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iomechlab
Inverse Problems and Optimization in Mechanics
Duke University

An Optimal Sensor Placement Approach for Damage Detection Under Frequency-Domain Structural Dynamics

EMI Conference 2022

Mark Chen

DOE-NEUP Project 19-16391: GuArDIAN: General Active Sensing for conDItion AssessmeNt

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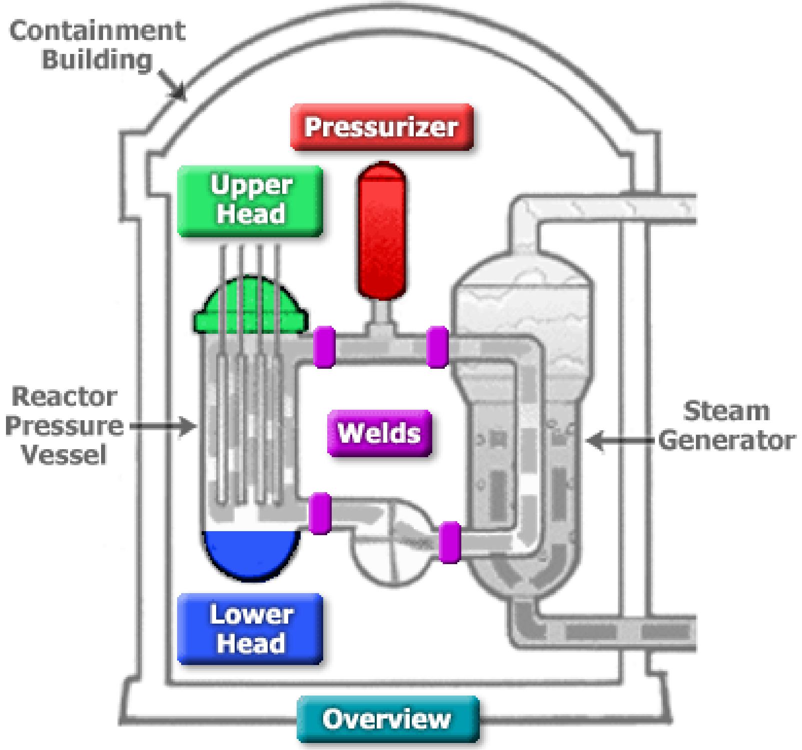
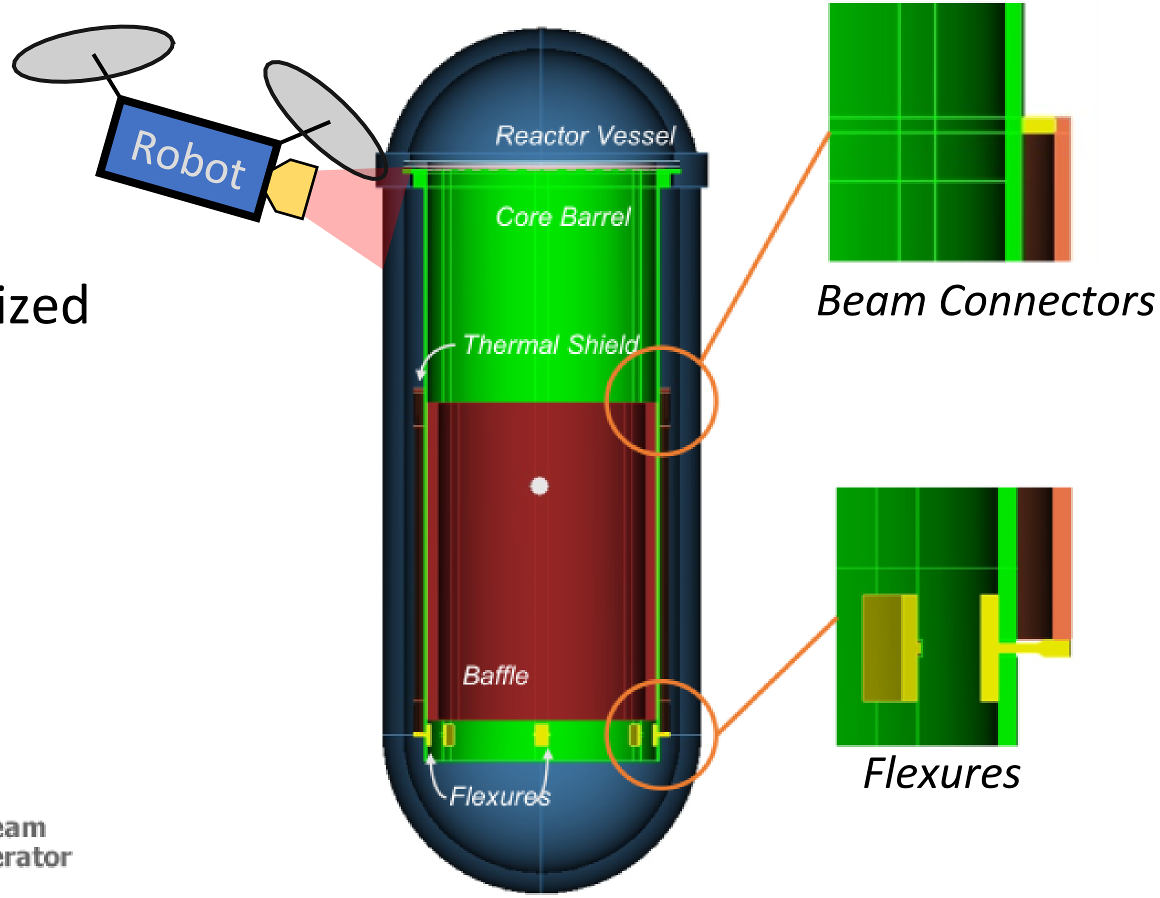
Motivation

