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Feedback-based quantum optimization

Alicia B. Magann, Kenneth M. Rudinger, Matthew D. Grace, Mohan Sarovar

Sandia National Laboratories

arXiv:2103.08619 & arXiv:2108.05945

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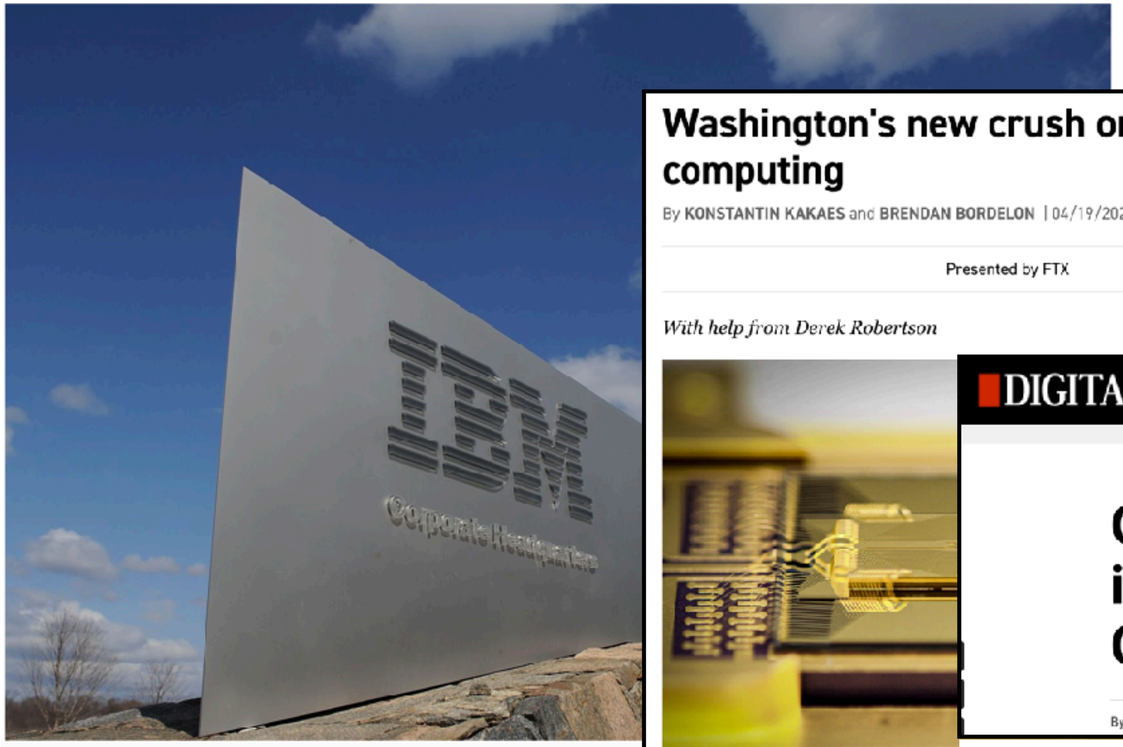
Quantum computing is coming

The Decade of Quantum Computing Is Upon Us, IBM Exec Says

With business uses poised to accelerate, CIOs need to start tinkering with platforms, forming work groups and looking for problems the technology could address, says IBM Research's Dario Gil



https://www

Washington's new crush on quantum computing

By KONSTANTIN KAKAES and BRENDAN BORDELON | 04/19/2022 04:00 PM EDT

Presented by FTX

With help from Derek Robertson

https://www.politico.com/newsletters/digital-future-daily/2022/04/19/washingtons-new-crush-on-quantum-computing-00026369

DIGITAL JOURNAL WORLD TECH & SCIENCE SOCIAL MEDIA BUSINESS ENTERTAINMENT LIFE SPORTS

Quantum Computing Technologies Market is Booming Worldwide with Airbus Group, Cambridge Quantum Computing, IBM, etc.

By Newsmantraa Published April 25, 2022

https://www.digitaljournal.com/pr/quantum-computing-technologies-market-is-booming-worldwide-with-airbus-group-cambridge-quantum-computing-ibm-etc

The Digital Transformation of Business | Digital Inclusion | Education, Skills and Learning

Why education must take a quantum leap

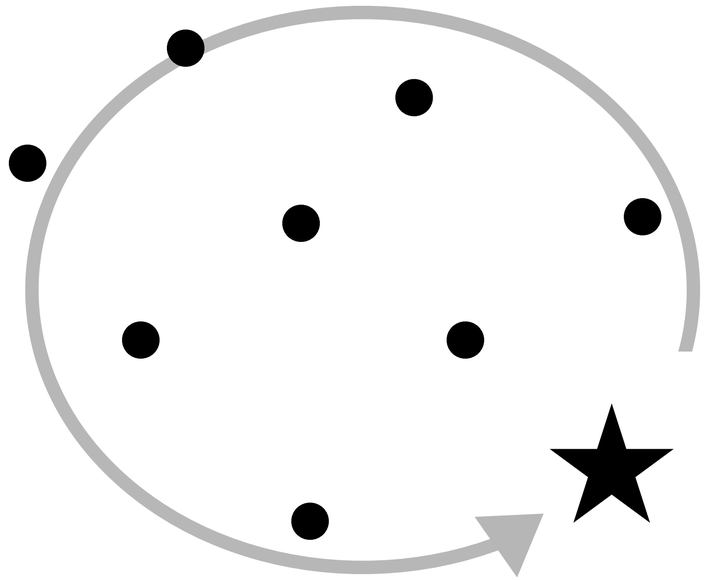
To truly embrace the full potential of quantum computing, quantum education and workforce development will be crucial. Image: REUTERS/Steve Marcus

https://www.weforum.org/agenda/2022/04/why-education-must-take-a-quantum-leap/

Quantum computing is coming

Combinatorial optimization

- Routing, scheduling, logistics
  - Example: traveling salesman



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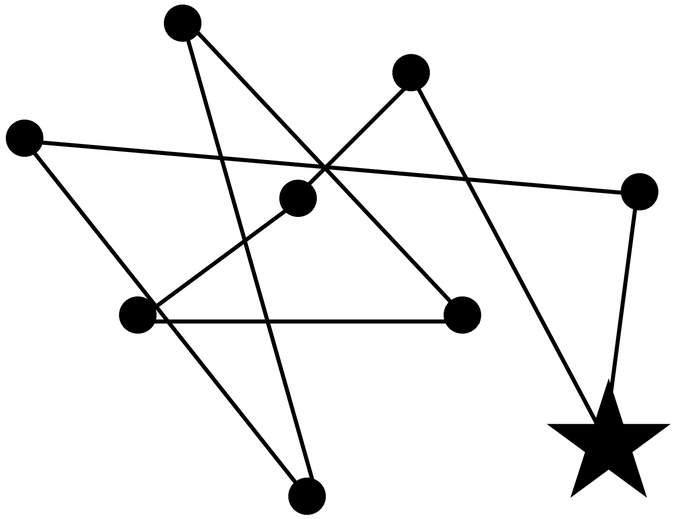

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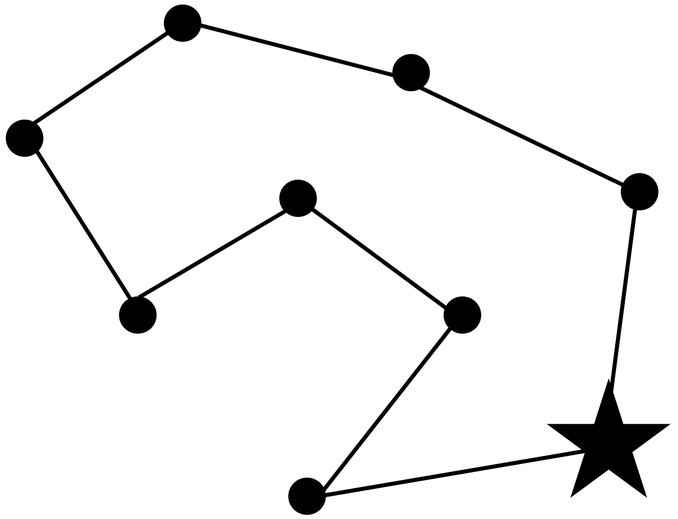

