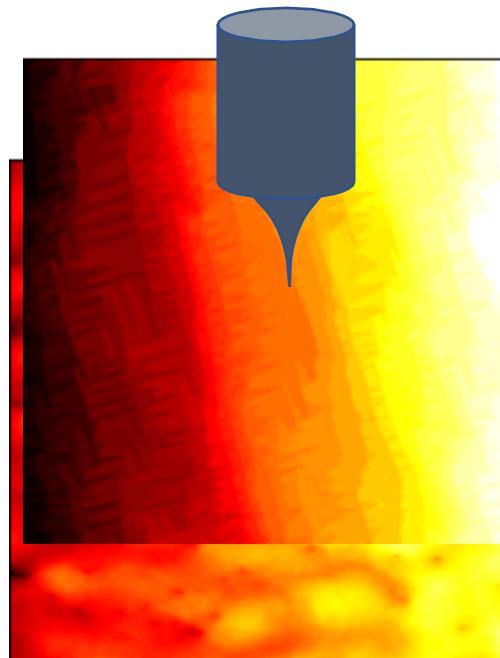
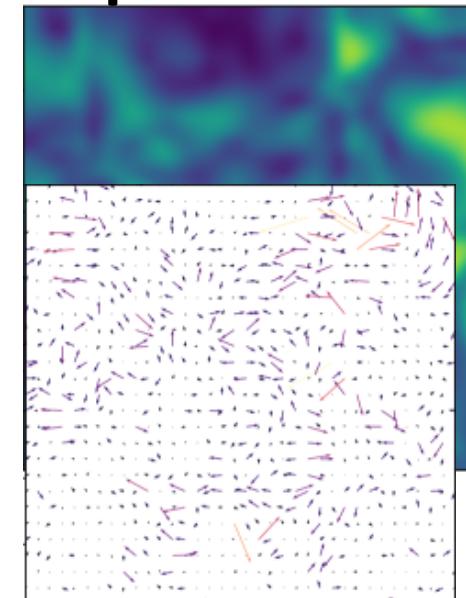


# Scanning tunneling microscopy-informed simulation of valley physics in single and double quantum dots



Project: Silver City



PI: Toby Jacobson

## Quantum Computing Program Review, July 2022

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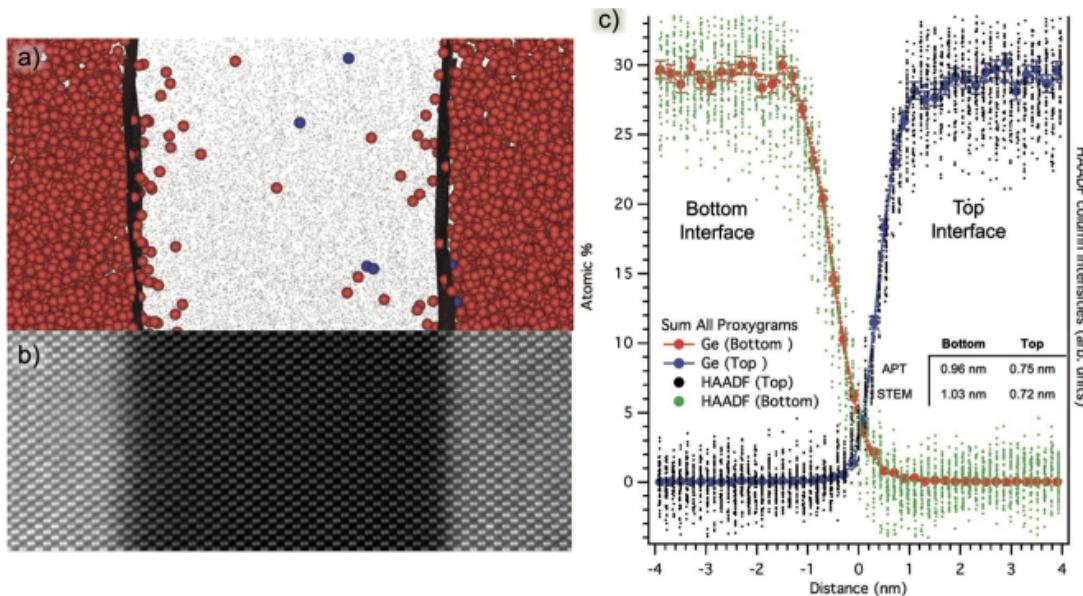
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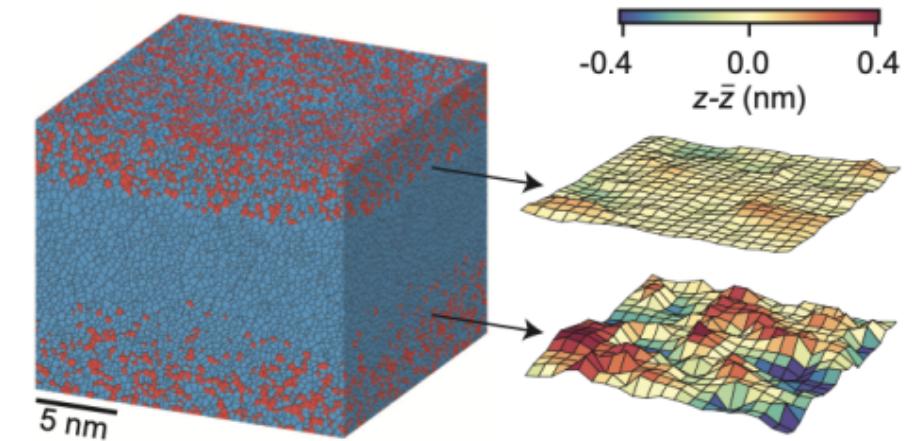


# Motivation

There's been much work recently to characterize SiGe/Si/SiGe wells, e.g.



[Dyck, et al. Adv. Mat. Interfaces 4, 1700622 (2017)]



[Wuetz, et al. arXiv:2112.09606]

**Q:** Given a realistic atomistic-level model for well structure, what are the consequences for quantum dot properties?

# STM measurements of interface topography

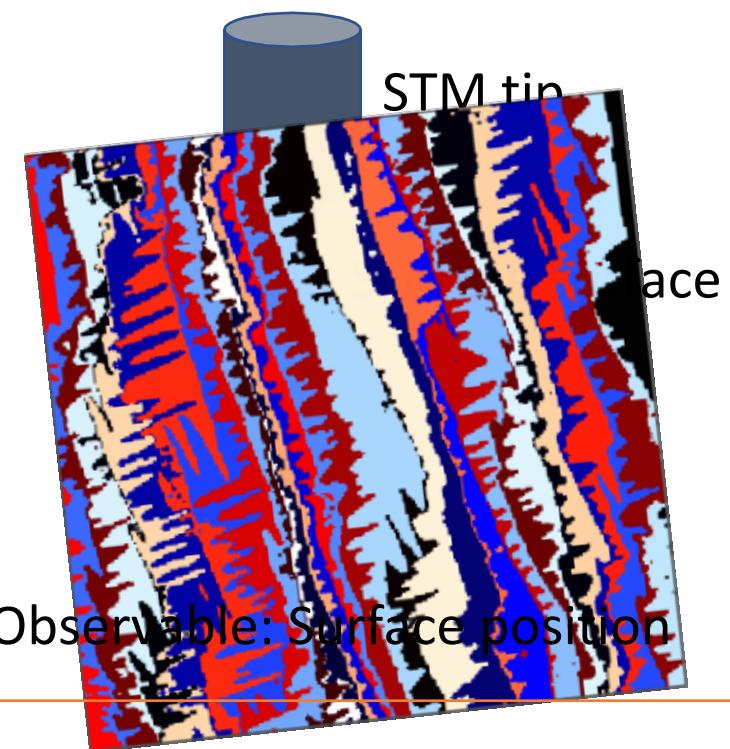
Scanning tunneling microscopy (STM) provides a means to characterize surface properties

