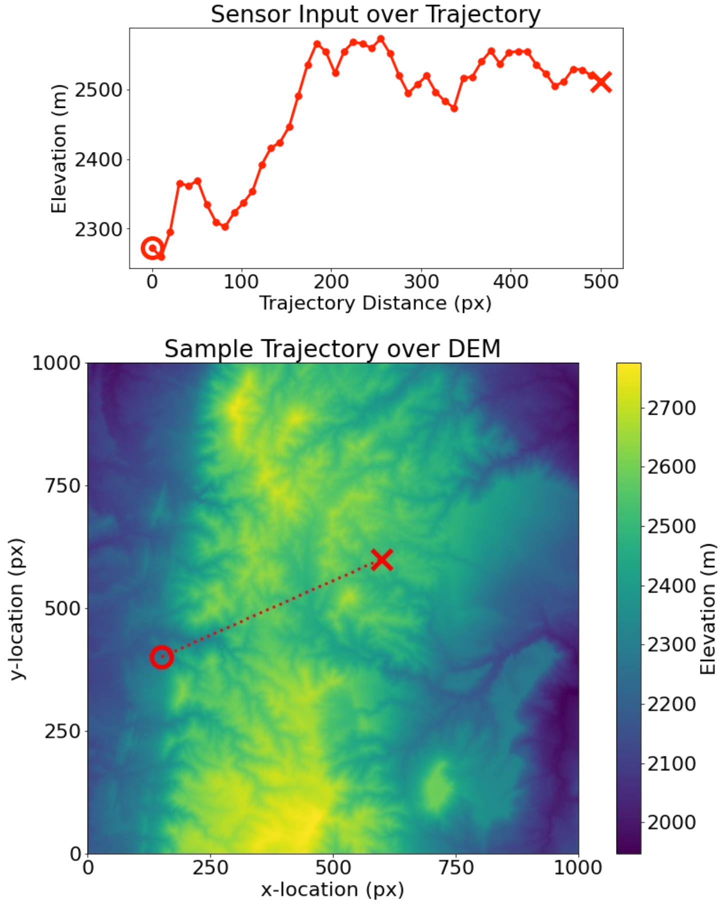
Encoding to Grid Module Phase Codes

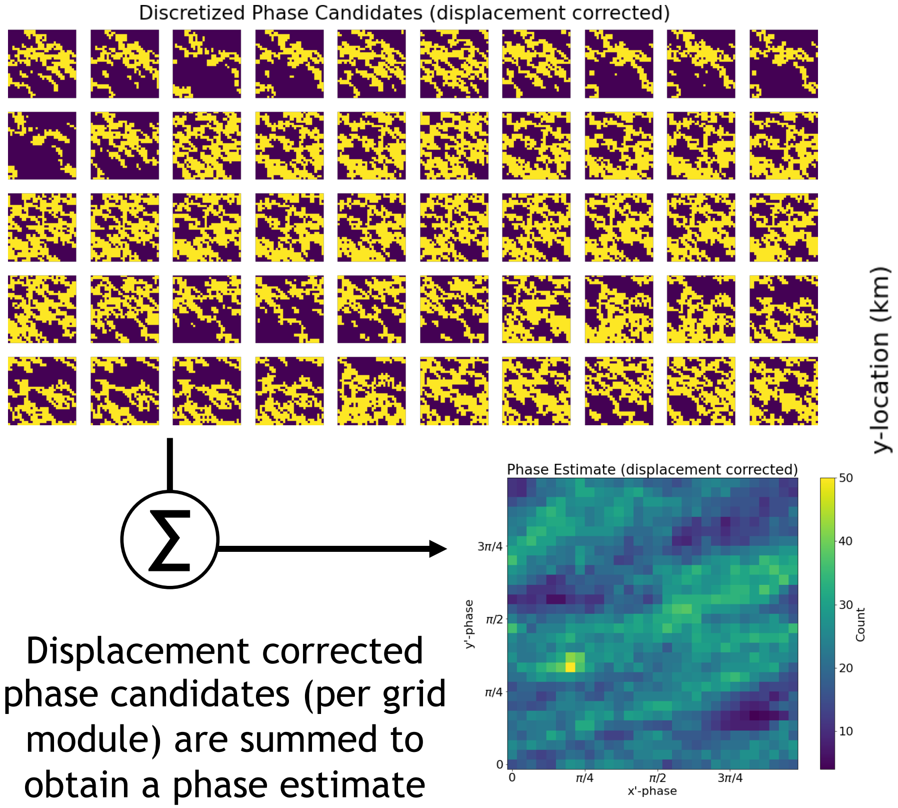
- We construct a distributed representation of the input space over the grid modules
  - Here, we transform locations associated with similar inputs (elevation contours) into phase space (per grid module) to obtain a dictionary of possible phase code candidates



