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Distributionally Robust Operating Reserve Demand Curves

Manuel Garcia

Sandia National Laboratories

mgarc19@sandia.gov

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Background

ORDCs:

Determine prices and dispatch levels for operating reserves.

Represent the expected cost of lost load w.r.t. the operating reserve.

Constructed using a *reserve error* probability distribution, which is assumed as Gaussian.

Economic dispatch problem:

$$\min_{(\mathbf{g},\mathbf{r})\in\mathcal{X},\,\mathbf{s}\in\mathcal{P}} \sum_{i\in\mathcal{V}} J_i(g_i, r_i)$$ **Generators' cost**

subject to

$$d_i - s_i - g_i = 0\ \forall i \in \mathcal{V}$$ **Power balance**

Economic dispatch problem with ORDCs:

$$\min_{(\mathbf{g},\mathbf{r})\in\mathcal{X},\,\mathbf{s}\in\mathcal{P},\,r\in\mathbb{R}_+} \sum_{i\in\mathcal{V}} J_i(g_i, r_i) + O(r)$$ **ORDC**

subject to

$$d_i - s_i - g_i = 0\ \forall i \in \mathcal{V}$$

$$r \leq \mathbf{1}^\top \mathbf{r}$$ **Reserve constraint**

This work proposes a Distributionally Robust (DR) ORDC using a robust representation of the reserve error distribution.

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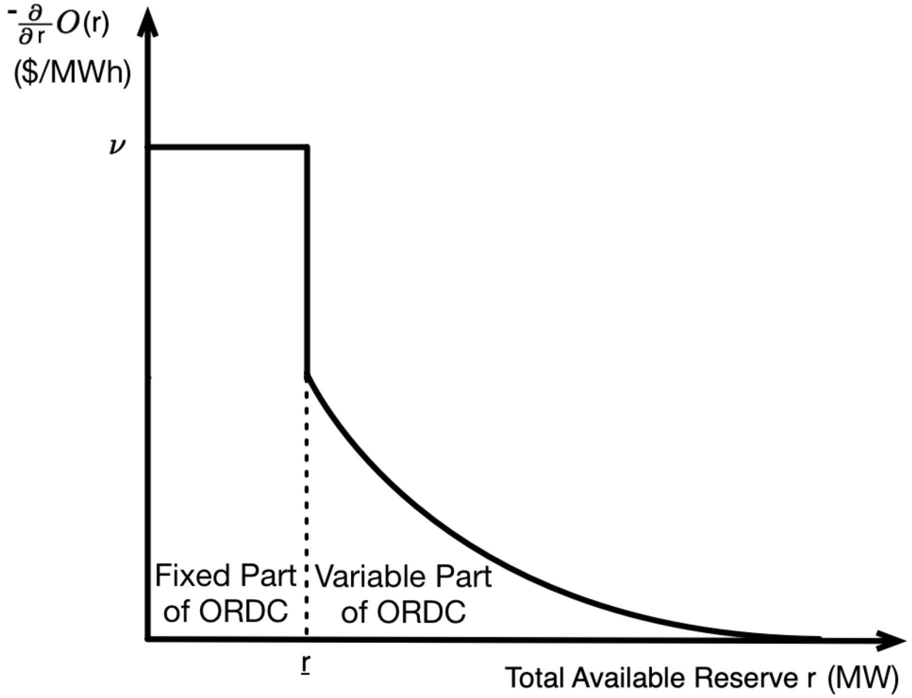


This work proposes a Distributionally Robust (DR) ORDC using a robust representation of the reserve error distribution.

ORDCs Formulation

Expected cost of lost load

$$C(\tilde{r}) := \nu \mathbb{E}^{\mathbb{P}}\left[\max\{\ell - \tilde{r}, 0\}\right]$$

ℓ reserve error (r.v.)

