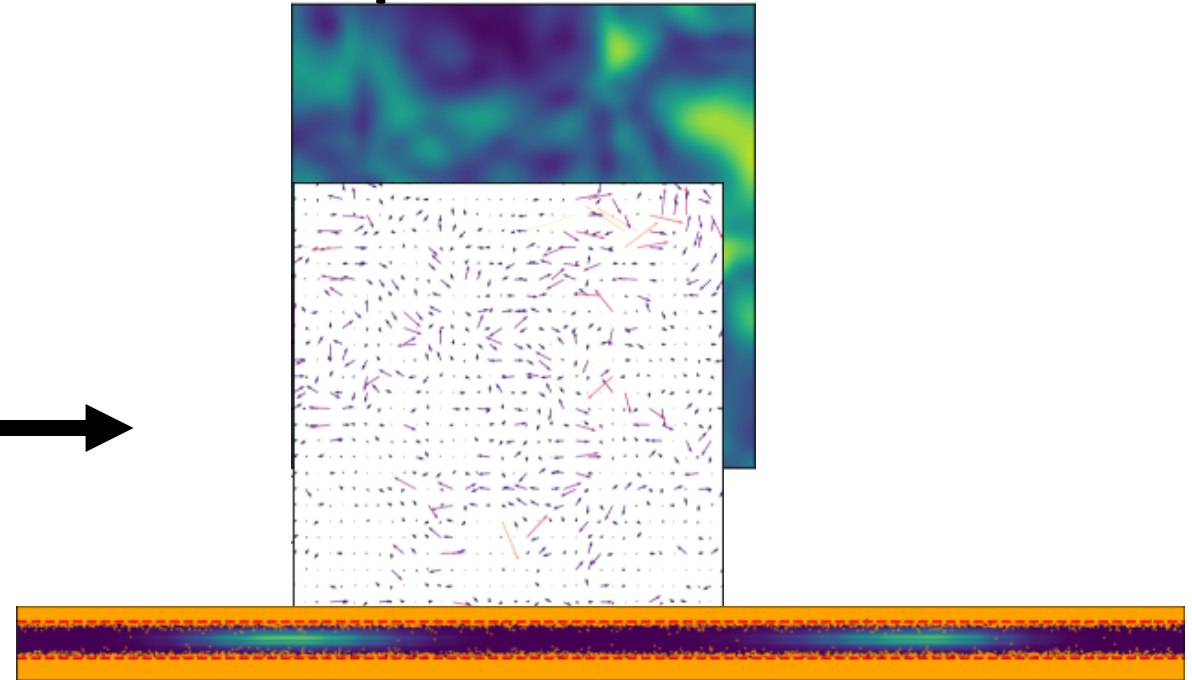
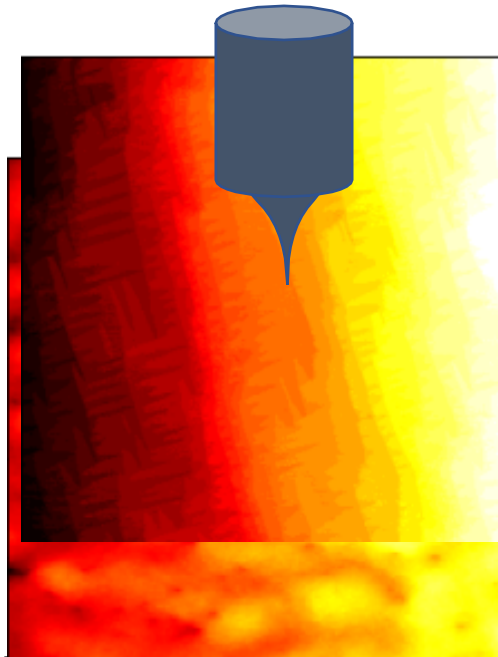


Scanning tunneling microscopy-informed simulation of valley physics in single and double quantum dots



Project: Silver City

PI: Toby Jacobson

Quantum Computing Program Review, July 2022

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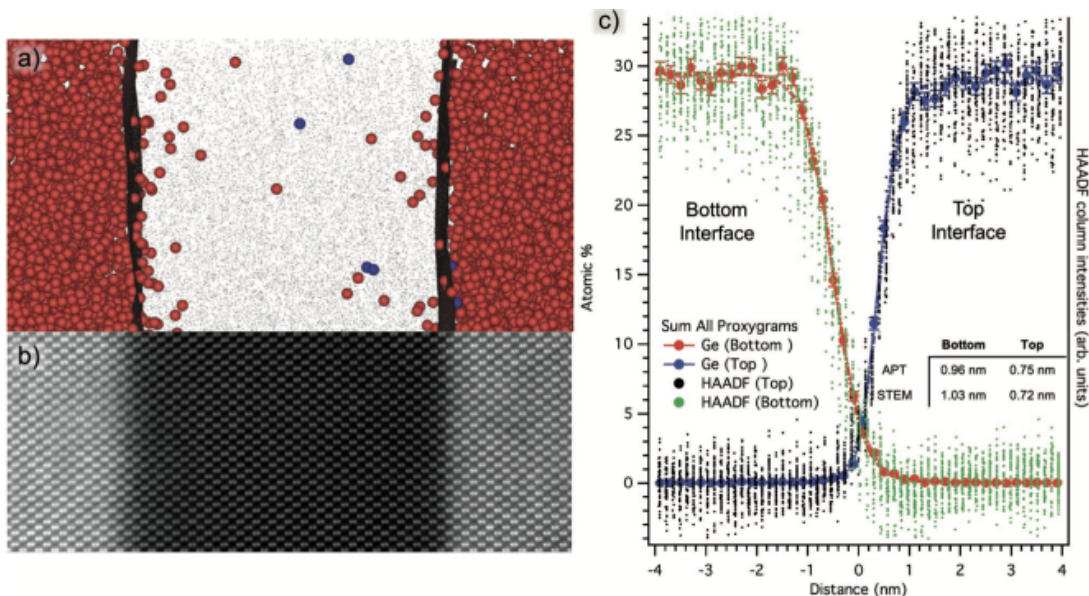
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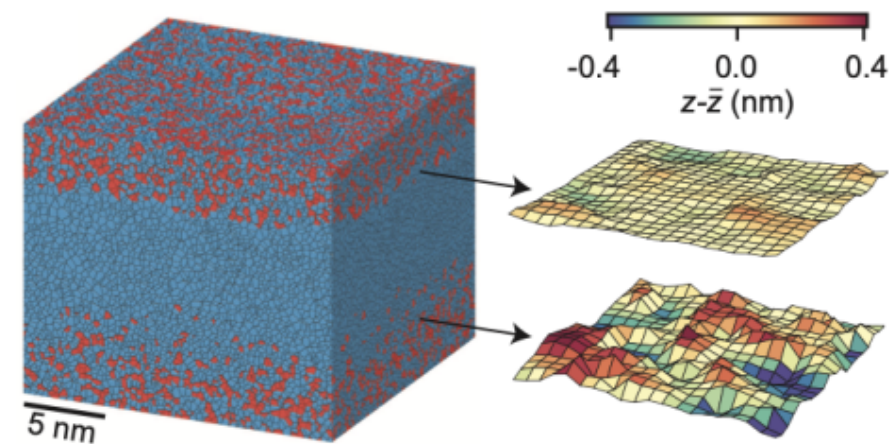


Motivation

There's been much work recently to characterize SiGe/Si/SiGe wells, e.g.



[Dyck, et al. Adv. Mat. Interfaces 4, 1700622 (2017)]



[Wuetz, et al. arXiv:2112.09606]

Q: Given a realistic atomistic-level model for well structure, what are the consequences for quantum dot properties?