Capability developed by Ezra Bussmann and collaborators (Sandia) under previous ARO/LPS-funded project

## Procedure:

- Grow SiGe substrate
- Image surface of SiGe substrate w/ STM (lower interface)
- Continue growing Si well (molecular beam epitaxy)
- Image surface of Si well w/ STM (upper interface)

## Cartoon of measurement



# STM measurements of interface topography

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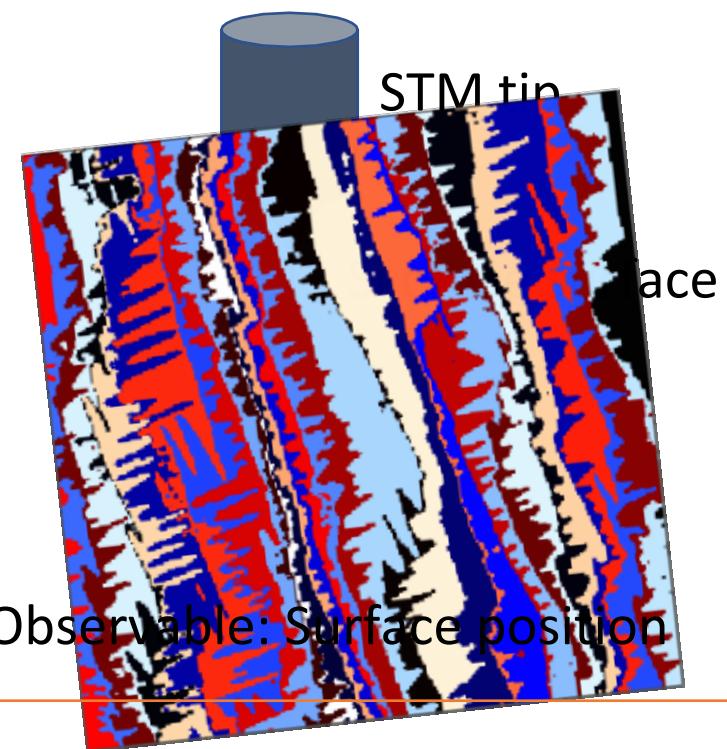
## Procedure:

- Grow SiGe substrate
- Image surface of SiGe substrate w/ STM (lower interface)
- Continue growing Si well (molecular beam epitaxy)
- Image surface of Si well w/ STM (upper interface)

## Result:

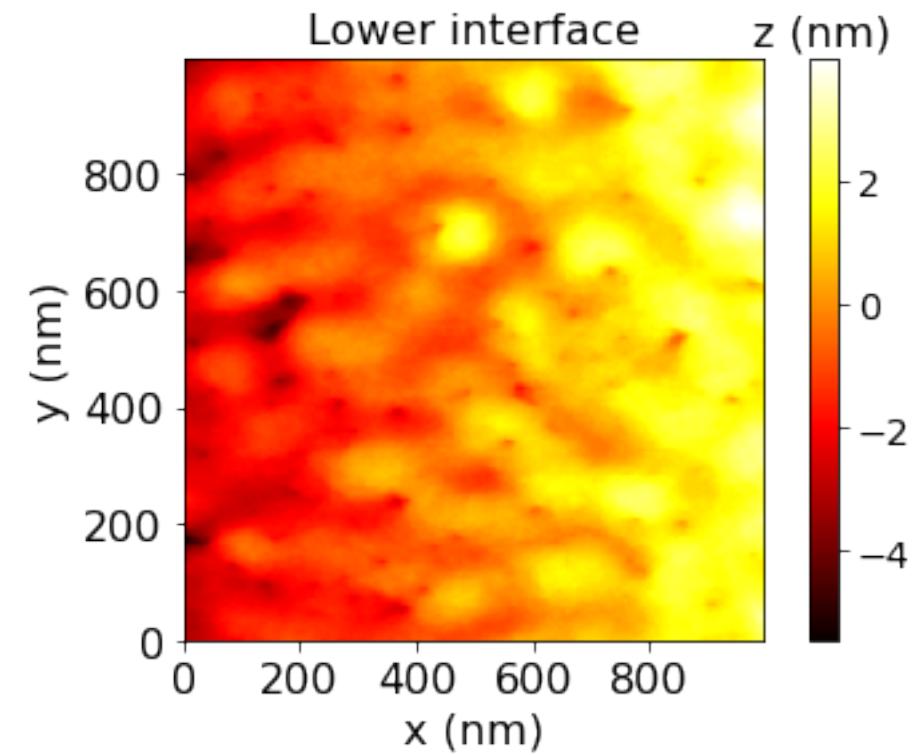
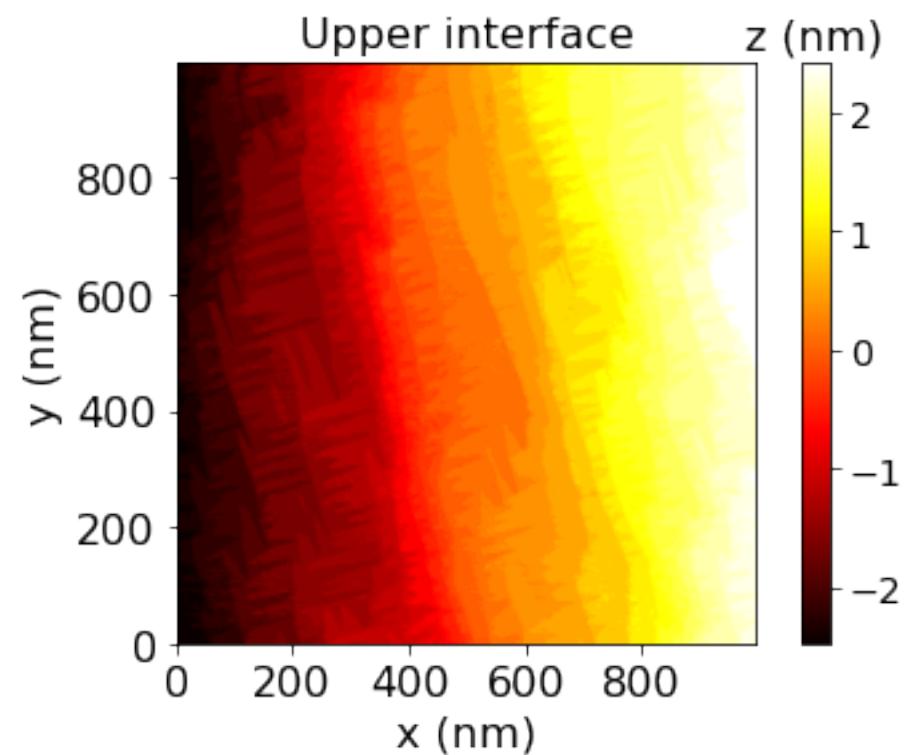
- Topography of both upper and lower interfaces of the Si well in the same device
- Use to generate an alloy disorder realization over a  $1\mu\text{m} \times 1\mu\text{m} \times 15\text{ nm}$  volume, assuming some interdiffusion length