$\tilde{r}$ available reserve

Minimum Contingency Shift

$$O(r) := \begin{cases} \nu(\underline{r} - r) + C(0) & 0 \le r \le \underline{r} \\ C(r - \underline{r}) & \underline{r} < r \end{cases}$$

$\tilde{r} = r - \underline{r}$

r operating reserve

Reserve Prices and the ORDC Derivative

$$\begin{aligned} \frac{\partial}{\partial r} O(r) = \frac{\partial}{\partial r} C(r - \underline{r}) &= \frac{\partial}{\partial r} \nu \mathbb{E}^{\mathbb{P}}[\max\{\ell - r + \underline{r}, 0\}] \\ &= \nu \mathbb{E}^{\mathbb{P}}[\frac{\partial}{\partial r} \max\{\ell - r + \underline{r}, 0\}] \\ &= -\nu \mathbb{E}^{\mathbb{P}}[\mathbf{1}\{\ell - r + \underline{r} \ge 0\}] \\ &= -\nu \mathbb{P}(\ell \ge r - \underline{r}) \end{aligned}$$

Reserve error probability distribution $\mathbb{P}$

A0: Gaussian Distribution

ERCOT assumes a Gaussian probability distribution. Mean and standard deviations are approximated by historical samples of the net-load forecast.

A1: Empirical Distribution

Support set $\hat{\Xi} = \{\hat{\ell}_i\}_{i=1}^{N}$

Empirical distribution $\hat{\mathbb{P}} = \frac{1}{N} \sum_{i=1}^{N} \delta_{\hat{\ell}_i}$

Expected cost of lost load

$$C(r) = \frac{\nu}{N} \sum_{i=1}^{N} \max\{\hat{\ell}_i - r, 0\}$$

A2: Distributionally Robust Representation

Considers the *worst-case* reserve error probability distribution that falls within a specified ball around the empirical distribution.

Distributionally Robust ORDC

Expected cost of lost load

$$C(r) := \nu \sup_{\mathbb{P}\in\mathcal{P}} \mathbb{E}^{\mathbb{P}}[\max\{\ell - r, 0\}]$$

ℓ reserve error (r.v.)

$\tilde{r}$ available reserve

Minimum Contingency Shift

$$O(r) := \begin{cases} \nu(\underline{r} - r) + C(0) & 0 \le r \le \underline{r} \\ C(r - \underline{r}) & \underline{r} < r \end{cases}$$

$\tilde{r} = r - \underline{r}$

r operating reserve

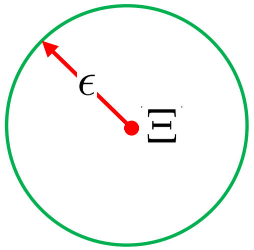
Reserve Prices and the DR ORDC Derivative

$$\frac{\partial}{\partial r} O(r) = \frac{\partial}{\partial r} C(r - \underline{r}) = \frac{\partial}{\partial r} \nu \mathbb{E}^{\mathbb{P}^\star}[\max\{\ell - r + \underline{r}, 0\}]$$
$$= -\nu \mathbb{P}^\star(\ell \ge r - \underline{r})$$

$\mathbb{P}^\star(\ell \ge r - \underline{r})$ is the LOLP of the optimal distribution

Specific Ambiguity Set

Wasserstein ball:



$$\mathcal{P} = \left\{\mathbb{P} \in \mathcal{M}(\Xi) : d_W(\hat{\mathbb{P}}, \mathbb{P}) \le \epsilon\right\}$$

$$\Xi := [\min(\hat{\ell}_i), \max(\hat{\ell}_i)]$$

$\mathcal{M}(\Xi)$ set of all distributions on the support set

Wasserstein distance metric:

$$d_W(\mathbb{P}_1, \mathbb{P}_2) := \inf_{\Pi} \int_{\Xi^2} |\ell_2 - \ell_1| \Pi(d\ell_2, d\ell_1)$$

st: Π is a joint distribution of ℓ_1 and ℓ_2 with marginals $\mathbb{P}_1$ and $\mathbb{P}_2$ respectively

DR-ORDC Formulation

Expected cost of lost load

$$C(r) := \nu \sup_{\mathbb{P}\in\mathcal{P}} \mathbb{E}^{\mathbb{P}}[\max\{\ell - r, 0\}]$$