STM measurements of interface topography

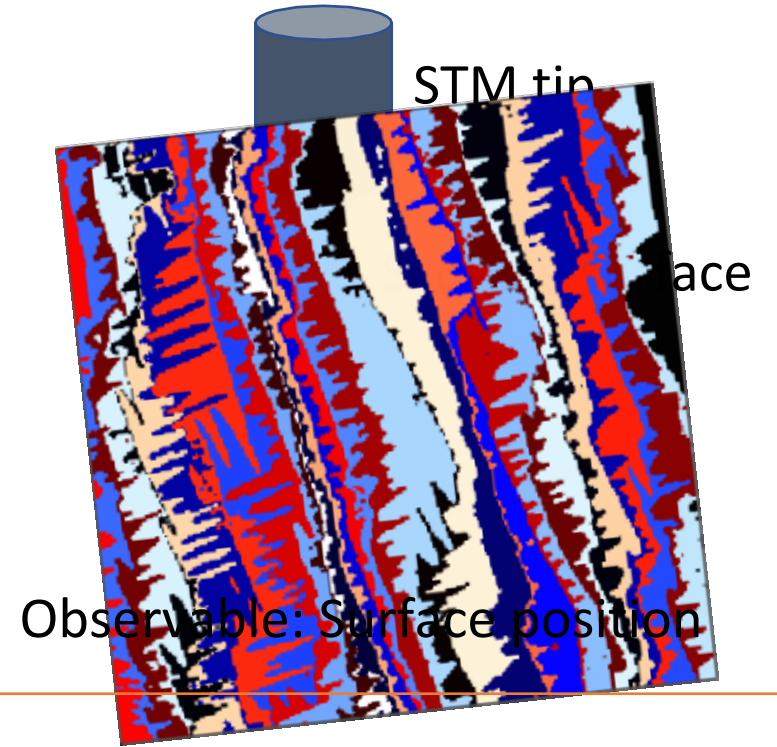
Scanning tunneling microscopy (STM) provides a means to characterize surface properties

Capability developed by Ezra Bussmann and collaborators (Sandia) under previous ARO/LPS-funded project

Procedure:

- Grow SiGe substrate
- Image surface of SiGe substrate w/ STM (lower interface)
- Continue growing Si well (molecular beam epitaxy)
- Image surface of Si well w/ STM (upper interface)

Cartoon of measurement



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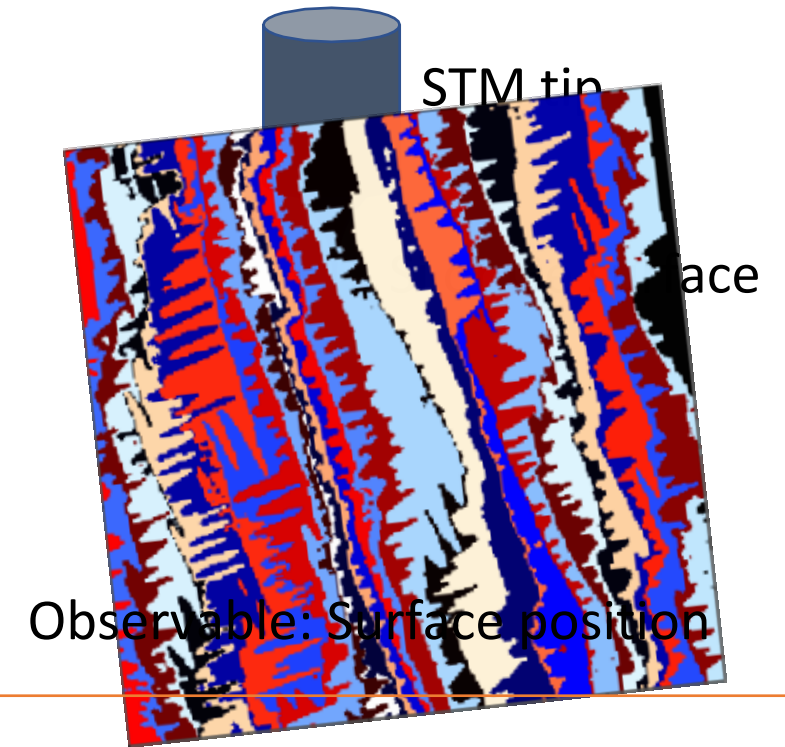
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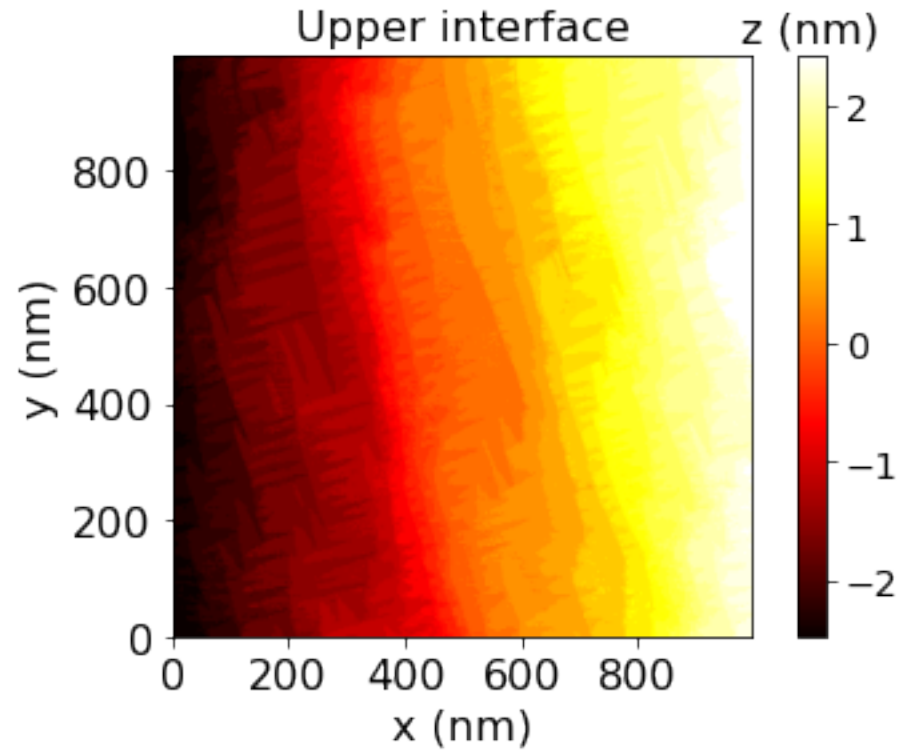
Result:

- Topography of both upper and lower interfaces of the Si well in the same device
- Use to generate an alloy disorder realization over a $1\mu\text{m} \times 1\mu\text{m} \times 15\text{ nm}$ volume, assuming some interdiffusion length

