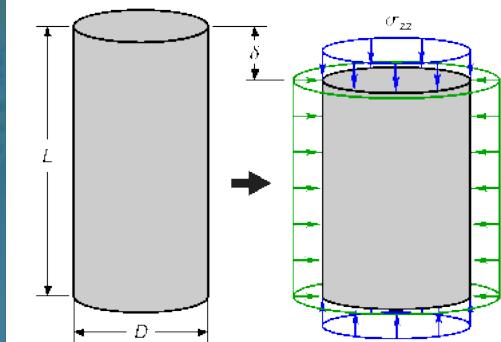
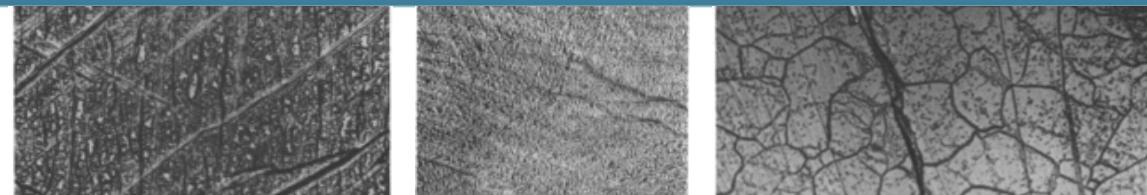




Sandia
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Laboratories

A New Constitutive Model for Rock Salt Viscoplasticity



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56th U.S. Rock Mechanics Symposium



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Background



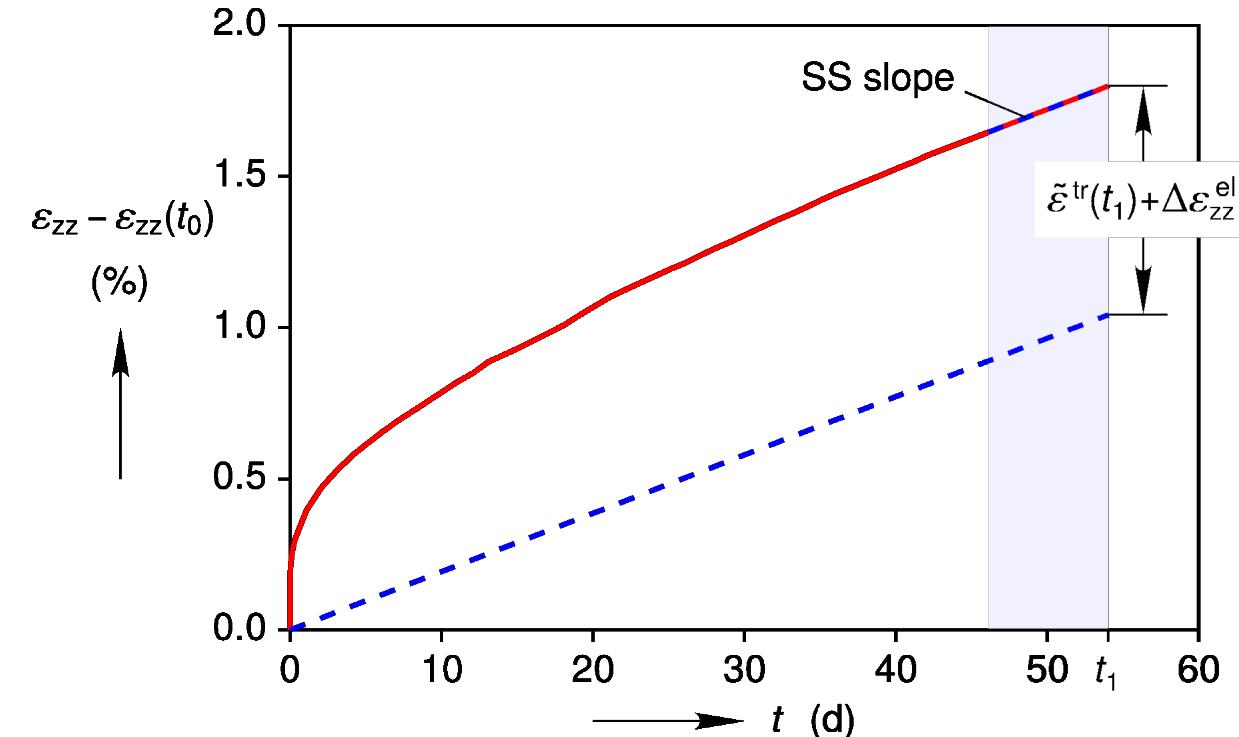
1. Dozens of existing salt constitutive models
 1. Phenomenological to micro-physically based
 2. Simple vs. full featured
2. My model priorities
 1. Phenomenological, yet motivated by micro-physical observations
 2. Conforms to the framework of Rational Thermodynamics
 3. Captures viscoplasticity, damage, and healing
 4. Captures viscoplastic hardening behavior
 1. Hardening transition from low to medium stresses (strain rates)
 2. Hardening transition from medium to high stresses (strain rates)
 3. Re-hardening during non-monotonic loading