Wasserstein ball ambiguity set

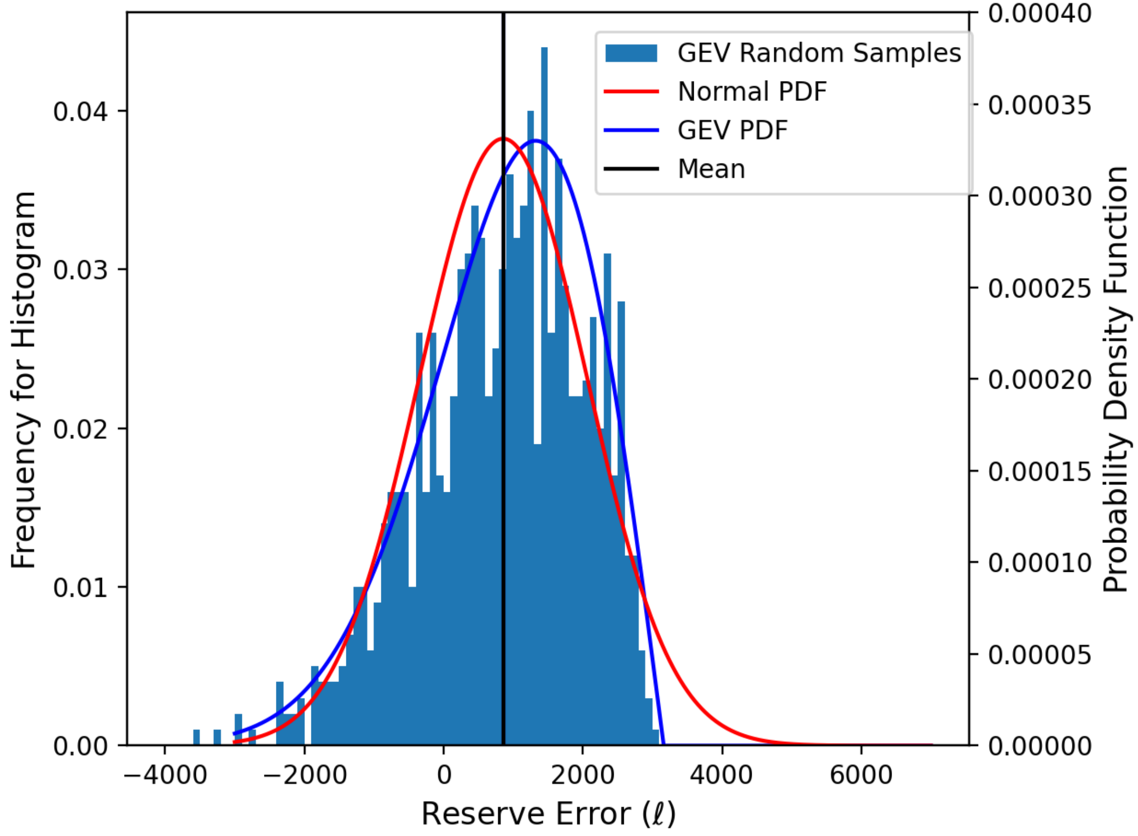
Convex optimization problem

$$C(r) = \min_{\substack{q_1\in\mathbb{R}^N, q_2\in\mathbb{R}^N \\ \alpha_1\in\mathbb{R}^N_+, \alpha_2\in\mathbb{R}^N_+}} \frac{1}{N}\sum_{i=1}^{N}(\alpha_{1i}\hat{\ell}_i - q_{1i} - \alpha_{1i} r)$$