- Monitoring/Qualification of Pressurized Water Reactors



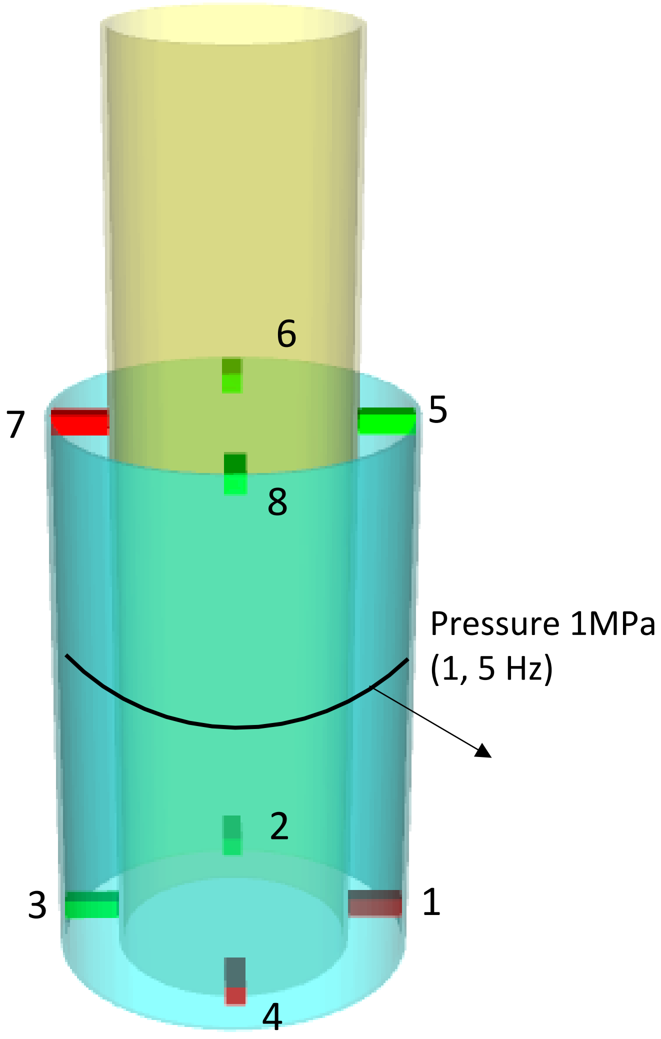
Pressurized Water Reactor (NRC.gov)



