

# Improving PIC-DSMC Simulations of RF Plasmas via Event Splitting

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Knowledge for Tomorrow

# Talk outline

- **Motivation**
- **PIC-DSMC overview**
  - Variable-weight DSMC
  - Event splitting
  - Overview of cross-sections
- **Numerical results**
  - 0-D
    - Xenon
    - Helium
  - 1-D
    - Helium
- **Conclusions**



# Introduction / Motivation

- Cannot apply fluid-based models in rarefied regimes without losing accuracy of modelling
- Particle-based methods (PIC-DSMC) are subject to stochastic noise
- This can affect accuracy of modelling of
  - Ionization
  - CEX collisions
  - Scattering events relevant to plume impingement, air-breathing EP intake design
  - Transient processes in general
- Thus, aim of current work is to present a method for **improved modelling of low-probability processes** within PIC-DSMC, apply to several benchmark problems to show benefits of method



# DSMC

## Basic ideas:

- Model (rarefied) gas by simulating motion of large number of computational particles
- Each particle represents  $F_{num} = const$  real-life molecules
- Separate convection, collision and acceleration steps
- Collisions performed stochastically
- NTC/MF/Bernoulli trial collision schemes: cost linear in number of particles  
(e.g.,  $F_{num}^* = F_{num}/10 \rightarrow 10x$  more particles  $\rightarrow 10x$  increase in cost)

## References:

- G. A. Bird, Molecular gas dynamics and the direct simulation of gas flows, 1994
- M. S. Ivanov, S. V. Rogasinsky, Russian Journal of numerical analysis, 1988
- S. K. Stefanov, SIAM Journ. Sci. Comp., 2011



# DSMC issues

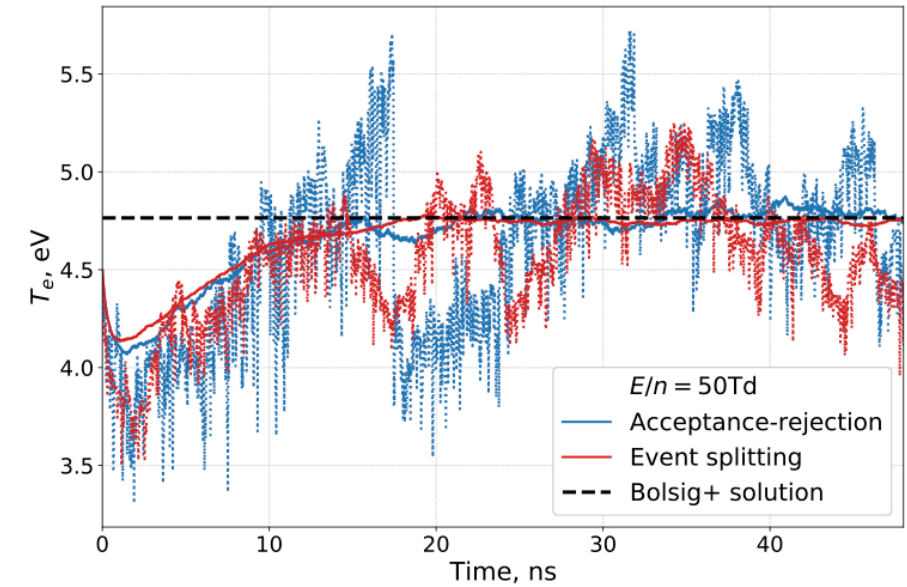
Stochasticity-related (aka “noise”):

- Low-speed flows
- Transient flows
- Coupling with CFD
- Particle-in-Cell (affects field solver)

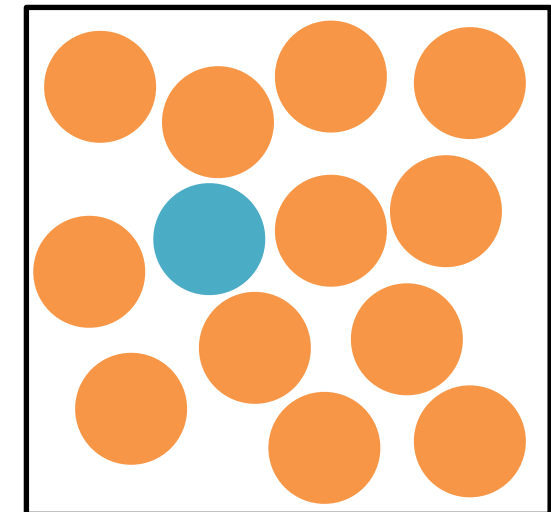
Fixed  $F_{num}$ -related (hard to resolve trace populations):

- Trace chemical species
- Excited internal states
- High-velocity distribution function tails

PIC-DSMC simulations can have **(almost) all** of the above!



Oblapenko et al., Journ. Comp. Phys., 2022





# Variable weight DSMC

Idea: solve fixed  $F_{num}$ -related problems by allowing computational particles to represent **arbitrary** amounts of real-life particles

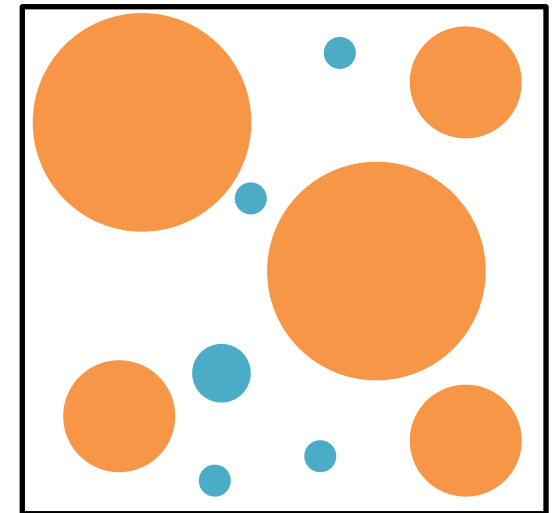
Resolves (mostly) issue of capturing trace populations

But new problems arise:

- Particles need to be split during collisions, which leads to growth in number of particles
- Controlling particle counts requires either conserving energy “on average” or particle merging
- Particle merging incurs additional computational cost, distorts velocity distribution function

## References:

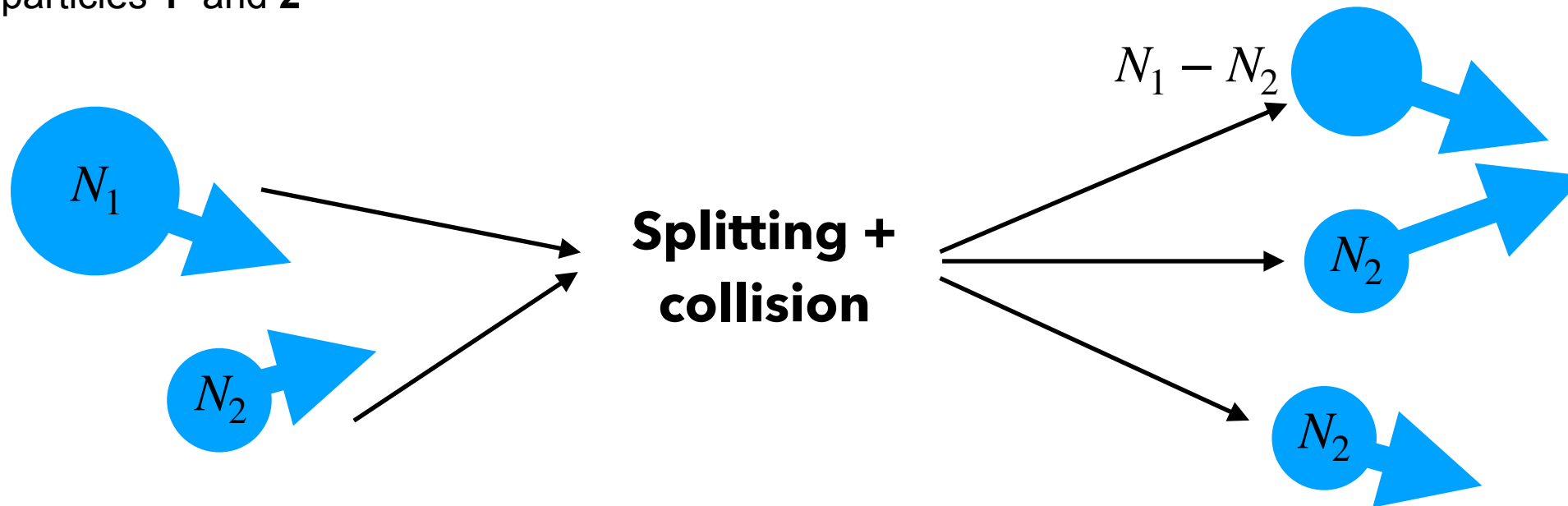
- I. D. Boyd, Journ. Thermophys. Heat Transf., 1996
- S. Rjasanow, W. Wagner, Journ. Comp. Phys., 1996
- S. J. Araki, R. S. Martin, Phys. Plasmas, 2020