## Cartoon of measurement



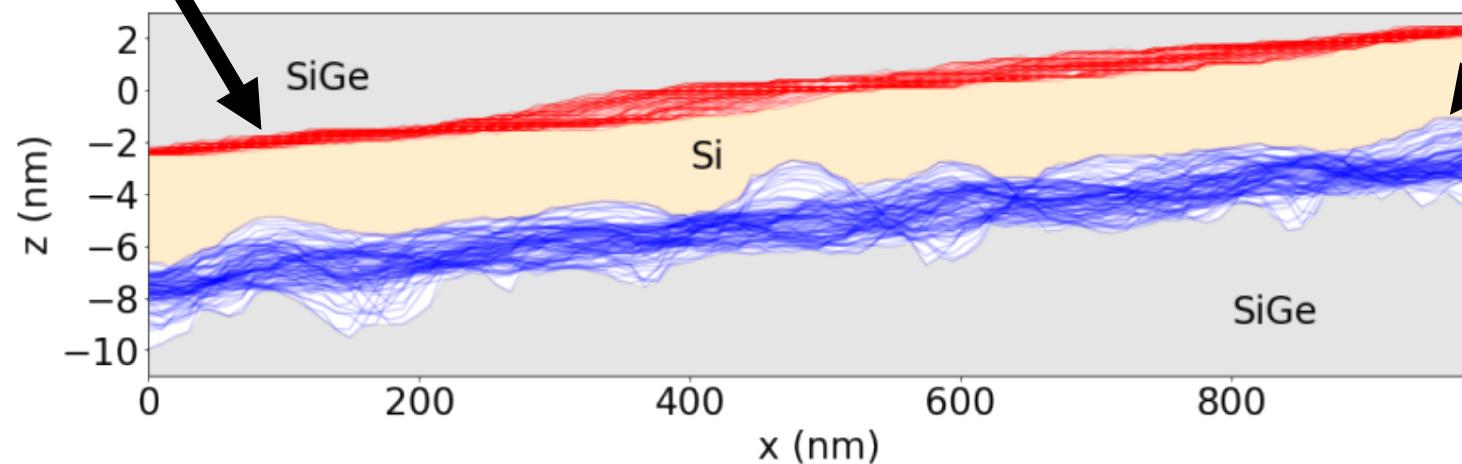
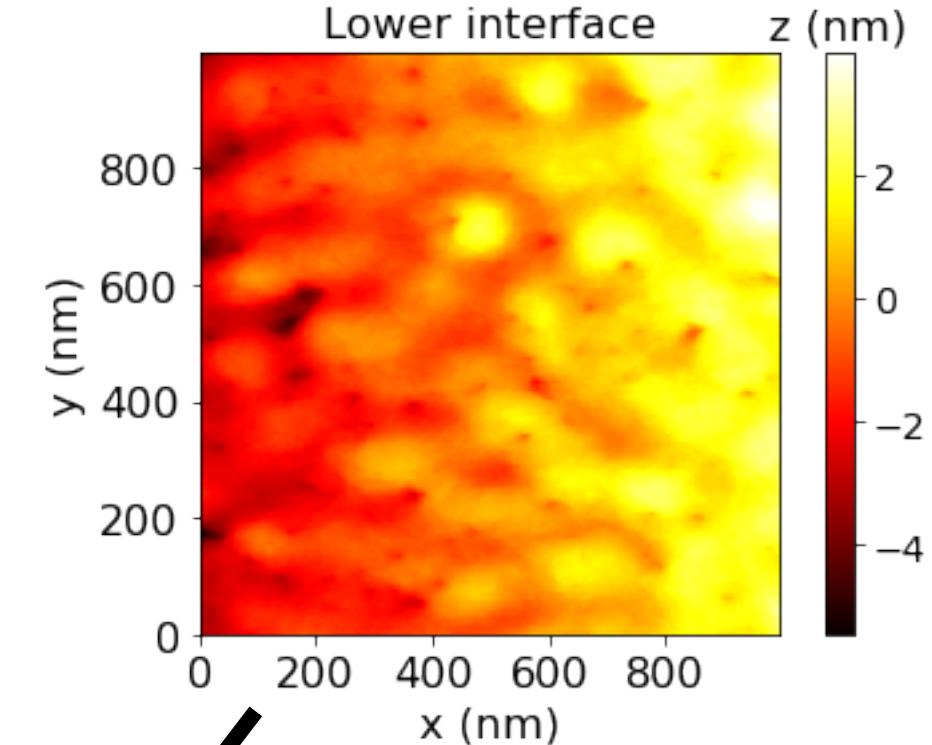
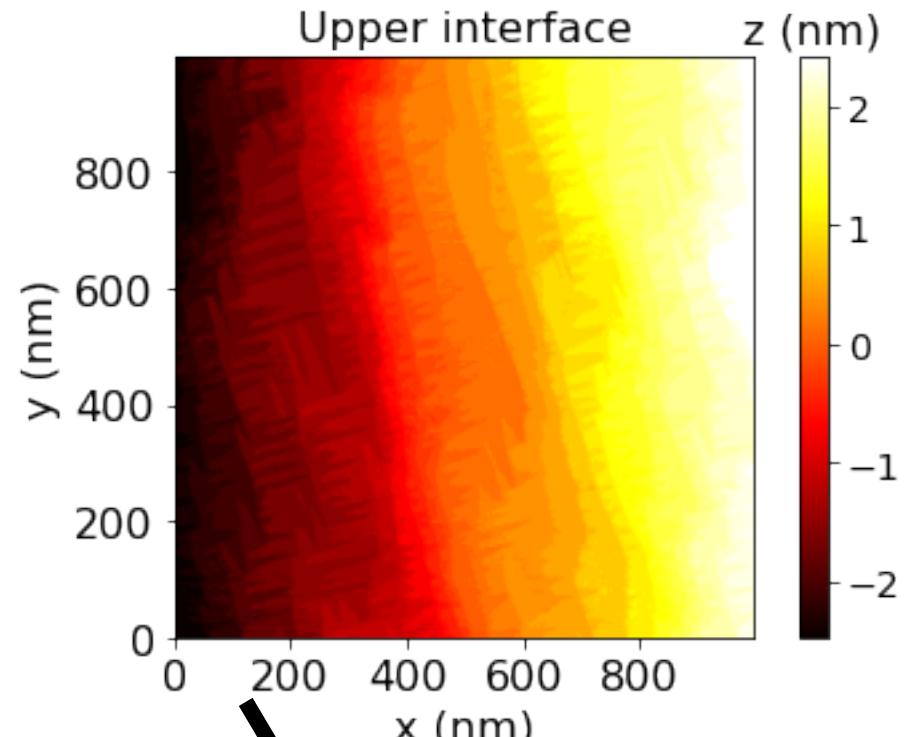
# STM measurements of interface topography

Data courtesy Fabián Peña and Ezra Bussmann (SNL)



# STM measurements of interface topography

Data courtesy Fabián Peña and Ezra Bussmann (SNL)



Note: Probably an extreme example of lower interface roughness due to unintended richer Ge level during growth