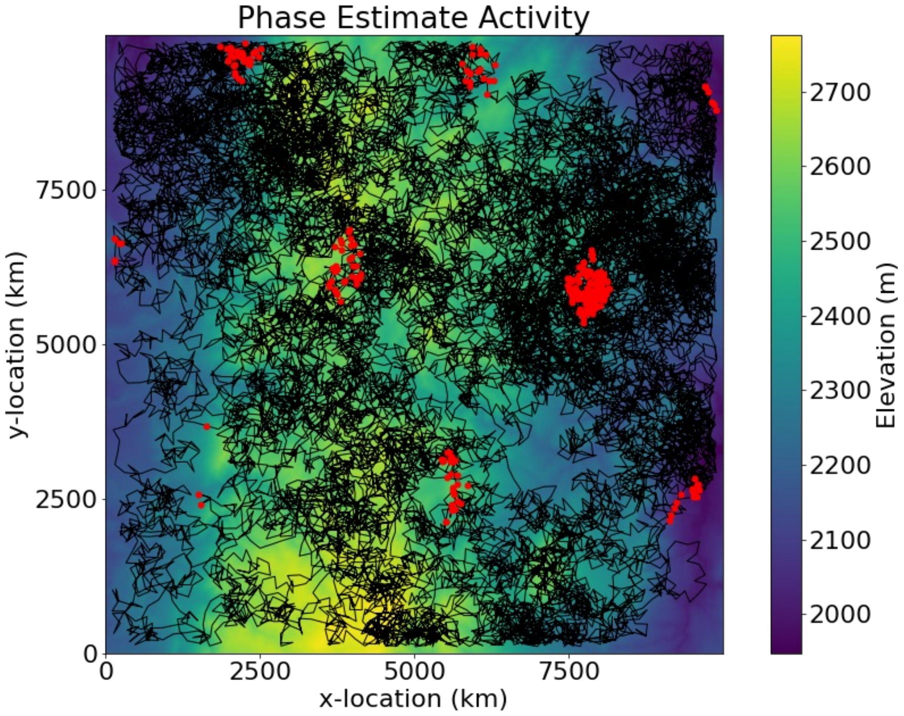
Encoding to Grid Module Phase Codes

- Spatial displacement corresponds to phase shifts and can be integrated with respect to a reference time/location from multiple measurements