Hardening Transition from Low to Medium Stresses (Strain Rates)

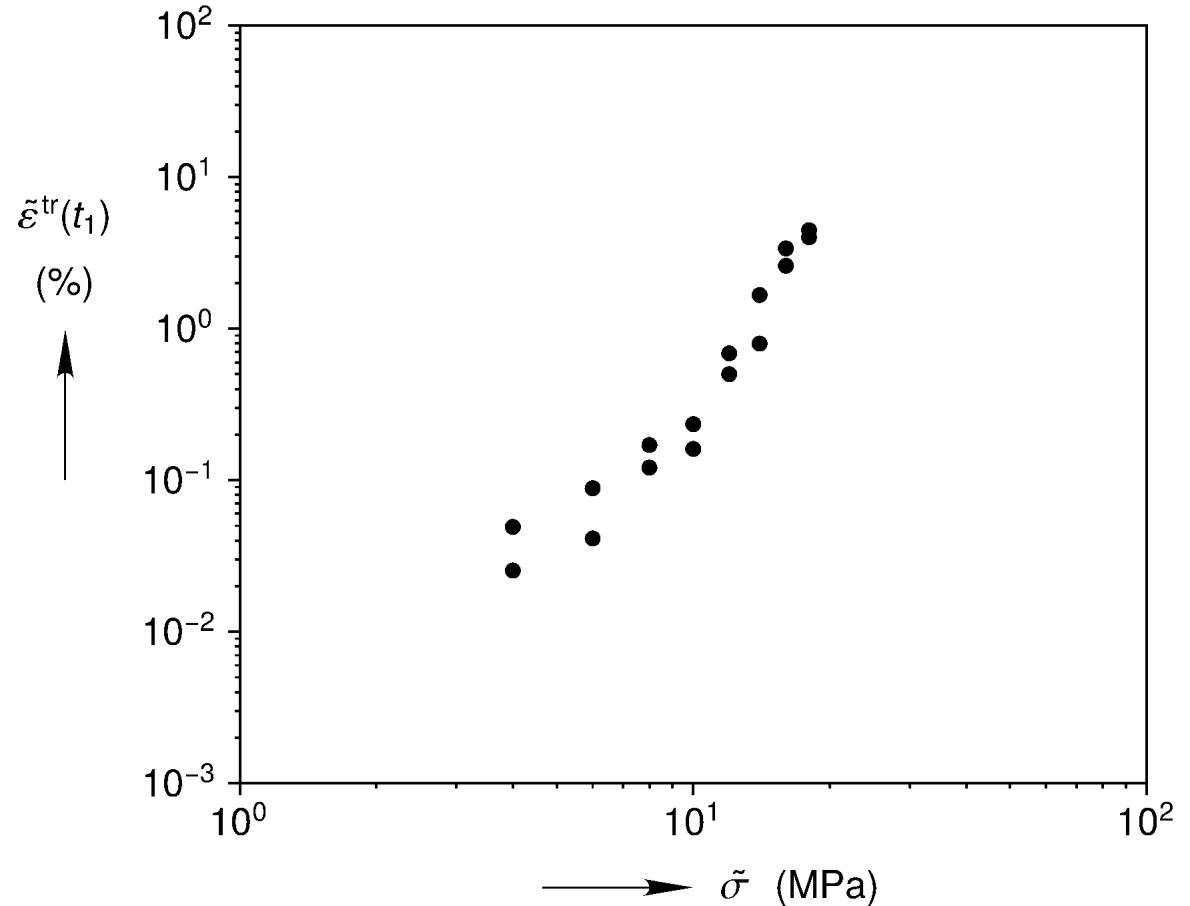


WIPP Salt, Constant Stress, Strain History

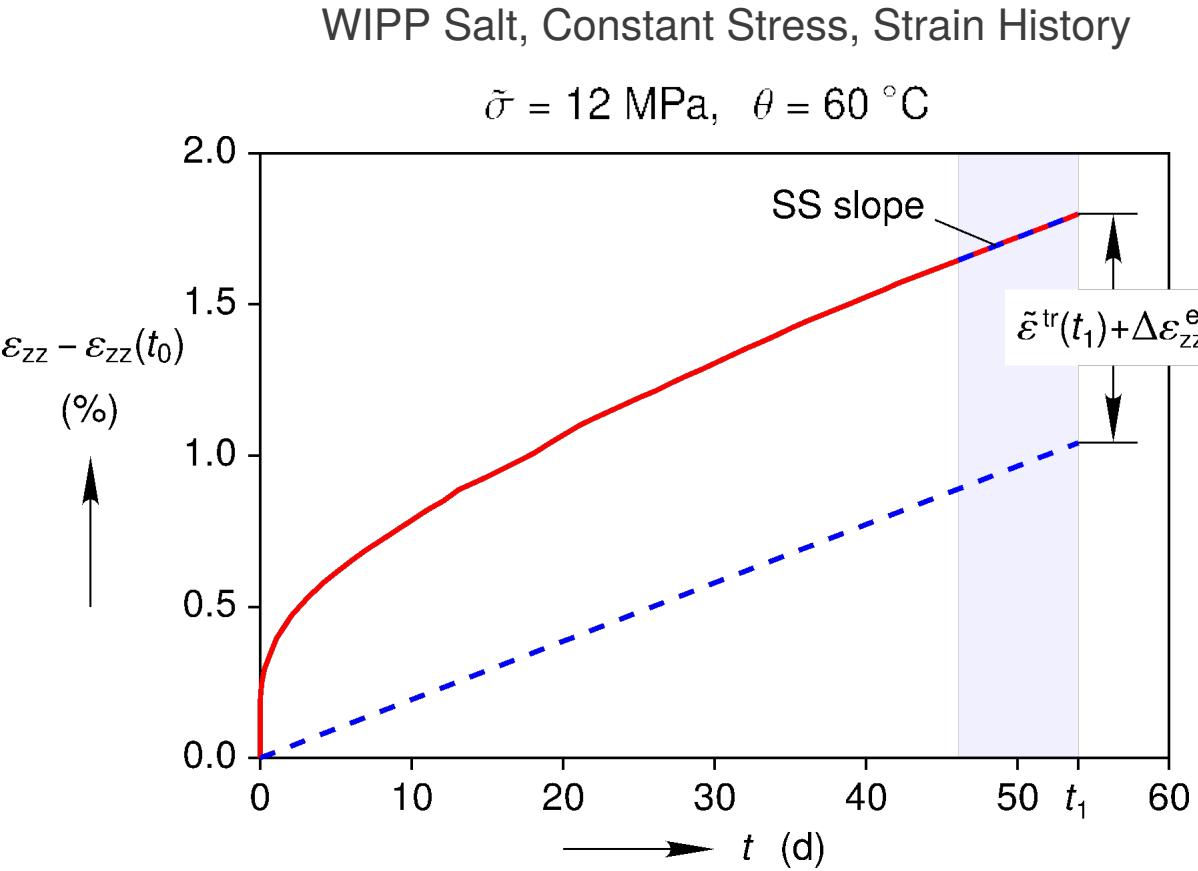
$$\tilde{\sigma} = 12 \text{ MPa}, \theta = 60^\circ\text{C}$$