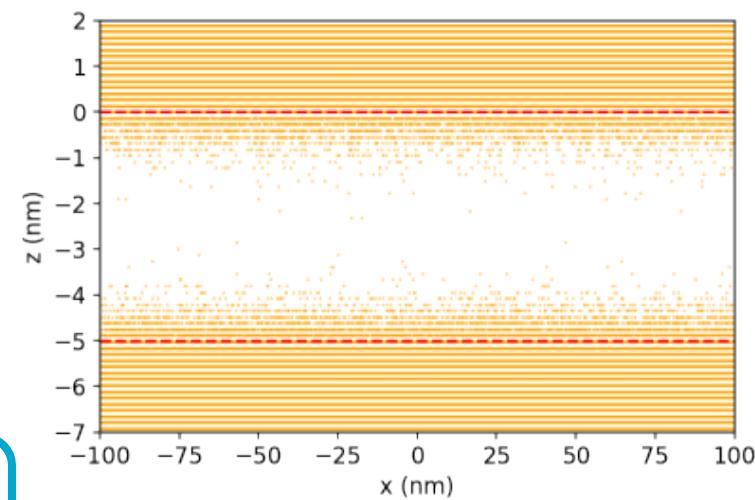
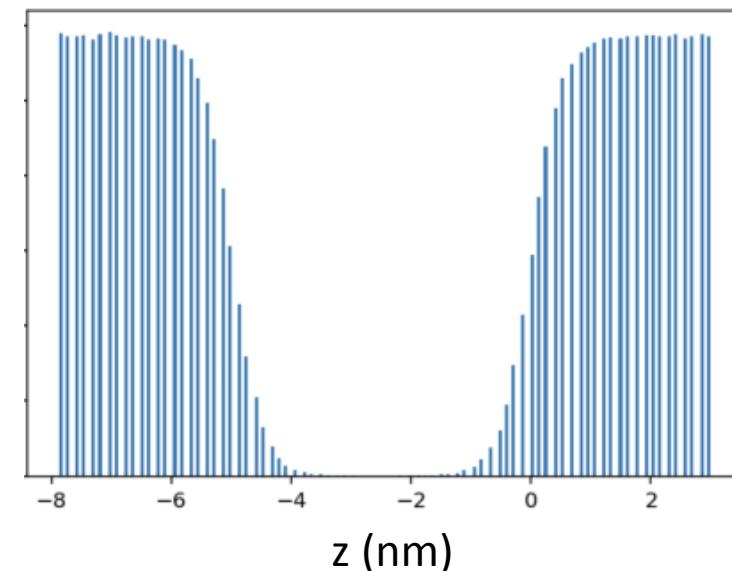
# Atomistic disorder model

- Use STM data or idealized model (“flat interface”) to define the interface locations in (x,y) plane
- For each interface, define a sigmoidal Ge concentration along z-axis, referenced to local interface location  $z_{\text{upper}}(x, y), z_{\text{lower}}(x, y)$
- Generate an alloy realization on diamond lattice for all atoms ( $\sim 10^9$  atoms) and down-select based on simulation volume
- Including both “step” structure and interdiffusion

• In the following, assuming  $4\tau=1$  nm based on TEM measurements of 25 nm thin lamellae [Bussmann]

- Consistent also with other APT and HAADF-STEM measurements:  
[Dyck, et al. Adv. Mat. Interfaces 4, 1700622 (2017)]  
[Wuetz, et al. arXiv:2112.09606]

Example Ge fraction histogram



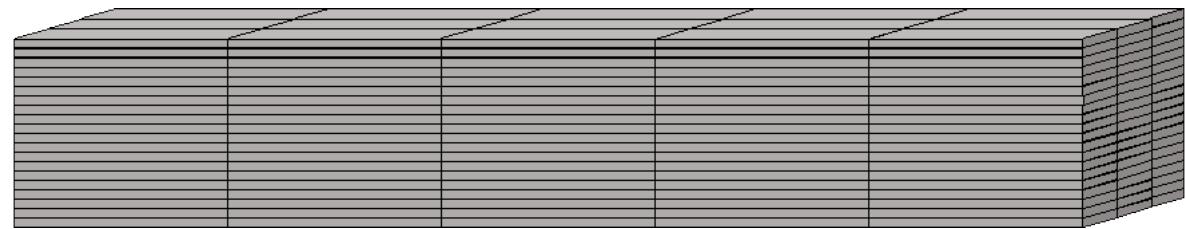
Sigmoidal interdiffusion distribution [Dyck, et al. (2017)]:

$$f(z) \propto 1/(1 + e^{-z/\tau})$$



## Model

- ***Mesh-based discretization***
  - Legendre polynomials on a hexahedral mesh (stack of bricks)
  - Taking up to 3<sup>rd</sup> order here (64-dim modal basis/mesh element)
  - Discontinuous Galerkin formulation in *Laconic* code developed at Sandia 
  - High-throughput simulations feasible: <1 hr/eigensolve on single core



Mesh highly refined along z-axis to resolve wavefunction tails near interfaces

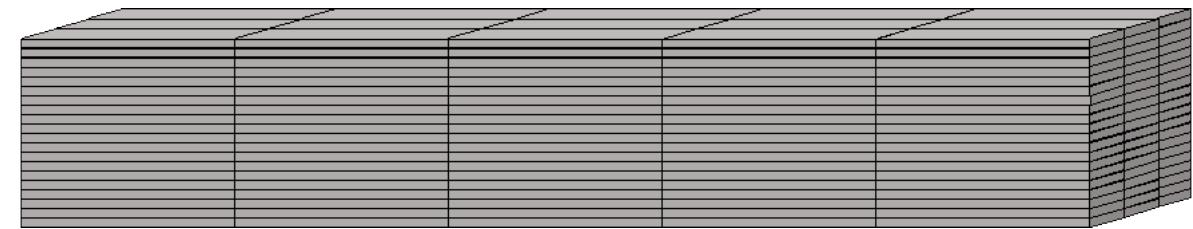
# Model

- **Mesh-based discretization**

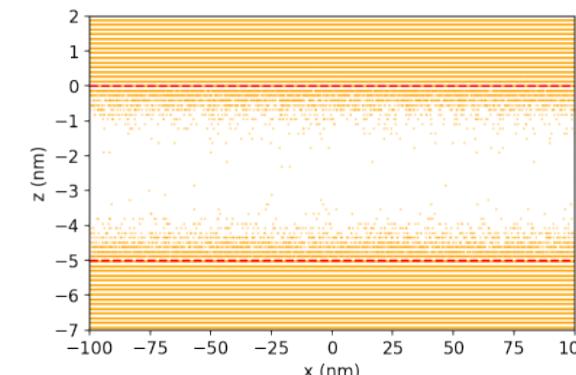
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- **Valley-orbit coupling:**

- Shindo-Nara equations [Shindo & Nara (1976)] for multi-valley EMT
- Plane wave representation of Bloch functions from DFT
- Atom-by-atom model for valley-orbit coupling: Each Ge atom contributes a **localized** repulsive potential scaled to agree with concentration-dependent Si → SiGe CB offset



Mesh highly refined along z-axis to resolve wavefunction tails near interfaces



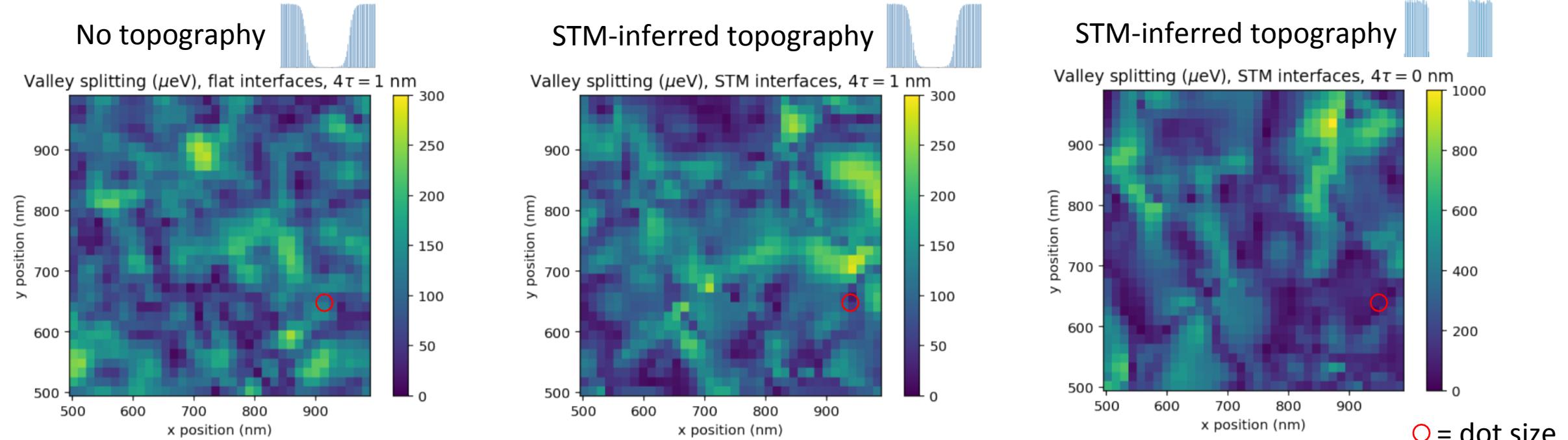
$$V_{\text{Ge},i}(\mathbf{r}) = \alpha_{\text{Ge}} \delta^{(3)}(\mathbf{r} - \mathbf{r}_i), \quad \alpha_{\text{Ge}} = 12 \text{ meV nm}^3$$

$$\langle \mathbf{r} | u_{\pm z} \rangle = \sum_{\mathbf{G}} c_{\mathbf{G}, \pm z} e^{i \mathbf{G} \cdot \mathbf{r}}$$

