SAND2022-8609C

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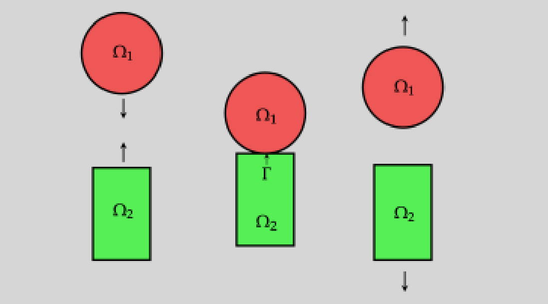
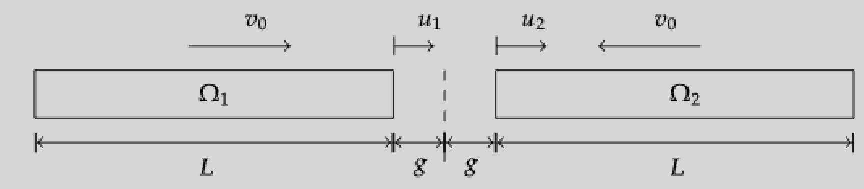
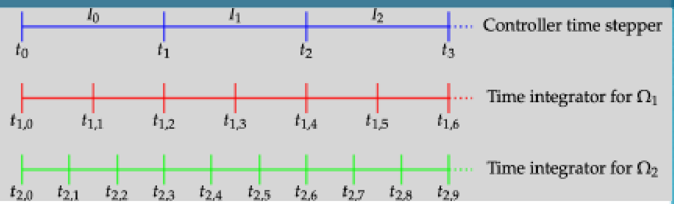
The Schwarz Alternating Method for Contact Mechanics







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15th World Congress on Computational Mechanics & 8th Asian Pacific Congress on Computational Mechanics, July 31st – August 5th 2022

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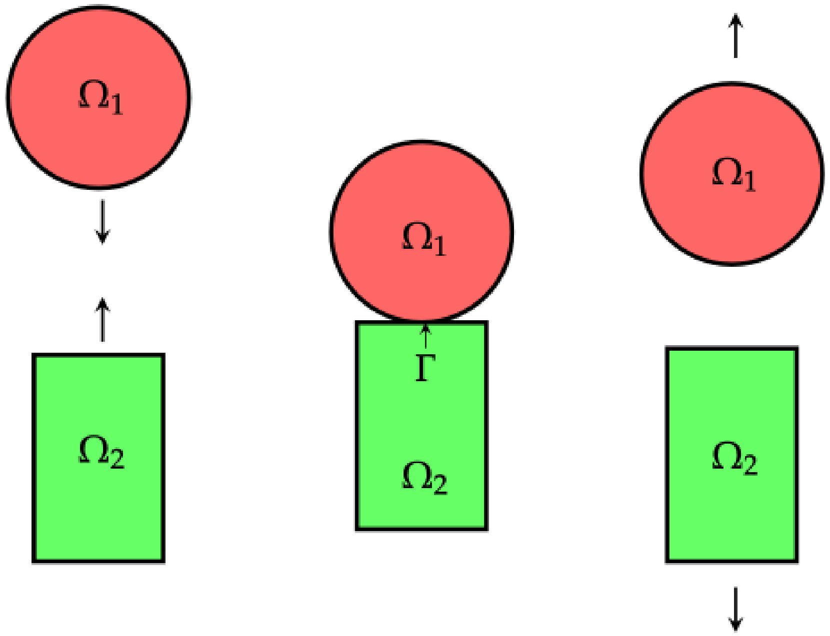
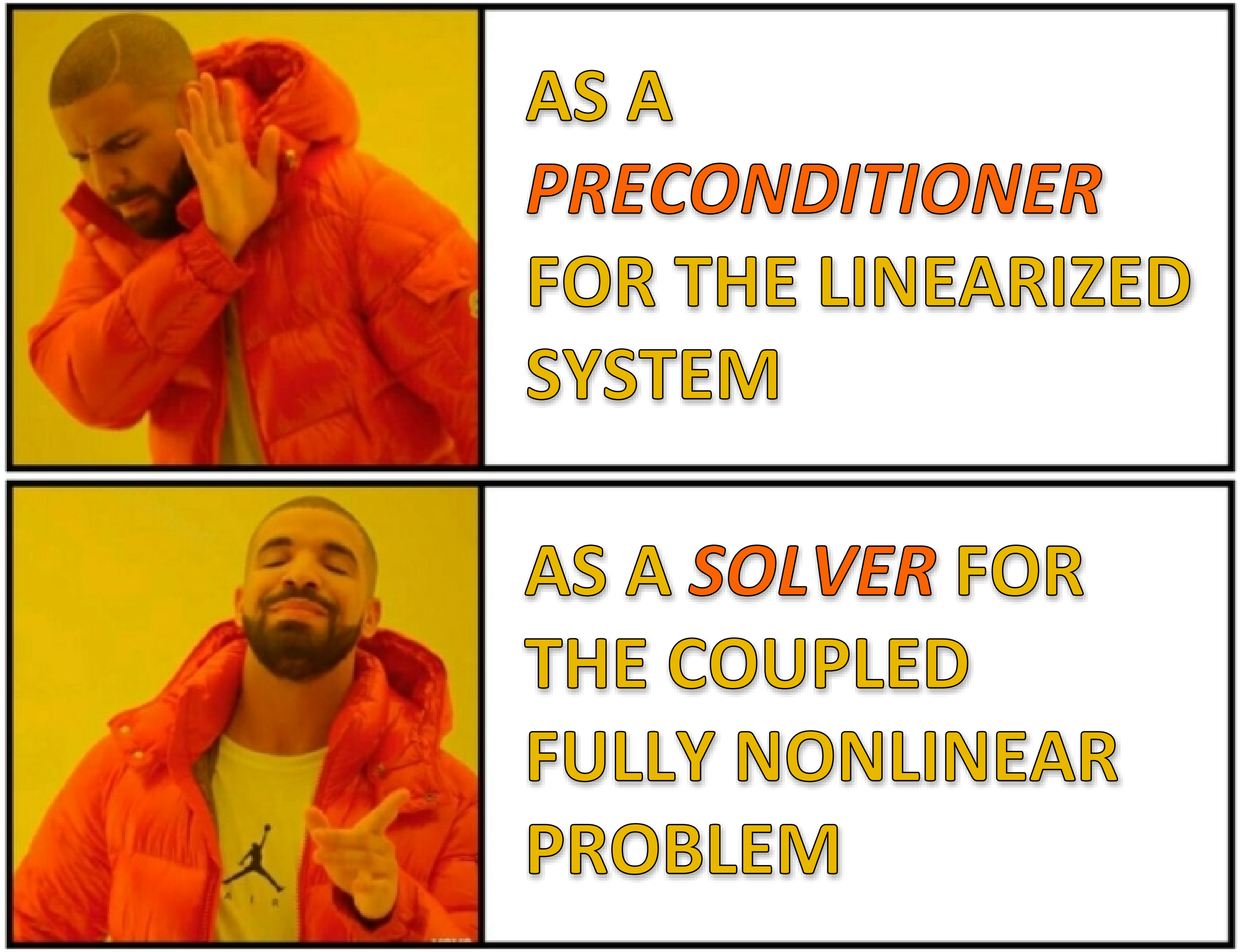
Outline

- Motivation
- Our previous Schwarz coupling work
- Non-overlapping Schwarz methods
- Non-overlapping Schwarz for contact
- Formulation
- A canonical 1D problem
- Comparison with traditional contact methods
- Conclusions and future work

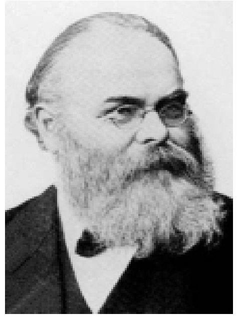


We Use the Schwarz Method ...

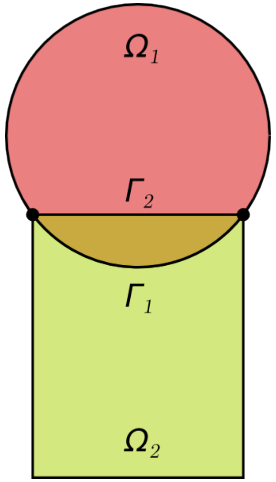


The Original Schwarz Method

- Proposed in 1870 by H. Schwarz for solving Laplace PDE on irregular domains.



Crux of Method: if the solution is known in regularly shaped domains, use those as pieces to iteratively build a solution for the more complex domain.



Basic Schwarz Algorithm

Initialize:

- Solve PDE by any method on Ω_1 w/ initial guess for Dirichlet BCs on Γ_1.