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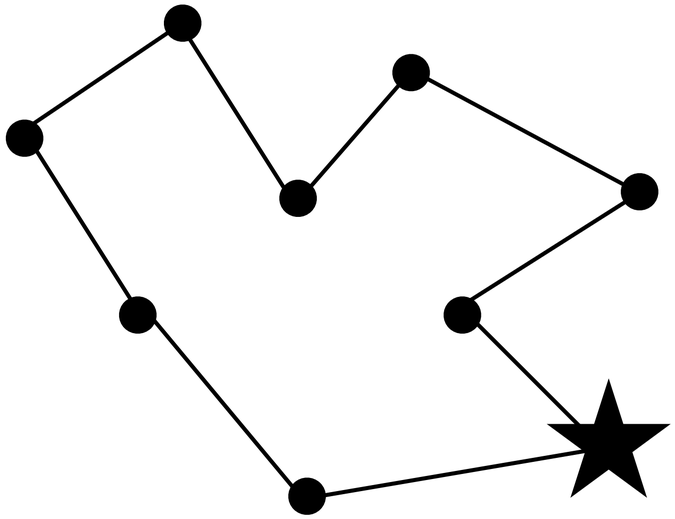
Combinatorial explosion



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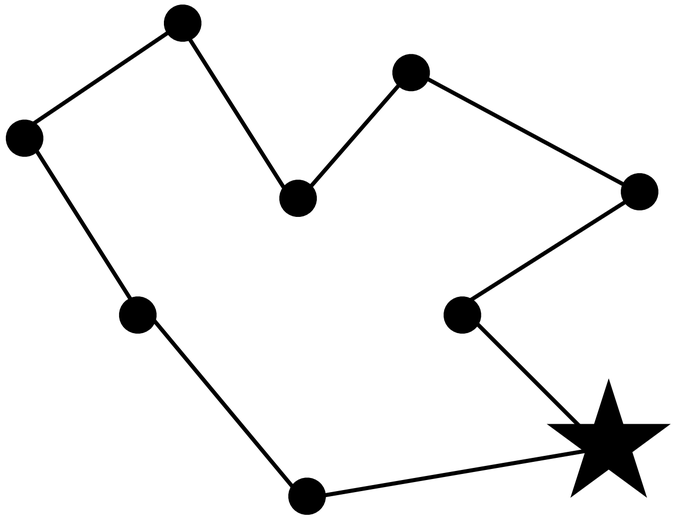


Can quantum resources help?

- Adiabatic/annealing strategies

T Kadowaki, H Nishimori, Quantum annealing in the transverse Ising model. *Physical Review E*, *58*(5), p.5355 (1998)

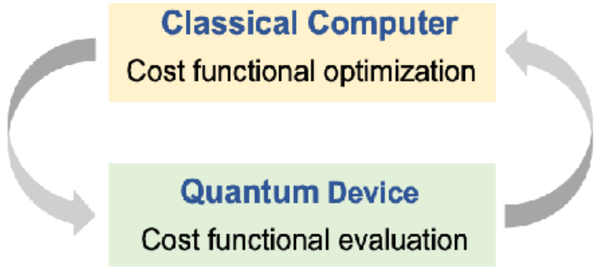
E Farhi, et. al., A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem. *Science*, *292*(5516), pp.472-475 (2001)

Can require considerable runtime

- More recently: Quantum Approximate Optimization Algorithm (QAOA)

E Farhi, J Goldstone, S Gutmann, A quantum approximate optimization algorithm, *arXiv:1411.4028* (2014)

Hybrid quantum-classical algorithm



Use classical processors in tandem with current error-prone quantum processors

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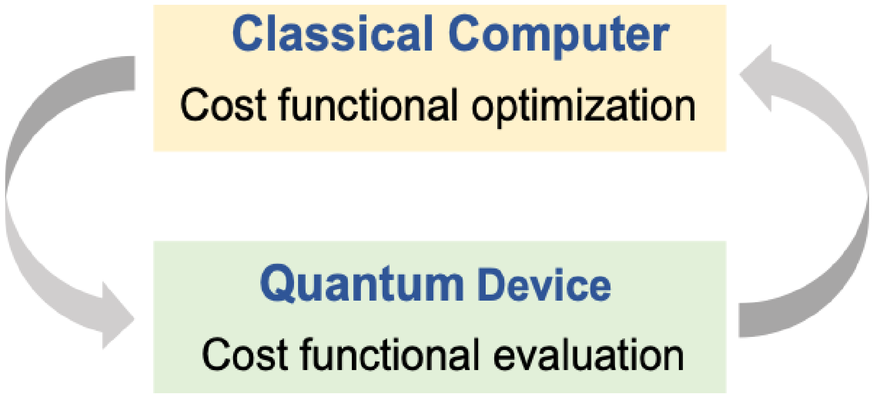


A Lucas, Ising formulations of many NP problems, *Frontiers in Physics* 2 (2014)

Encode combinatorial optimization problem

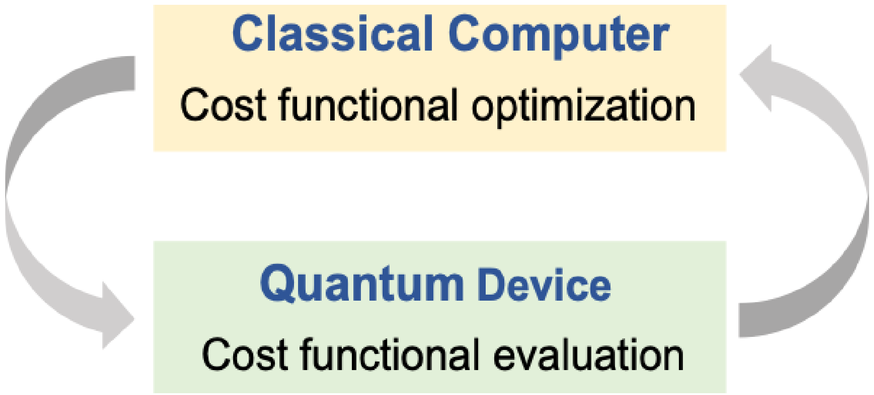
$$\min_{\vec{\theta}} J(\vec{\theta})$$

Cost function quantifies how well we solve combinatorial optimization problem under consideration

- Can be estimated by first performing quantum computation, and then measuring qubits

Quantum computing is coming

Hybrid quantum-classical algorithm



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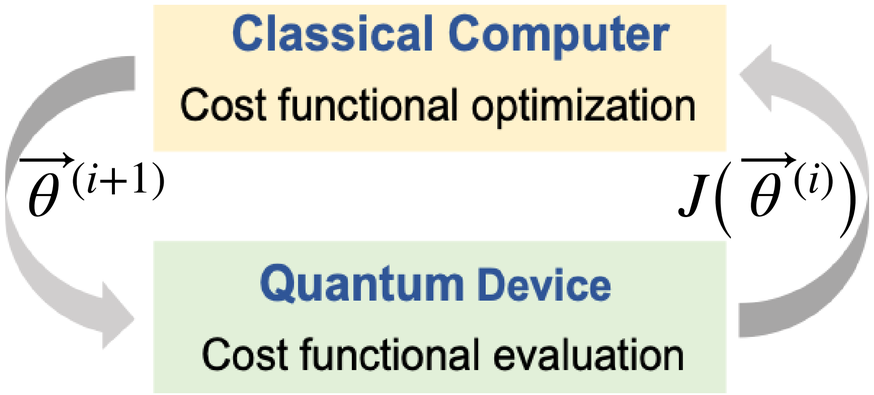