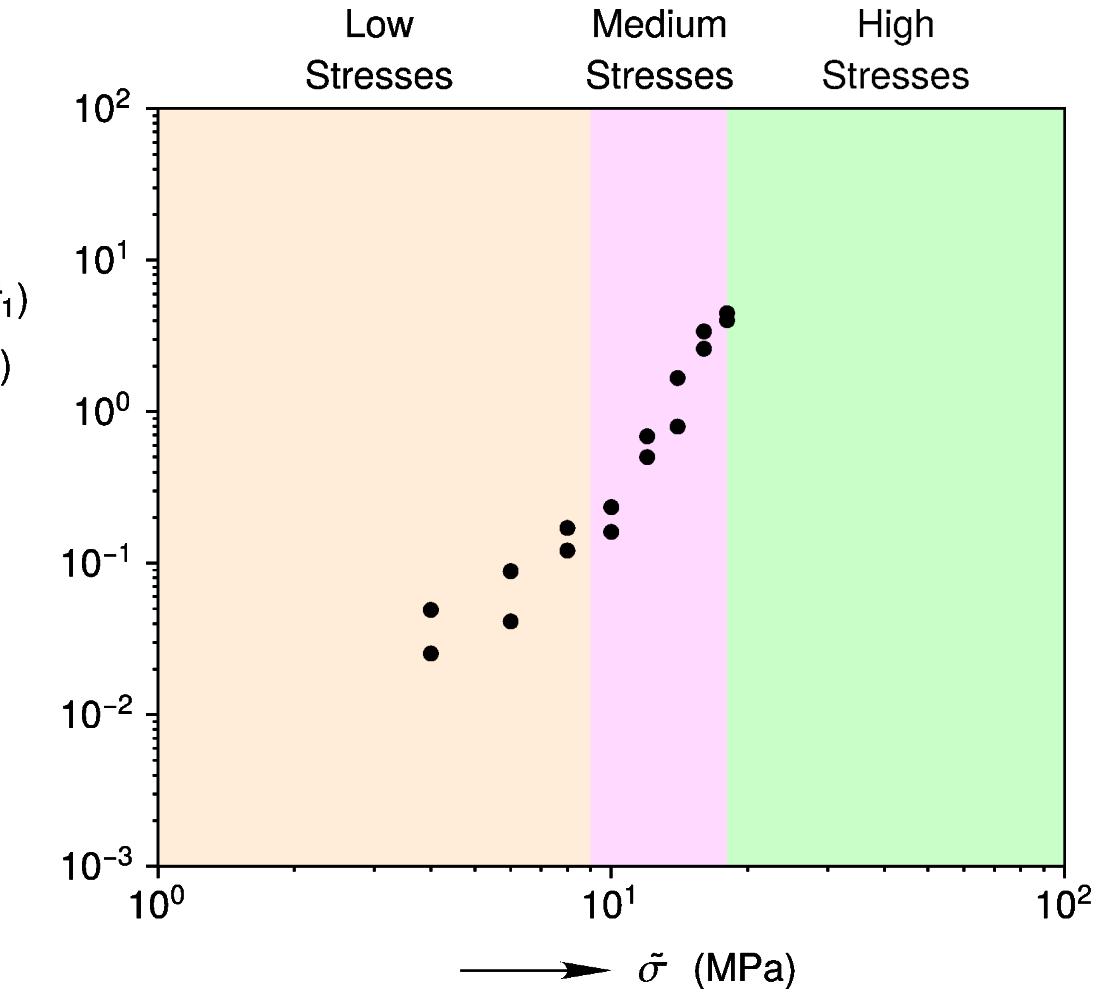
Transient Strain After 50 days at $\theta = 60^\circ\text{C}$



Hardening Transition from Low to