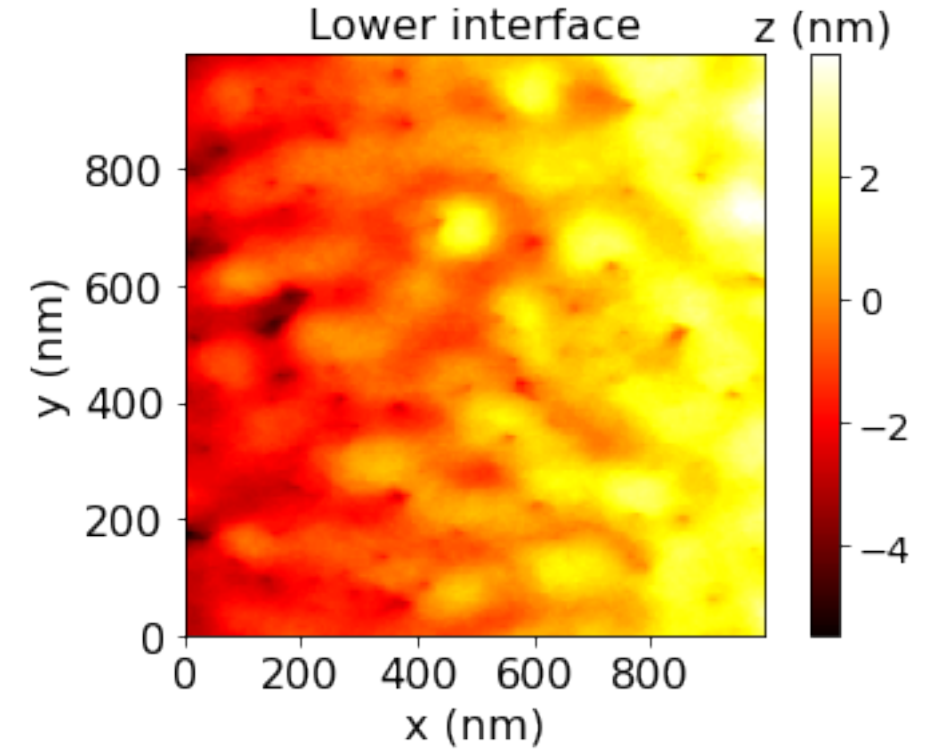
Cartoon of measurement



STM measurements of interface topography

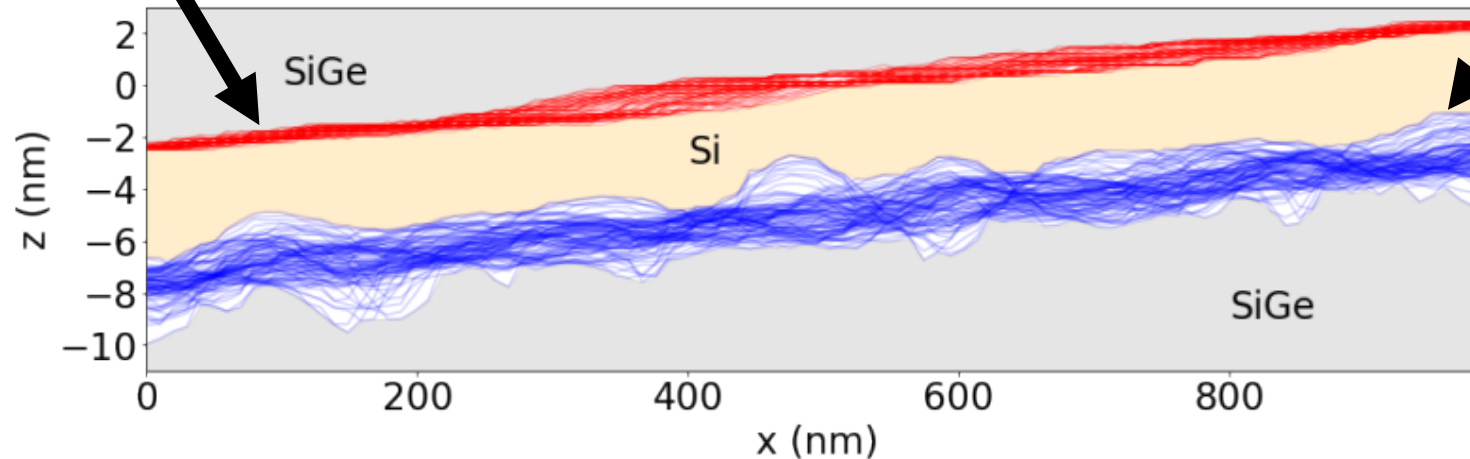
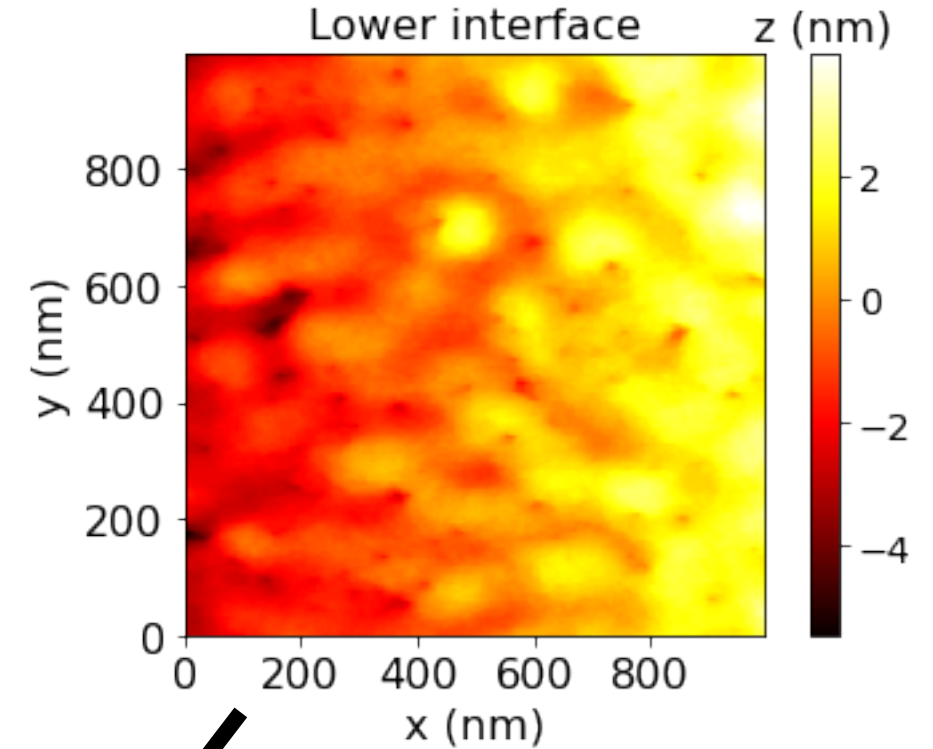
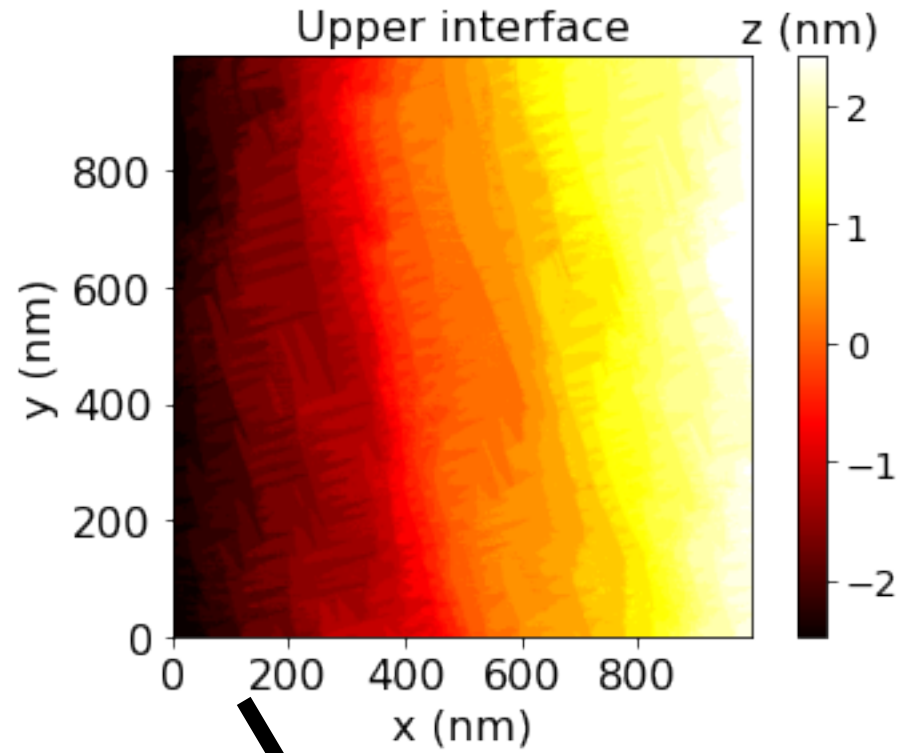


Data courtesy Fabián Peña and Ezra Bussmann (SNL)



STM measurements of interface topography

Data courtesy Fabián Peña and Ezra Bussmann (SNL)