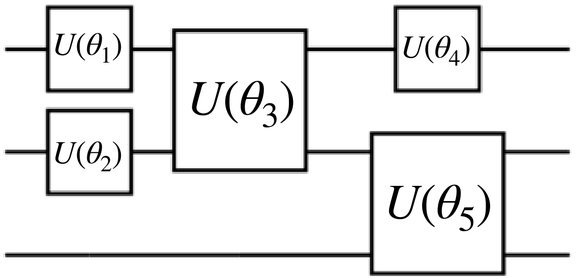

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Use classical computer to iteratively search for the values of $\theta_1, \theta_2, \ldots$ that minimize $J(\vec{\theta})$

Quantum computing is coming

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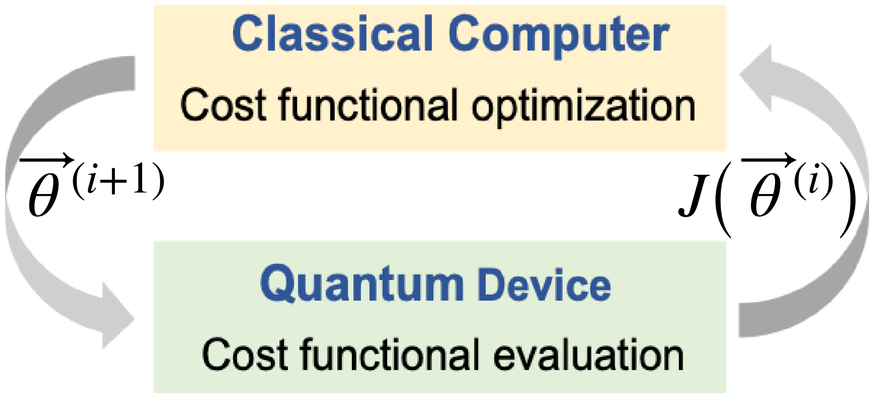

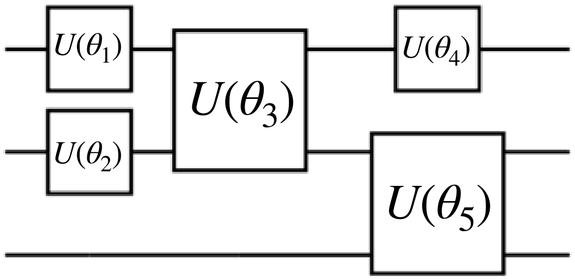

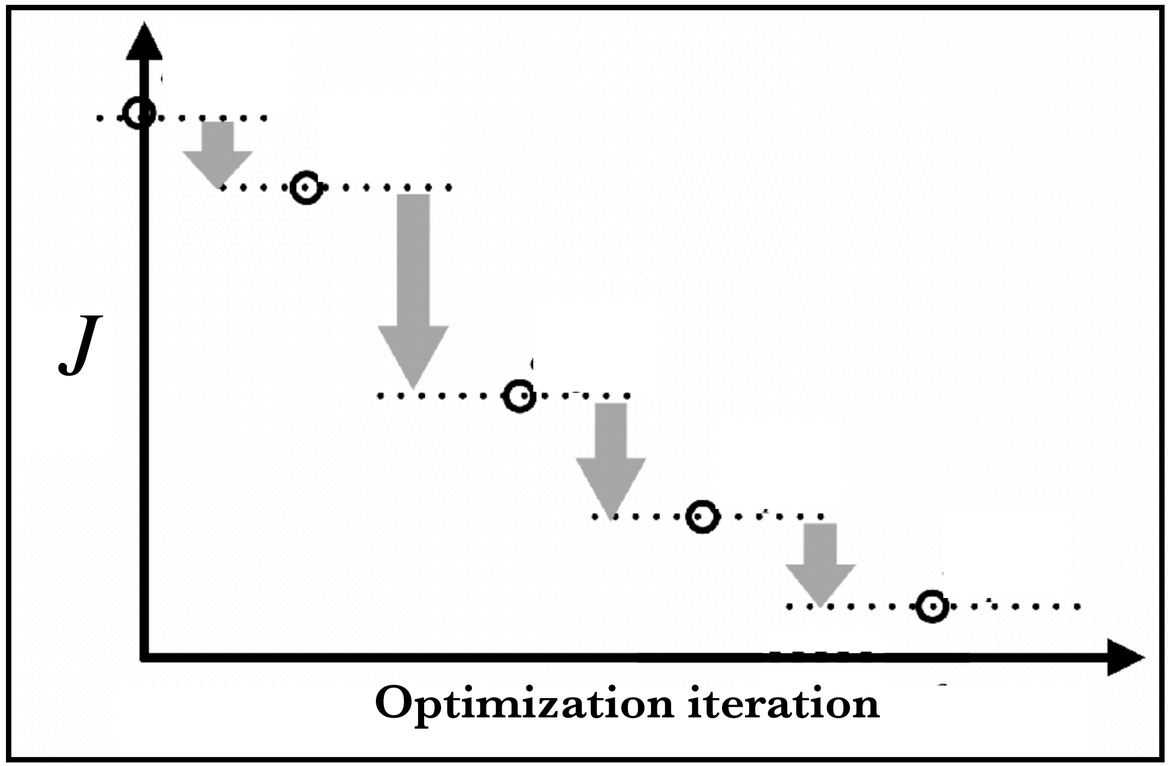

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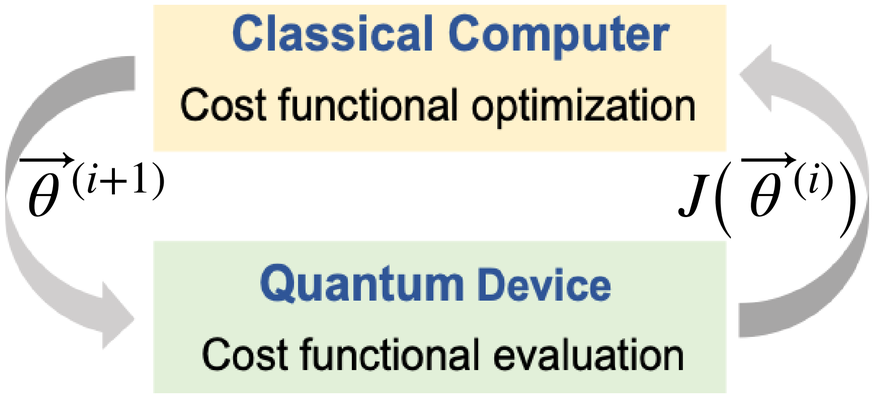

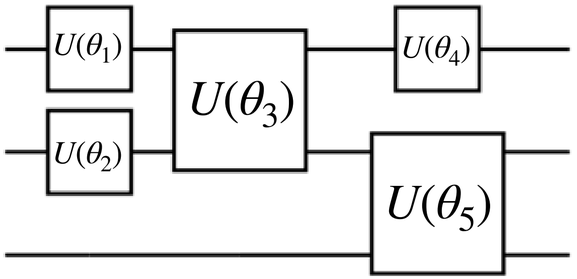

Bottleneck:

Searching for the optimal values of parameters $\theta_1, \theta_2, \ldots$

This work:

Incorporate the use of measurement-based feedback to remove this bottleneck

AB Magann, KM Rudinger, MD Grace, M Sarovar, Feedback-based quantum optimization, *arXiv:2103.08619* (2021)

AB Magann, KM Rudinger, MD Grace, M Sarovar, Lyapunov control-inspired strategies for quantum combinatorial optimization, *arXiv:2108.05945* (2021)

- Feedback-based ALgorithm for Quantum OptimizatioN, or "FALQON"

New framework for hybrid quantum-classical algorithms, inspired by quantum control theory

AB Magann, C Arenz, MD Grace, TS Ho, RL Kosut, JR McClean, HA Rabitz, M Sarovar "From pulses to circuits and back again: A quantum optimal control perspective on variational quantum algorithms." *PRX Quantum* 2 (2020)