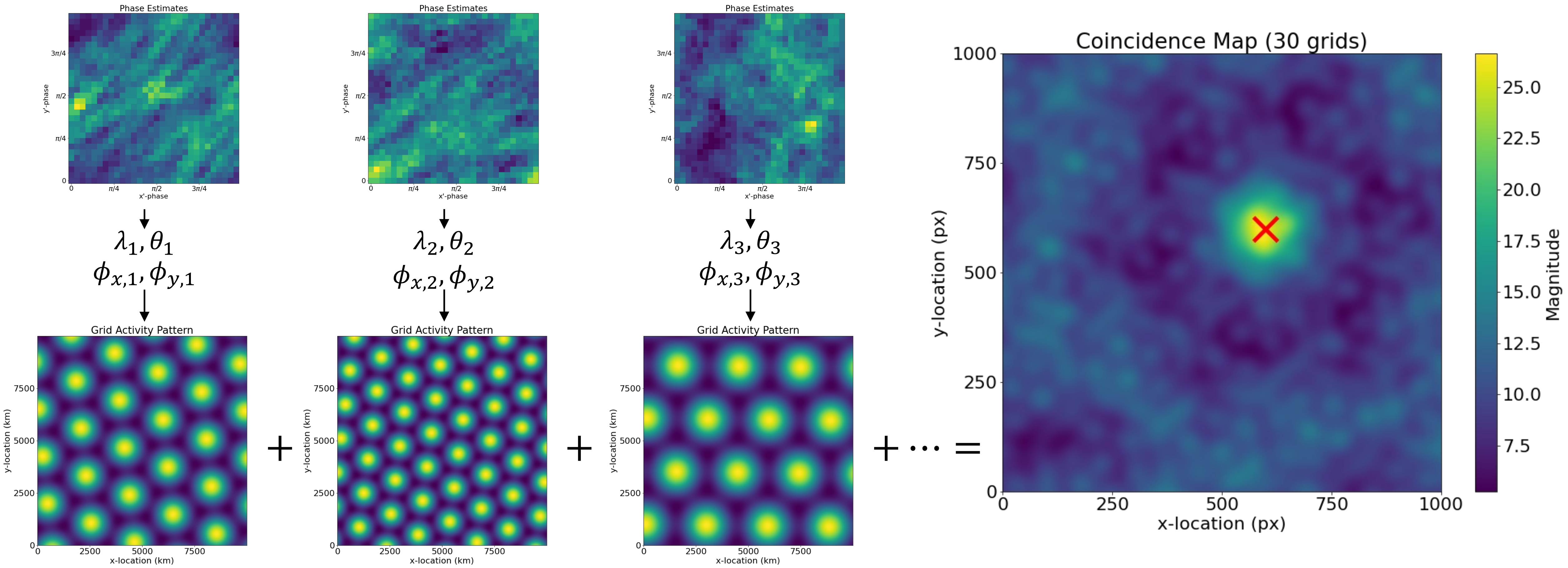
Displacement corrected phase candidates (per grid module) are summed to obtain a phase estimate



Locations over a random walk corresponding to consistent phase estimates for a given grid module

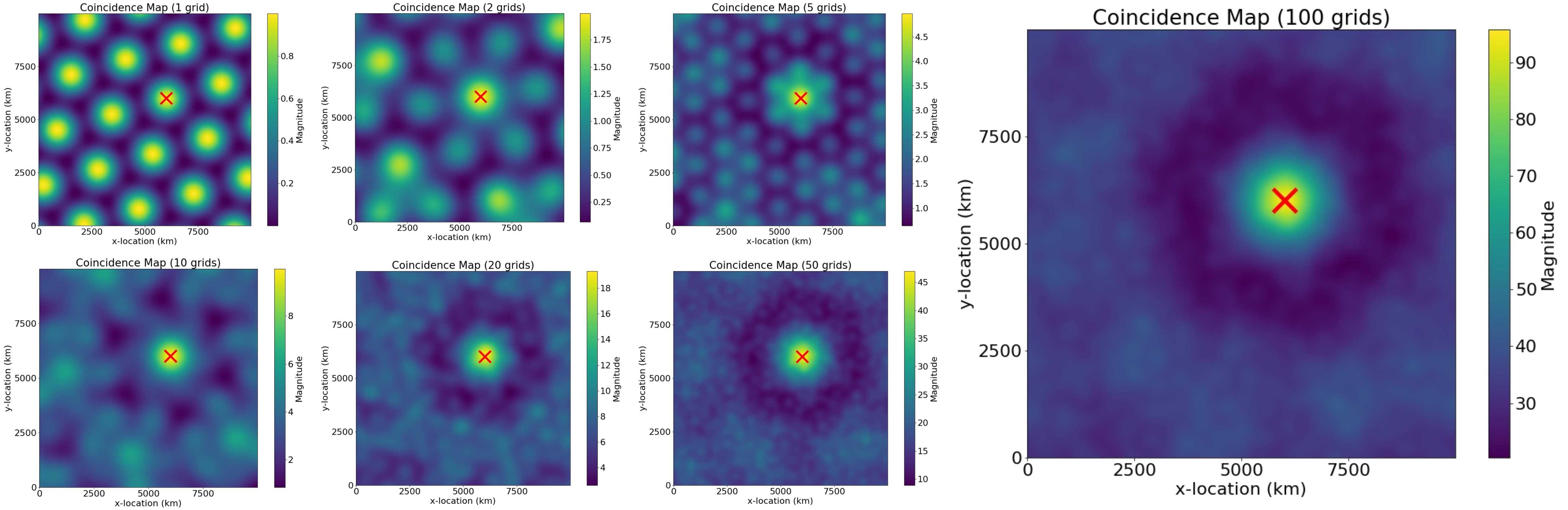
Decoding to the Location Estimate

- By summing the grid cell activations corresponding to grid module phase codes, we can compute a "coincidence map" to find where they may uniquely intersect