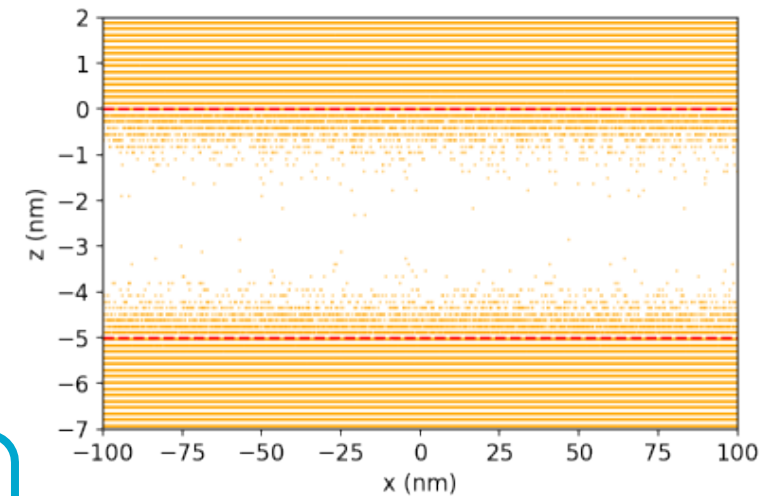
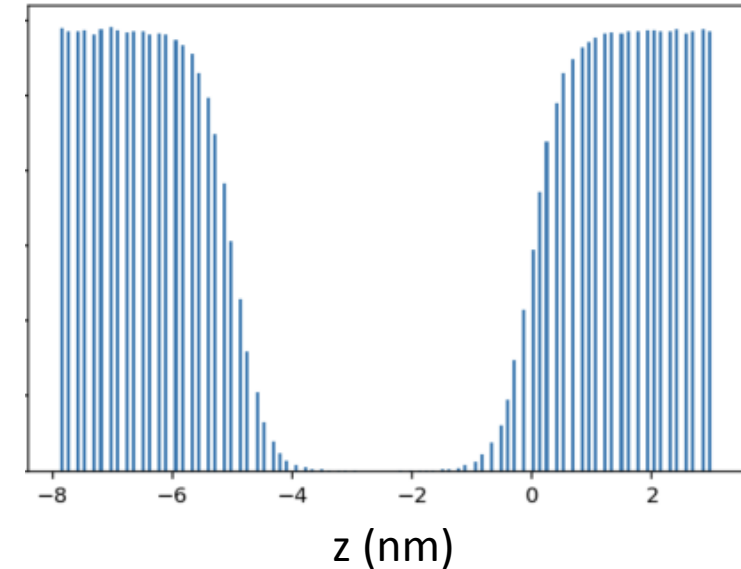


Note: Probably an extreme example of lower interface roughness due to unintended richer Ge level during growth

Atomistic disorder model

- Use STM data or idealized model (“flat interface”) to define the interface locations in (x,y) plane
 - For each interface, define a sigmoidal Ge concentration along z-axis, referenced to local interface location $z_{\text{upper}}(x, y), z_{\text{lower}}(x, y)$
 - Generate an alloy realization on diamond lattice for all atoms ($\sim 10^9$ atoms) and down-select based on simulation volume
 - Including both “step” structure and interdiffusion
-
- In the following, assuming $4\tau=1$ nm based on TEM measurements of 25 nm thin lamellae [Bussmann]
 - Consistent also with other APT and HAADF-STEM measurements:
[Dyck, et al. Adv. Mat. Interfaces 4, 1700622 (2017)]
[Wuetz, et al. arXiv:2112.09606]

Example Ge fraction histogram




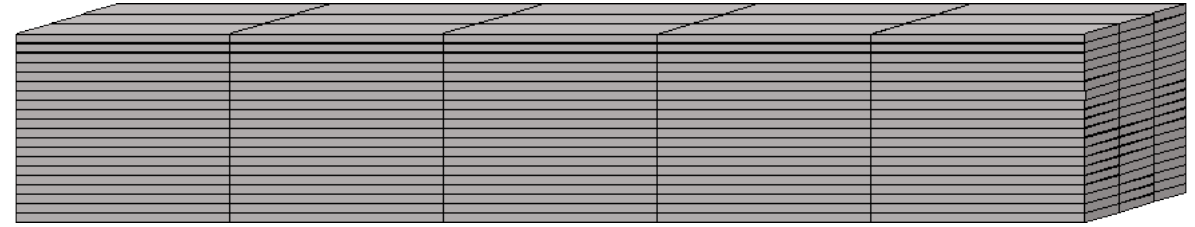
Sigmoidal interdiffusion distribution [Dyck, et al. (2017)]:

$$f(z) \propto 1/(1 + e^{-z/\tau})$$



Model

- ***Mesh-based discretization***
 - Legendre polynomials on a hexahedral mesh (stack of bricks)
 - Taking up to 3rd order here (64-dim modal basis/mesh element)
 - Discontinuous Galerkin formulation in *Laconic* code developed at Sandia 
 - High-throughput simulations feasible: <1 hr/eigensolve on single core



Mesh highly refined along z-axis to resolve wavefunction tails near interfaces



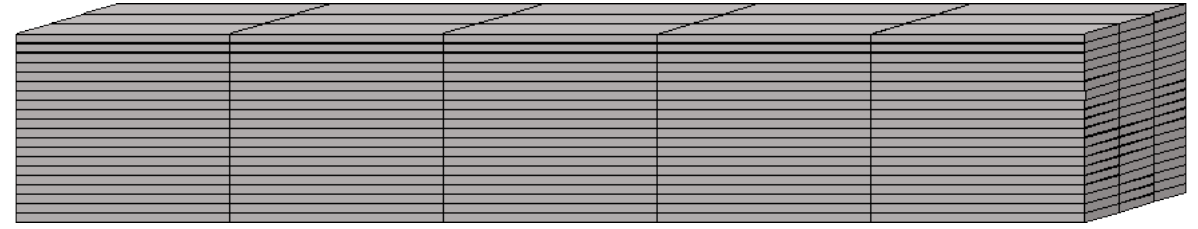
Model

- **Mesh-based discretization**

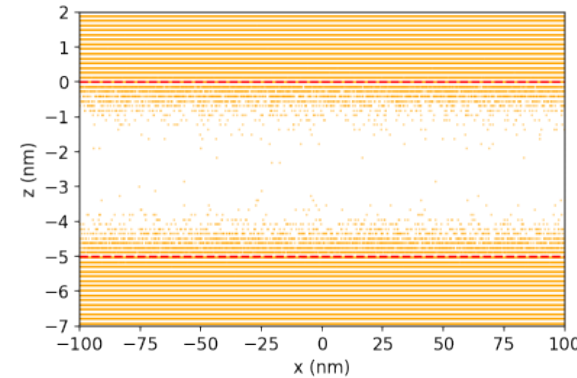
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- High-throughput simulations feasible: <1 hr/eigensolve on single core

- **Valley-orbit coupling:**

- Shindo-Nara equations [Shindo & Nara (1976)] for multi-valley EMT
- Plane wave representation of Bloch functions from DFT
- Atom-by-atom model for valley-orbit coupling: Each Ge atom contributes a **localized** repulsive potential scaled to agree with concentration-dependent Si → SiGe CB offset