## Particle splitting schematic

If particle **1** represents  $N_1$  molecules, particle **2** represents  $N_2$  molecules (and  $N_1 > N_2$ ), then during collisions only  $N_2$  molecules actually collide

Have to split particle **1** into two particles **1'** and **1''** with weights  $N_2$ ,  $N_1 - N_2$ , collide particles **1'** and **2**

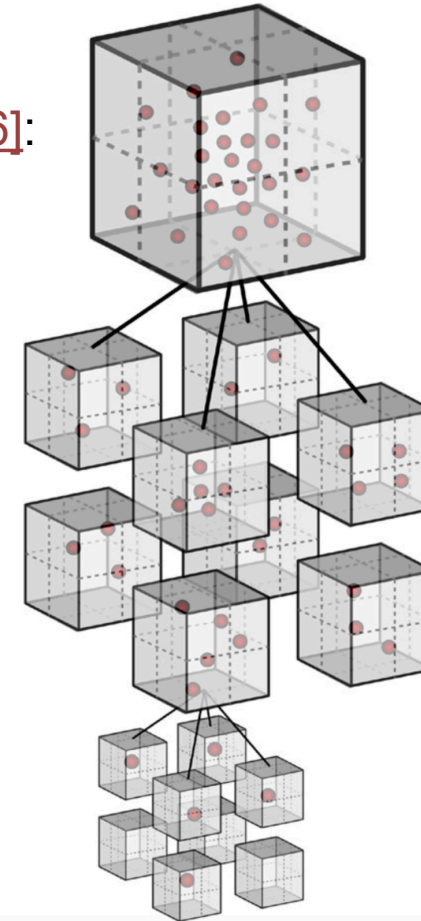


# Merging schematic

## One of the (many) possible approaches

Octree merging [[R. Martin, J.-L. Cambier, Journ. Comp. Phys., 2016](#)]:

- Divide velocity space into octants
- Subdivide octants based on number density inside
- In each suboctant, can replace  $N$  ( $>2$ ) particles with 2 particles (need 2 particles for conservation)
- Continue subdivision until target # of particles is reached
- Cost is  $\mathcal{O}(n)$  (depends on particle sorting algorithm)



R. Martin, J.-L. Cambier, Journ. Comp. Phys., 2016



# Event splitting

## Outline

- Suppose we have  $N_p$  possible processes that can occur during a collision (e.g. elastic collision, ionization reaction, vibrational transition, etc.), and the corresponding probabilities are  $\{p_i\}_{i=1}^{N_p}$
- Standard DSMC “all-or-nothing” approach: sample process type based on  $\{p_i\}_{i=1}^{N_p}$  and model **only** that collision process
- But we’re doing particle splitting anyway, so what if we split particles proportionally to  $\{p_i\}_{i=1}^{N_p}$  and simulate **all** possible collision outcomes?

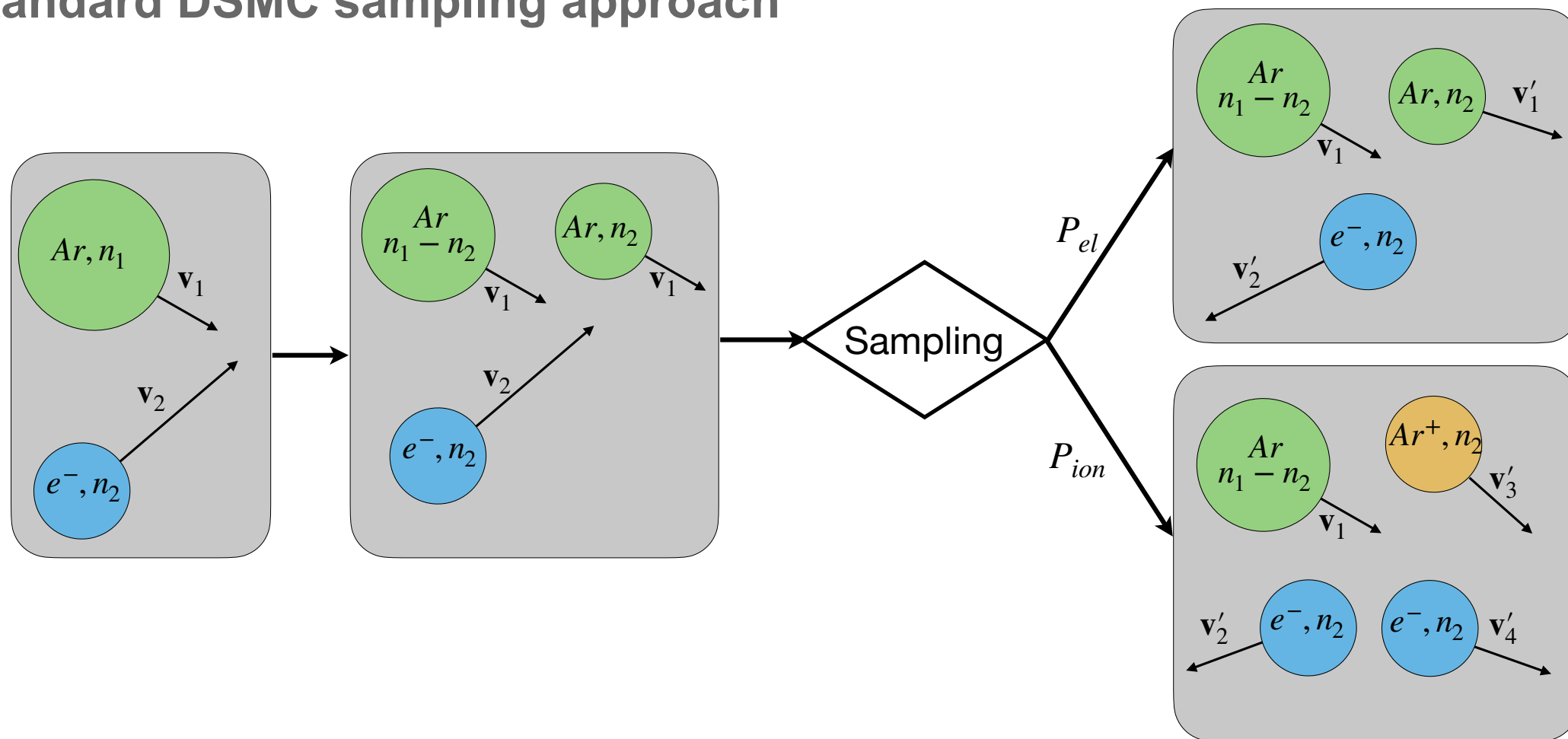
This is what we call “**event splitting**” (similar reasoning also be applied to boundary conditions and scattering)