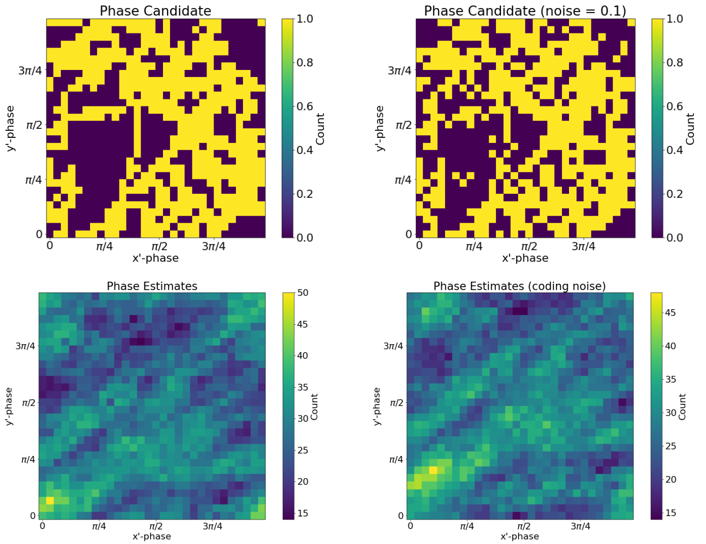
Decoding to the Location Estimate

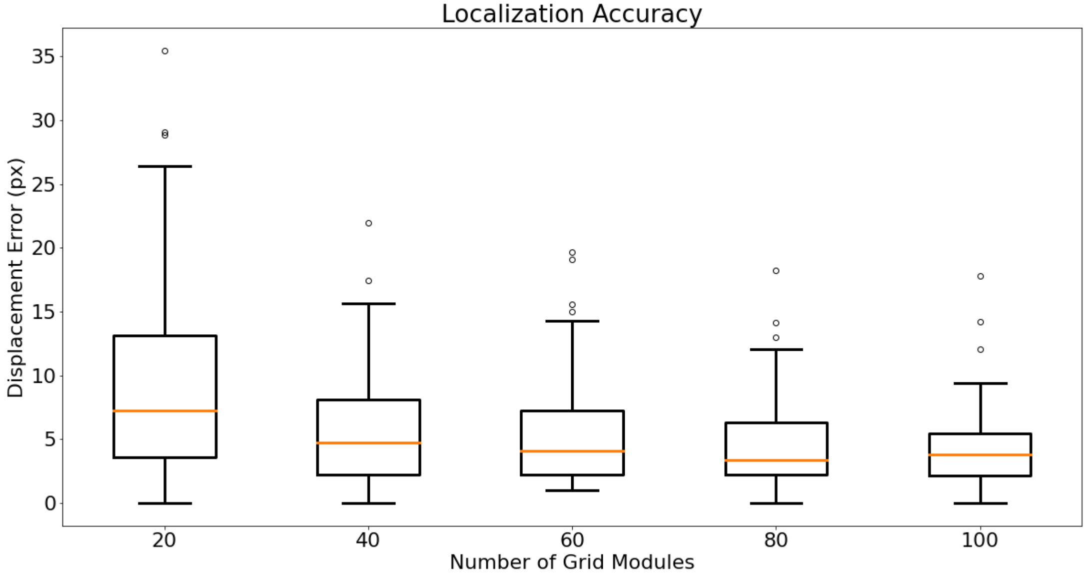
- This computation is scalable, where only a subset of grid modules is required for successful decoding, and grid modules can be encoded/trained independently

