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Distributionally Robust Operating Reserve Demand Curves

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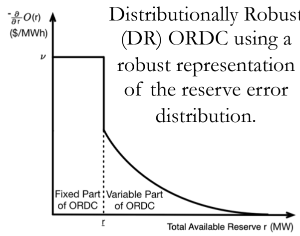
Introduction - ORDC



This work proposes a Distributionally Robust (DR) ORDC using a robust representation of the reserve error distribution.

Economic dispatch problem with ORDCs:

$$\min_{(\mathbf{g},\mathbf{r})\in\mathcal{X},\,\mathbf{s}\in\mathcal{P},\,r\in\mathbb{R}_+} \sum_{i\in\mathcal{V}} J_i(g_i,r_i) + \underbrace{O(r)}_{\text{ORDC}}$$

subject to

$$d_i - s_i - g_i = 0 \;\forall i \in \mathcal{V}$$

$$\underbrace{r \le \mathbf{1}^\top \mathbf{r}}_{\text{Reserve constraint}}$$

Determine prices and dispatch levels for operating reserves.

Represent the expected cost of lost load w.r.t. the operating reserve.

Constructed using a reserve error probability distribution, which is assumed as Gaussian.

Reserve Error Probability

Expected cost of lost load

$$C(\tilde{r}) := \nu\mathbb{E}^{\mathbb{P}}[\max\{\ell - \tilde{r}, 0\}]$$

Minimum Contingency Shift

$$O(r) := \begin{cases} \nu(\underline{r} - r) + C(0) & 0 \le r \le \underline{r} \\ C(r - \underline{r}) & \underline{r} < r \end{cases}$$

A0: Gaussian Distribution

ERCOT assumes a Gaussian probability distribution. Mean and standard deviations are approximated by historical samples of the net-load forecast.

A1: Empirical Distribution

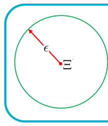
Support set	Empirical distribution	Expected cost of lost load
$\hat{\Xi} = \{\hat{\ell}_i\}_{i=1}^N$	$\hat{\mathbb{P}} = \frac{1}{N}\sum_{i=1}^{N}\delta_{\hat{\ell}_i}$	$C(r) = \frac{\nu}{N}\sum_{i=1}^{N}\max\{\hat{\ell}_i - r, 0\}$

A2: Distributionally Robust Representation

Considers the worst-case reserve error probability distribution that falls within a specified ball around the empirical distribution.

DR ORDC Formulation

Specific Ambiguity Set



Wasserstein ball

$$\mathcal{P} = \left\{\mathbb{P} \in \mathcal{M}(\Xi) : d_W(\hat{\mathbb{P}}, \mathbb{P}) \le \epsilon\right\}$$

$$\Xi := [\min(\hat{\ell}_i), \max(\hat{\ell}_i)]$$

$\mathcal{M}(\Xi)$ set of all distributions on the support set

Wasserstein distance metric

$$d_W(\mathbb{P}_1, \mathbb{P}_2) := \inf_{\Pi} \int_{\Xi^2} |\ell_2 - \ell_1| \Pi(d\ell_2, d\ell_1)$$

st: Π is a joint distribution of ℓ_1 and ℓ_2 with marginals $\mathbb{P}_1$ and $\mathbb{P}_2$ respectively

Reserve Prices and the ORDC Derivative

$$\begin{aligned} \frac{\partial}{\partial r}O(r) = \frac{\partial}{\partial r}C(r - \underline{r}) &= \frac{\partial}{\partial r}\nu\mathbb{E}^{\mathbb{P}}[\max\{\ell - r + \underline{r}, 0\}] \\ &= \nu\mathbb{E}^{\mathbb{P}}[\tfrac{\partial}{\partial r}\max\{\ell - r + \underline{r}, 0\}] \\ &= -\nu\mathbb{E}^{\mathbb{P}}[\mathbf{1}\{\ell - r + \underline{r} \ge 0\}] \\ &= -\nu\mathbb{P}(\ell \ge r - \underline{r}) \end{aligned}$$

Reserve Prices and the DR ORDC Derivative

$$\begin{aligned} \frac{\partial}{\partial r}O(r) = \frac{\partial}{\partial r}C(r - \underline{r}) &= \frac{\partial}{\partial r}\nu\mathbb{E}^{\mathbb{P}^*}[\max\{\ell - r + \underline{r}, 0\}] \\ &= -\nu\mathbb{P}^*(\ell \ge r - \underline{r}) \end{aligned}$$

$\mathbb{P}^*(\ell \ge r - \underline{r})$ is the LOLP of the optimal distribution

Expected cost of lost load defined by the worst-case distribution

$$C(r) := \nu \sup_{\mathbb{P}\in\mathcal{P}} \mathbb{E}^{\mathbb{P}}[\max\{\ell - r, 0\}]$$

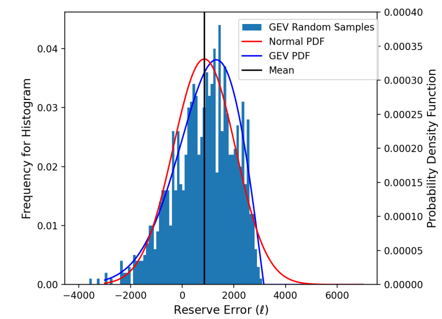
Convex optimization problem

$$C(r) = \min_{\substack{q_1\in\mathbb{R}^N,\, q_2\in\mathbb{R}^N \\ \alpha_1\in\mathbb{R}_+^N,\, \alpha_2\in\mathbb{R}_+^N}} \frac{1}{N}\sum_{i=1}^{N}(\alpha_{1i}\hat{\ell}_i - q_{1i} - \alpha_{1i}r)$$

$$\frac{1}{N}\sum_{i=1}^{N}|q_{1i}| + |q_{2i}| \le \epsilon$$

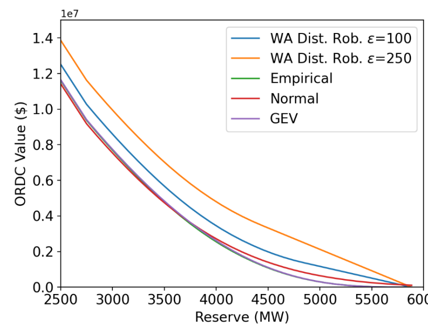
$$\alpha_{1i} + \alpha_{2i} = 1 \qquad \forall i \in [1, N]$$

$$\alpha_{ji}L \le \hat{\ell}_i\alpha_{ji} - q_{ji} \le \alpha_{ji}U \quad \forall i \in [1, N]\; \forall j \in [1, 2]$$



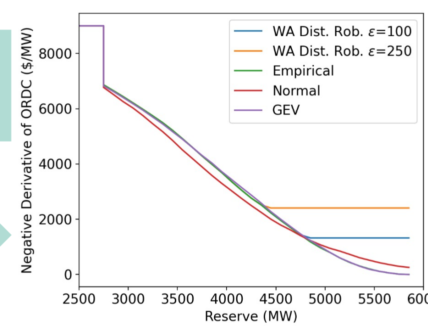
Reserve error probability distributions

Results



ORDC value using different reserve error probability distributions.

Negative gradient of the ORDC (reserve Price).



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