Iterate until convergence:

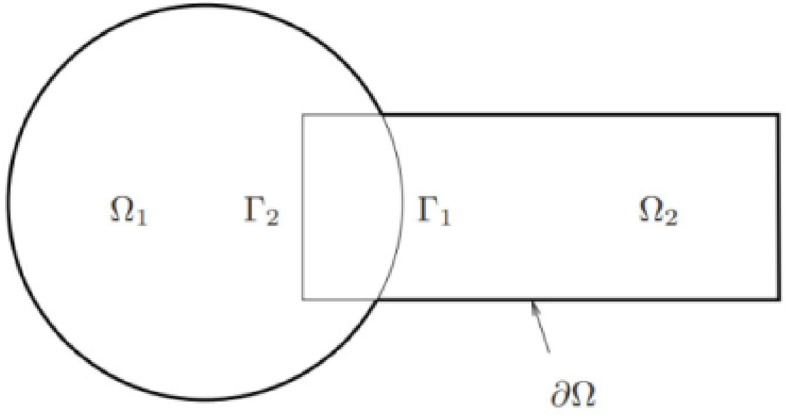
- Solve PDE by any method (can be different than for Ω_1) on Ω_2 w/ Dirichlet BCs on Γ_2 that are the values just obtained for Ω_1.
- Solve PDE by any method (can be different than for Ω_2) on Ω_1 w/ Dirichlet BCs on Γ_1 that are the values just obtained for Ω_2.

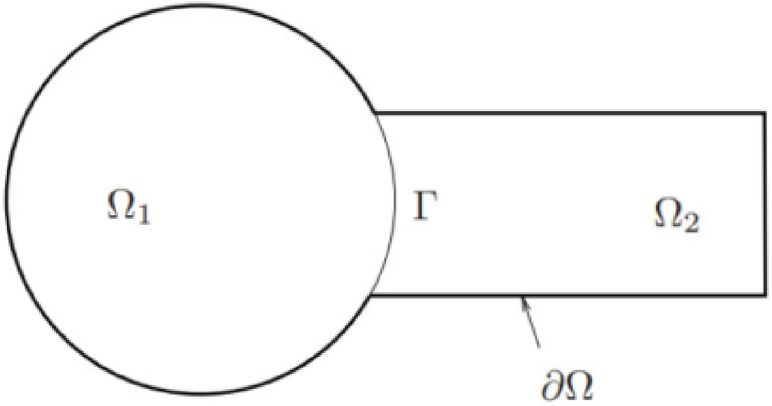
- Schwarz alternating method most commonly used as a ***preconditioner*** for Krylov iterative methods to solve linear algebraic equations.

Novel idea: using the Schwarz alternating as a ***solution method*** for solving multiscale partial differential equations (PDEs).

Schwarz with and without Overlap

- Schwarz with overlap
  - The original Schwarz method
  - Simple transmission with Dirichlet BCs
  - Requires overlap for convergence
  - Amount of overlap affects convergence
- Schwarz without overlap
  - Does not require overlap
  - Requires Dirichlet-Neumann or Robin BCs
  - It does not converge using D-D BCs
  - But D-D works in contact!



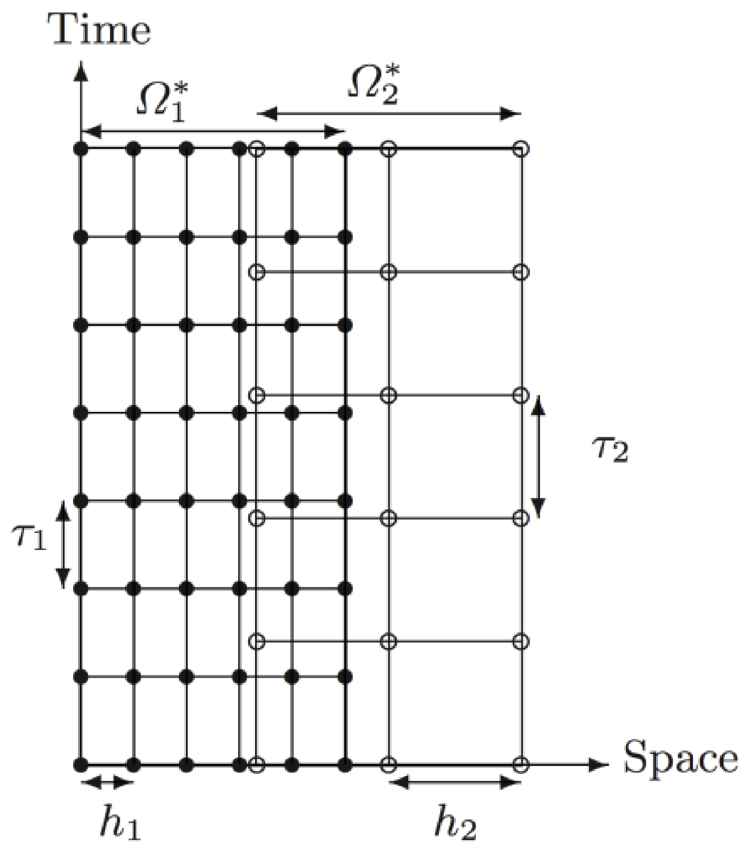


The Schwarz Method for Dynamics

- In the literature the Schwarz method is applied to dynamics by using ***space-time discretizations***.

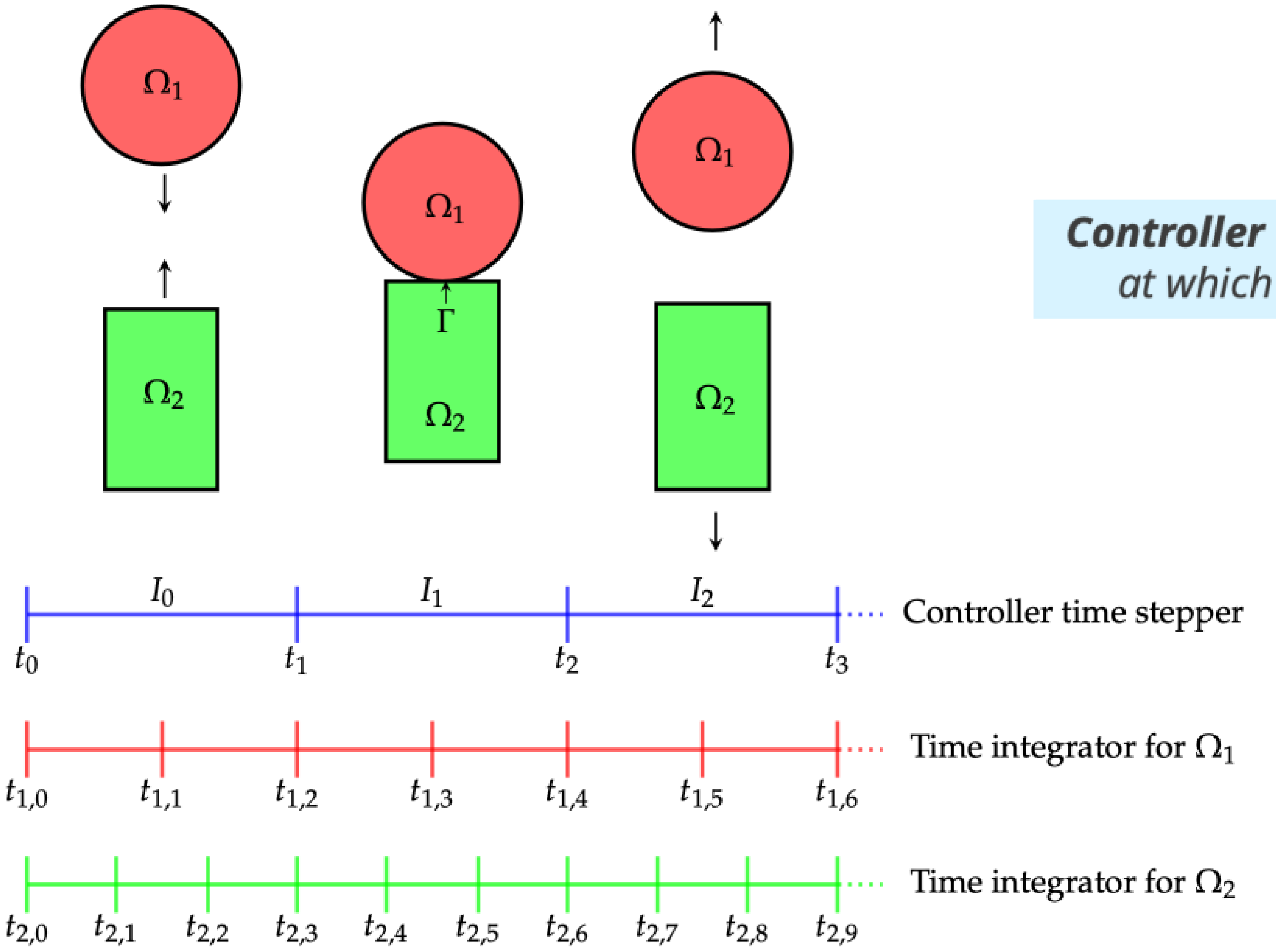
Pro ☺: Can use ***non-matching*** meshes and time-steps (see right figure).

Con ☹: ***Unfeasible*** given the design of our current codes and size of simulations.



Overlapping non-matching meshes and time steps in dynamics.