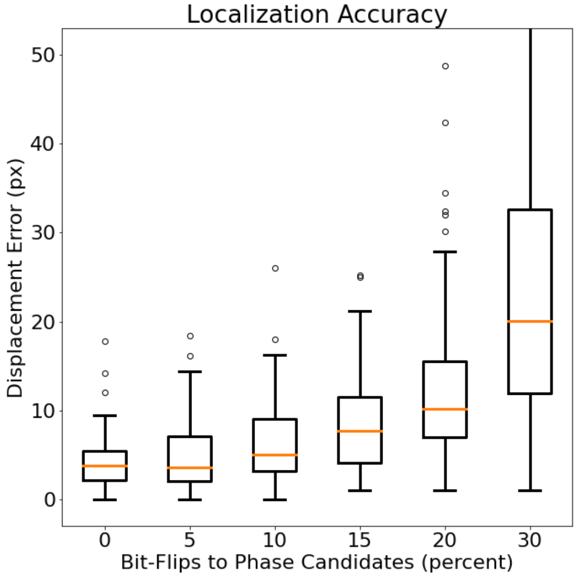


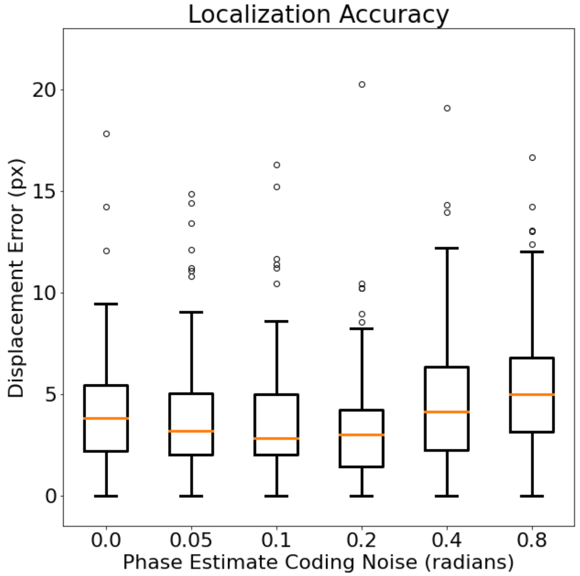
Experimental Results

- We performed Monte Carlo simulations over arbitrary trajectories and computed displacement error
- We also performed robustness analysis on process noise as relevant to a potential neuromorphic implementation
  - E.g. bit-flips in the stored map, coding noise in the phase estimate
  - Redundancy through the use of multiple grid modules results in graceful degradation of location estimates



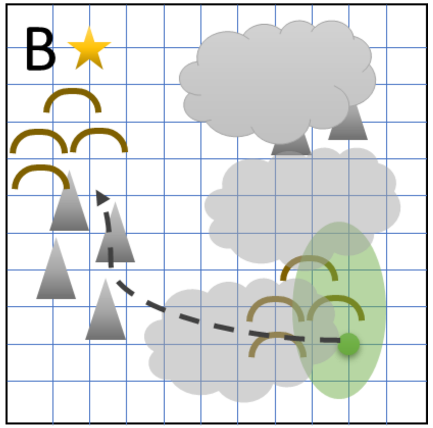






Summary and Future Work

- We developed a neuro-inspired model of distributed grid-based computation to localize from a set of elevation inputs
  - Applied algorithm to simulations on navigation-based datasets (DEMs)
  - Robustness analysis to process noise as relevant to a potential neuromorphic implementation
- Current and Future Work
  - Translating the linear algebra formulation onto a spiking implementation
  - Analysis of tradeoff spaces (e.g. computation, storage costs, robustness)
  - Learning/training phase candidates from data (e.g. mapping part of SLAM)
  - Adaptation of localization algorithm to different datasets, sensor and noise models, and integration with filters (e.g. EKF update)



Goal: leverage neuro-inspired strategies in support of intelligent navigation

Backup: Representing Locations Uniquely

- To represent locations uniquely, we need the phase code dimensions to be orthogonal
  - This is achieved by performing an affine/shear-like transformation per grid module
  - With period and orientation fixed per grid module, the phase corresponds to the offset of the corresponding grid cell that is maximally active at the encoded location
  - This is computed using the modulo operator in the orthogonalized space

Affine transform

$$(x_i', y_i') = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i + \frac{\pi}{6}) \\ \sin(\theta_i) & \cos(\theta_i + \frac{\pi}{6}) \end{bmatrix} (x, y)$$

Modulo operation

$$\phi_x^i = x_i' \bmod \lambda_i$$

$$\phi_y^i = y_i' \bmod \lambda_i$$

Sample grid cell activation transformed into orthogonalized space (and thresholded image for clarity)