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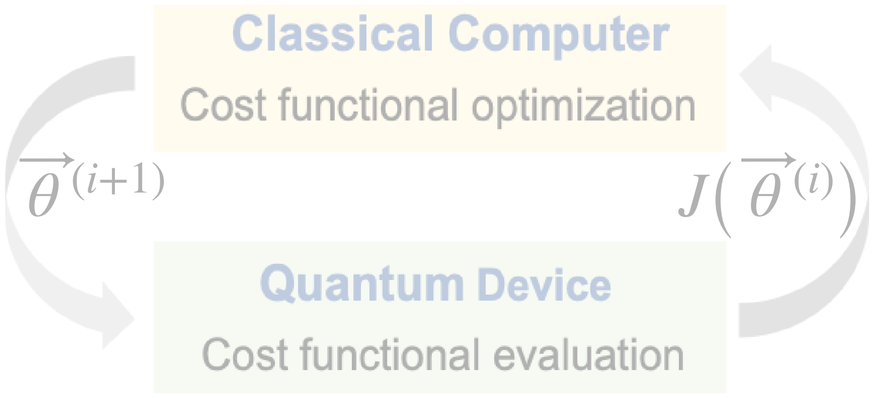

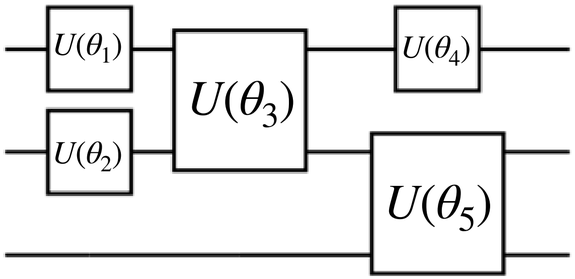

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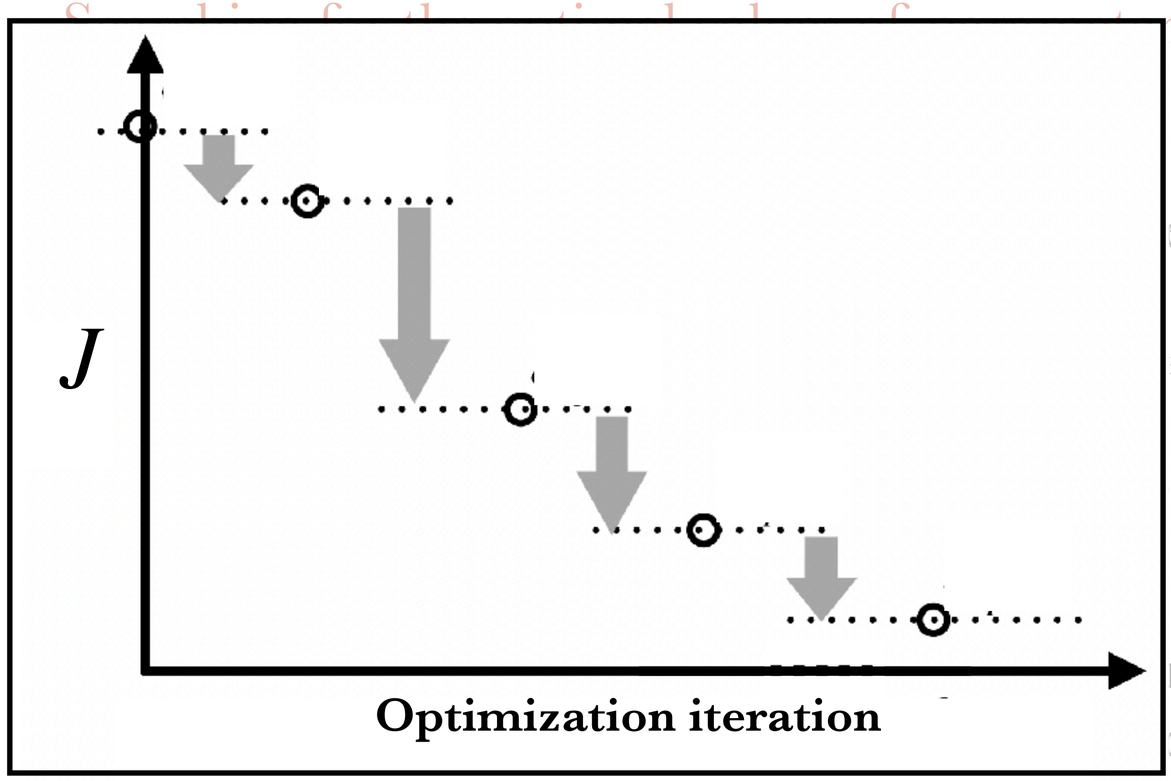

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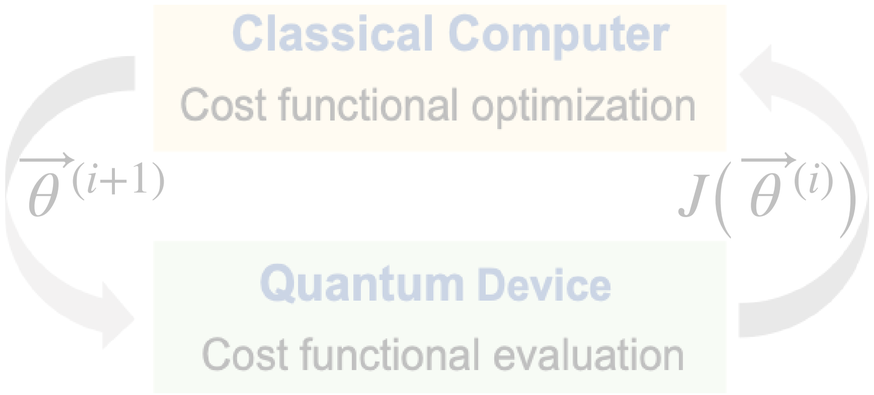