↓

$$\langle u_{\pm z} | \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) | u_{\pm z} \rangle \approx 1$$
$$\langle u_{\pm z} | \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) | u_{\mp z} \rangle \approx 1$$

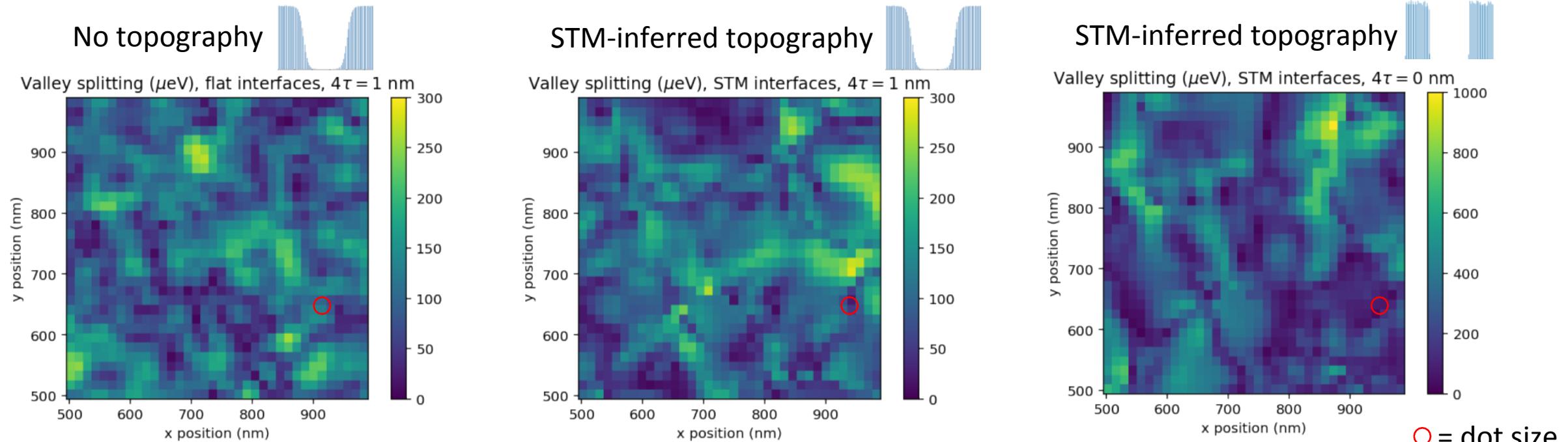
# Single quantum dot rastered across the surface: Valley splitting



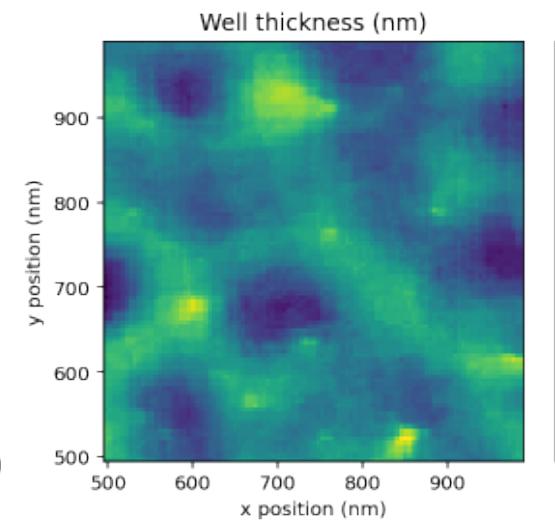
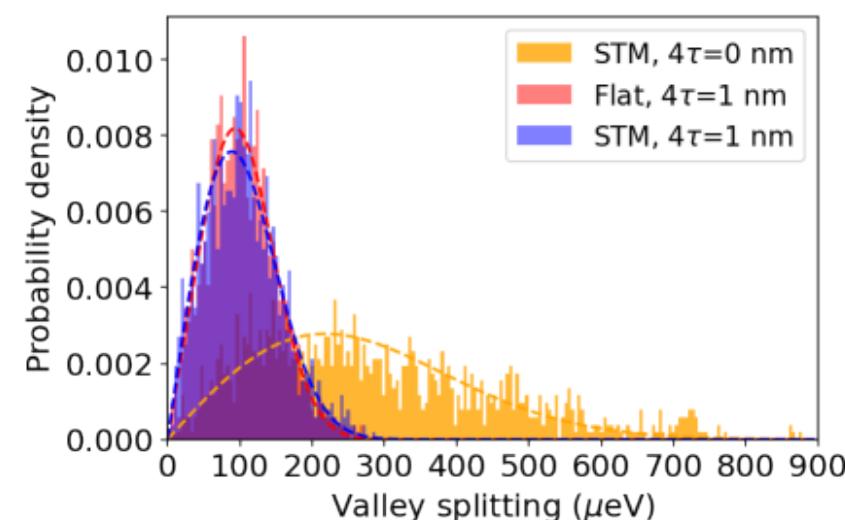
- 1.5 meV symmetric harmonic  $x$ - $y$  confinement



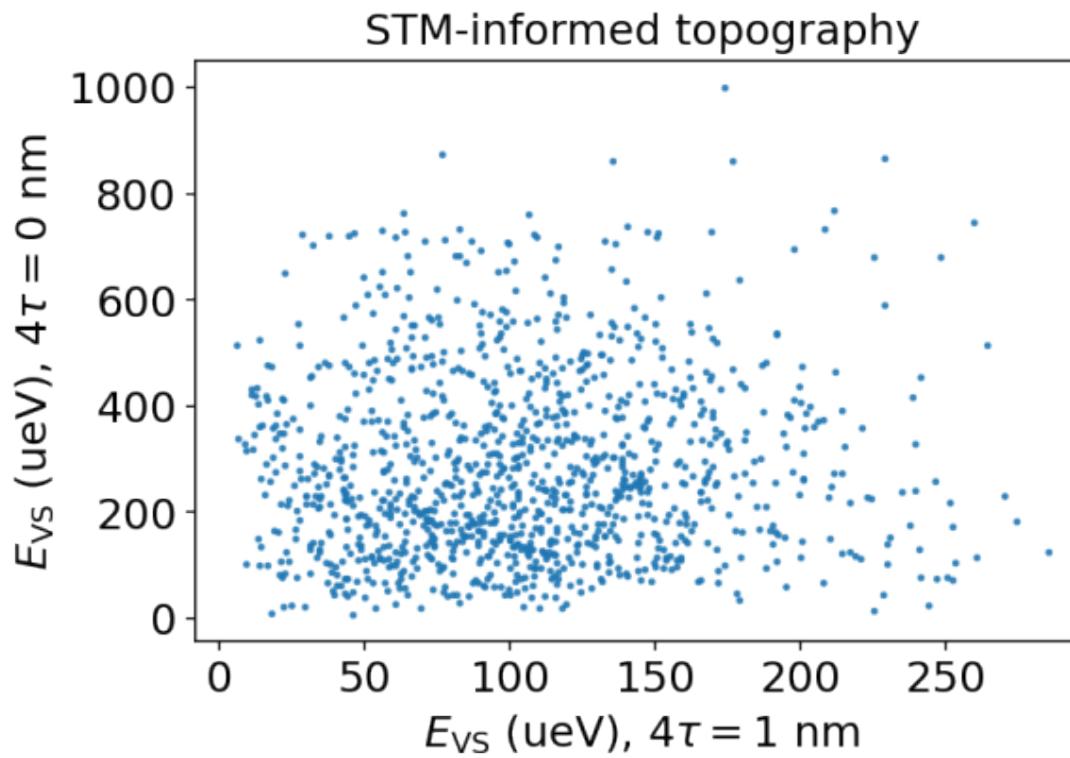
# Single quantum dot rastered across the surface: Valley splitting