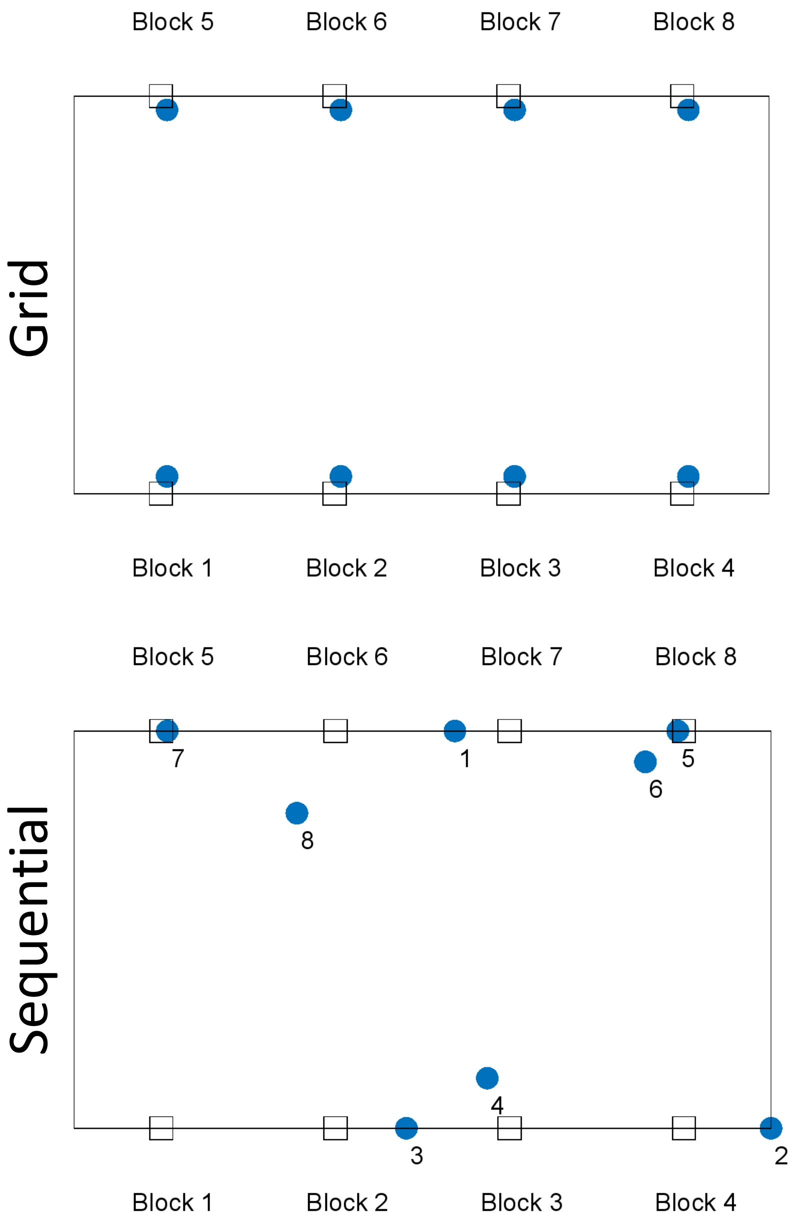
Key Components:

- Structure Modeling (forward)
- Damage Estimator (inverse)
- Optimal Experimental Design (OED)

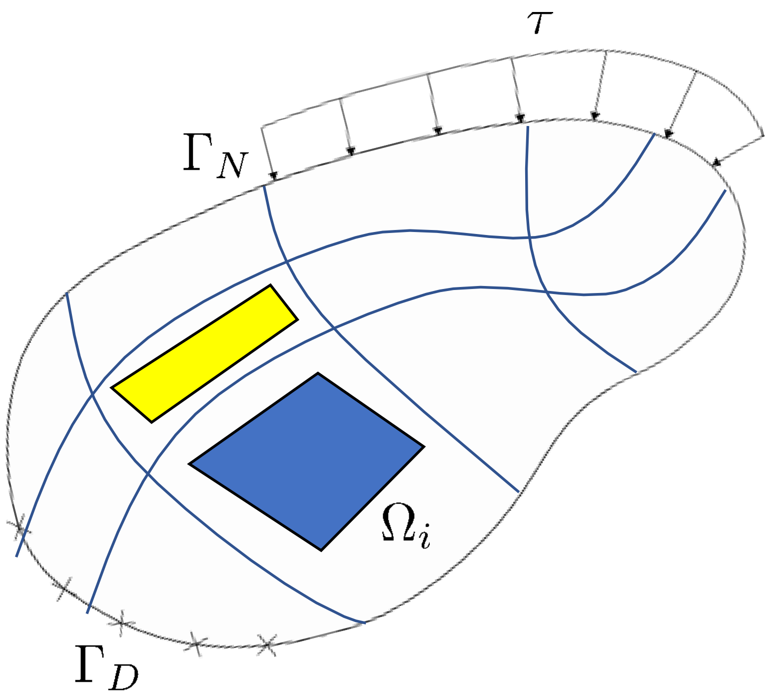
Reactor Vessel Internals



Simplified Reactor Vessel



Modeling Reactor Vessel (Forward)



Component Modeling:

$$\Omega = \bigcup_{i=1}^{m} \Omega_i$$

- Assume linear elasticity (frequency domain) governing PDE
- Assume simple isotropic material penalty parameter

Damage Parameter: $\theta \in [0, 1]$

Forward System:

$$[K(\boldsymbol{\theta}) - \omega^2 M(\boldsymbol{\theta})]\boldsymbol{u} = \boldsymbol{f}$$

$$\Longrightarrow \; [H(\boldsymbol{\theta})]\boldsymbol{u} = \boldsymbol{f}$$

Obtaining a Damage Estimator (Inverse)