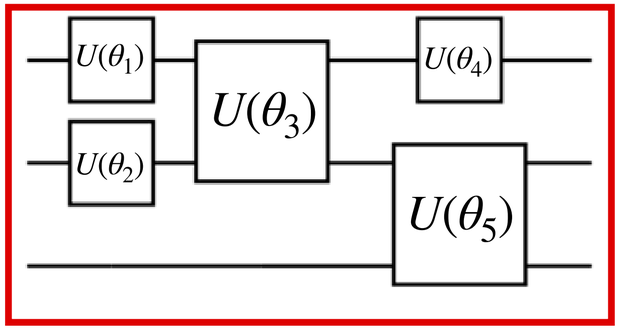

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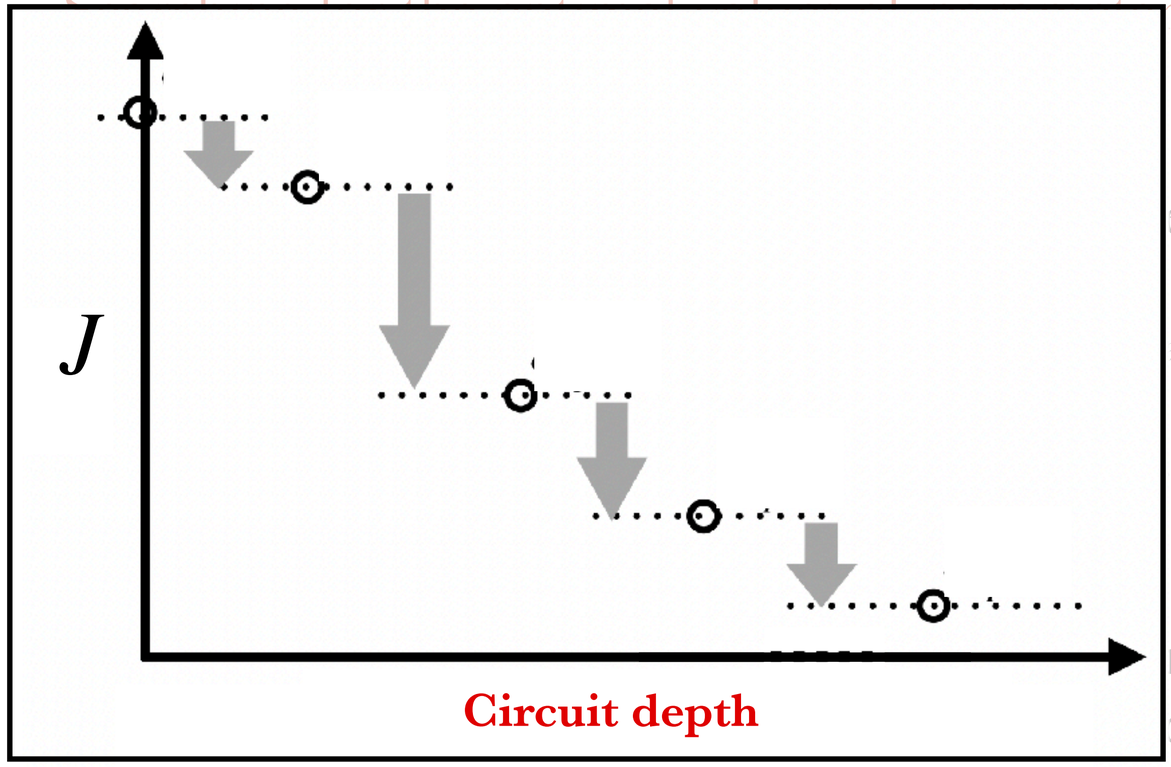

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Feedback-based quantum optimization

<u>Continuous-time control perspective</u>

Consider a quantum system whose dynamics are governed by

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle \qquad \text{with} \qquad H(t) = H_p + H_d\beta(t)$$

Scalar, time-dependent control function

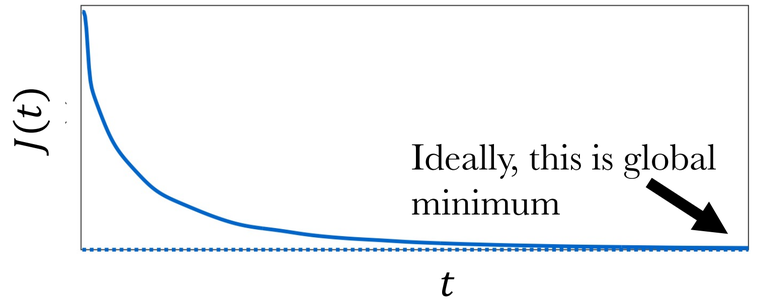
"problem" Hamiltonian

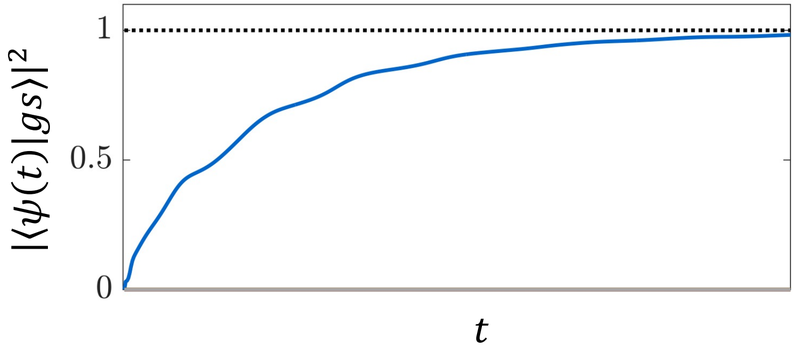
"driver" Hamiltonian

Objective: design $\beta(t)$ to drive the system to a state that minimizes

$$J(t) = \langle\psi(t)|H_p|\psi(t)\rangle$$

Illustration of the principle





Quantum Lyapunov control principle:
Design $\beta(t)$ to ensure $J(t)$ is strictly decreasing over time by enforcing $\frac{d}{dt}J(t) \leq 0$ at all times

AB Magann, KM Rudinger, MD Grace, M Sarovar, Feedback-based quantum optimization, *arXiv:2103.08619* (2021)
AB Magann, KM Rudinger, MD Grace, M Sarovar, Lyapunov control-inspired strategies for quantum combinatorial optimization, *arXiv:2108.05945* (2021)

Feedback-based quantum optimization

$$\frac{d}{dt}J(t) \leq 0$$

The left-hand side is given by,

$$\frac{d}{dt}J(t) = \frac{d}{dt}\langle\psi(t)\,|\,H_p\,|\,\psi(t)\rangle = i\langle\psi(t)\,|\,[H_d, H_p]\,|\,\psi(t)\rangle\beta(t)$$

Define $A(t) \equiv \langle\psi(t)\,|\,i[H_d, H_p]\,|\,\psi(t)\rangle$

Then, $\frac{d}{dt}J(t) = A(t)\beta(t)$

We choose the following control law

$$\beta(t) = -A(t) \quad \Rightarrow \quad \frac{d}{dt}J(t) = -\big(A(t)\big)^2$$

Implementation using alternating operator QAOA ansatz

One "layer" given by

$$|\psi_k\rangle = e^{-i\beta_k H_d \Delta t} e^{-iH_p \Delta t}\,|\psi_{k-1}\rangle = U_d(\beta_k)U_p\,|\psi_{k-1}\rangle$$

Such that full circuit has the form

$$U_d(\beta_\ell)U_p \ldots U_d(\beta_3)U_p U_d(\beta_2)U_p U_d(\beta_1)U_p$$

Set β values per feedback law $\beta_k = -A_{k-1}$

"Feedback-based quantum optimization"

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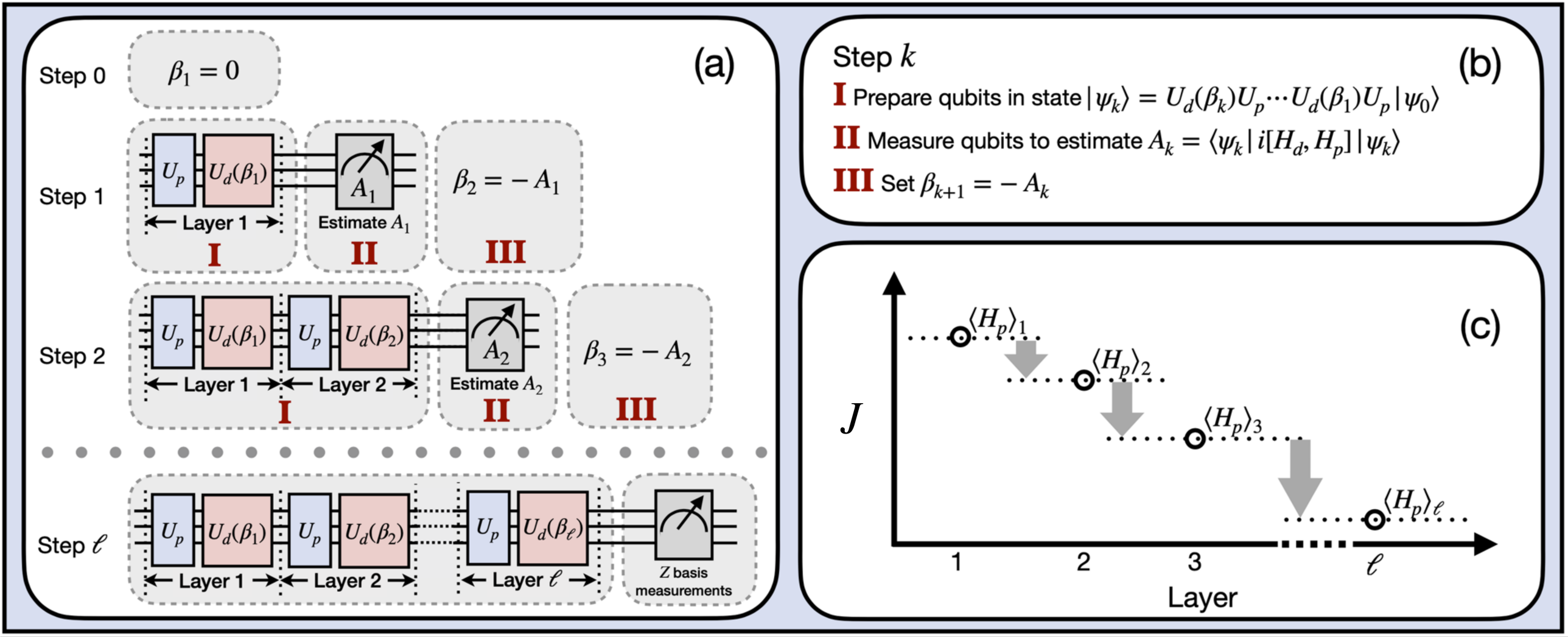