Mesh highly refined along z-axis to resolve wavefunction tails near interfaces



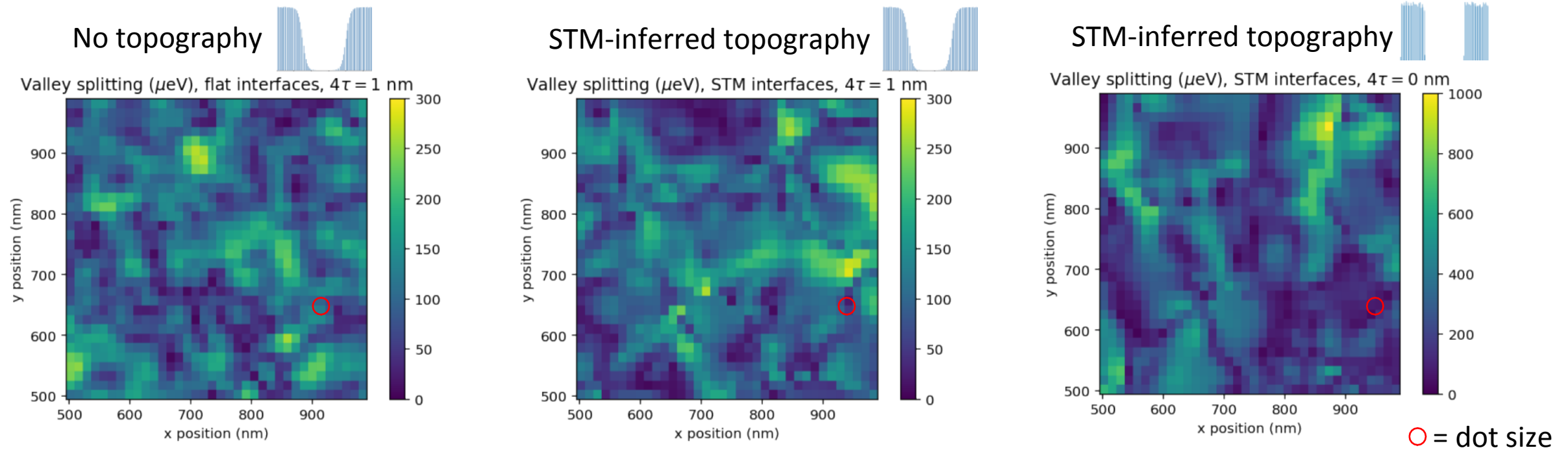
$$V_{\text{Ge},i}(\mathbf{r}) = \alpha_{\text{Ge}} \delta^{(3)}(\mathbf{r} - \mathbf{r}_i), \quad \alpha_{\text{Ge}} = 12 \text{ meV nm}^3$$

$$\langle \mathbf{r} | u_{\pm z} \rangle = \sum_{\mathbf{G}} c_{\mathbf{G}, \pm z} e^{i\mathbf{G} \cdot \mathbf{r}}$$

↙

$$\langle u_{\pm z} | \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) | u_{\pm z} \rangle \approx 1$$
$$\langle u_{\pm z} | \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) | u_{\mp z} \rangle \approx 1$$

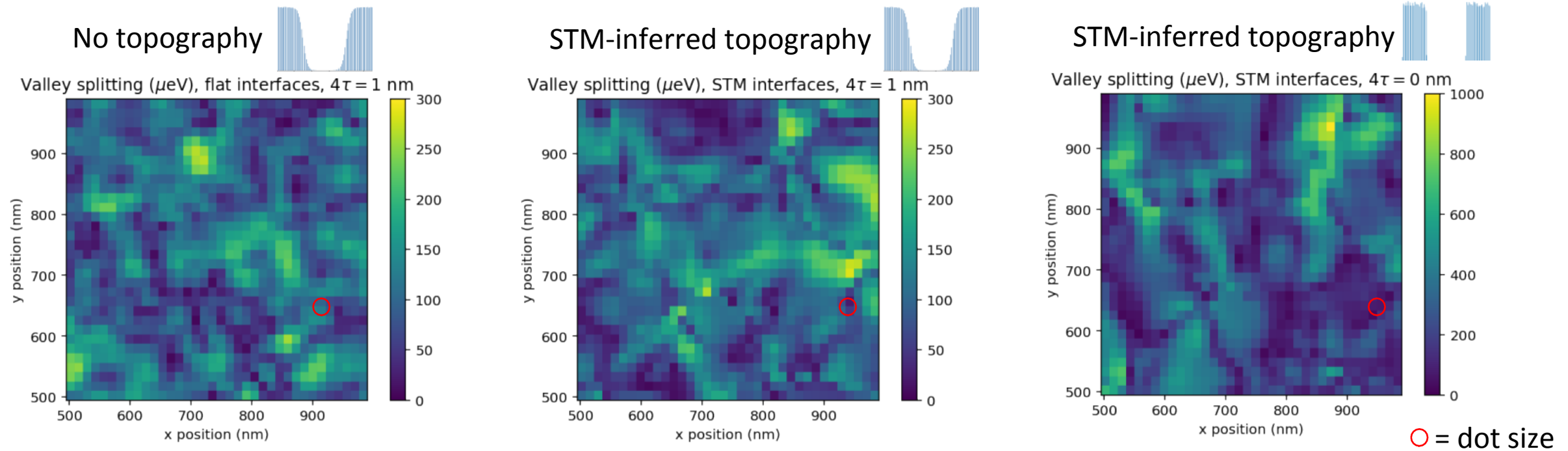
Single quantum dot rastered across the surface: Valley splitting



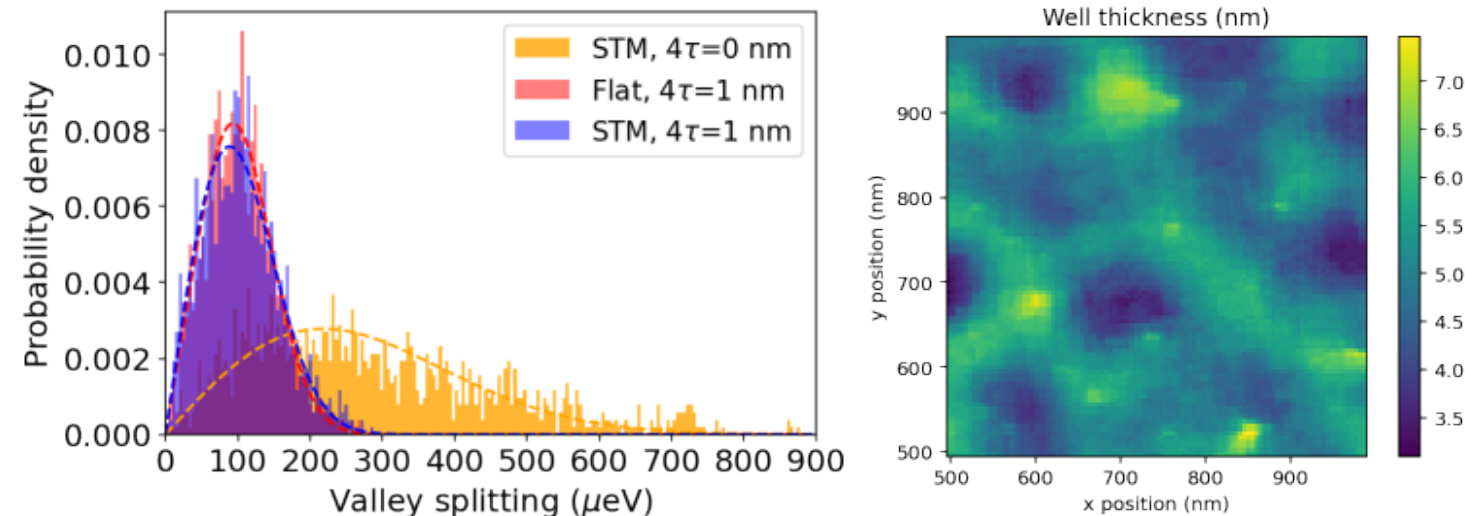
- 1.5 meV symmetric harmonic x-y confinement