Data Model: $\boldsymbol{y} = H(\boldsymbol{\theta})^{-1}\boldsymbol{f} + \boldsymbol{\epsilon}, \quad \epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$

Observations: $Q(\boldsymbol{d}) = \text{diagonal matrix}; \quad d_i \in \{0, 1\}$

Notation:

$\boldsymbol{u},\ \boldsymbol{y},\ \boldsymbol{\epsilon},\ \boldsymbol{d} \in \mathbb{R}^n$

$Q,\ H \in \mathbb{R}^{n \times n}$

$\boldsymbol{\theta} \in \mathbb{R}^m$

$$\begin{aligned} \hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{2}\|Q(\boldsymbol{d})(\boldsymbol{u} - \boldsymbol{y})\|^2 + \mathcal{R}(\boldsymbol{\theta}) \\ s.t. \quad \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{\theta}) = H(\boldsymbol{\theta})\boldsymbol{u} - \boldsymbol{f} = \boldsymbol{0} \\ \boldsymbol{\theta} \in [0, 1] \end{aligned}$$

(Aquino 2019)

Linearizing the Damage Estimation Problem

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{2} \|Q(\boldsymbol{d})(\boldsymbol{u} - \boldsymbol{y})\|^2 + \mathcal{R}(\boldsymbol{\theta})$$
$$s.t. \quad \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{\theta}) = H(\boldsymbol{\theta})\boldsymbol{u} - \boldsymbol{f} = \boldsymbol{0}$$

(Nonlinear IP)

Jacobian Matrix: $\boldsymbol{J}(\boldsymbol{\theta})_{ij} := \frac{\partial u_i}{\partial \theta_j}$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{\delta\theta}$$
$$\boldsymbol{y} = \boldsymbol{y}_0 + \boldsymbol{\delta y}$$

$$\boldsymbol{\delta y} = \boldsymbol{J}(\boldsymbol{\theta}_0)\boldsymbol{\delta\theta}$$

$$\hat{\boldsymbol{\theta}}(\boldsymbol{\theta}, \boldsymbol{y}) = \boldsymbol{\theta}_0 + \underset{\boldsymbol{\theta} \in \mathcal{Z}}{\operatorname{argmin}} \frac{1}{2} \|Q(\boldsymbol{d})(\boldsymbol{\delta y} - J(\boldsymbol{\theta}_0)\boldsymbol{\delta\theta})\|^2 + \frac{1}{2} \|B(\boldsymbol{\theta}_0 + \boldsymbol{\delta\theta})\|^2$$

(Linearized IP)

Least-Squares Solution: $\boldsymbol{\delta\theta} = (\boldsymbol{J}^T Q \boldsymbol{J} + B^T B)^{-1} (\boldsymbol{J}^T Q \boldsymbol{\delta y} - B^T B \boldsymbol{\theta}_0)$

Optimal Experimental Design

- Control the **variance** in our parameters; define the Fisher Information Matrix (FIM):

$$\mathbf{FIM} := \boldsymbol{J}^T Q \boldsymbol{J}$$

- Consider an unregularized least-squares solution:

$$\mathrm{Var}[\boldsymbol{\delta\theta}] = \mathrm{Var}[(\mathbf{FIM})^{-1} \boldsymbol{J}^T Q \boldsymbol{\delta y}] = \sigma^2 \mathbf{FIM}^{-1}$$

- Employ Various Optimality Criteria: A, D, E criteria

(Tenorio 2010)

$$\underset{\boldsymbol{d}}{\mathrm{argmin}} \quad \lambda_{\max}[\mathbf{FIM}(\boldsymbol{d})^{-1}]$$
$$\text{s.t. } 0 \leq d_i \leq 1$$
$$\mathbf{1}^T \boldsymbol{d} = 1$$

Sequential OED (SOED) Algorithm

Algorithm 1 Sequential Optimal Experimental Design

procedure SOED
  $k \leftarrow 0$
  $\epsilon_k \leftarrow 1e6, \boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_0, \boldsymbol{d}_k \leftarrow \boldsymbol{0}$ ▷ Initialize parameters
  while $k <$ MAX ITER and $\epsilon_k >$ TOL **do**
    $k \leftarrow k + 1$
    $\boldsymbol{d}_k \leftarrow OED(\boldsymbol{\theta}_{k-1})$ ▷ Solve OED problem
    $\boldsymbol{\theta}_k \leftarrow IP(\boldsymbol{d}_k)$ ▷ Solve Inverse problem
    $\epsilon_k \leftarrow \frac{\|\boldsymbol{\theta}_k - \boldsymbol{\theta}_{k-1}\|_2}{\|\boldsymbol{\theta}_{k-1}\|_2}$ ▷ Stopping criterion
  return $\boldsymbol{d}_k, \boldsymbol{\theta}_k$