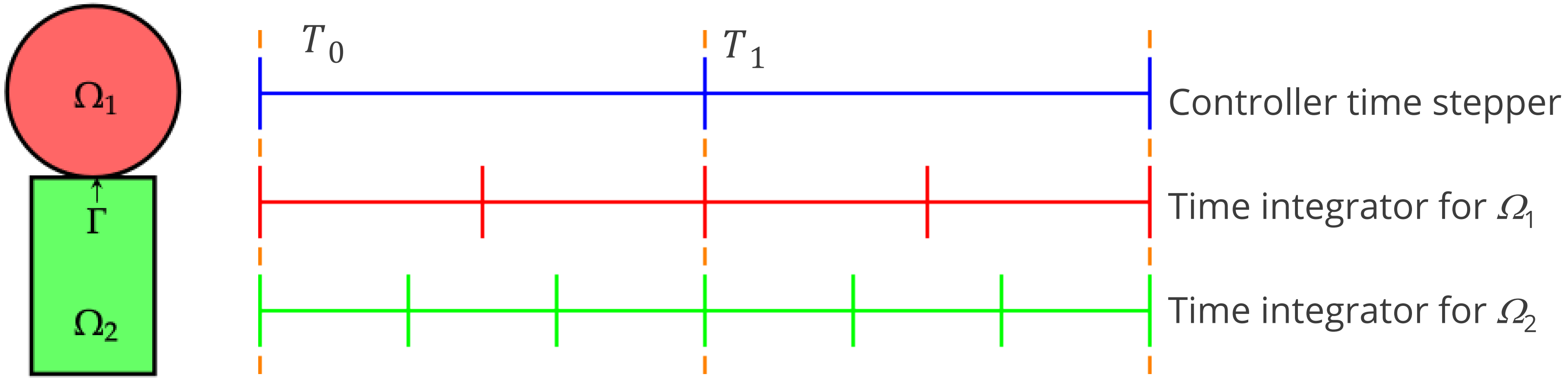
Algorithm for Dynamics and Schwarz Contact



Controller time stepper: *defines global ΔTs at which subdomains are synchronized*

Can use ***different integrators*** with ***different time steps*** within each domain!

Algorithm for Schwarz Contact



Step 0: Initialize $i = 0$ (controller time index).

Step 1: Advance Ω_1 solution from time T_i to time T_{i+1} using time-stepper in Ω_1 with time-step Δt_1, using solution in Ω_2 interpolated to Γ to apply a DBC or NBC.

Step 2: Advance Ω_2 solution from time T_i to time T_{i+1} using time-stepper in Ω_2 with time-step Δt_2, using solution in Ω_1 interpolated to Γ to apply a DBC or NBC.

Step 3: Check for convergence at time T_{i+1}.

- If unconverged, return to Step 1.
- If converged, set $i = i + 1$ and return to Step 1.

Formulation of the Solid Mechanics Problem

Potential Energy: $T(\dot{\varphi}) := \frac{1}{2}\int_{\Omega} \rho_0 \dot{\varphi} \cdot \dot{\varphi} \ \mathrm{d}V,$

Kinetic Energy: $V(\varphi) := \int_{\Omega} A(\boldsymbol{F}, \boldsymbol{Z}) \ \mathrm{d}V - \int_{\Omega} \rho_0 \boldsymbol{B} \cdot \varphi \ \mathrm{d}V - \int_{\partial_{\boldsymbol{T}}\Omega} \boldsymbol{T} \cdot \varphi \ \mathrm{d}S,$

Lagrangian: $L(\varphi, \dot{\varphi}) := T(\dot{\varphi}) - V(\varphi),$

Action functional: $S[\varphi] := \int_I L(\varphi, \dot{\varphi}) \ \mathrm{d}t.$

Euler-Lagrange Equations:

$$\begin{aligned} \mathrm{Div}\, \boldsymbol{P} + \rho_0 \boldsymbol{B} &= \rho_0 \ddot{\varphi} \quad \text{in} \quad \Omega \times I, \\ \varphi(\boldsymbol{X}, t_0) &= \boldsymbol{x}_0 \quad \text{in} \quad \Omega, \\ \dot{\varphi}(\boldsymbol{X}, t_0) &= \boldsymbol{v}_0 \quad \text{in} \quad \Omega, \\ \varphi(\boldsymbol{X}, t) &= \chi \quad \text{on} \quad \partial_{\varphi}\Omega \times I, \\ \boldsymbol{P}\boldsymbol{N} &= \boldsymbol{T} \quad \text{on} \quad \partial_{\boldsymbol{T}}\Omega \times I. \end{aligned}$$

Traditional Formulation of the Contact Problem

Indicator function for admissible set:

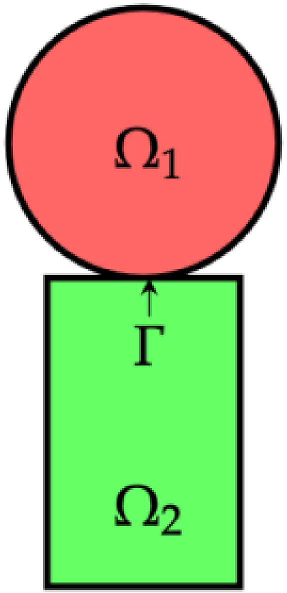
$$I_{\mathcal{C}}(\boldsymbol{\varphi}) := \begin{cases} 0, & \text{if } \boldsymbol{\varphi} \in \mathcal{C}, \\ \infty, & \text{if } \boldsymbol{\varphi} \notin \mathcal{C}, \end{cases}$$

Augmented potential energy with contact constraint :

$$V(\boldsymbol{\varphi}) := \int_{\Omega} A(\boldsymbol{F}, \boldsymbol{Z}) \ \mathrm{d}V - \int_{\Omega} \rho_0 \boldsymbol{B} \cdot \boldsymbol{\varphi} \ \mathrm{d}V + \int_{\Omega} I_{\mathcal{C}}(\boldsymbol{\varphi}) \ \mathrm{d}V - \int_{\partial_{\boldsymbol{T}}\Omega} \boldsymbol{T} \cdot \boldsymbol{\varphi} \ \mathrm{d}S.$$

- The contact constraint is enforced strictly or approximately.
- Lagrange multiplier methods enforce it strictly.
- Penalty methods enforce it approximately.

Schwarz Formulation of the Contact Problem



$$
\begin{cases}
\operatorname{Div} \boldsymbol{P}^{(n)} + \rho_0 \boldsymbol{B} = \rho_0 \ddot{\boldsymbol{\varphi}}^{(n)}, & \text{in} \quad \Omega_1 \times I_k, \\
\boldsymbol{\varphi}^{(n)}(\boldsymbol{X}, t) = \boldsymbol{\chi}, & \text{on} \quad \partial_{\varphi}\Omega_1 \times I_k, \\
\boldsymbol{\varphi}^{(n)}(\boldsymbol{X}, t) = P_{\Omega_2 \to \Gamma}[\boldsymbol{\varphi}^{(n-1)}(\Omega_2, t_k)], & \text{on} \quad \Gamma \times I_k, \\
\boldsymbol{P}^{(n)} \boldsymbol{N} = \boldsymbol{T}, & \text{on} \quad [\partial_{\boldsymbol{T}}\Omega_1 \cup \Gamma] \times I_k,
\end{cases}
$$

$$
\begin{cases}
\operatorname{Div} \boldsymbol{P}^{(n)} + \rho_0 \boldsymbol{B} = \rho_0 \ddot{\boldsymbol{\varphi}}^{(n)}, & \text{in} \quad \Omega_2 \times I_k, \\
\boldsymbol{\varphi}^{(n)}(\boldsymbol{X}, t) = \boldsymbol{\chi}, & \text{on} \quad [\partial_{\varphi}\Omega_2 \cup \Gamma] \times I_k, \\
\boldsymbol{P}^{(n)} \boldsymbol{N} = \boldsymbol{T}, & \text{on} \quad \partial_{\boldsymbol{T}}\Omega_2 \times I_k, \\
\boldsymbol{P}^{(n)} \boldsymbol{N} = P_{\Omega_1 \to \Gamma}[\boldsymbol{T}^{(n)}(\Omega_1, t_k)], & \text{on} \quad \Gamma \times I_k,
\end{cases}
$$

- This is the D-N variant of the method, known to converge.
- The D-D, N-N method does not converge in theory.
- But this variant works for contact.

$$
\begin{aligned}
\boldsymbol{\varphi}^{(n)}(\boldsymbol{X}, t_k) &= \boldsymbol{x}_k^{(i)}, \quad \text{in} \quad \Omega_i \\
\dot{\boldsymbol{\varphi}}^{(n)}(\boldsymbol{X}, t_k) &= \boldsymbol{v}_k^{(i)}, \quad \text{in} \quad \Omega_i,
\end{aligned}
$$

Schwarz Algorithm for Contact

$k \leftarrow 0$
repeat ▷ controller time stepper
  Check contact criteria ▷ defined in Section 3.1
  if contact detected **then**
    $\varphi(\Omega, t_k) \leftarrow$ solution of Algorithm 2 in $\Omega \times I_k$ ▷ contact enforcement
  else
    $\varphi(\Omega, t_k) \leftarrow$ solution of (9) in $\Omega \times I_k$ ▷ no contact
  end if
  $k \leftarrow k + 1$
until $k = N$ ▷ N is the total number of steps

Algorithm 1: Full simulation workflow with Schwarz-based contact enforcement for the specific case of two subdomains.

