

Accurate Hellmann-Feynman forces with optimized atom-centered Gaussian basis sets

Machine Learned First Principles Forces

- Increasing demand of accurate forces for very large systems:
Proteins, Twisted Bilayer graphene
- Machine learning models can train on forces for *small pieces* and predict accurate forces on *large systems*
 - AGNI (Botu *et al.* J. Phys. Chem 2017)

- **Hang-up**: ML models for force require expensive first principles force calculations for training data

Hellmann-Feynman Theorem

$$\vec{F}_I^E = \frac{-\partial \langle \Psi(\{\vec{R}\}) | \hat{H} | \Psi(\{\vec{R}\}) \rangle}{\partial \vec{R}_I} = \vec{F}_I^{HF} + \vec{F}_I^{Pulay}$$

$$\vec{F}_I^{HF} = -\langle \Psi(\{\vec{R}\}) | \frac{\partial \hat{H}}{\partial \vec{R}_I} | \Psi(\{\vec{R}\}) \rangle \quad \text{Depends only on } \rho$$

$$\vec{F}_I^{Pulay} = -\langle \Psi(\{\vec{R}\}) | \hat{H} | \frac{\partial \Psi(\{\vec{R}\})}{\partial \vec{R}_I} \rangle$$

Solution: If we can reduce Pulay term, only need to train **ML density model** for forces!

Optimized Hellmann-Feynman Basis Sets: σ NZHF

- Rico *et al.* J Comput. Chem. (2007) for Slater-type orbitals
- For a single slater determinant wavefunction, a Gaussian basis that can reproduce its own nuclear gradients has small Pulay forces

Technique:

1. Begin with a starting basis set, which we will improve: $\sigma N Z$; $N = S, D, T$ (Ema *et al.* 2022, ArXiv:2207.02782)
2. Compute the distance Δ_λ between the basis set space and nuclear derivative space
3. If $\Delta_\lambda < \epsilon$, a chosen value ($\epsilon = 10^{-3}$), stop
4. Else, add additional primitives to the starting basis set, return to step 2

We built bases for C, N, O, F, P, S, Cl with three different composition:
 $\sigma SZHF$, $\sigma DZHF$, $\sigma TZHF$

arXiv:2207.03587, Submitted to JCTC
 $\sigma NZHF$ bases on Basis Set Exchange

Accurate Forces on H₂O Clusters and DNA Fragments

- Compare analytic and HF gradients using DFT PBE0, σ NZHF, (aug)-cc-pvNZ and (aug)-pcseg-N bases
- Configurations sampled from room temperature MD
H₂O: 10 molecule clusters, 50 configurations
DNA: 35 atom fragments, 60 structures