- 1.5 meV symmetric harmonic  $x$ - $y$  confinement
- Decent fit to Rice distribution for valley splitting [Wuetz, et al. (2021)]
- Interdiffusion length  $4\tau$  has a strong effect on valley splitting
- Atomic steps/topography appear to be less important than interdiffusion



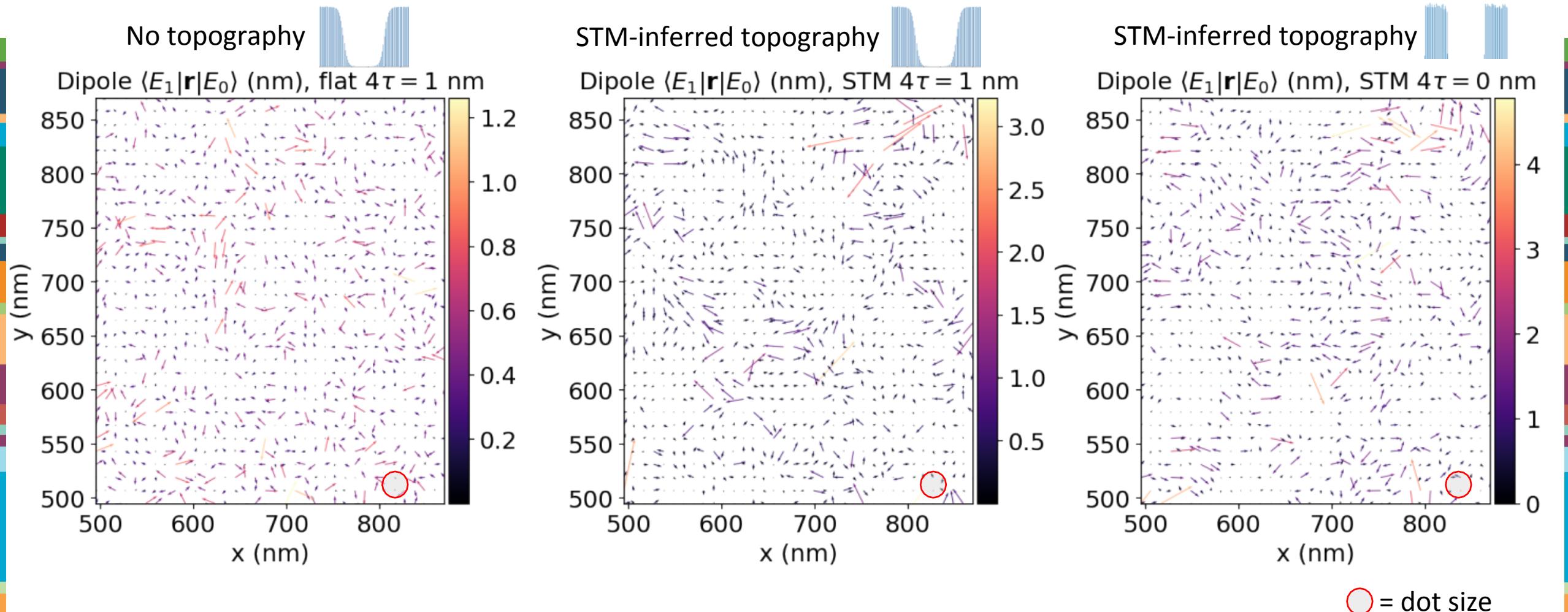
# Single quantum dot rastered across the surface: Valley splitting



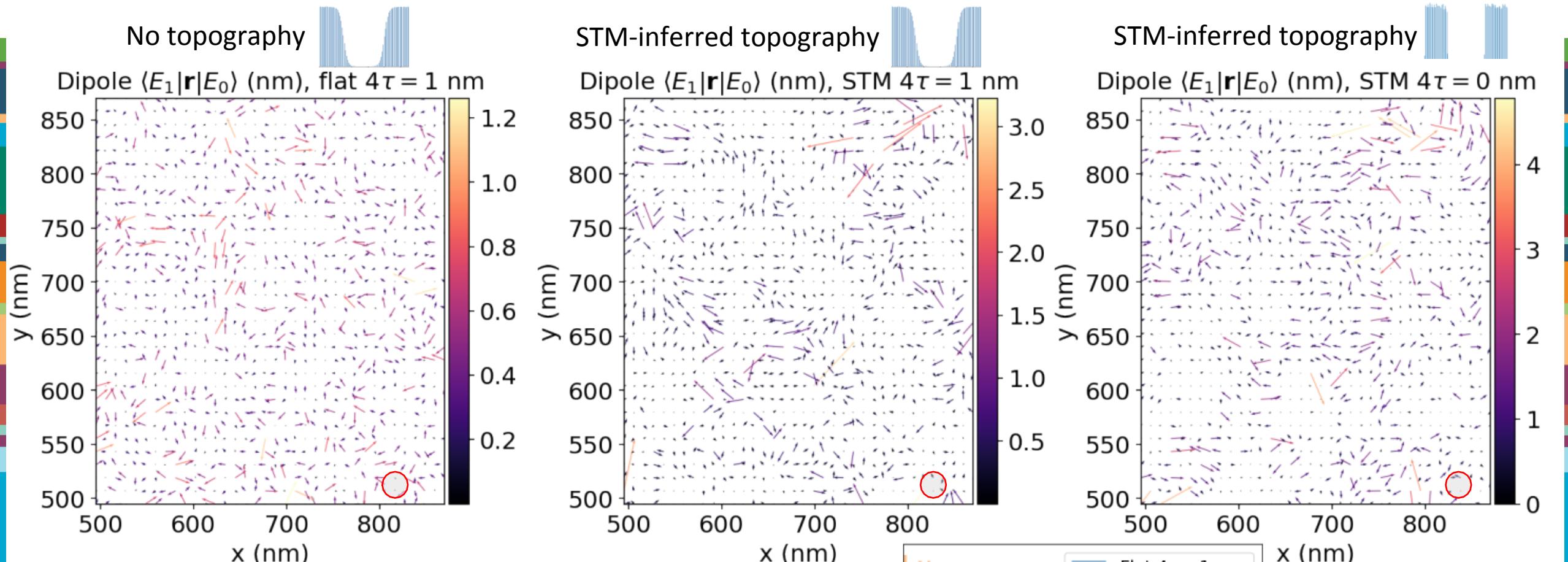
Each point: Same x-y confinement coordinate

- Valley splitting uncorrelated between different interdiffusion lengths  $4\tau$  for same interface topography
- Interdiffusion appears to be the dominant driver of valley splitting