## References:

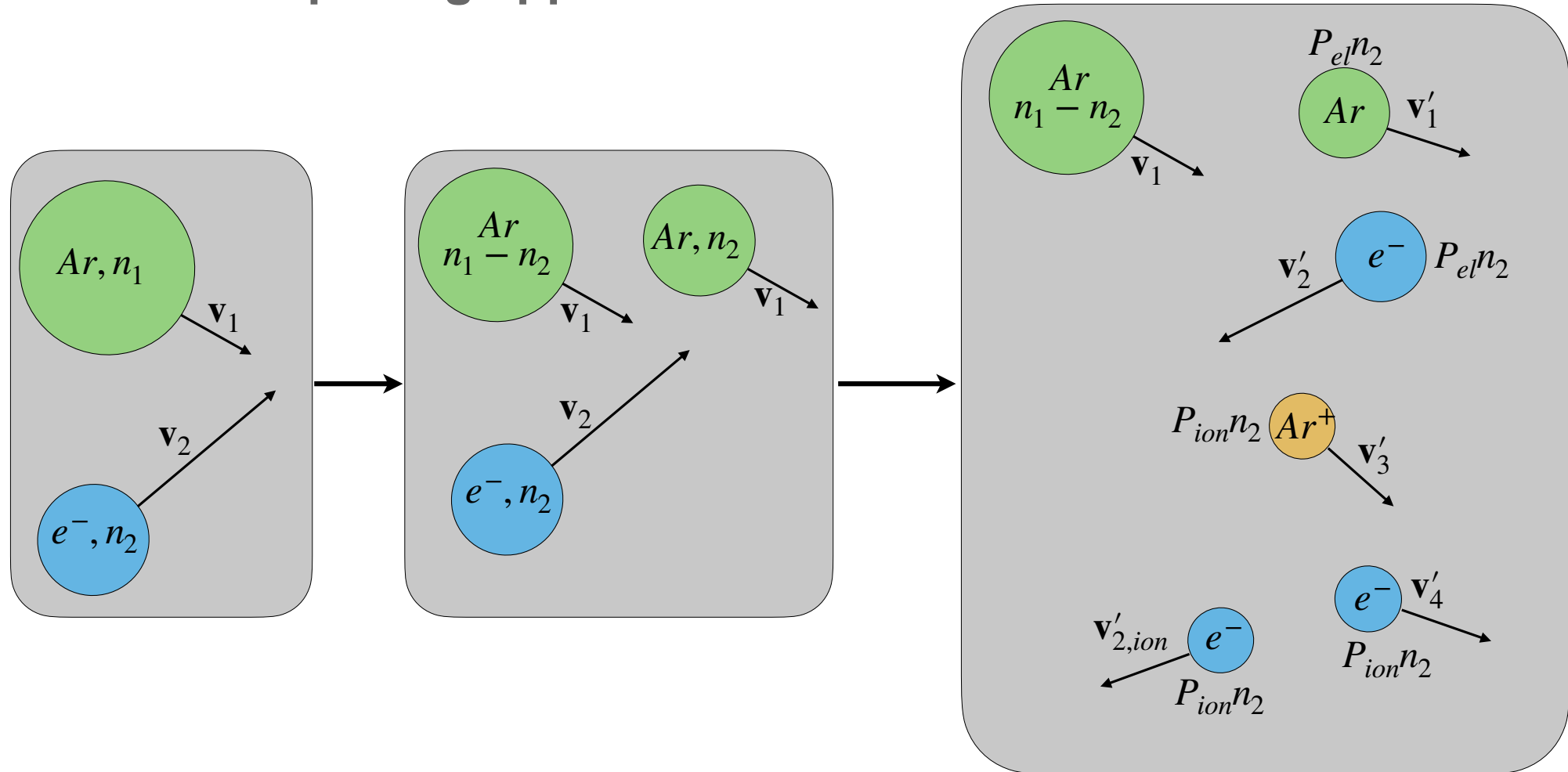
- G. Oblapenko et al., Journ. Comp. Phys., 2022
- G. Oblapenko et al., Scitech 2022 Proceedings, 2022



# Standard DSMC sampling approach



# New DSMC event splitting approach



# Event splitting motivation

## Why do event splitting?

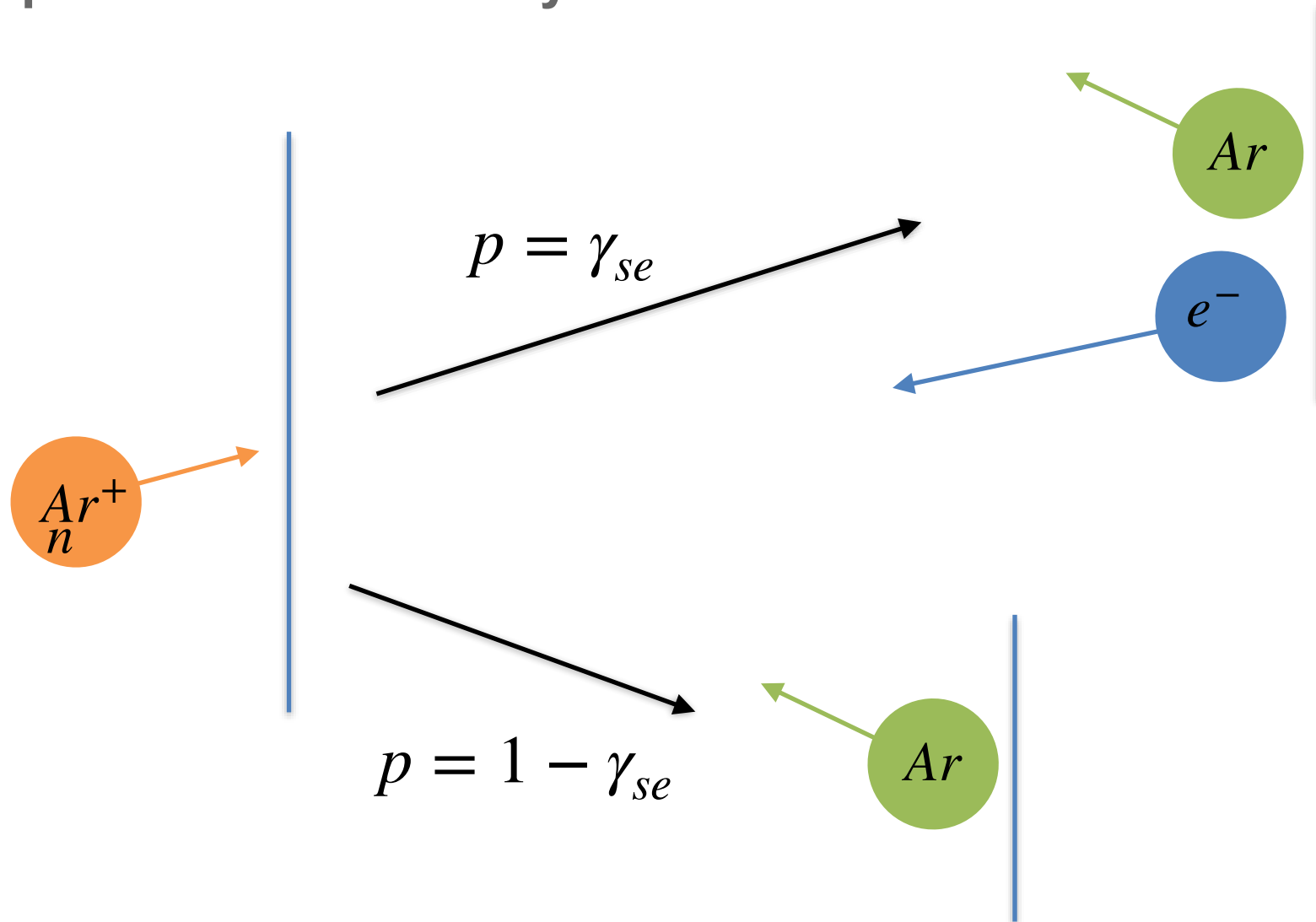
- Improve simulation of low-probability processes
- Reduce need for particle cloning, since we create more particles during a collision step

## Possible cons of the event splitting approach?

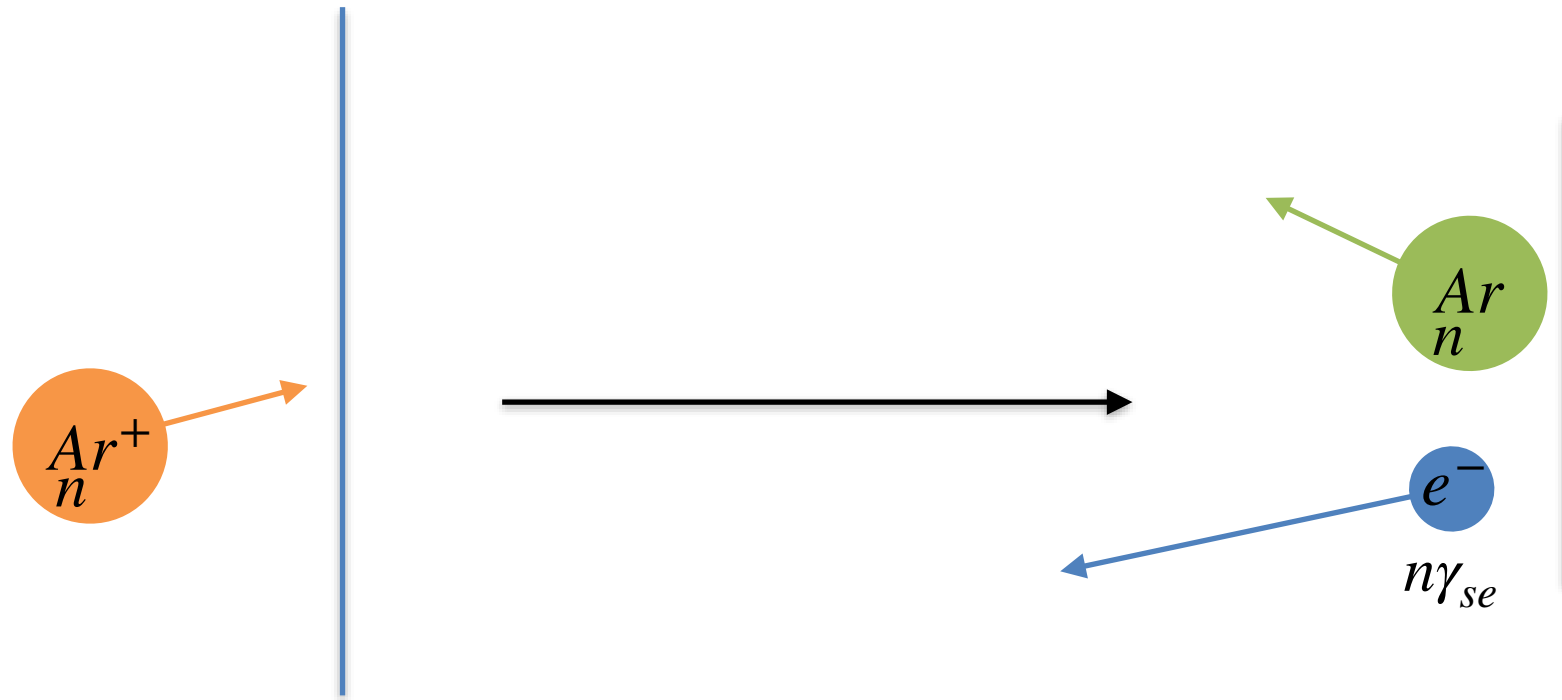
- Increased computational cost
  1. Need to simulate  $N_p$  scattering events for each collision instead of 1  
Example:  $Ar - e^-$  collisions with electronic excitation:  
31 excitation reactions + ionization + elastic scattering = 33 processes!
  2. More particles produced → more frequent merging required (also causes distortion in VDF)  
Explored in more detail for excitation reactions in [\[Oblapenko et al., Scitech 2022 Proceedings, 2022\]](#)