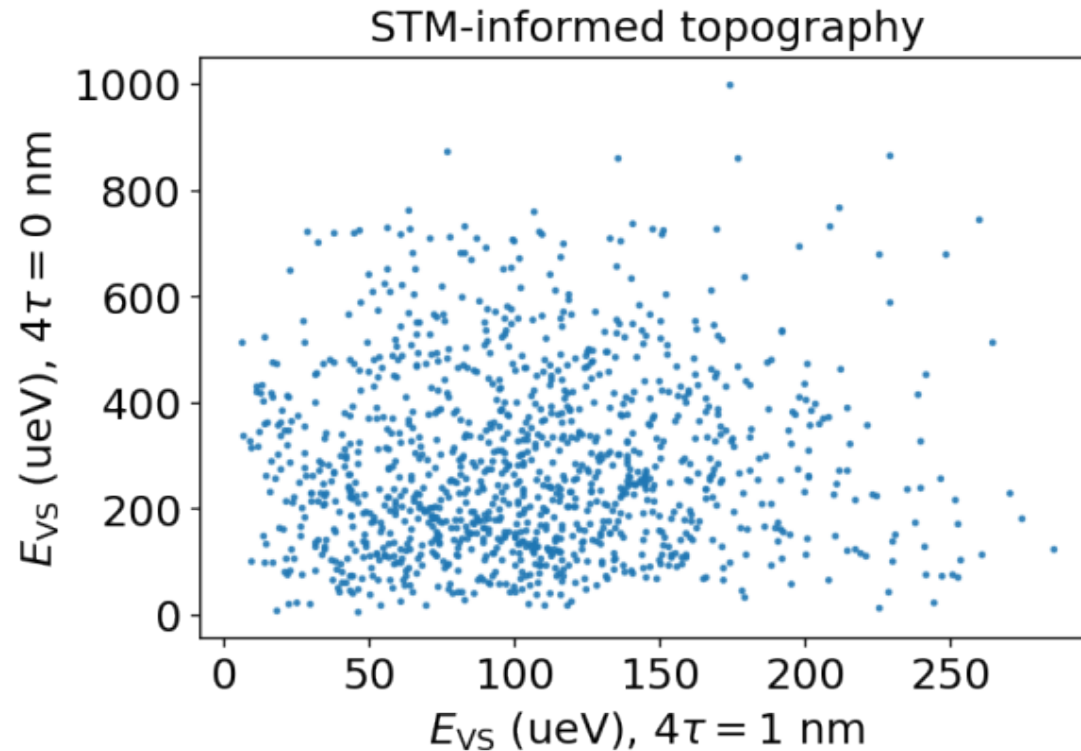
Single quantum dot rastered across the surface: Valley splitting



- 1.5 meV symmetric harmonic x-y confinement
- Decent fit to Rice distribution for valley splitting [Wuetz, et al. (2021)]
- Interdiffusion length 4τ has a strong effect on valley splitting
- Atomic steps/topography appear to be less important than interdiffusion



Single quantum dot rastered across the surface: Valley splitting

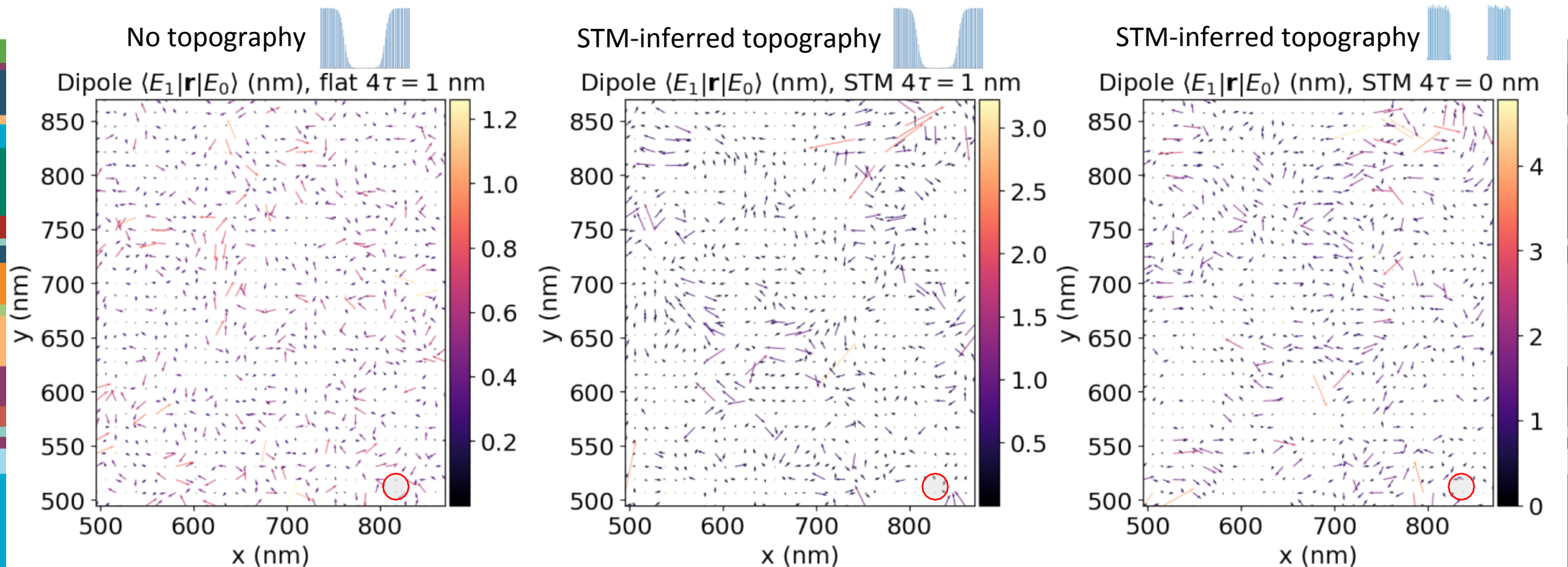


Each point: Same x-y confinement coordinate

- Valley splitting uncorrelated between different interdiffusion lengths 4τ for same interface topography
- Interdiffusion appears to be the dominant driver of valley splitting



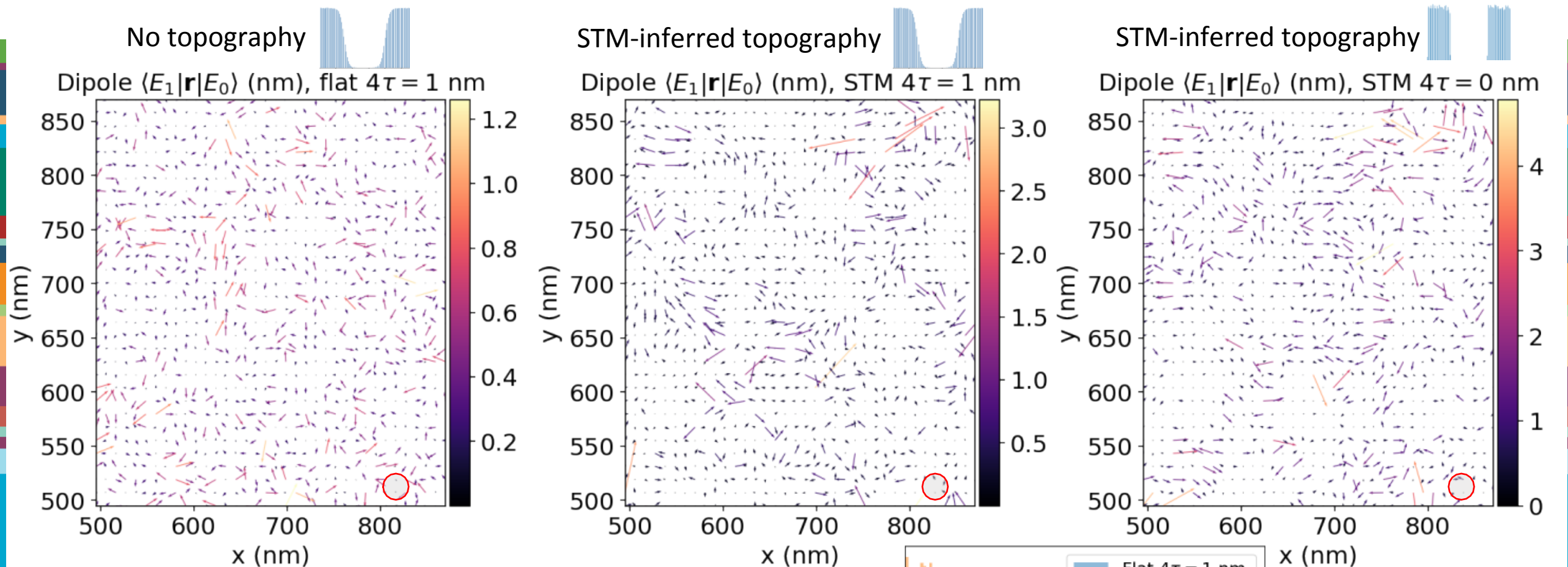
Single quantum dot rastered across the surface: Inter-valley dipole