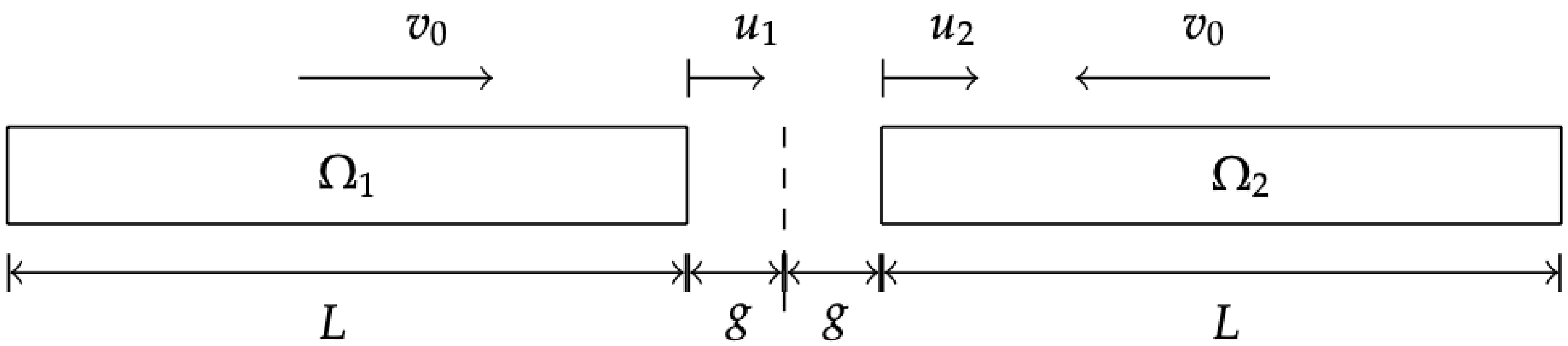
Contact criteria:

- **Overlap**: interpenetration of subdomains
- **Compression**: Positive normal traction
- **Persistence**: Was in contact previous step

$n \leftarrow 1$
repeat ▷ Schwarz loop
  for i from 1 to 2 **do** ▷ subdomain loop
    $\varphi^{(n)}(\Omega_i, t_k) \leftarrow \boldsymbol{x}_k^{(i)}$ ▷ position IC
    $\dot{\varphi}^{(n)}(\Omega_i, t_k) \leftarrow \boldsymbol{v}_k^{(i)}$ ▷ velocity IC
    if $i = 1$ **then** ▷ first subdomain
      $\varphi^{(n)}(\partial_\varphi \Omega_1, I_k) \leftarrow \boldsymbol{\chi}$ ▷ regular Dirichlet BC
      $\varphi^{(n)}(\Gamma, I_k) \leftarrow P_{\Omega_2 \to \Gamma}[\varphi^{(n-1)}(\Omega_1, I_k)]$ ▷ Schwarz Dirichlet BC
      $\boldsymbol{PN} \leftarrow \boldsymbol{T}$ on $[\partial_{\boldsymbol{T}} \Omega_1 \cup \Gamma] \times I_k$ ▷ regular traction BC
      $\varphi(\Omega_1, I_k) \leftarrow$ solution of (14) ▷ solve dynamic problem in $\Omega_1 \times I_k$
    else ▷ second subdomain
      $\varphi^{(n)}([\partial_\varphi \Omega_2 \cup \Gamma], I_k) \leftarrow \boldsymbol{\chi}$ ▷ regular Dirichlet BC
      $\boldsymbol{PN} \leftarrow \boldsymbol{T}$ on $\partial_{\boldsymbol{T}} \Omega_2 \times I_k$ ▷ regular traction BC
      $\boldsymbol{PN} \leftarrow P_{\Omega_1 \to \Gamma}[\boldsymbol{T}^{(n)}(\Omega_2, t_k)]$ ▷ Schwarz traction BC
      $\varphi(\Omega_2, I_k) \leftarrow$ solution of (15) ▷ solve dynamic problem in $\Omega_2 \times I_k$
    end if
  end for
  $n \leftarrow n + 1$
until converged

Algorithm 2: The Schwarz alternating method for contact enforcement during a controller time interval I_k for the specific case of two subdomains.

A Canonical 1D Problem – 2 Colliding Elastic Bars



Position and velocity of left contact point:

$$x(t) = \begin{cases} -g + v_0(t - t_0), & t < t_{\text{imp}}, \\ 0, & t_{\text{imp}} \le t \le t_{\text{rel}}, \\ -v_0(t - t_{\text{rel}}), & t > t_{\text{rel}}, \end{cases} \quad v(t) = \begin{cases} v_0, & t < t_{\text{imp}}, \\ 0, & t_{\text{imp}} \le t \le t_{\text{rel}}, \\ -v_0, & t > t_{\text{rel}}, \end{cases}$$

Contact force: $f_{\text{contact}} = v_0 \sqrt{E\rho} A,$

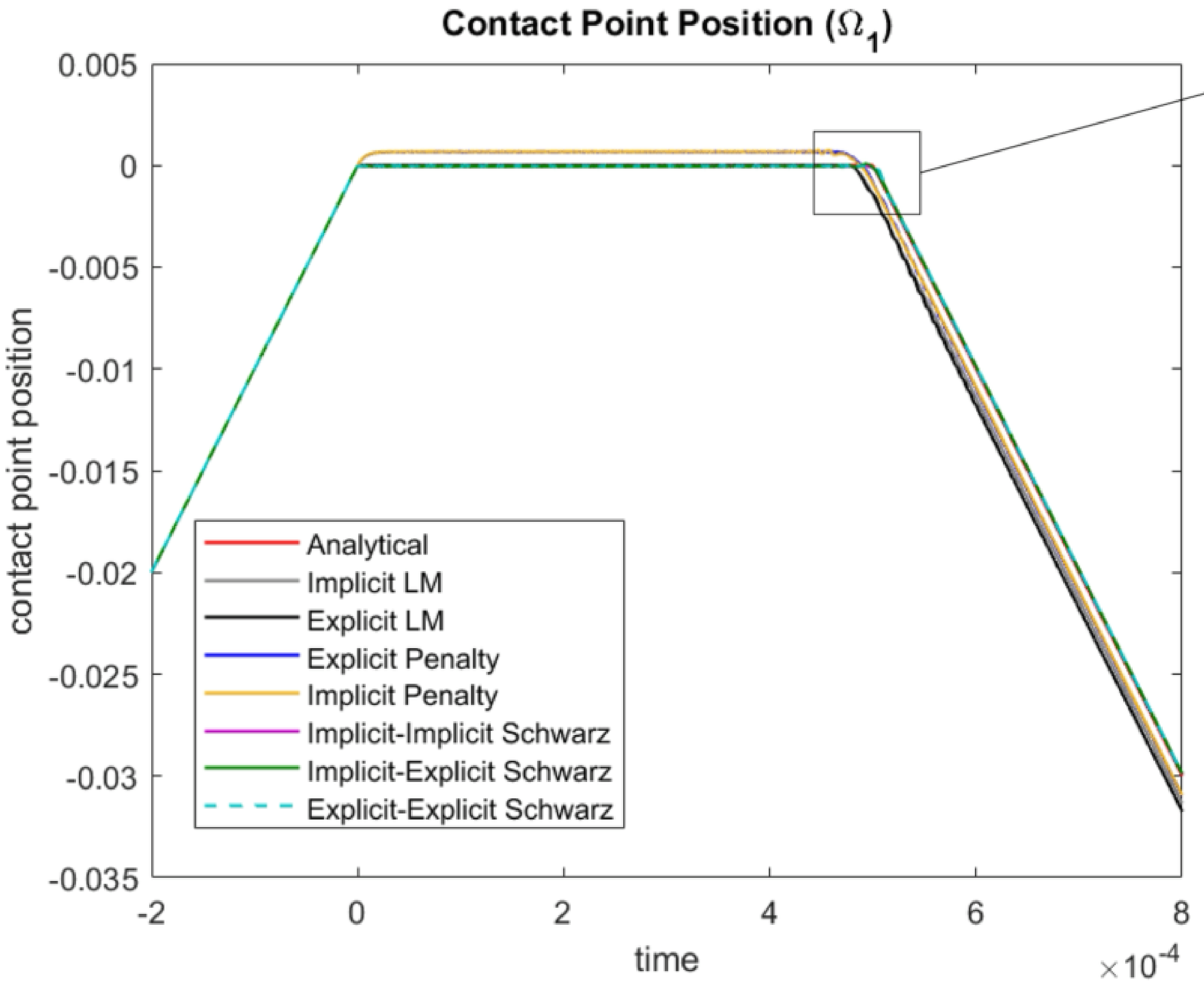
Impact & release times:

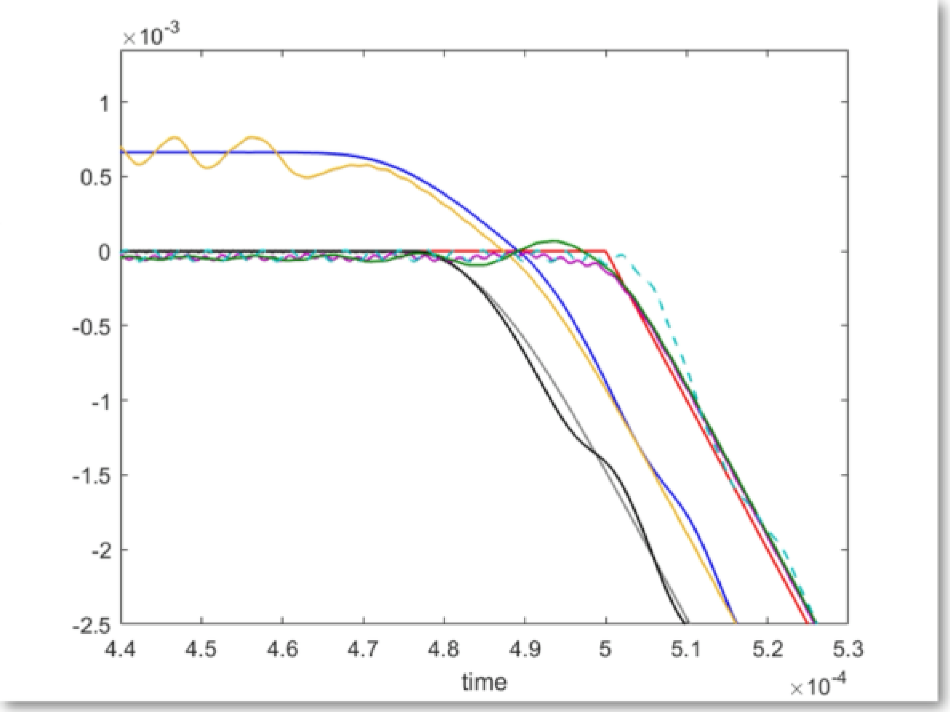
$$t_{\text{imp}} = t_0 + \frac{g}{v_0}, \quad t_{\text{rel}} = t_{\text{imp}} + 2L\sqrt{\frac{\rho}{E}},$$

Comparison of Results

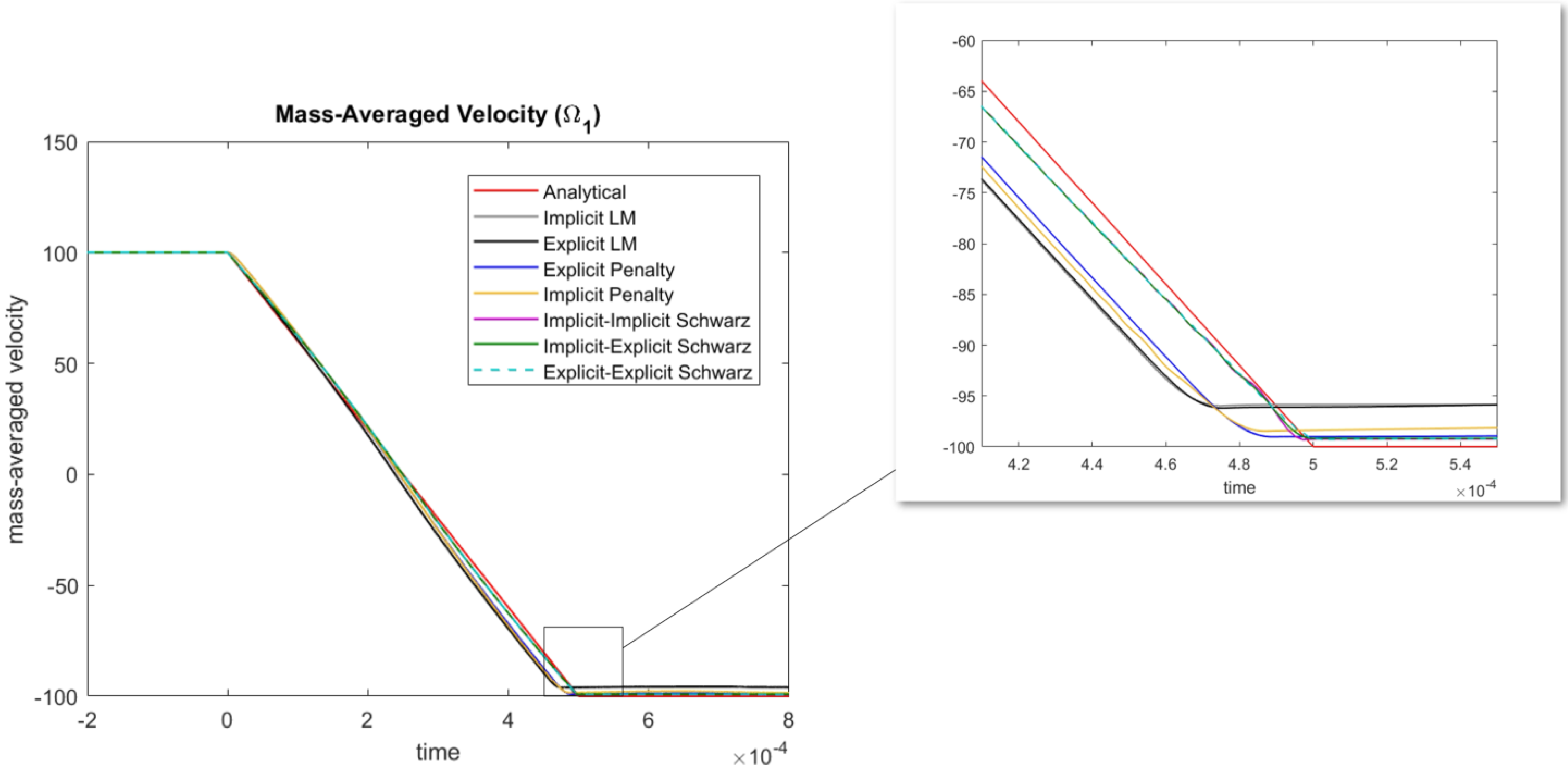
- Analytic solution
- Lagrange multiplier method with implicit time integration
- Lagrange multiplier method with explicit time integration
- Penalty method with implicit time integration
- Penalty method with explicit time integration
- Schwarz method with implicit-implicit integration
- Schwarz method with implicit-explicit time integration
- Schwarz method with explicit-explicit time integration