## Example of application to boundary conditions

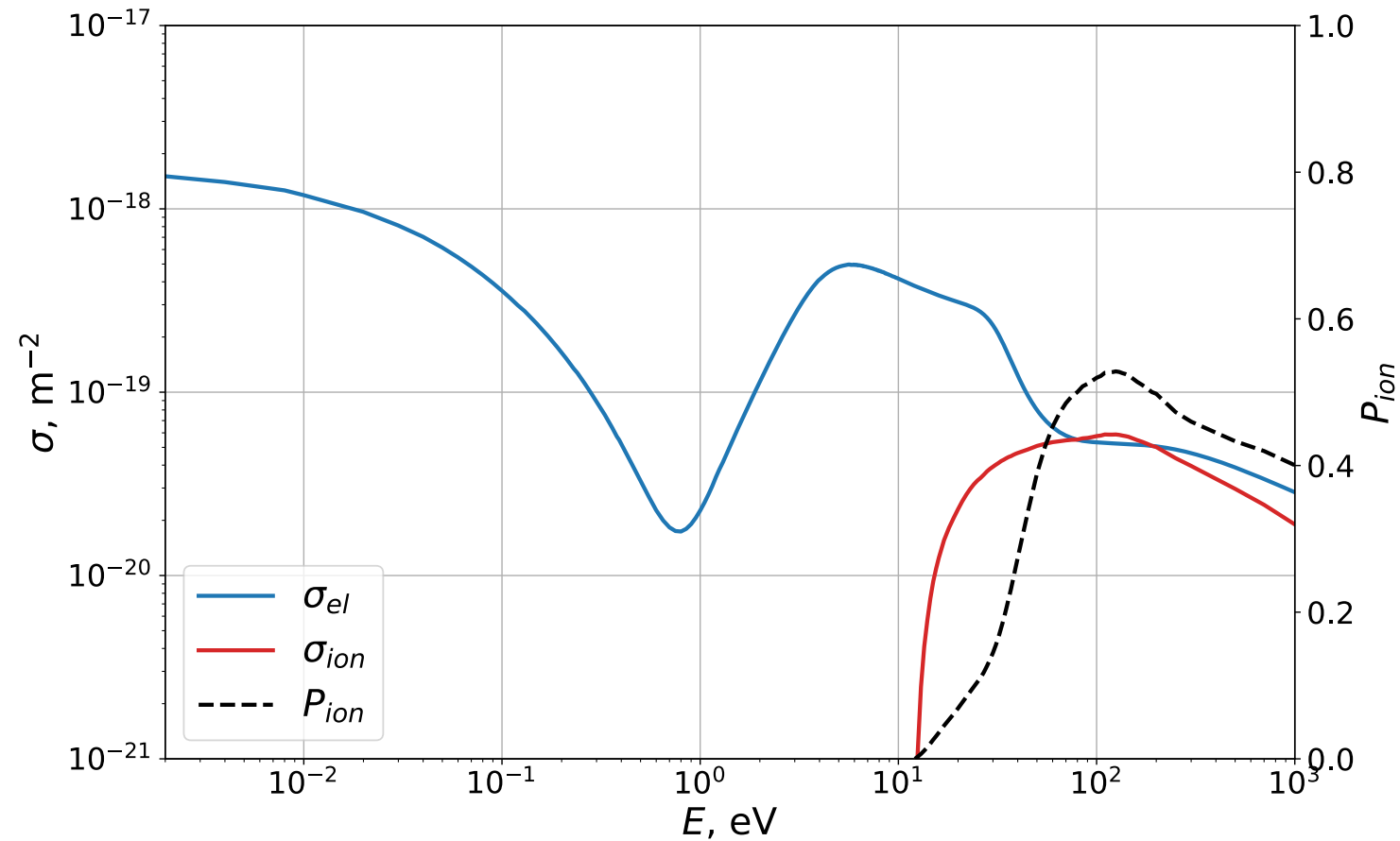


# Example of application to boundary conditions





# Electron-neutral collision cross-sections (xenon)



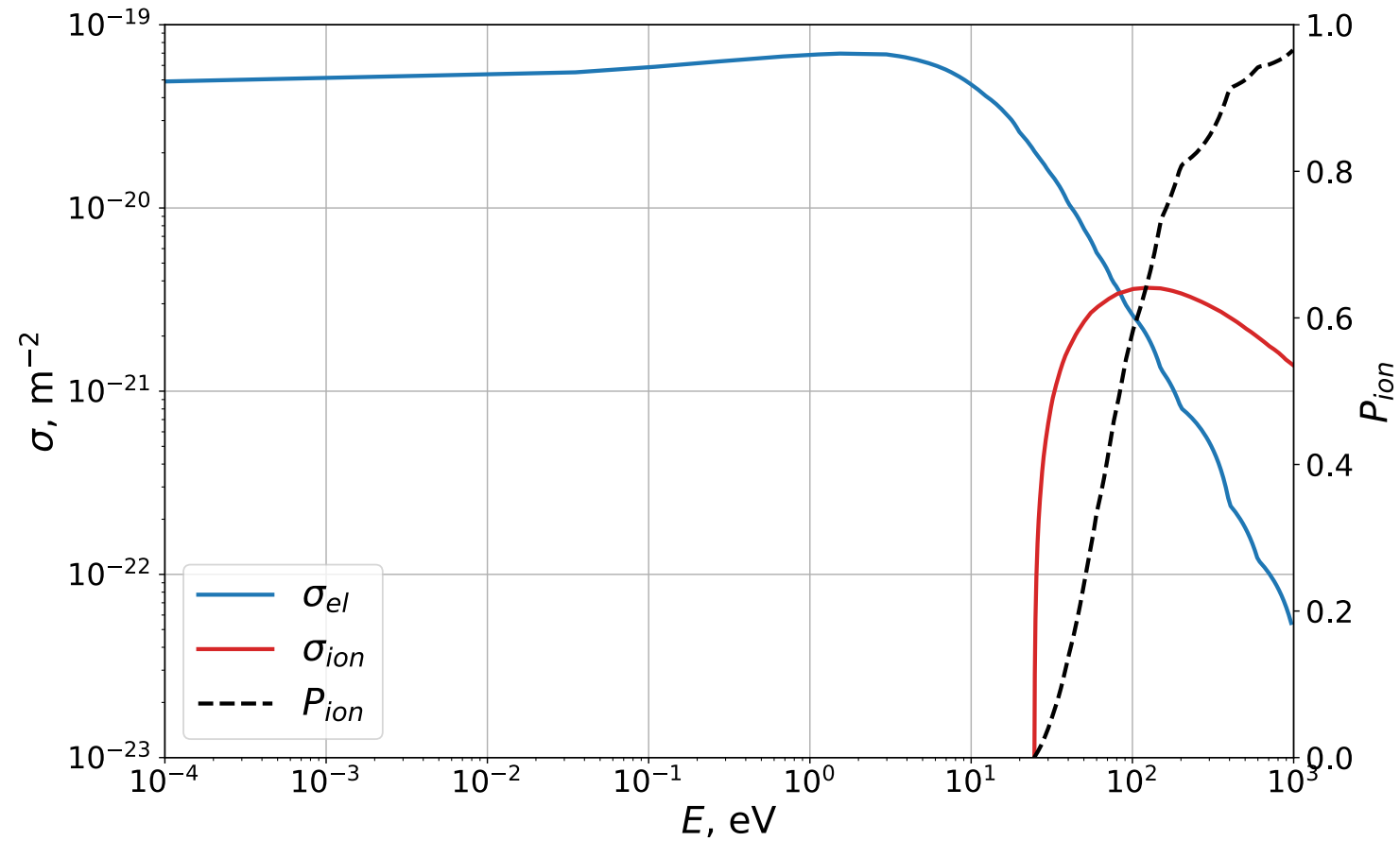
COP dataset for elastic scattering: R. McEachran, A. Stauffer, Eur. Phys. J. D, 2014-2015