- $\boldsymbol{\theta}_k$: (estimated) damage parameters per iteration
- $\boldsymbol{d}_k$: sensor locations (observations) per iteration

Numerical Example: Two Cylinders

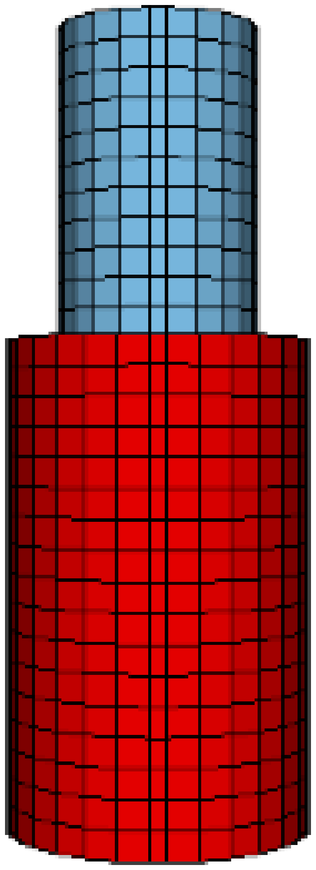
- Pressure applied on outer surface at **1 Hz**
- Inner bottom cylinder fixed; 1% Gaussian noise added

- Parameters:
    - Regularization: Double Well Potential
    - Budget: 20 Sensors
    - Number of MC trials: 30

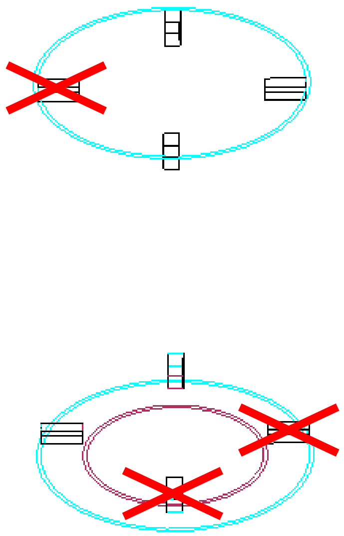
- Metrics:

$$\text{Relative Iter Difference} := \frac{\|\theta_i - \theta_{i-1}\|_2}{\|\theta_{i-1}\|_2}$$

$$\text{Relative Solution Error} := \frac{\|\theta_i - \theta_{true}\|_2}{\|\theta_{true}\|_2}$$