Medium Stresses (Strain Rates)



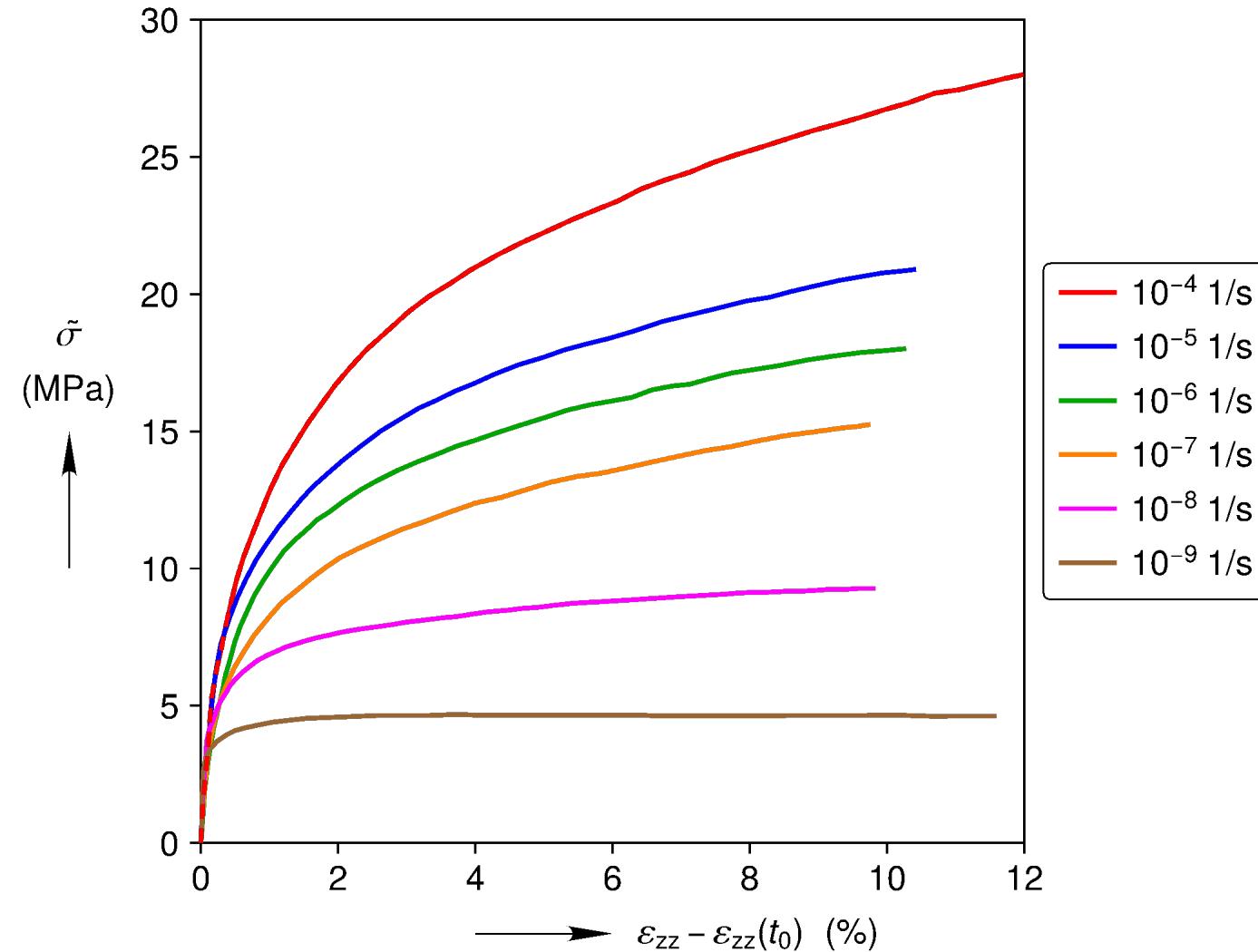
Transient Strain After 50 days at $\theta = 60^\circ\text{C}$