$$\frac{1}{N}\sum_{i=1}^{N}|q_{1i}| + |q_{2i}| \leq \epsilon$$

$$\alpha_{1i} + \alpha_{2i} = 1 \quad \forall i \in [1, N]$$

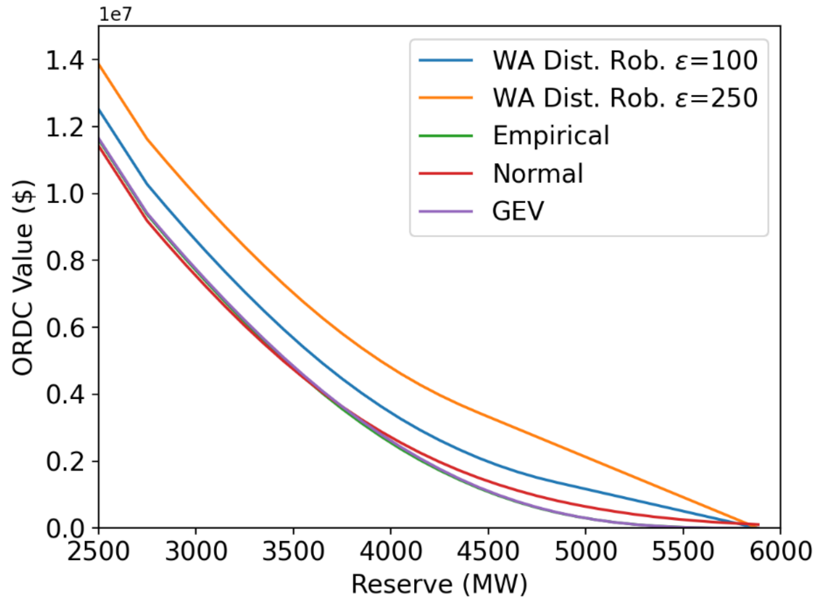
$$\alpha_{ji} L \leq \hat{\ell}_i \alpha_{ji} - q_{ji} \leq \alpha_{ji} U \quad \forall i \in [1, N]\ \forall j \in [1, 2]$$



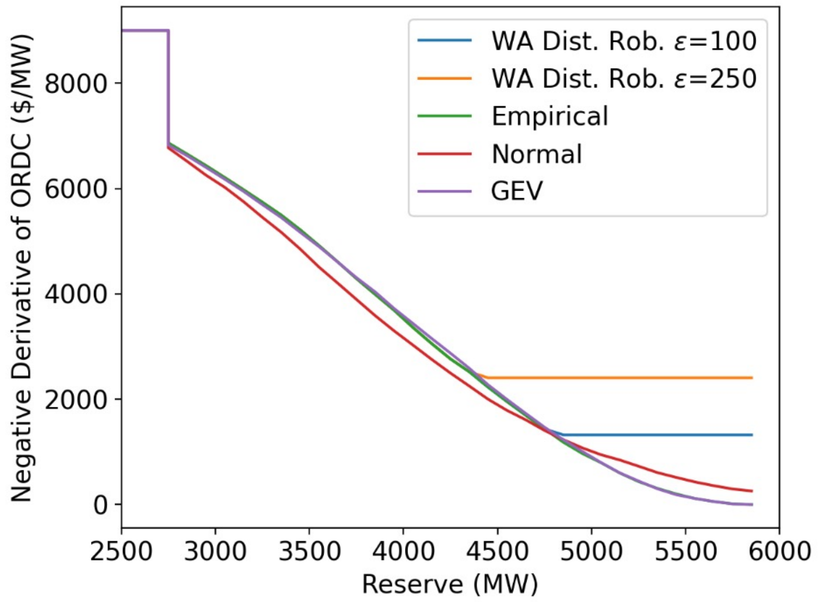
Reserve error probability distributions

Numerical Results

Difference between our proposed ORDCs and the ORDC used by ERCOT, which assumes the reserve error follows a Gaussian distribution.



The ORDC value using different reserve error probability distributions.



The negative gradient of the ORDC (reserve Price) under different reserve error probability distributions.

Conclusions/Recommendations

DR ORDC represents the expected cost of lost load evaluated using the worst-case reserve error probability distribution that falls within the Wasserstein ball around the empirical distribution.

DR ORDC is the optimal value of an LP that can be embedded into a UC or ED problem.

We use an ambiguity set in the form of a Wasserstein ball around the empirical distribution.

The degree of robustness can be adjusted using the Wasserstein ball radius.

DR ORDC conservatively appr. the tails of the reserve error distribution by raising prices at high operating reserve levels.

Numerical results illustrate how the DR ORDC is a more accurate representation of the historical reserve error data when the Wasserstein ball radius is chosen to be zero and how the Wasserstein ball radius can be raised (lowered) to make the DR ORDC more (less) conservative.