Hayashi dataset for ionization: M. Hayashi, Technical Report

Scattering model: Okhrimovskyy et al., Phys. Rev. E, 2002



# Electron-neutral collision cross-sections (helium)



Biagi dataset: L. Alves et al., J. Phys. D: Appl. Phys, 2013

Scattering model: isotropic

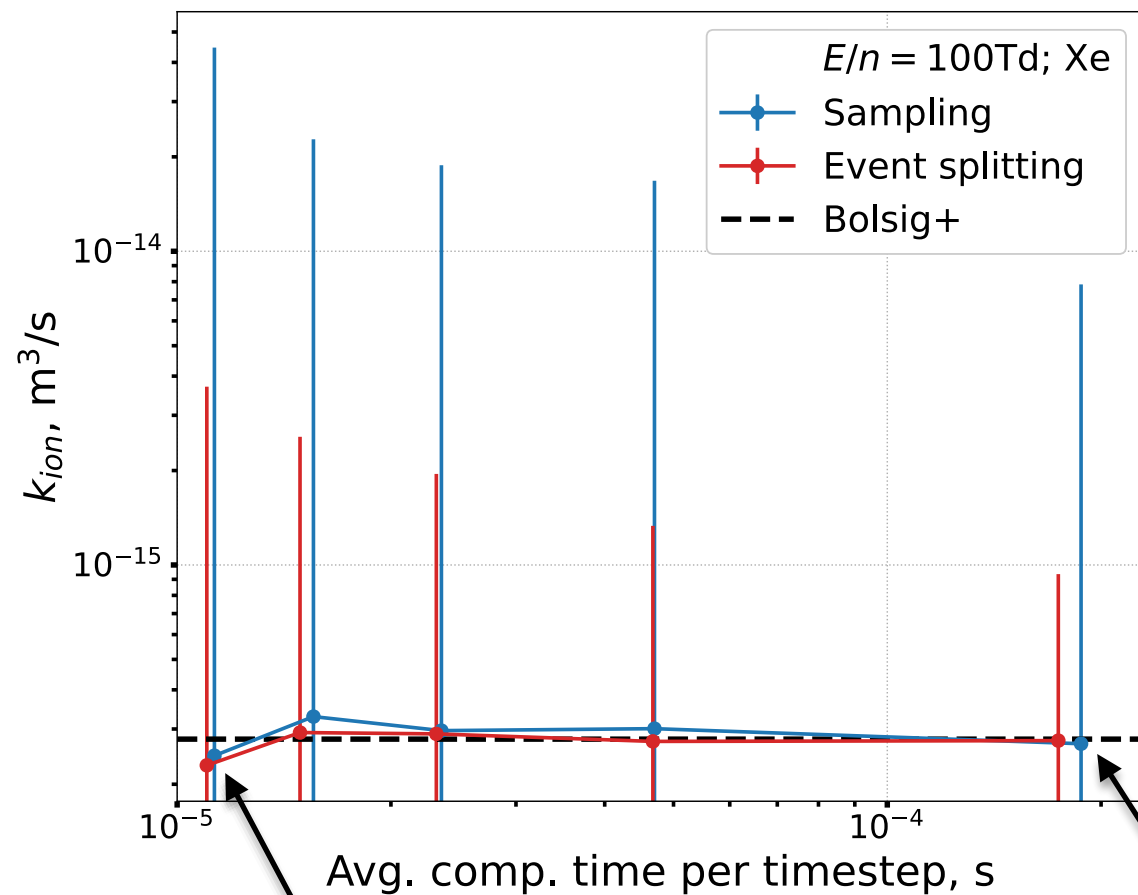


## 0-D test cases

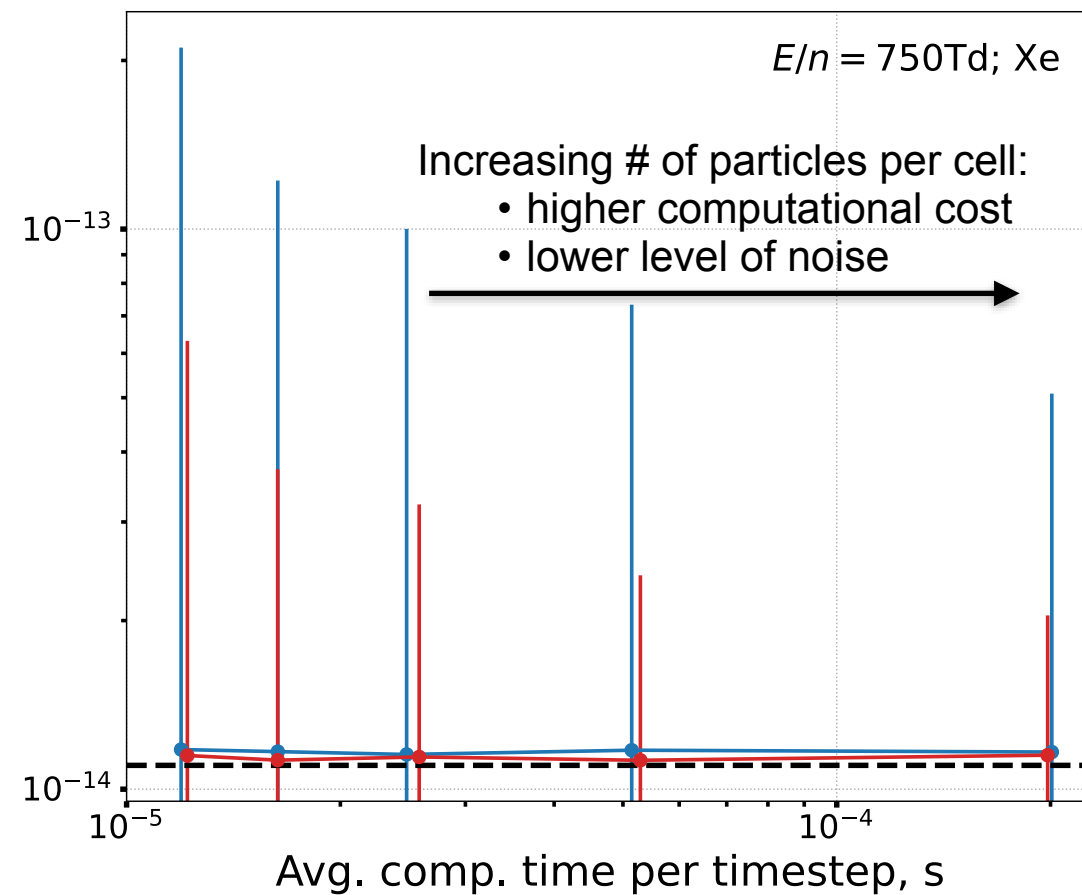
- $\text{Xe}/\text{Xe}^+/\text{e}^-$  or  $\text{He}/\text{He}^+/\text{e}^-$  mixture, initialized with small population of ions and electrons
- Accelerated by a constant electric field
- After initial transient period, gas reaches quasi-steady state (characterized by constant ionization rate coefficient)
- Steady-state dependent only on
  1. Value of reduced electric field ( $E/N$ )
  2. Processes considered
  3. Cross-sections
- Can gather statistics for the instantaneous ionization rate coefficient
- Can compare to Bolsig+ solver