Contact Point Position

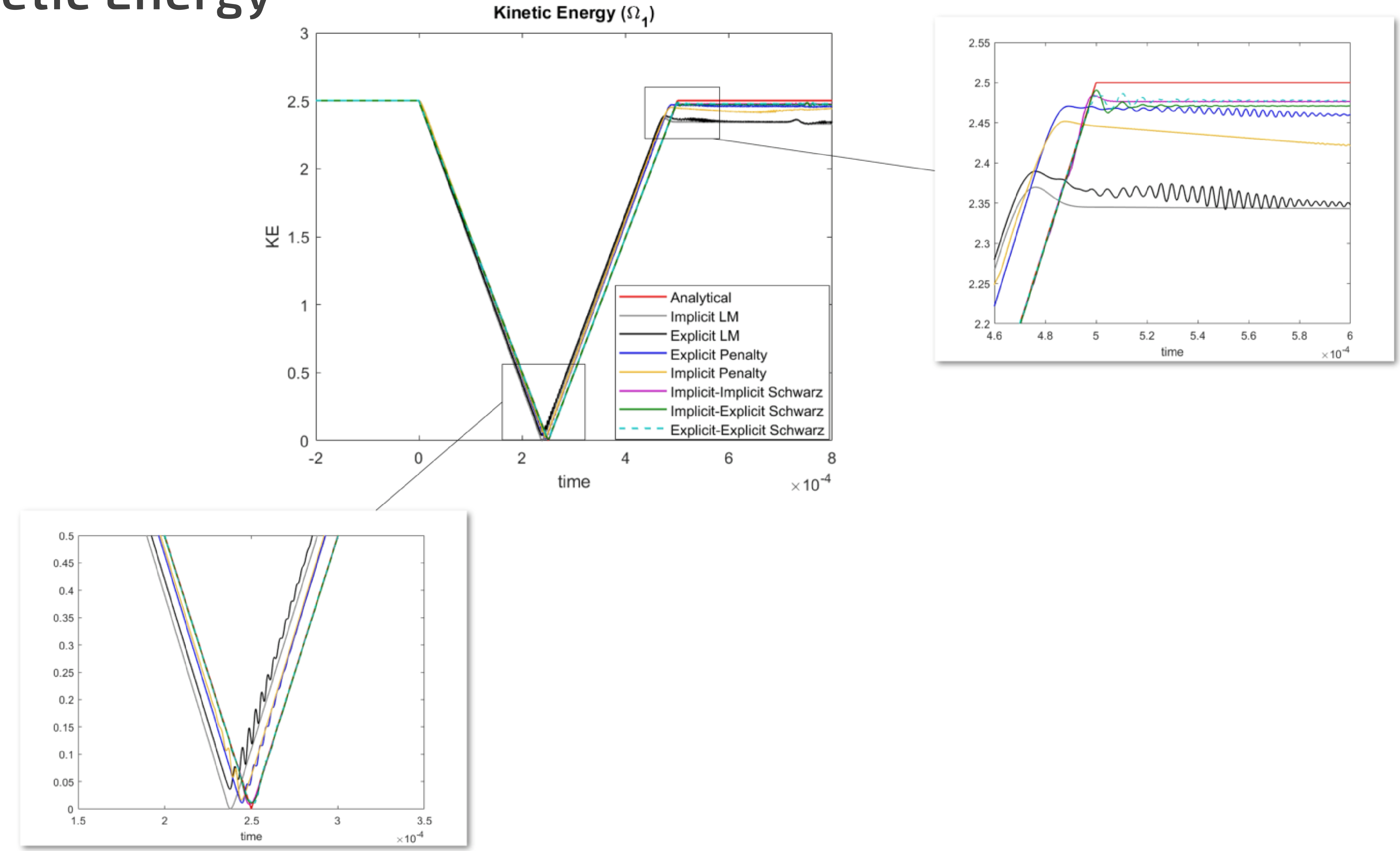




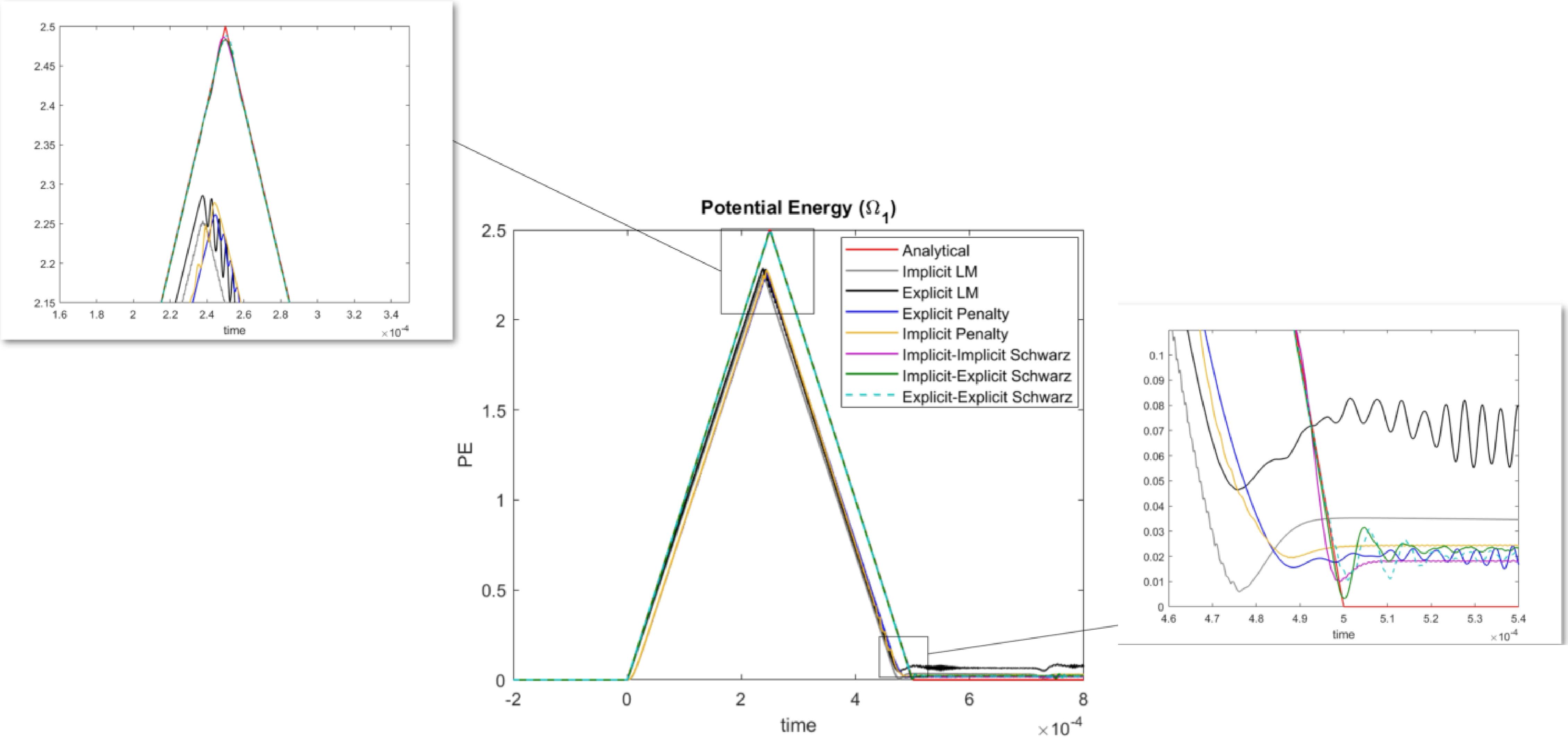
Mass-Averaged Velocity



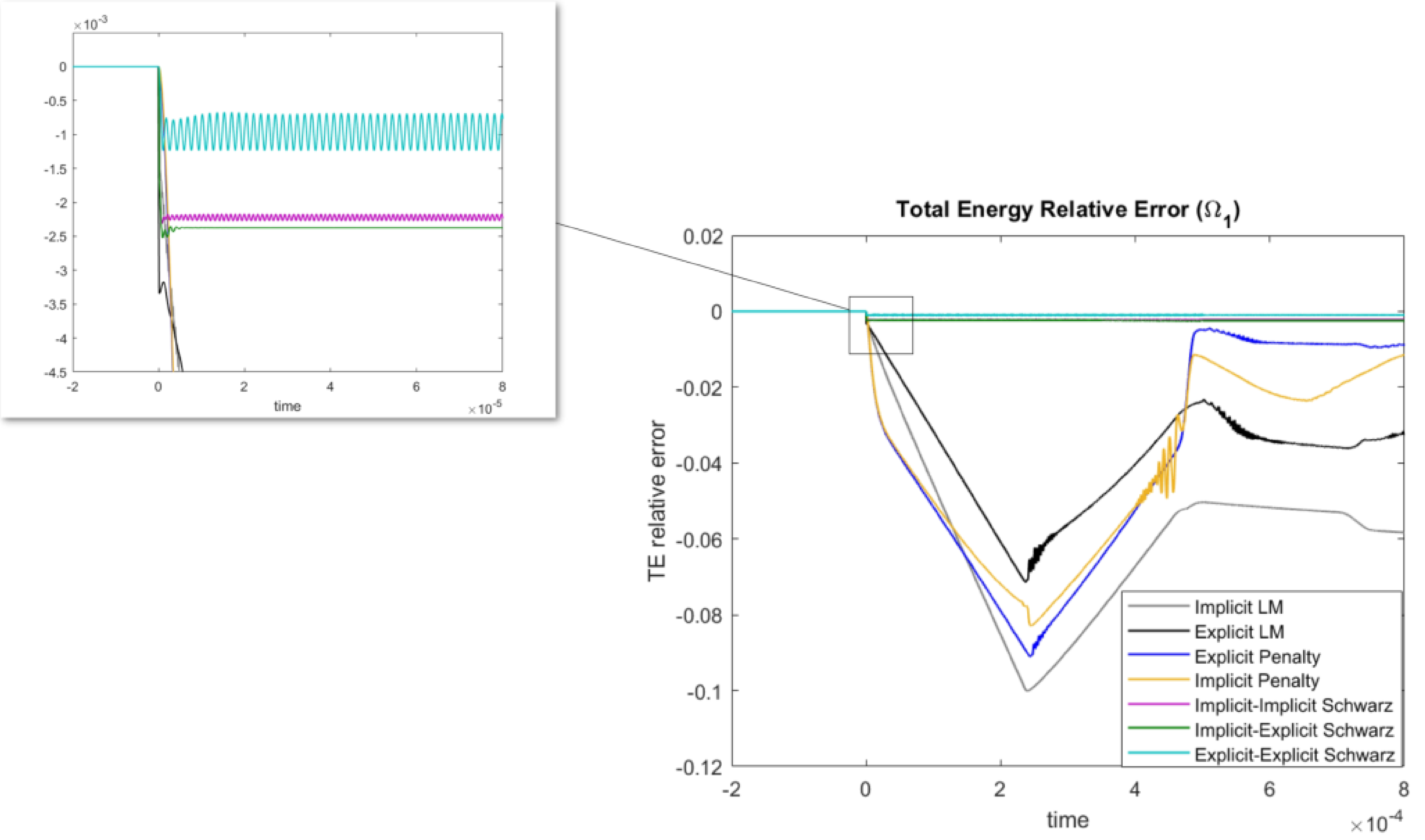
Kinetic Energy



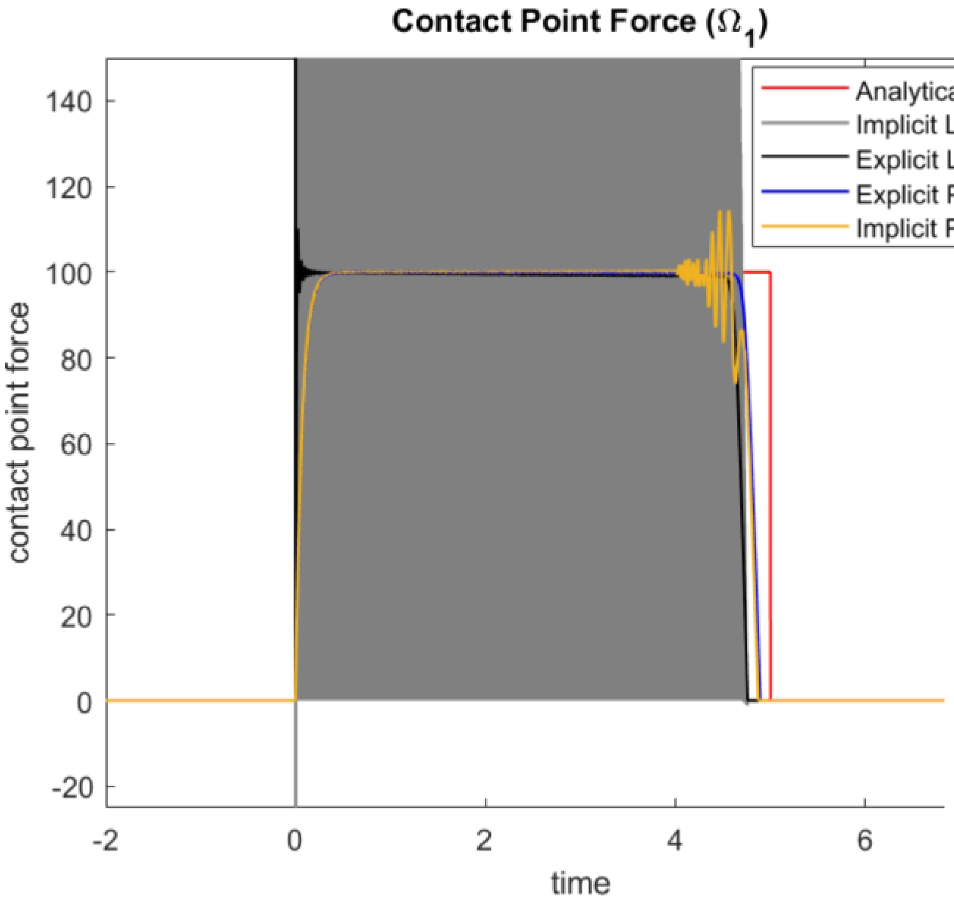
Potential Energy

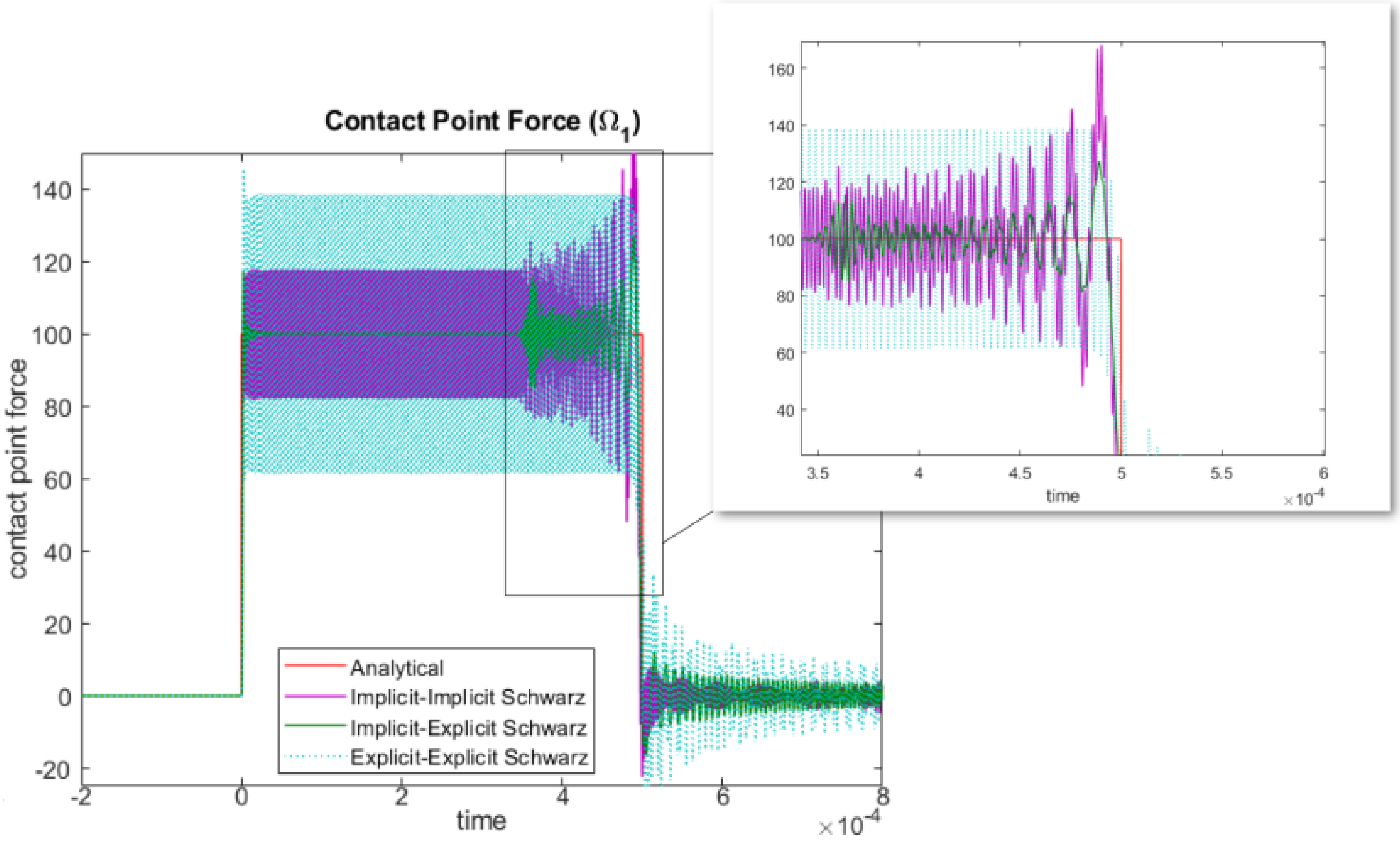