Hardening Transition from Medium to High Strain Rates (Stresses)



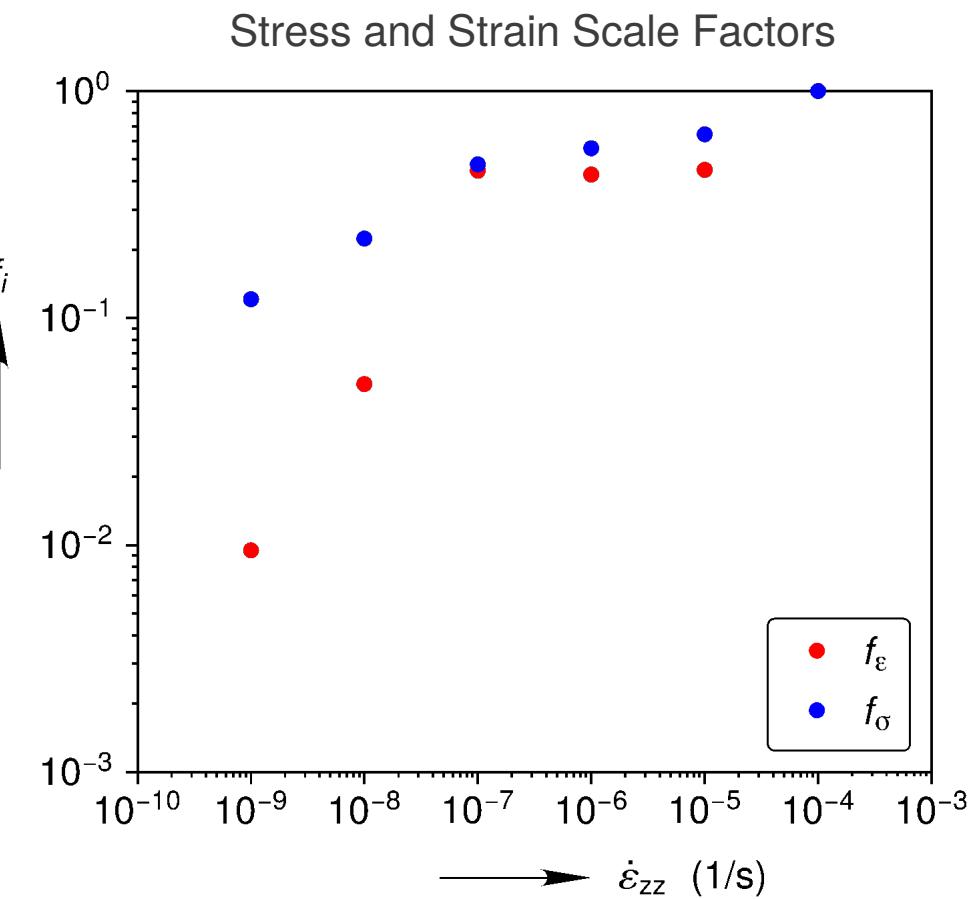
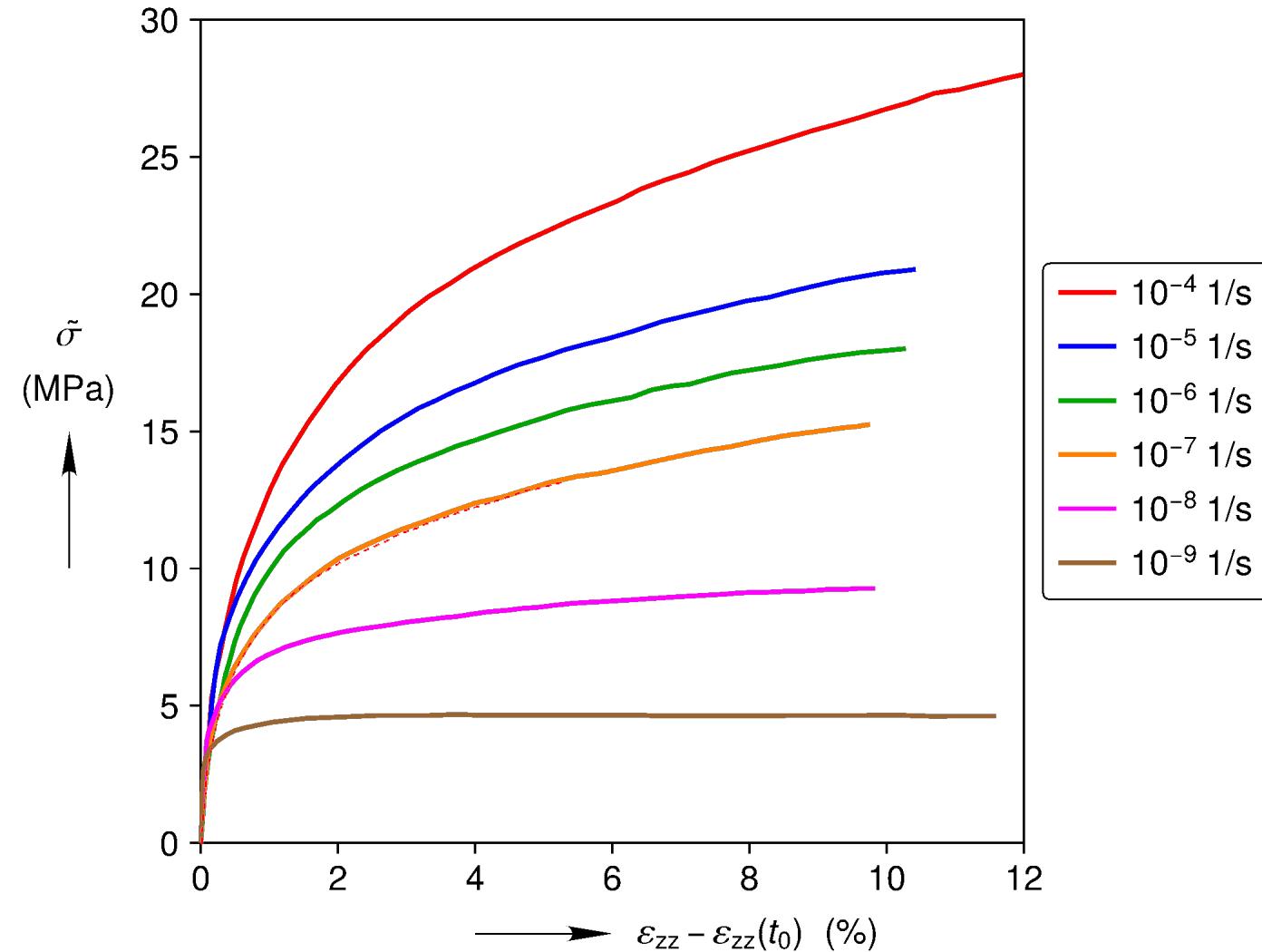
Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$



Hardening Transition from Medium to High Strain Rates (Stresses)