## 0-D test case: Xenon

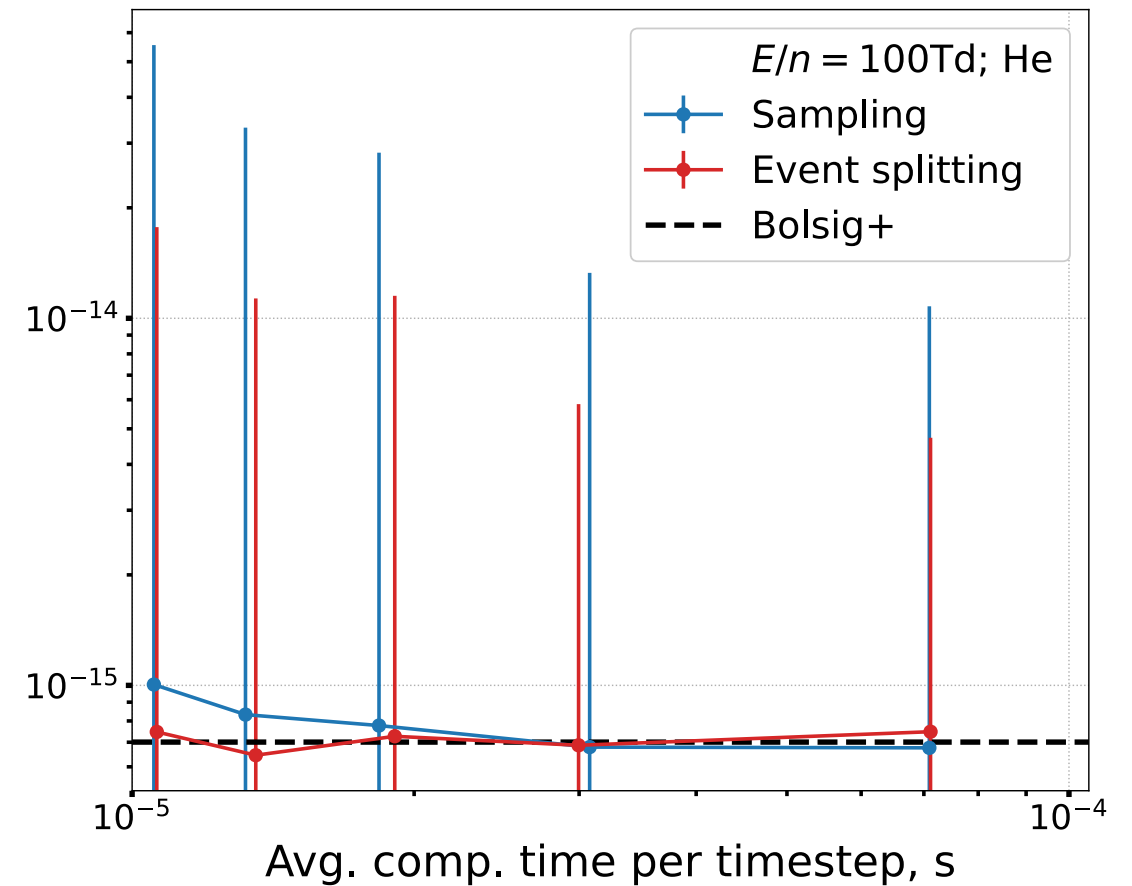
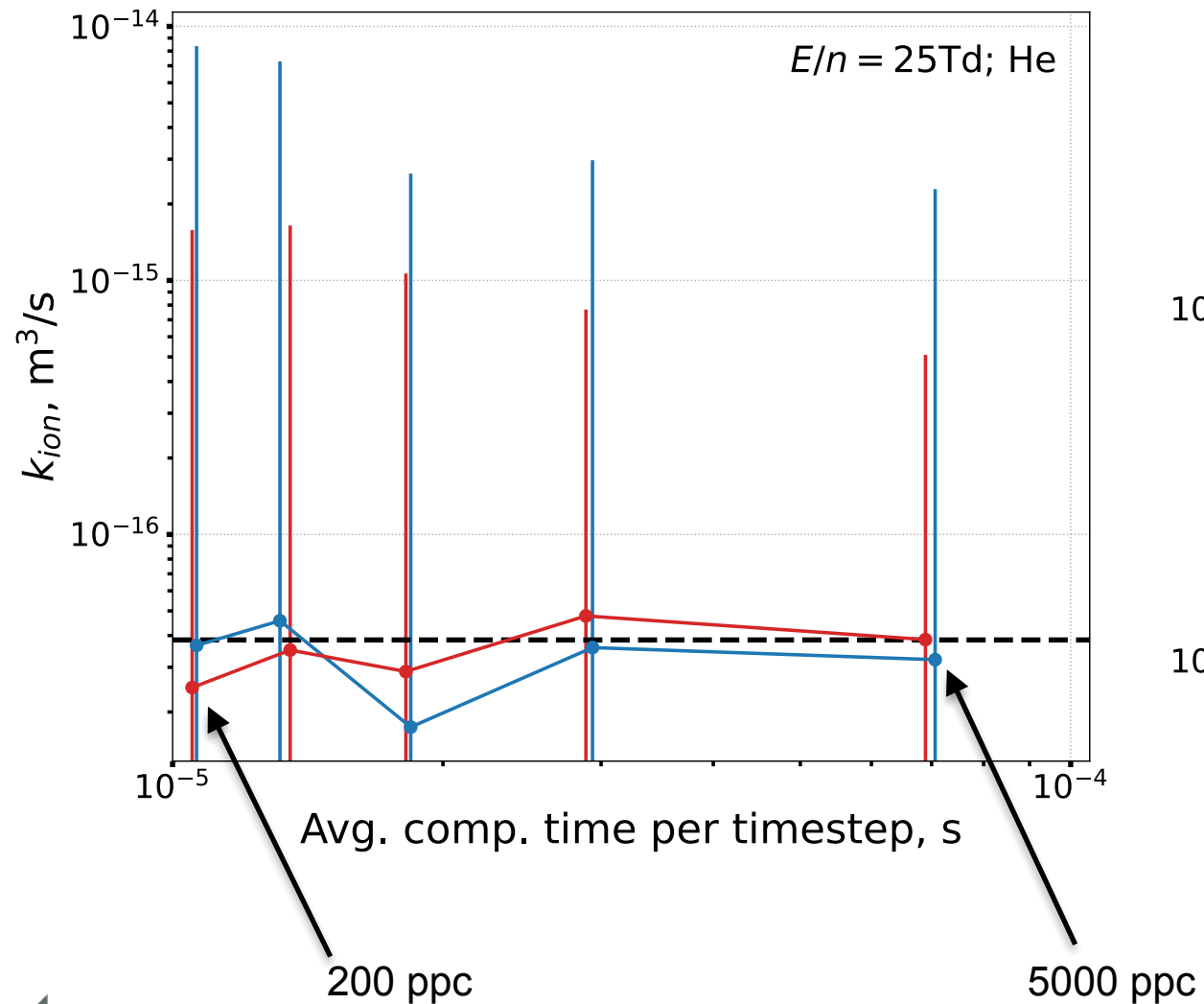