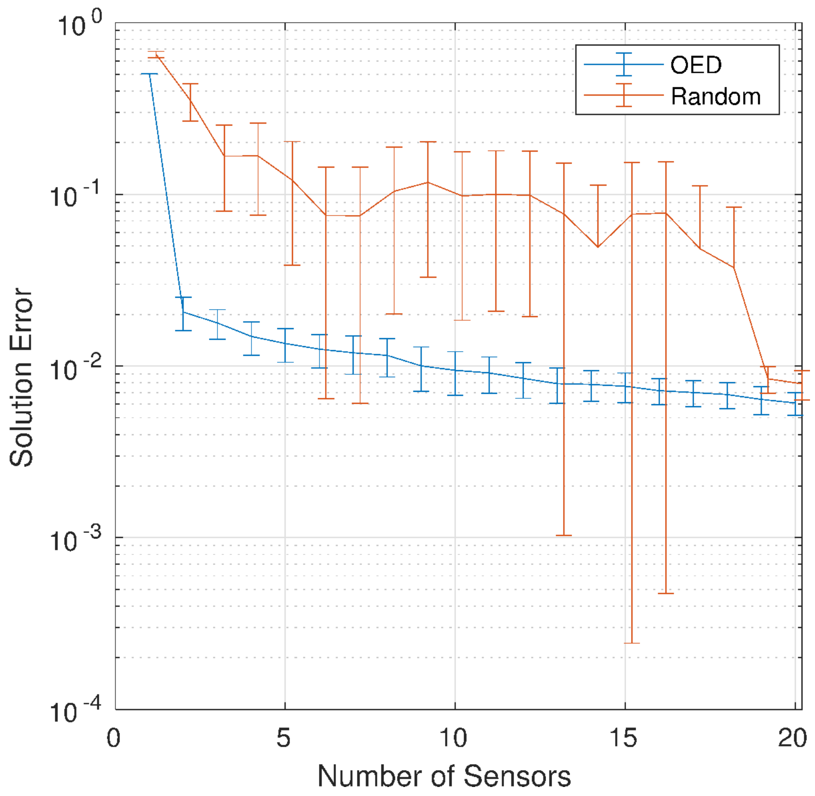
Cylinder Mesh

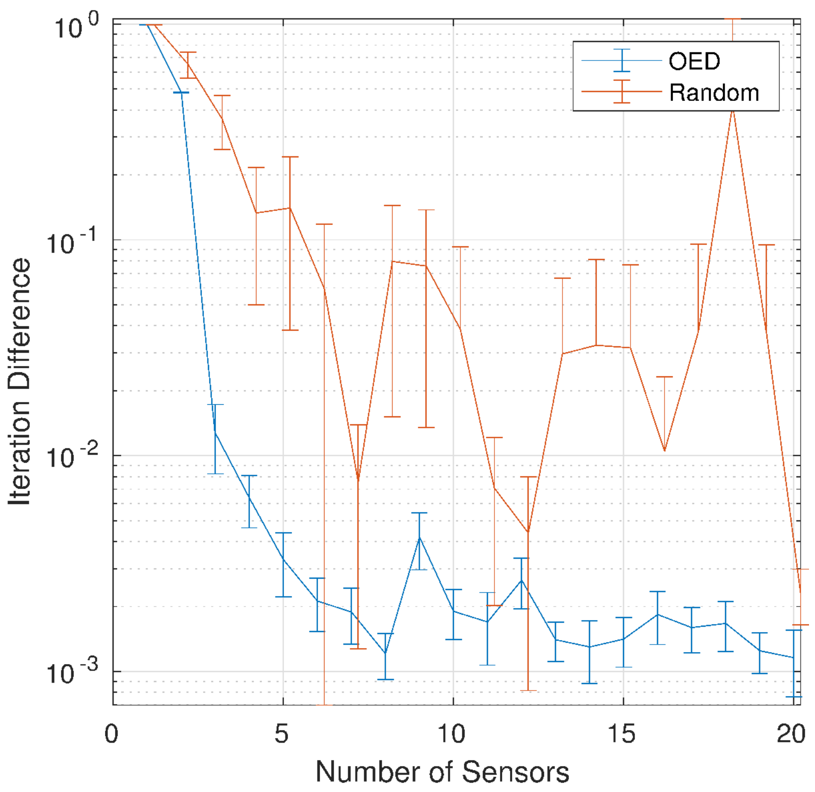


Example damage case

SOED Results

- Compare SOED with a random-walk sensor placement scheme





Modified Error in Constitutive Equations

- Helmholtz-type problems contain singularities (e.g. at natural frequencies) which can force damage parameter estimators to be at local minima

- Goal: Convexify the damage ID problem

$$(\boldsymbol{u}^*, \boldsymbol{\sigma}^*; \boldsymbol{C}^*) = \underset{(\boldsymbol{u},\boldsymbol{\sigma},\boldsymbol{C}) \in \mathcal{U}\times\mathcal{S}\times\mathcal{C}}{\operatorname{argmin}} \; \underline{\Pi(\boldsymbol{u},\boldsymbol{\sigma};\boldsymbol{C})} + \underline{\kappa\Gamma(\boldsymbol{u},\boldsymbol{\sigma})}$$

Model Error: $\Pi(\boldsymbol{u},\boldsymbol{\sigma};\boldsymbol{C}) := \frac{1}{2}\int_\Omega (\boldsymbol{\sigma} - \boldsymbol{C} : \varepsilon[\boldsymbol{u}]) : \boldsymbol{C}^{-1} : \overline{(\boldsymbol{\sigma} - \boldsymbol{C} : \varepsilon[\boldsymbol{u}])} d\Omega$

Data Misfit: $\Gamma(\boldsymbol{u},\boldsymbol{\sigma}) := \frac{1}{2}\|\boldsymbol{u} - \boldsymbol{y}\|^2$

(Banerjee 2013)