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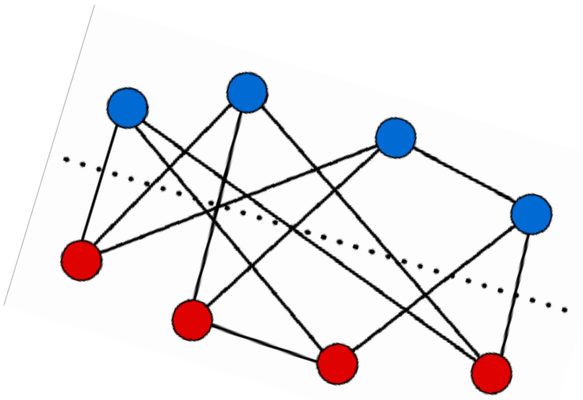
Applications to MaxCut



The aim of MaxCut is to identify bipartition of a graph that maximizes the number of edges crossing the two sets.

QAOA sampling cost: $N_{samp} \geq mq(1+2\ell) = O(mq\ell)$

This work: $N_{samp} \leq m(1+2\ell(d+1)) = O(md\ell)$

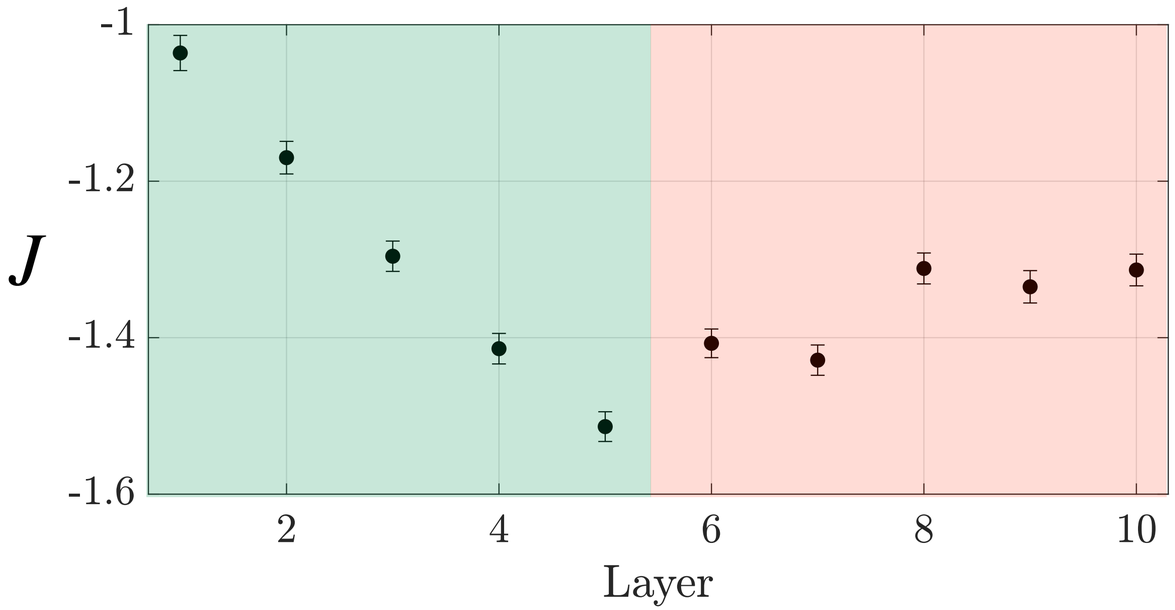
m: # samples to evaluate single expectation value

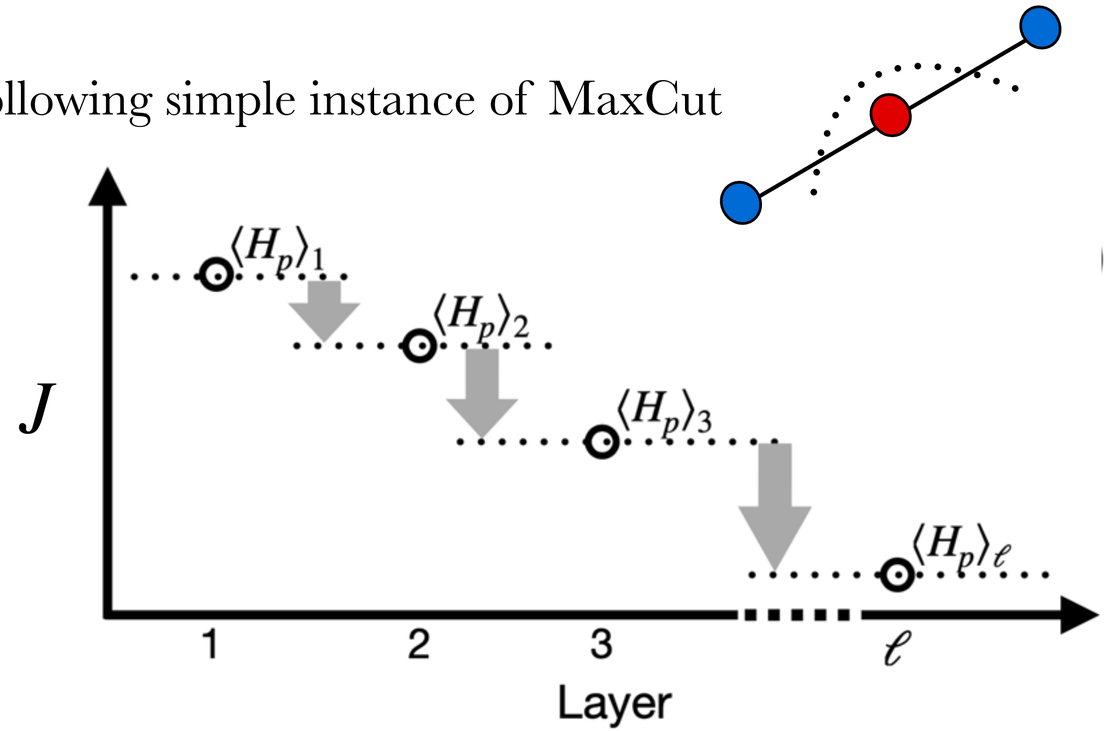
q: # classical optimization iterations

ℓ: # layers

d: degree of graph

Performed hardware demonstration on ibmq_manila for the following simple instance of MaxCut