200 ppc



5000 ppc

## 0-D test case: Helium



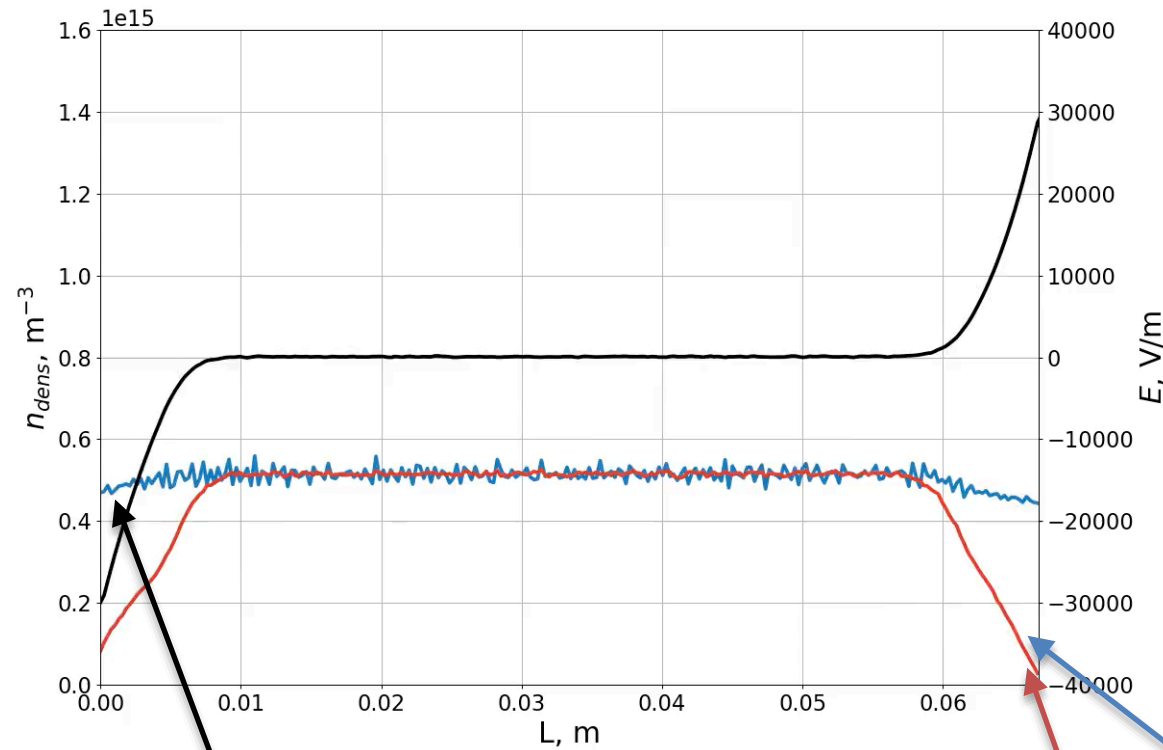
# 1-D test case

- Helium CCRF plasma
- Based on case 2 from set of benchmark cases by Turner et al. [M.M. Turner et al., *Phys Plasmas*, 2013]
- 13.56 MHz frequency, 200 V voltage, 13.3 Pa pressure
- 6.7 cm domain, 256 cells, 800 timesteps per RF cycle; cycle-averaged quantities
- **Main QOI: Cycle-averaged  $k_{ion}$**
- Particles per cell (approximately): varied from 55 to 550
- Not considered:
  - Excitation reactions
  - Neutral-ion collisions