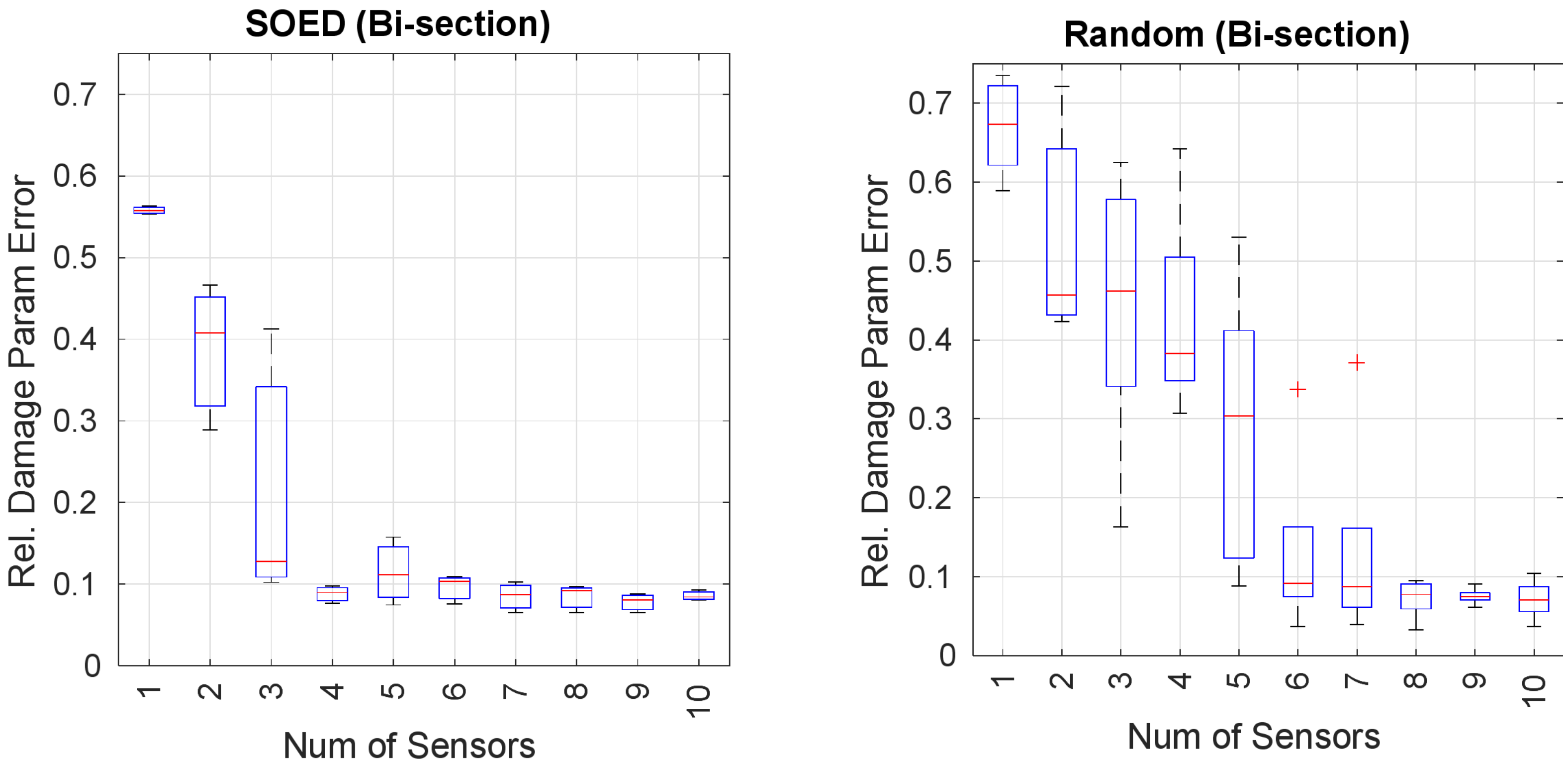
Sequential OED (SOED) Algorithm with MECE

Algorithm 2 Sequential Optimal Experimental Design with MECE

procedure SOED
  $k \leftarrow 0$
  $\epsilon_k \leftarrow 1e6, \boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_0, \boldsymbol{d}_k \leftarrow \mathbf{0}$ ▷ Initialize parameters
  while k <MAX ITER and ϵ_k >TOL **do**
    $k \leftarrow k + 1$
    $\boldsymbol{d}_k \leftarrow OED(\boldsymbol{\theta}_{k-1})$ ▷ Solve OED problem
    $\boldsymbol{\theta}_k \leftarrow IP\_MECE^{\dagger}(\boldsymbol{d}_k)$ ▷ Solve Inverse problem (MECE)
    $\epsilon_k \leftarrow \frac{\|\boldsymbol{\theta}_k - \boldsymbol{\theta}_{k-1}\|_2}{\|\boldsymbol{\theta}_{k-1}\|_2}$ ▷ Stopping criterion
  return $\boldsymbol{d}_k, \boldsymbol{\theta}_k$