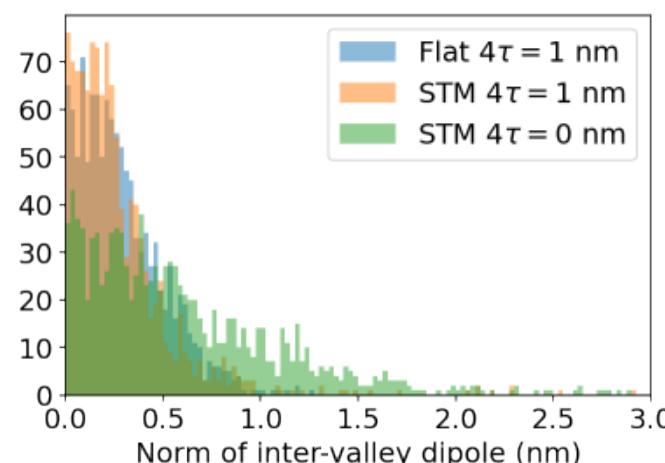
# Single quantum dot rastered across the surface: Inter-valley dipole



# Single quantum dot rastered across the surface: Inter-valley dipole



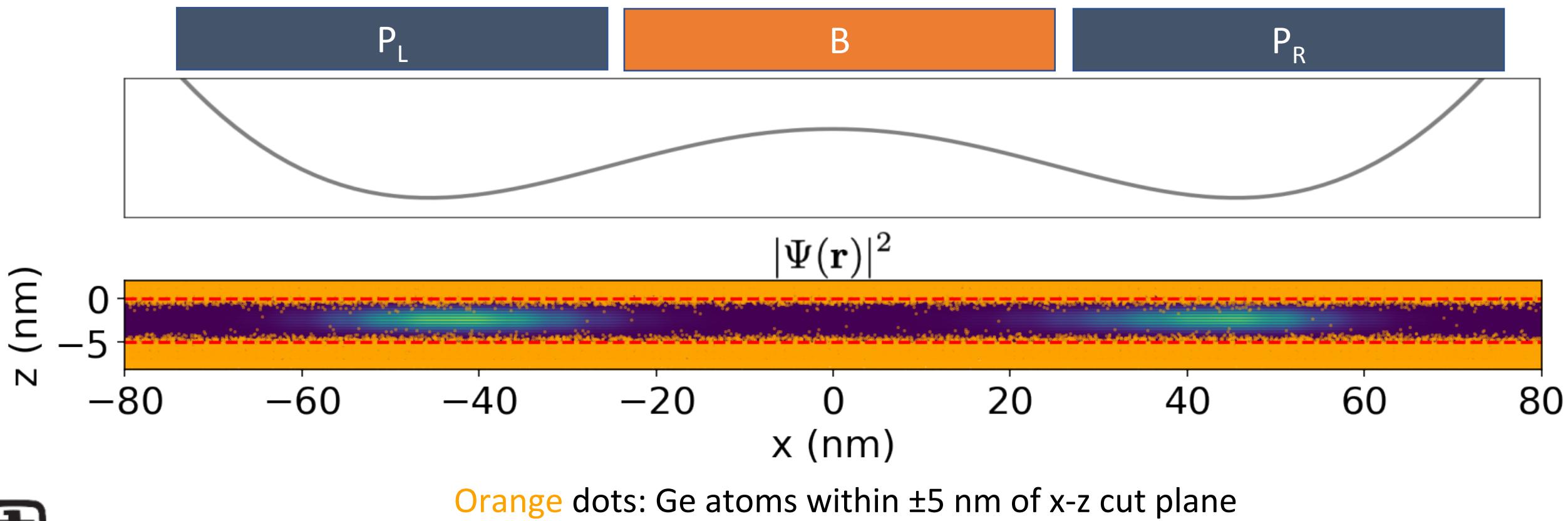
- Inter-valley dipole should relate to spin-orbit coupling properties, e.g. spin-valley coupling [Yang, et al. Nat. Comm. 4, 2069 (2013)]
- Generally larger dipole in presence of crisp interface topography



○ = dot size

## Double quantum dot

- Assuming “benchmark” double well potential of [Anderson, et al. arXiv:2203.00082]
- 5 nm well
- Interface topographies: flat or STM-inferred
- Interdiffusion length,  $4\tau$ : 0 or 1 nm

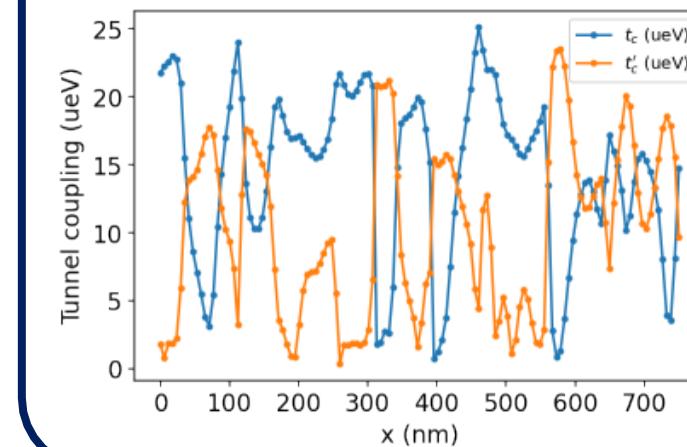
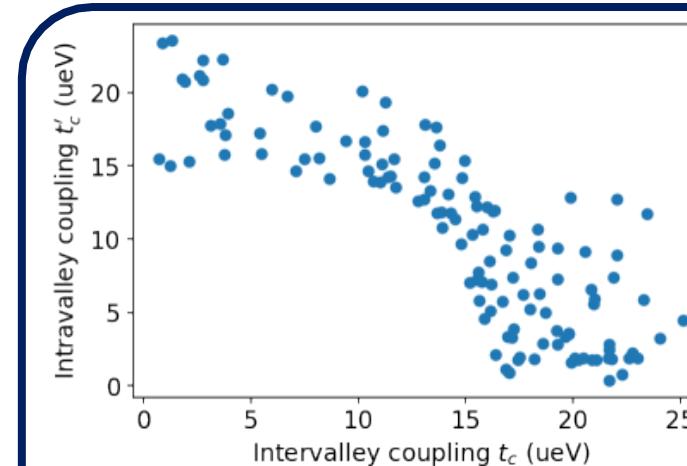
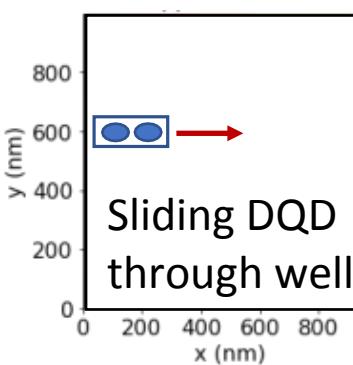


# Double quantum dot: Flat interfaces

- Compute position dipole and infer four-level Hamiltonian
- Fits nicely to the 4-level double dot model of [Borjans, et al. PRX Quantum 2, 020309 (2021)]

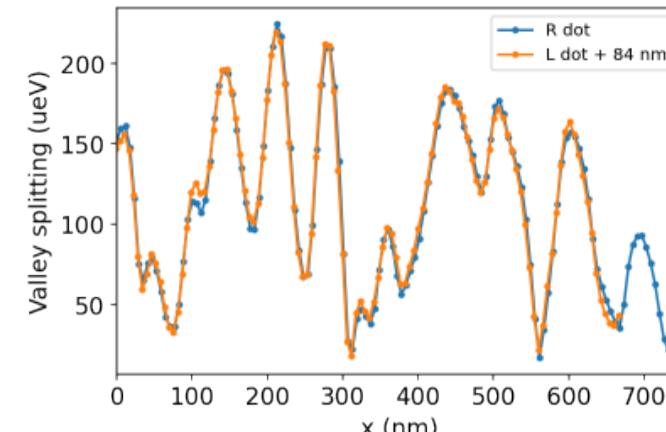
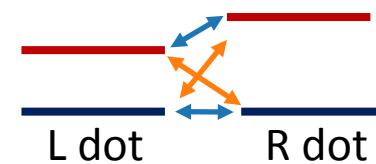
$$H = \begin{pmatrix} \frac{\epsilon}{2} & 0 & t_c & t'_c \\ 0 & \frac{\epsilon}{2} + E_{VS,L} & t'_c & t_c \\ t_c^* & t_c'^* & -\frac{\epsilon}{2} & 0 \\ t_c'^* & t_c^* & 0 & -\frac{\epsilon}{2} + E_{VS,R} \end{pmatrix}$$