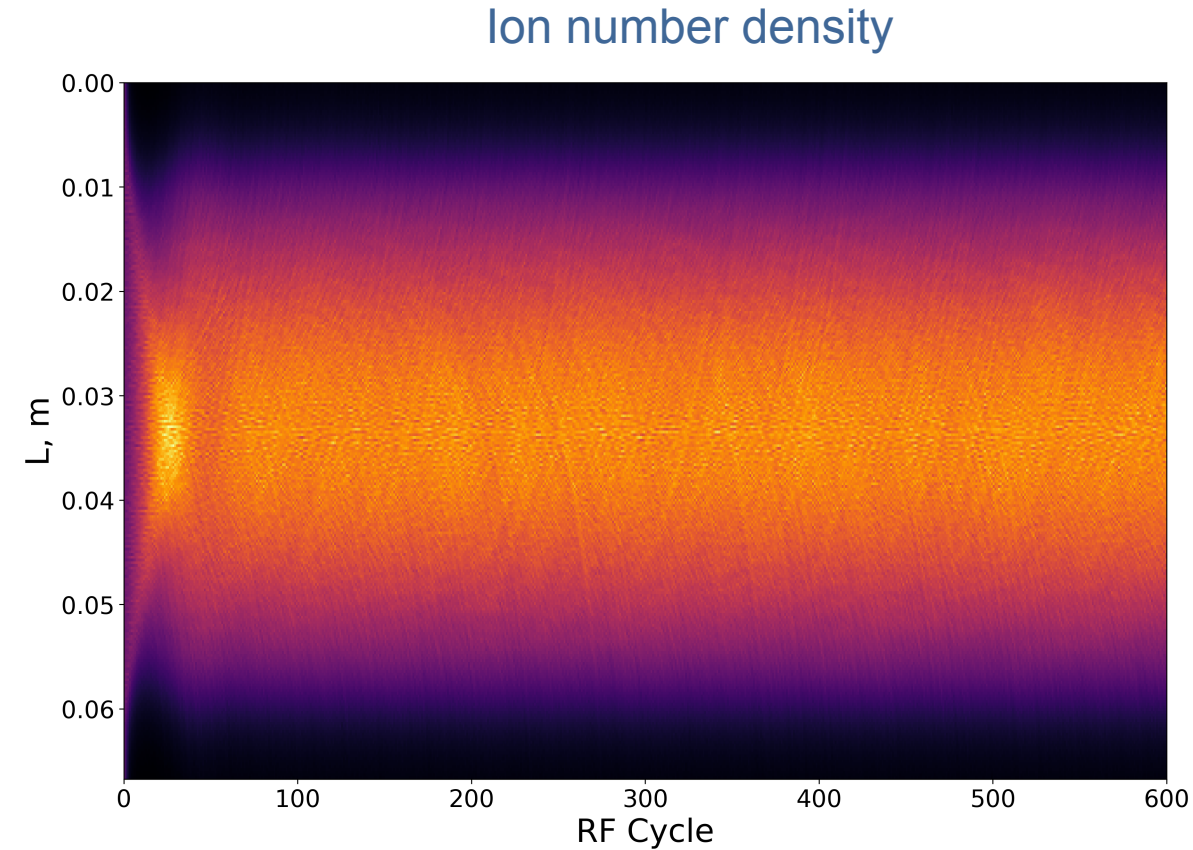




# 1-D test case: ion density, electric field



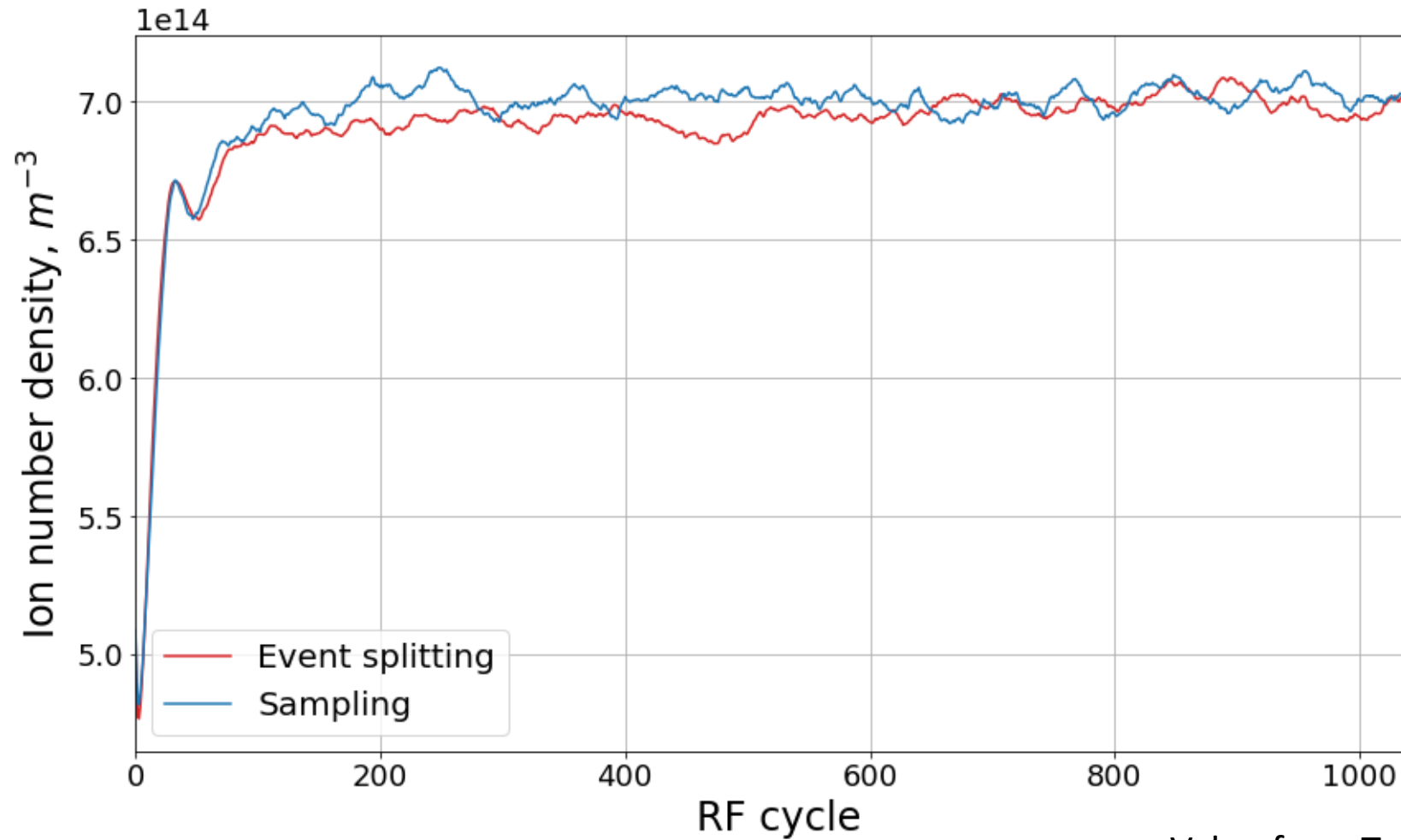
Electric field



Ion number density

Electron number density

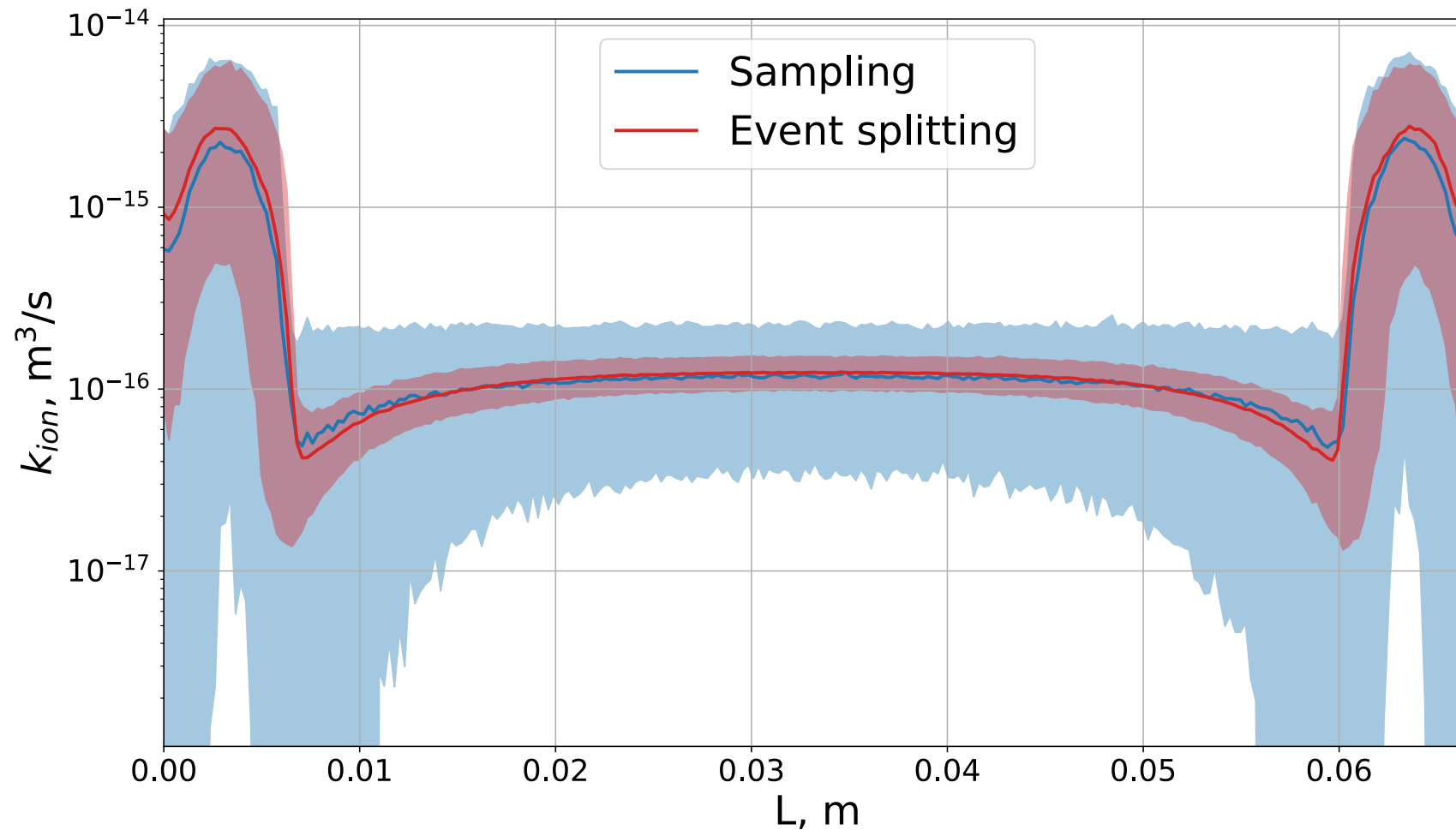
# 1-D test case: total ion number density



Value from Turner et al.:  $8.28 \cdot 10^{14}$



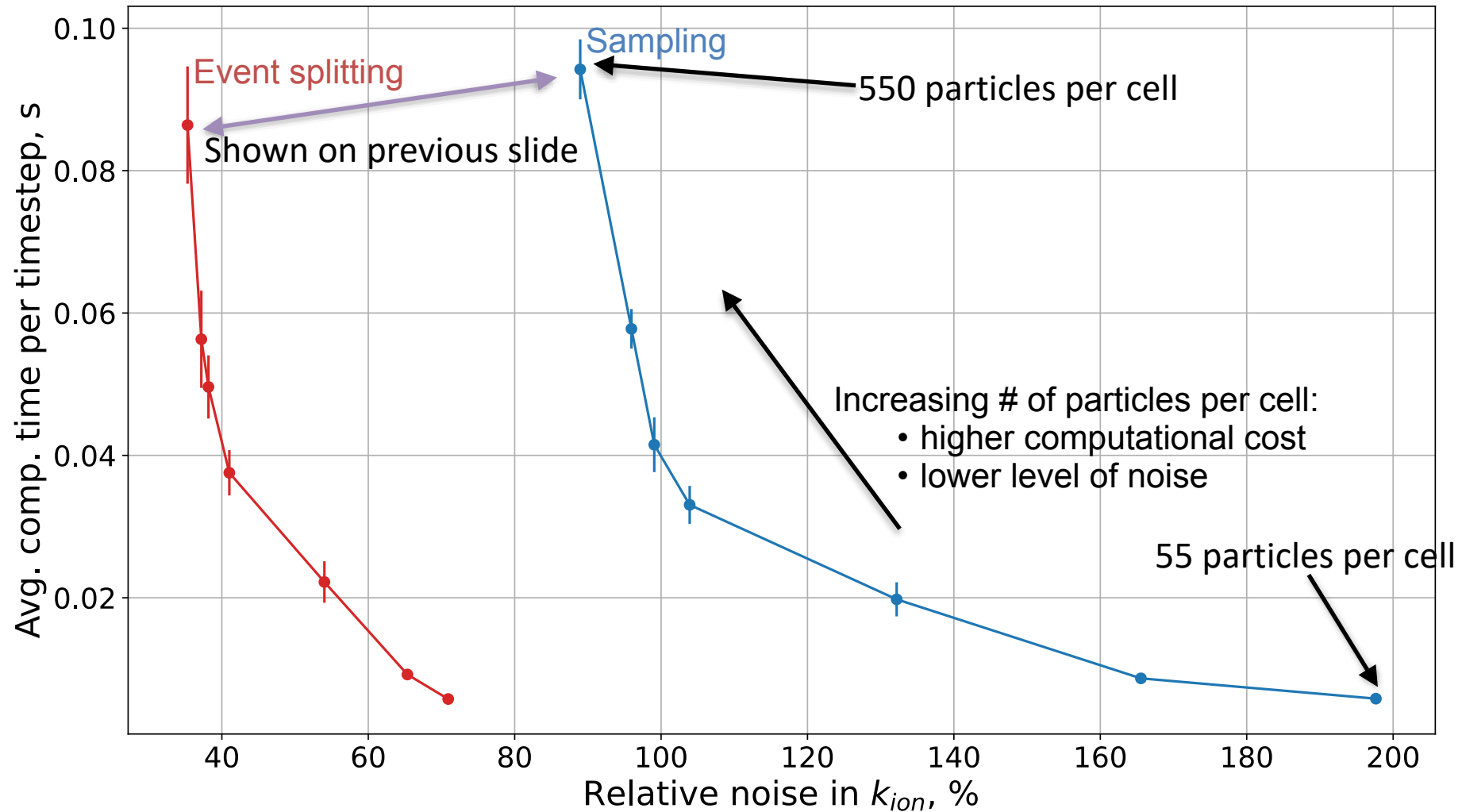
## 1-D test case: ionization rate coefficient



- **Significant reduction in noise** throughout whole domain
- **Computational cost not affected!**



# 1-D test case: error in ionization rate coefficient



# Conclusions

- Recently developed **event splitting scheme** tested for xenon and helium plasmas
- Shows significant **improvement** over standard DSMC collision scheme in **xenon**
- 1-D simulations of CCRF helium plasma show **2.5-fold reduction of noise** in ionization rate coefficient for same level of computational cost
- **Planned extensions:**
  - CEX collisions
  - Scattering events



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