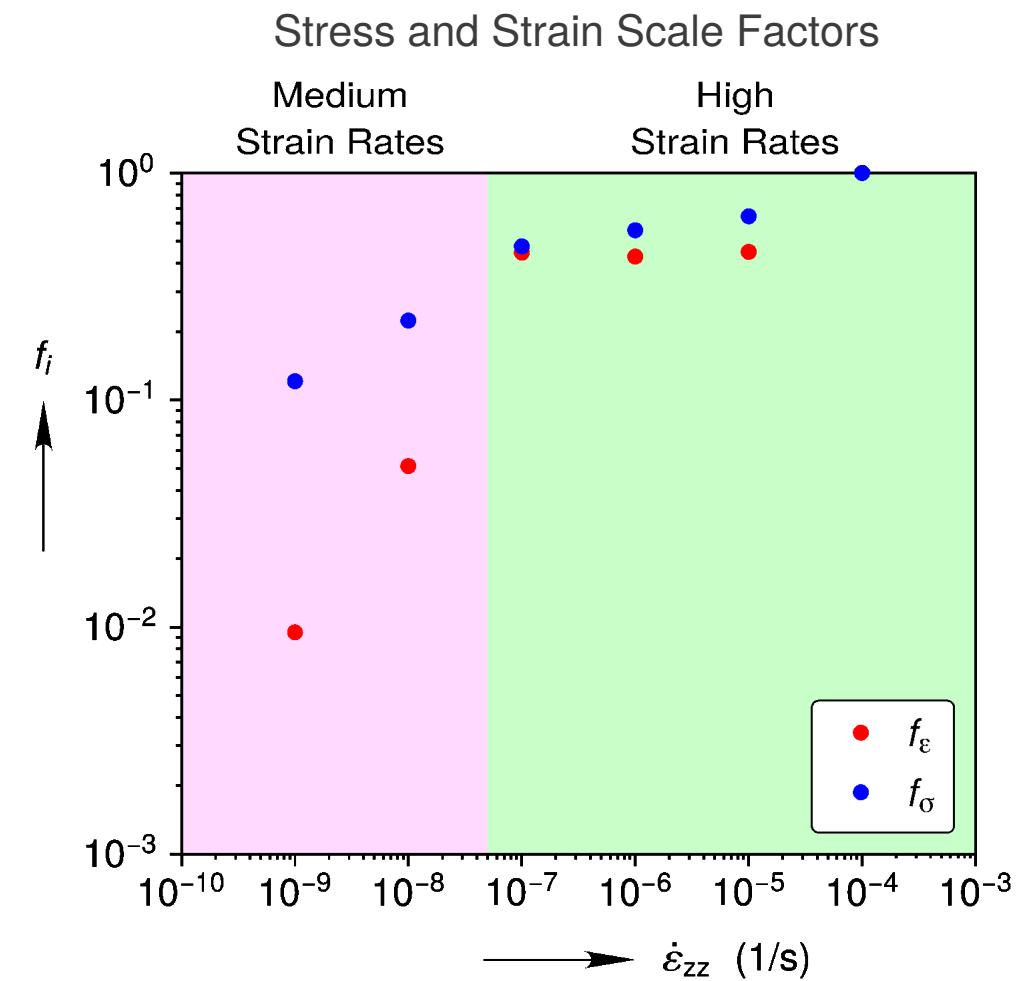
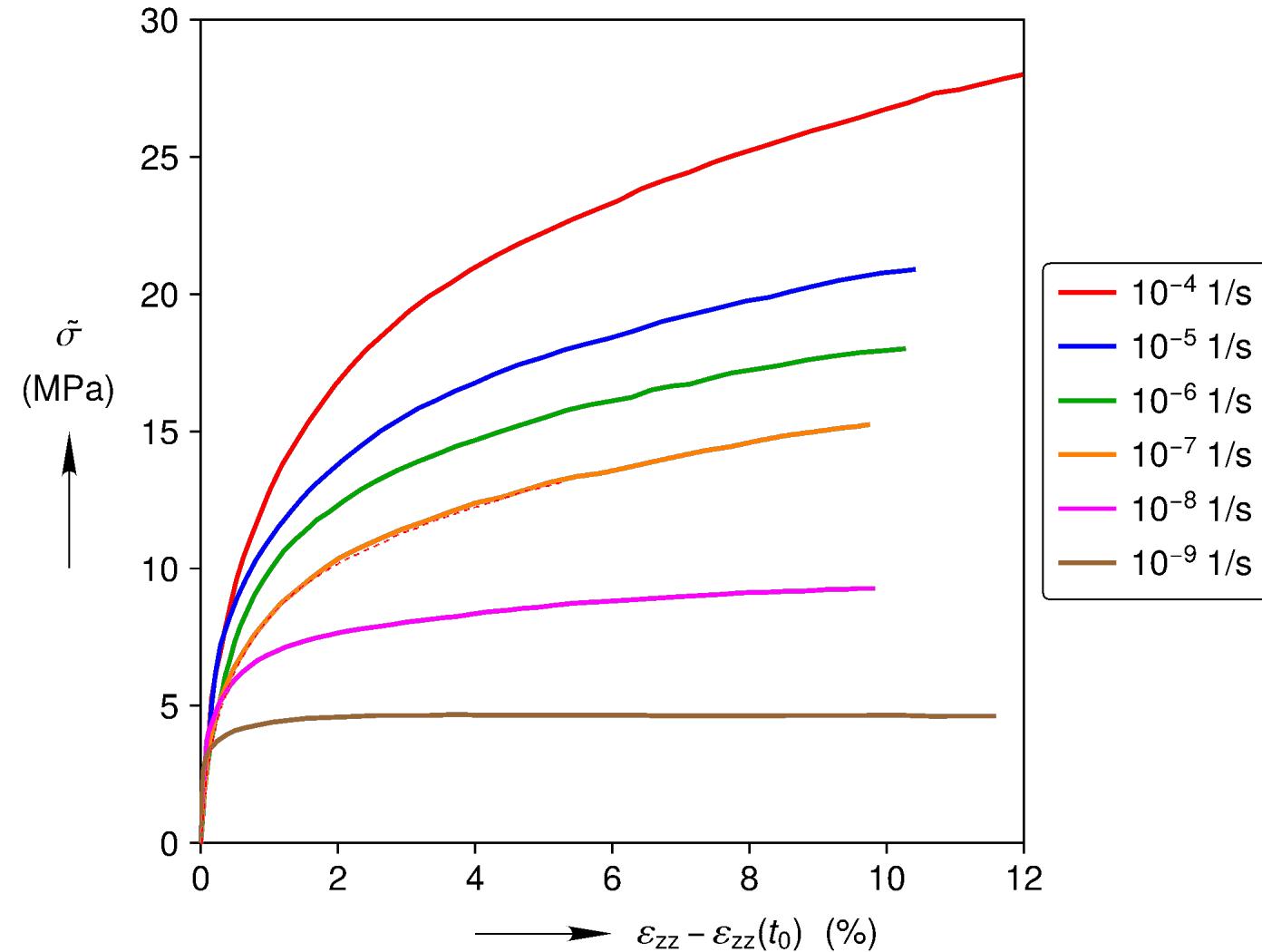
Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$