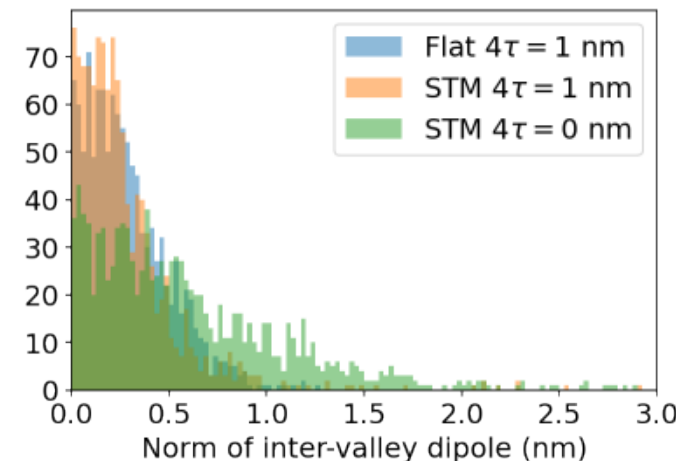
○ = dot size



Single quantum dot rastered across the surface: Inter-valley dipole



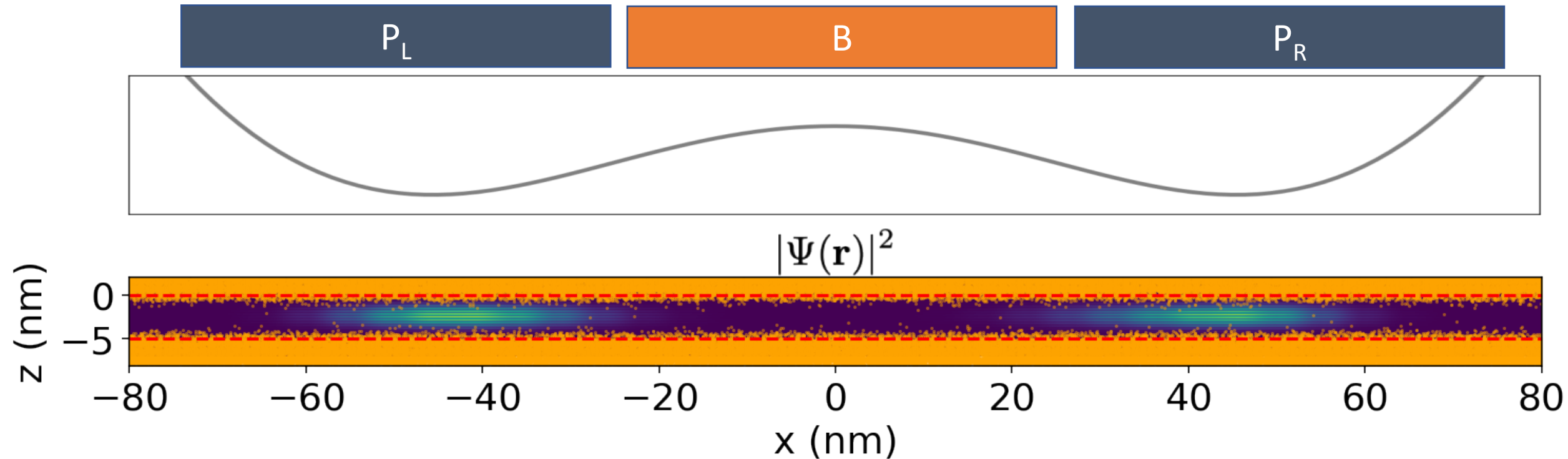
- Inter-valley dipole should relate to spin-orbit coupling properties, e.g. spin-valley coupling [Yang, et al. Nat. Comm. 4, 2069 (2013)]
- Generally larger dipole in presence of crisp interface topography



○ = dot size

Double quantum dot

- Assuming “benchmark” double well potential of [Anderson, et al. arXiv:2203.00082]
- 5 nm well
- Interface topographies: flat or STM-inferred
- Interdiffusion length, 4τ : 0 or 1 nm



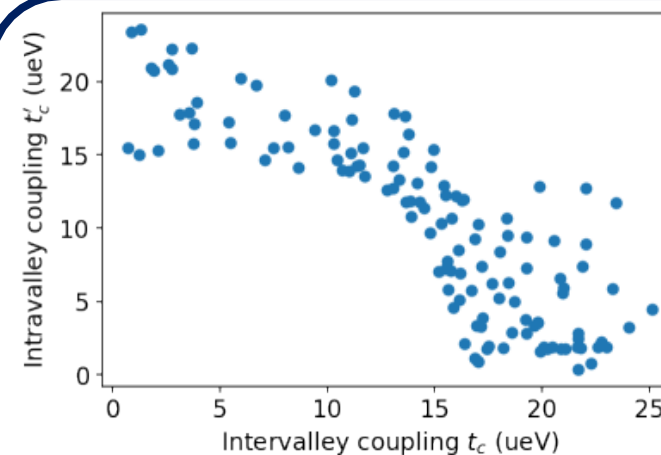
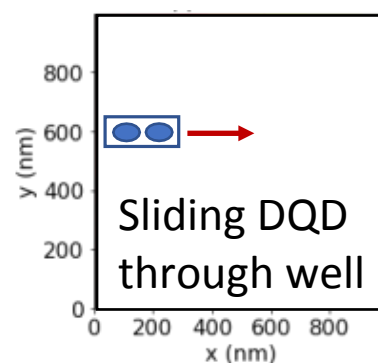
Orange dots: Ge atoms within ± 5 nm of x - z cut plane

Double quantum dot: Flat interfaces

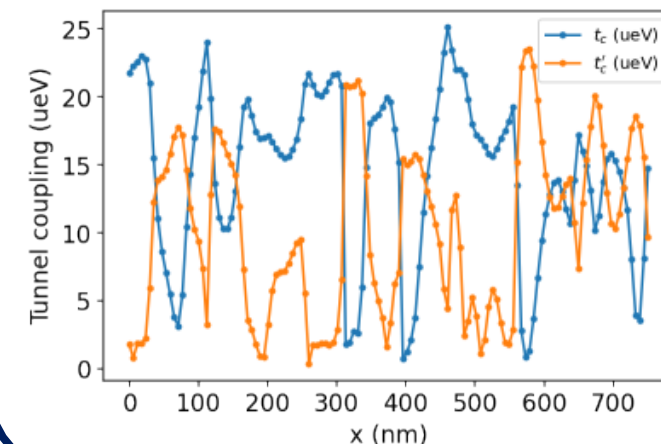
- Compute position dipole and infer four-level Hamiltonian
- Fits nicely to the 4-level double dot model of [Borjans, et al. PRX Quantum 2, 020309 (2021)]

$$H = \begin{pmatrix} \frac{\epsilon}{2} & 0 & t_c & t'_c \\ 0 & \frac{\epsilon}{2} + E_{VS,L} & t'_c & t_c \\ t_c^* & t'_c^* & -\frac{\epsilon}{2} & 0 \\ t'_c^* & t_c^* & 0 & -\frac{\epsilon}{2} + E_{VS,R} \end{pmatrix}$$

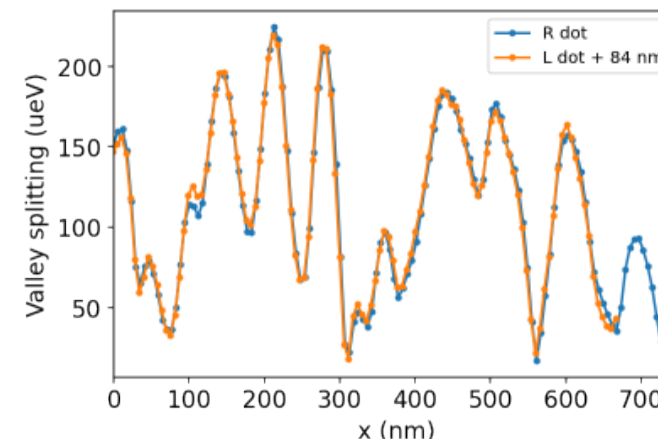
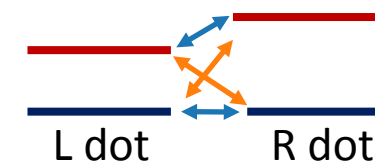
- Extract:
 - Valley splittings $E_{VS,L}, E_{VS,R}$
 - Intra-/inter-valley tunnel couplings t_c, t'_c
 - Detuning offset ϵ_0 due to alloy disorder