Total Energy

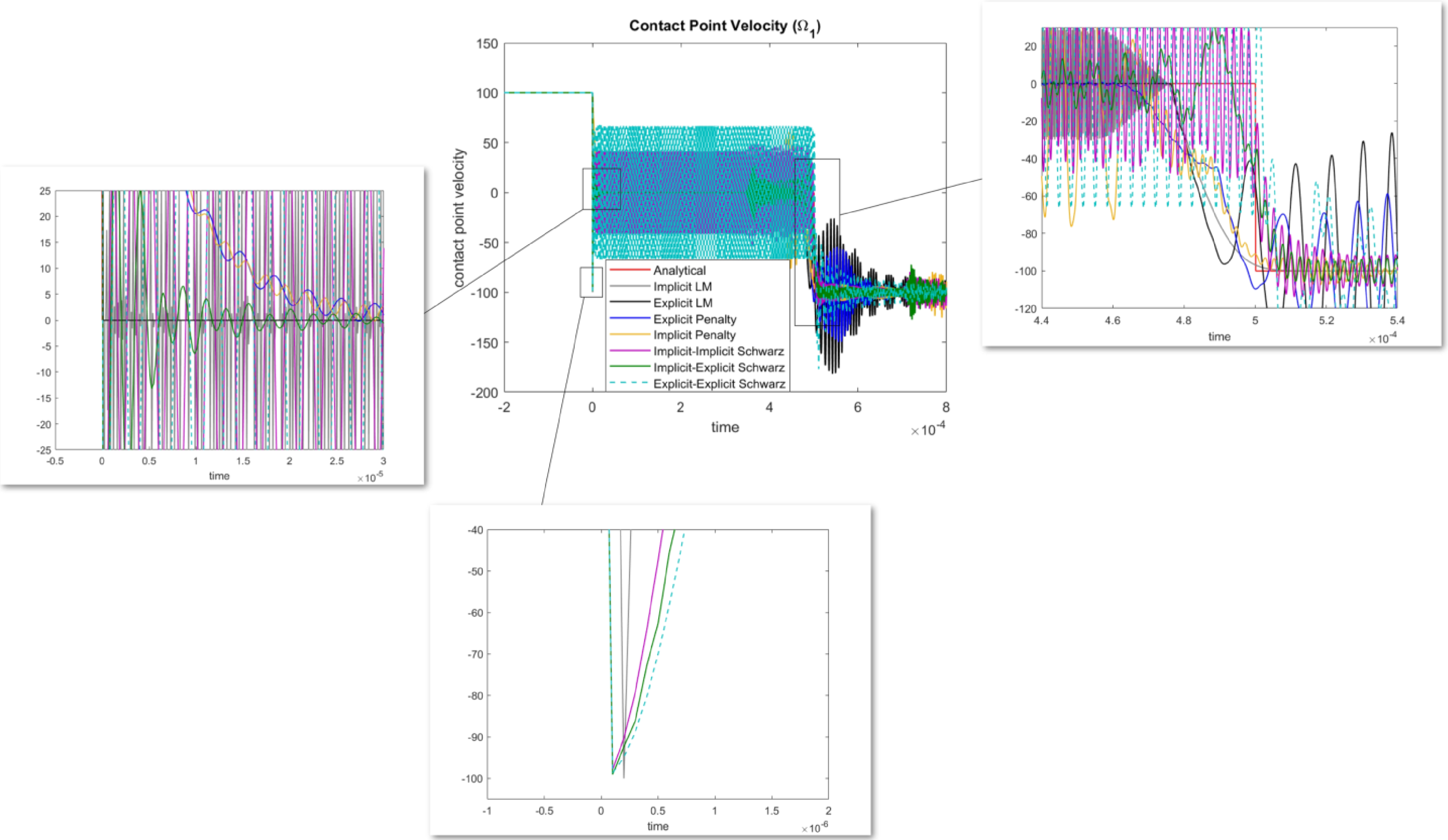


Contact Force



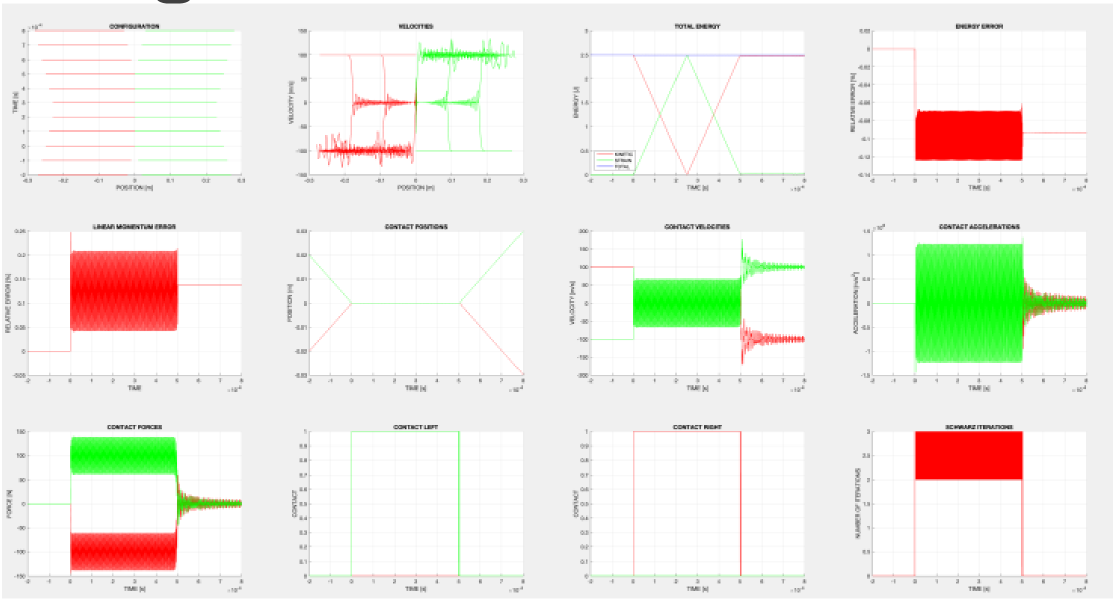


Contact Velocity

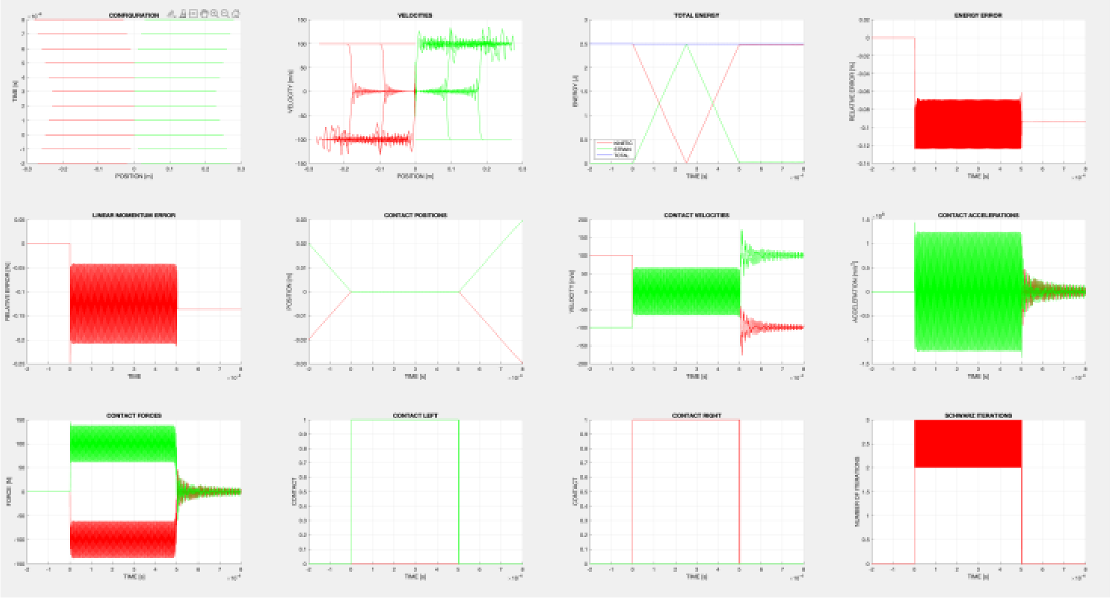


Mitigation of Chatter

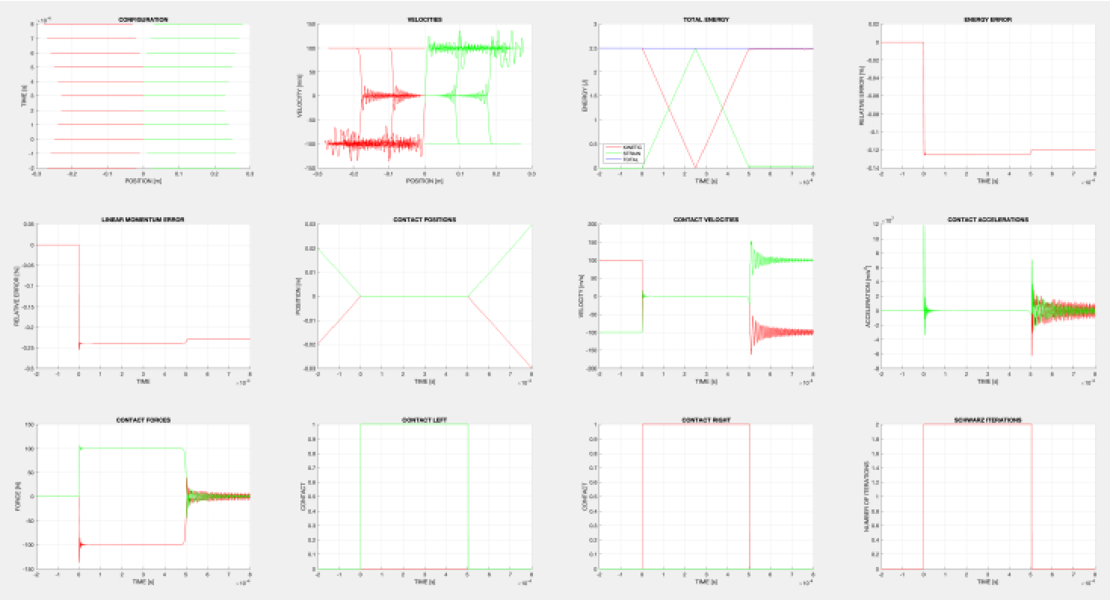
DN



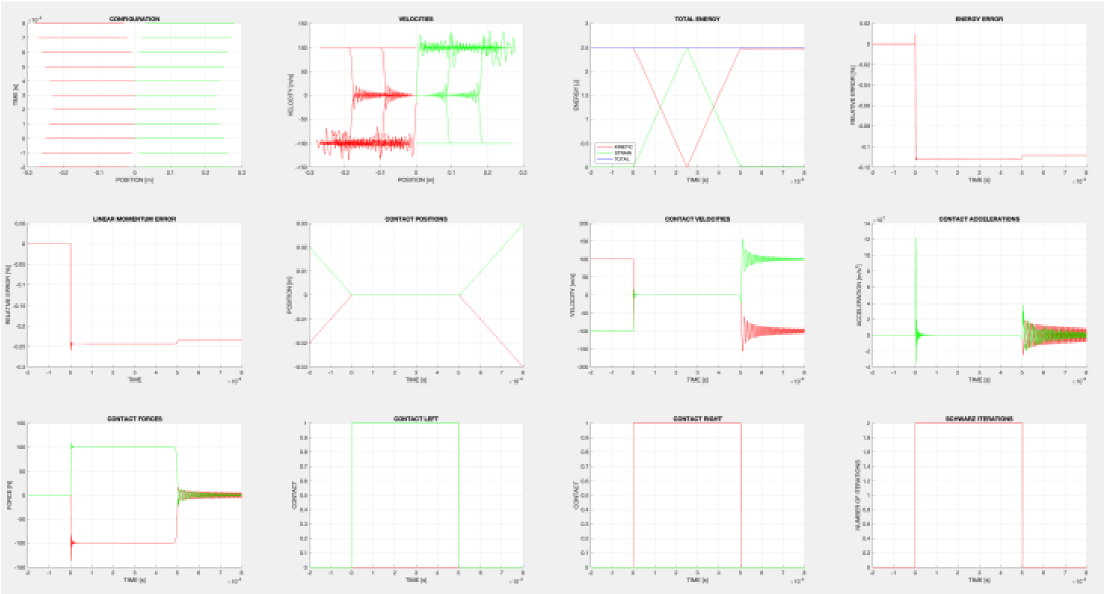
ND



DD



NN



Conclusions and Future Work

- Schwarz contact works surprisingly well
- Very promising results
- Warrants extension to 2D and 3D
- Use specialized integrators for traditional methods
- Very recent chatter mitigation
- Investigate implications of DD-NN variants