- Extract:
  - Valley splittings  $E_{VS,L}, E_{VS,R}$
  - Intra-/inter-valley tunnel couplings  $t_c, t'_c$
  - Detuning offset  $\epsilon_0$  due to alloy disorder



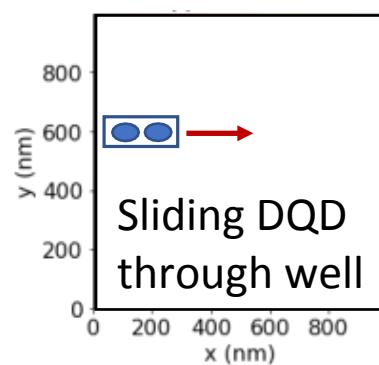
Intra-/inter-valley tunnel couplings are anticorrelated, as expected

$t_c$  = intravalley  
 $t'_c$  = intervalley

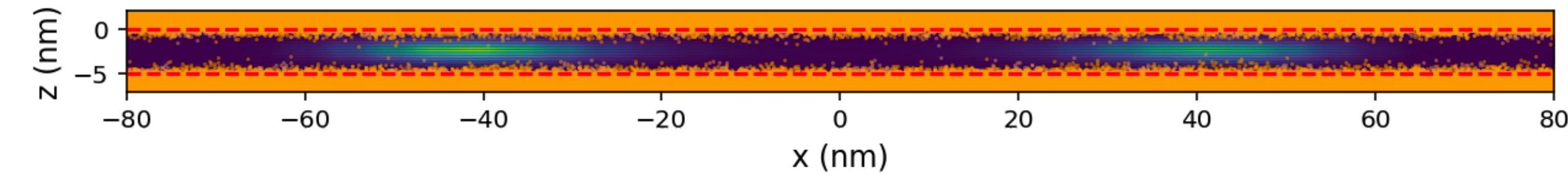
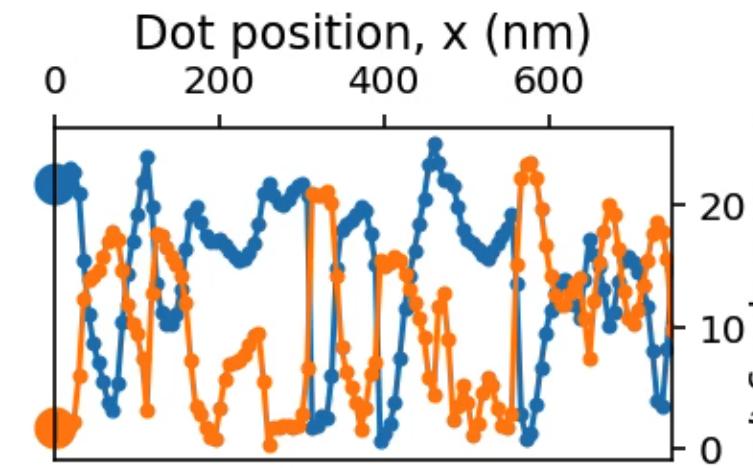
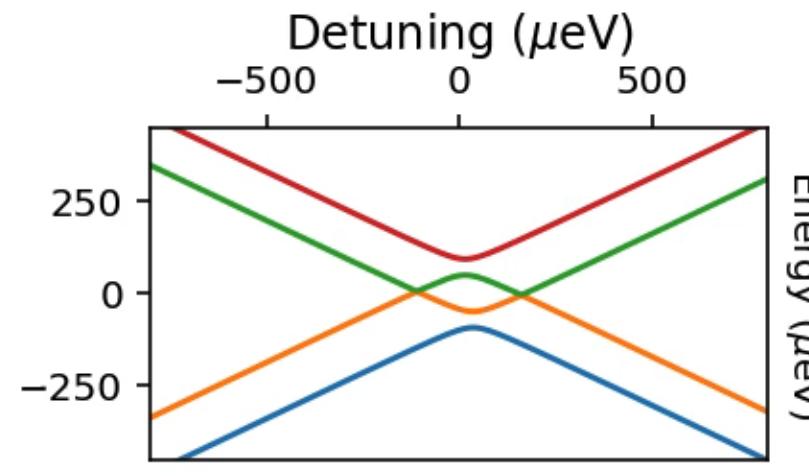
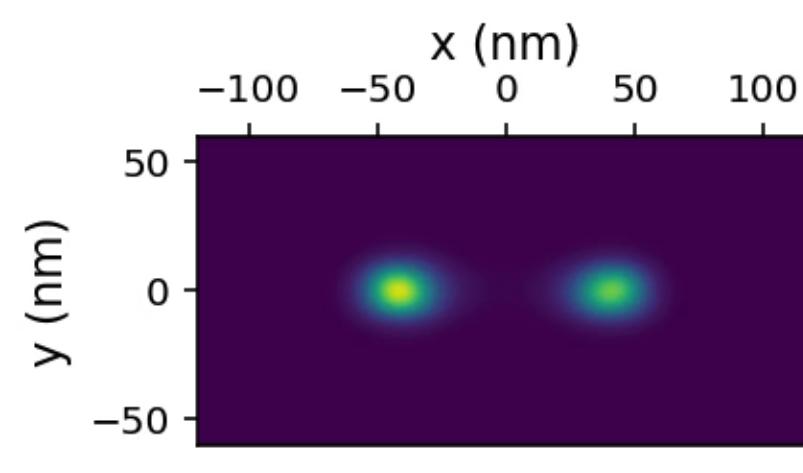


$E_{VS}$  consistent with single-dot behavior, as expected

## Double quantum dot: Flat interfaces

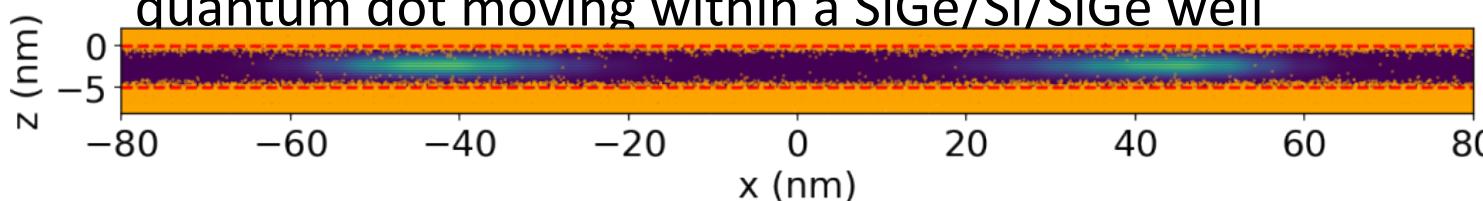


$t_c$  = intra-valley tunnel coupling  
 $t'_c$  = inter-valley tunnel coupling



For STM-informed interfaces, alloy disorder-induced detuning variation is much more dramatic (approximately 10x)

- We've developed machinery for simulating large-scale atomistic alloy disorder within multi-valley effective mass theory based on information about interface topography
  - Able to use data from e.g. APT, STM characterization of atomic-scale disorder
- For significant interdiffusion between Si and SiGe layers, **valley splitting** and **inter-valley dipole** statistics primarily governed by diffusion rather than interface topography
- We've characterized inter- and intra-valley tunnel coupling variation for a double quantum dot moving within a SiGe/Si/SiGe well



## Acknowledgements:

- Ezra Bussmann (Sandia)
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- Andrew Baczewski (Sandia)