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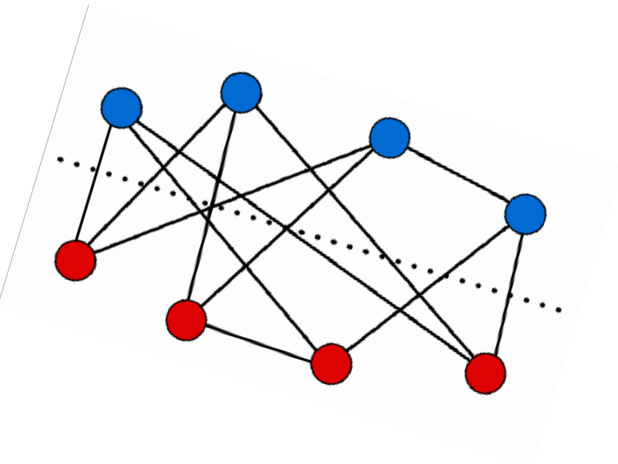
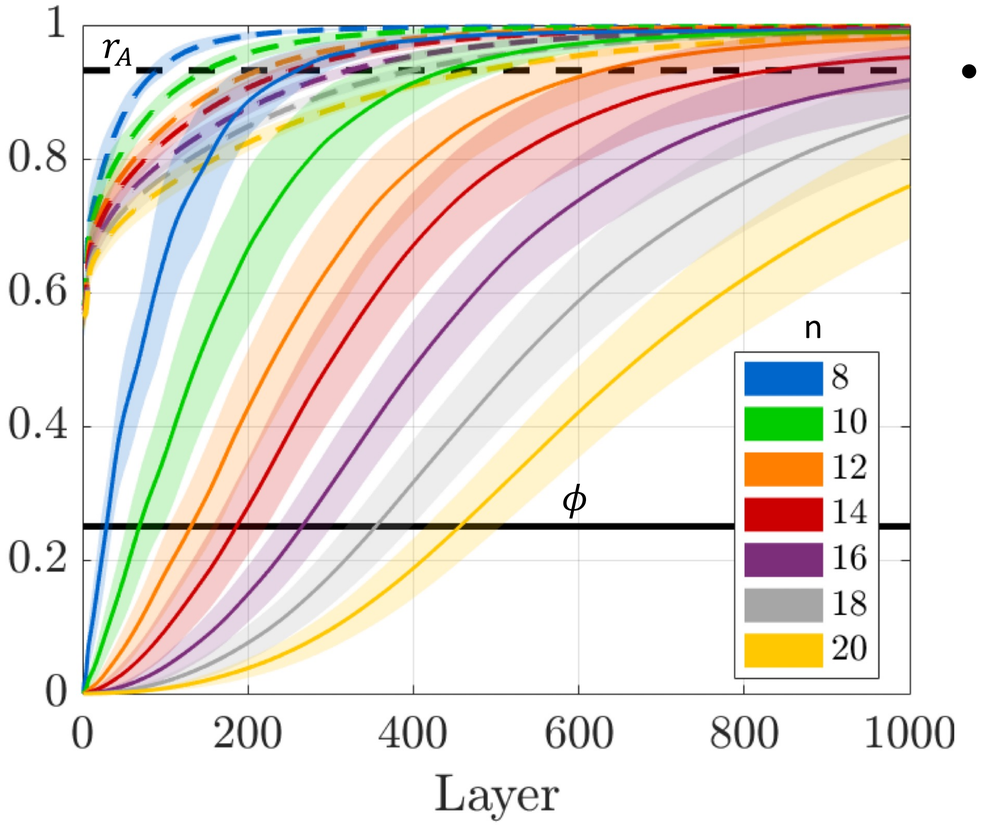
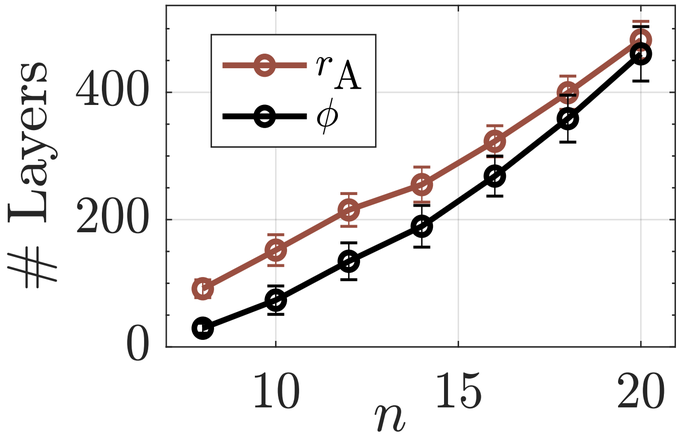
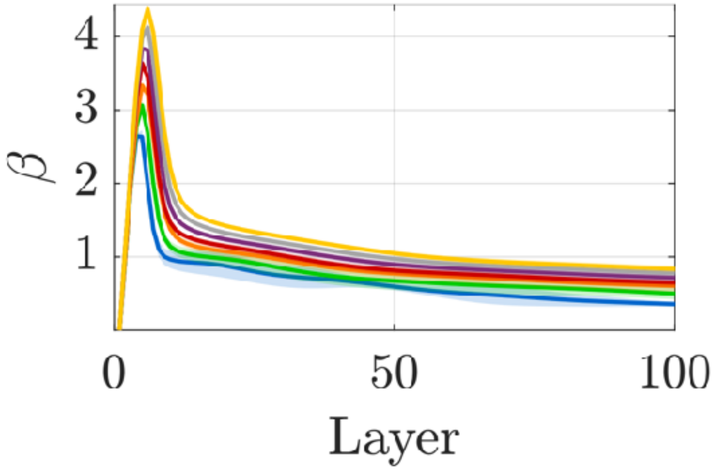


Performed a variety of numerical analyses exploring the performance of FALQON towards MaxCut on connected, 3-regular graphs

- We consider two figures of merit:
  - the probability ϕ of measuring $|gs\rangle$
  - the approximation ratio $r_A = \dfrac{\langle H_p \rangle}{\min \langle H_p \rangle}$







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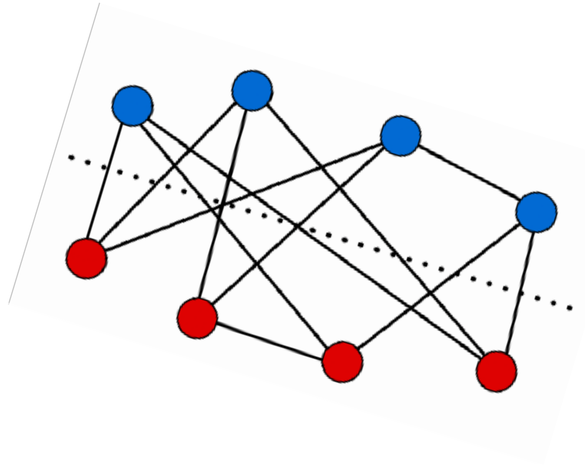