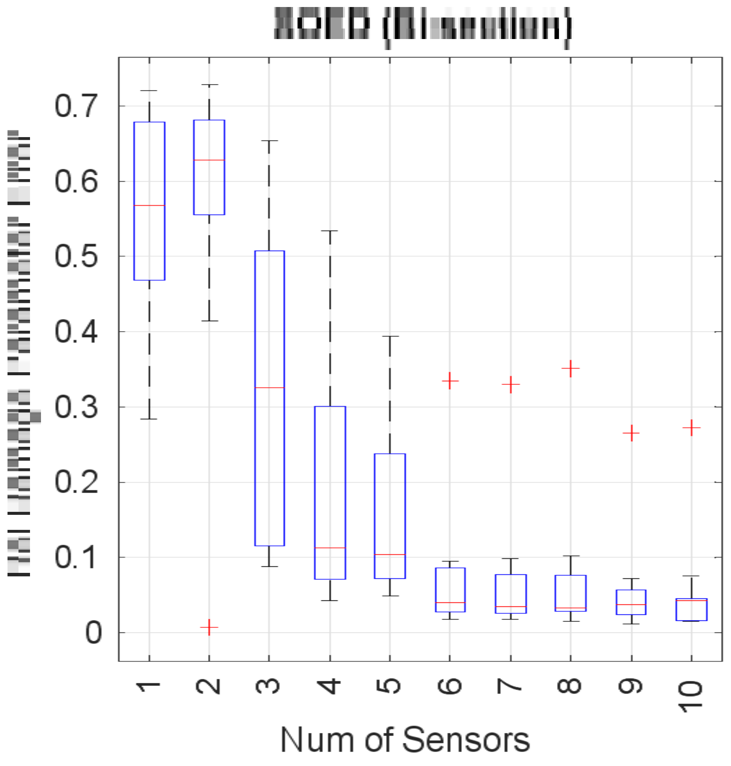
- $\boldsymbol{\theta}_k$: (estimated) damage parameters per iteration
- $\boldsymbol{d}_k$: sensor locations (observations) per iteration
- †: MECE penalty parameter chosen by bisection or continuation

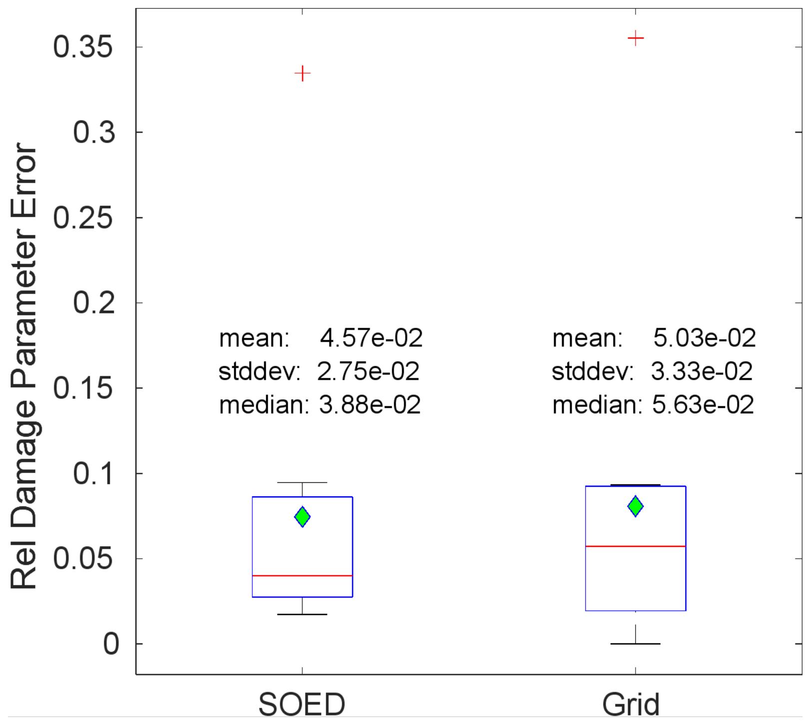
SOED with MECE Performance Comparison



- Compare performance of SOED against ergodically-placed sensors
- Operating Frequency: 5Hz
- Noise Level: 5% Gaussian

SOED Performance (Multiple Damage Cases)





- 10 Damage Cases

- 5Hz Frequency
- 5% Noise

- SOED: 6 sensors
- Grid: 8 sensors at damage locations

Concluding Remarks

- Formulated a linearized damage estimator inverse problem and used a linear optimal experimental design framework (E-optimality). Created a sequential OED framework

- Enhanced the damage estimator using the Modified Error in Constitutive Equation functional

- Evaluated the sequential OED framework by considering number of needed sensors and damage estimator error

- Demonstrated that sequential OED can perform model-specific damage localization

Acknowledgements and Selected References

- DOE-NEUP Project 19-16391: GuArDIAN: General Active Sensing for condition AssessmeNt
- Chandler Smith (Sierra/Inverse) and the Sierra/SD Team for code development support

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