Intra-/inter-valley tunnel couplings are anticorrelated, as expected



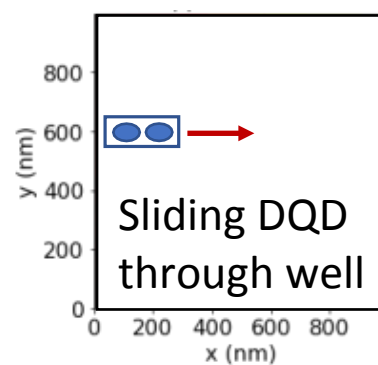
t_c = intravalley
 t'_c = intervalley



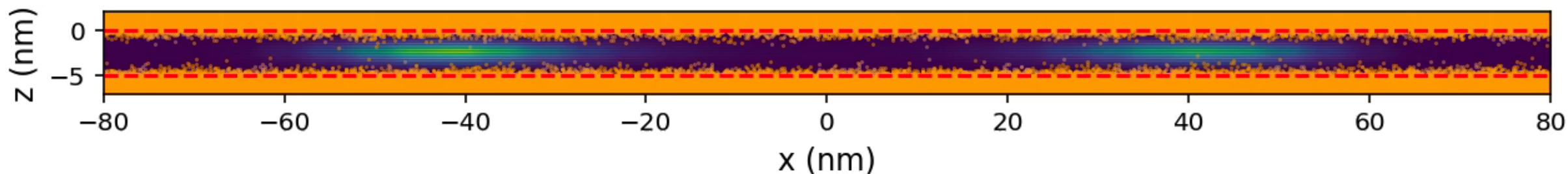
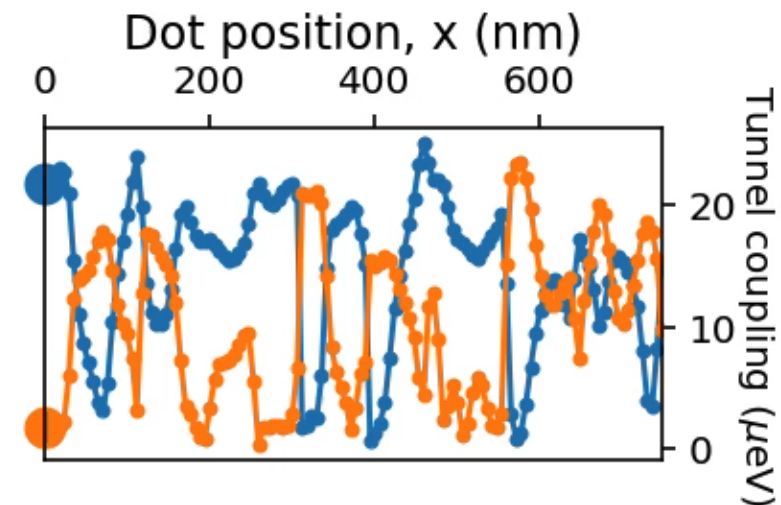
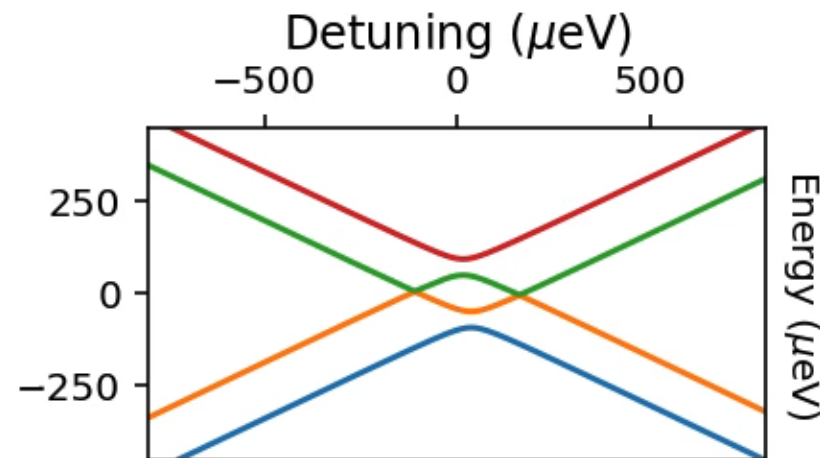
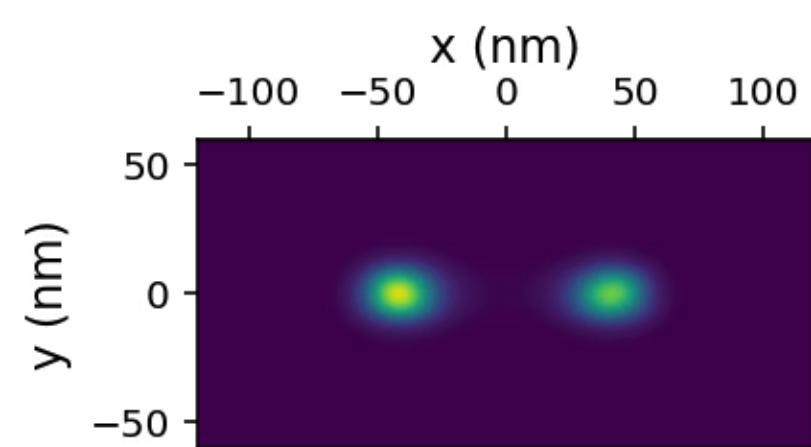
E_{VS} consistent with single-dot behavior, as expected



Double quantum dot: Flat interfaces



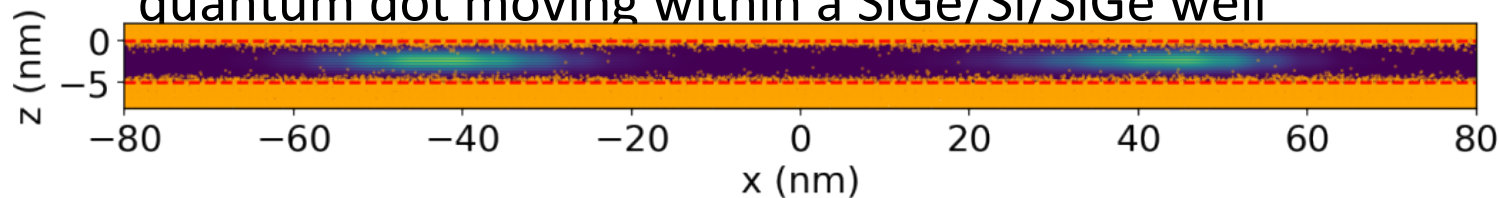
t_c = intra-valley tunnel coupling
 t'_c = inter-valley tunnel coupling



For STM-informed interfaces, alloy disorder-induced detuning variation is much more dramatic (approximately

Summary

- We've developed machinery for simulating large-scale atomistic alloy disorder within multi-valley effective mass theory based on information about interface topography
 - Able to use data from e.g. APT, STM characterization of atomic-scale disorder
- For significant interdiffusion between Si and SiGe layers, **valley splitting** and **inter-valley dipole** statistics primarily governed by diffusion rather than interface topography
- We've characterized inter- and intra-valley tunnel coupling variation for a double quantum dot moving within a SiGe/Si/SiGe well



Acknowledgements:

- Ezra Bussmann (Sandia)
- Fabián Peña (formerly at Sandia)
- Mitchell Brickson (UNM)
- Andrew Baczewski (Sandia)