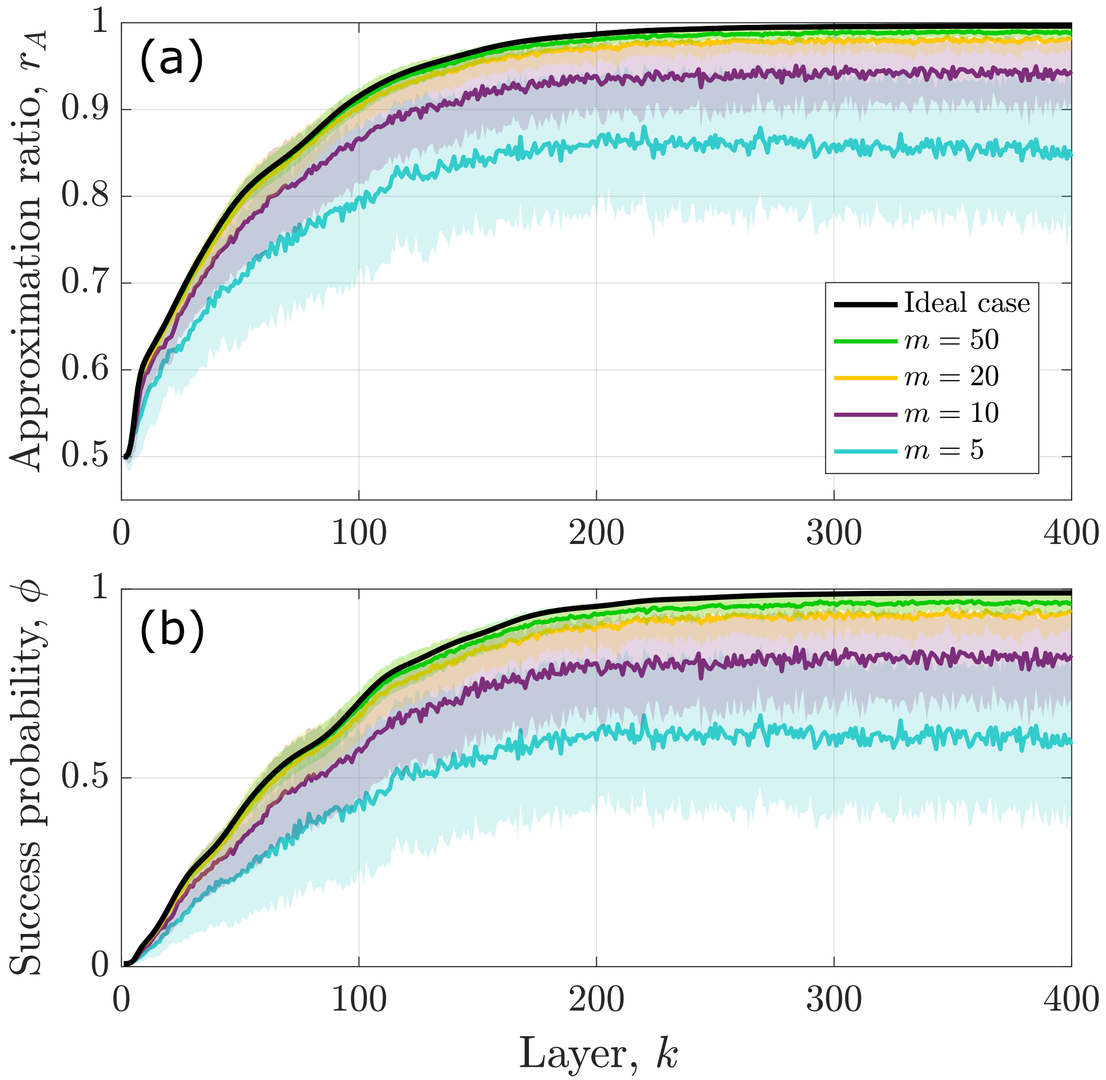

m = # samples to estimate A_k at each layer

We also explored the performance of our protocol in the presence of sampling noise in our estimate of A, that directly infects β through $\beta_k = A_{k-1}$

Find robustness to sampling noise, due to freedom in control law for satisfying

$$\frac{d}{dt}J(t) \leq 0$$

i.e., we chose $\beta(t) = -A(t)$ such that $\frac{d}{dt}J(t) = -\left(A(t)\right)^2$, but the Lyapunov condition is satisfied as long as β is the correct sign

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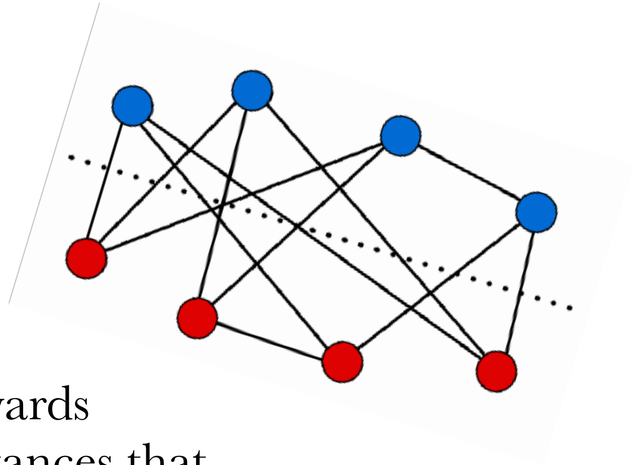
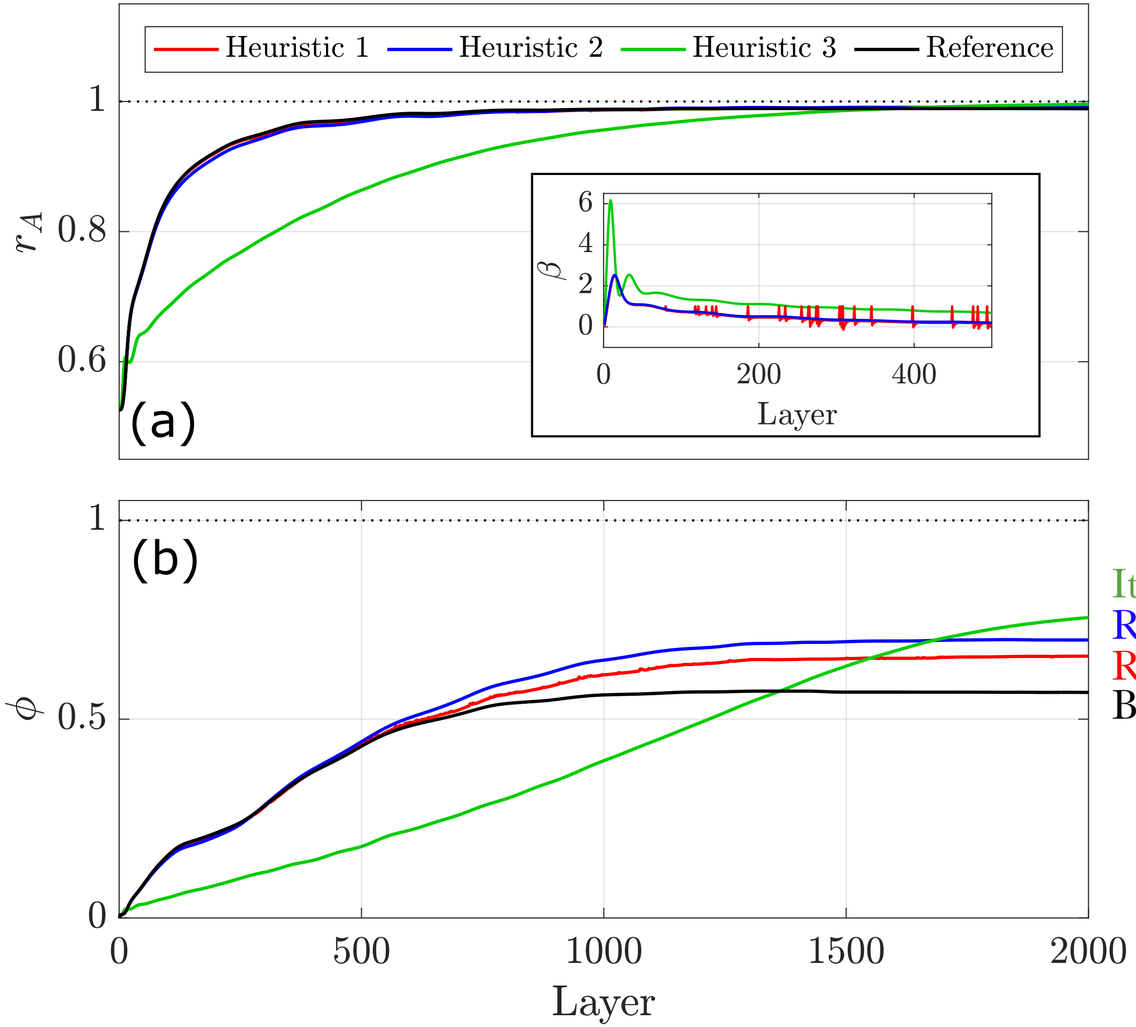
Applications to MaxCut





We also explored applications towards weighted MaxCut, and found instances that did not converge

For such cases, we introduced a series of modifications that can be included that improve performance

Iterative updates to perturbation
Reference field perturbation in β
Random perturbations in β
Base case

Summary

- Introduced a feedback-based algorithm for combinatorial optimization on quantum computers
- Feedback from qubit measurements is used to sequentially assign values to circuit parameters
- This eliminates the need to search for optimal parameters
- Presented results of hardware demonstration on IBM quantum processor
- Presented numerical illustrations involving MaxCut on regular graphs to investigate convergence, scalability, and robustness

Thank you

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