Hardening Transition from Medium to High Strain Rates (Stresses)



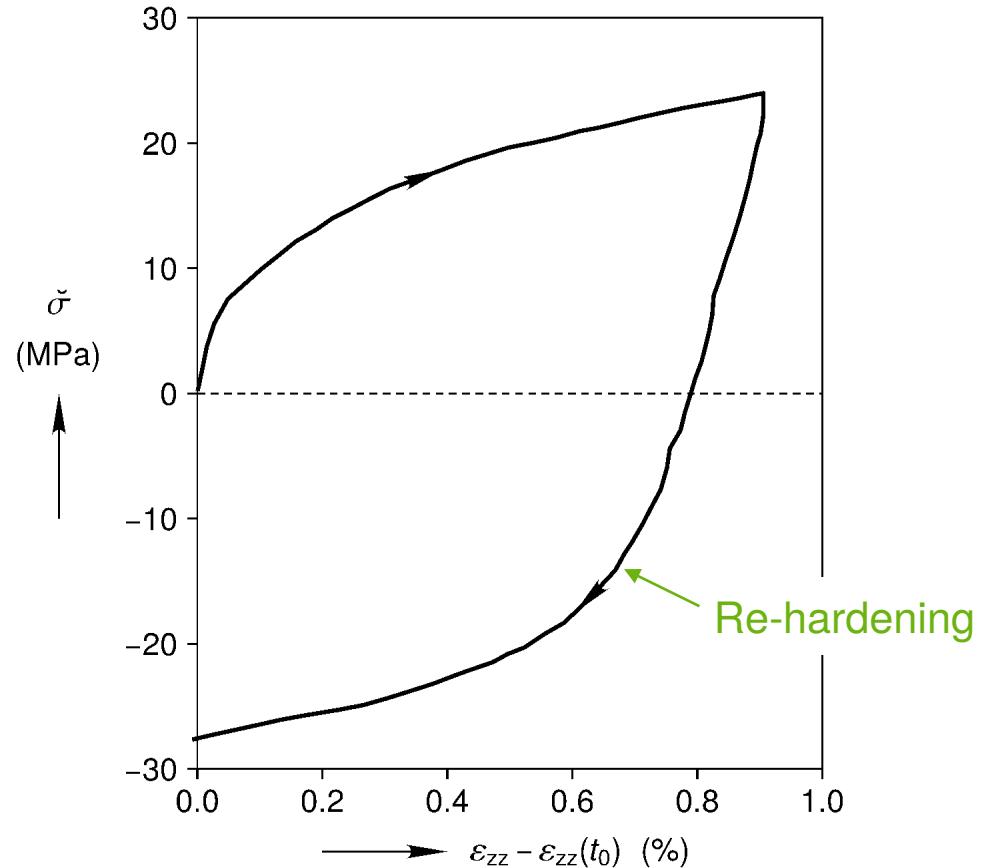
Avery Island Salt, Stress vs. Strain Curves at $\theta = 100^\circ\text{C}$



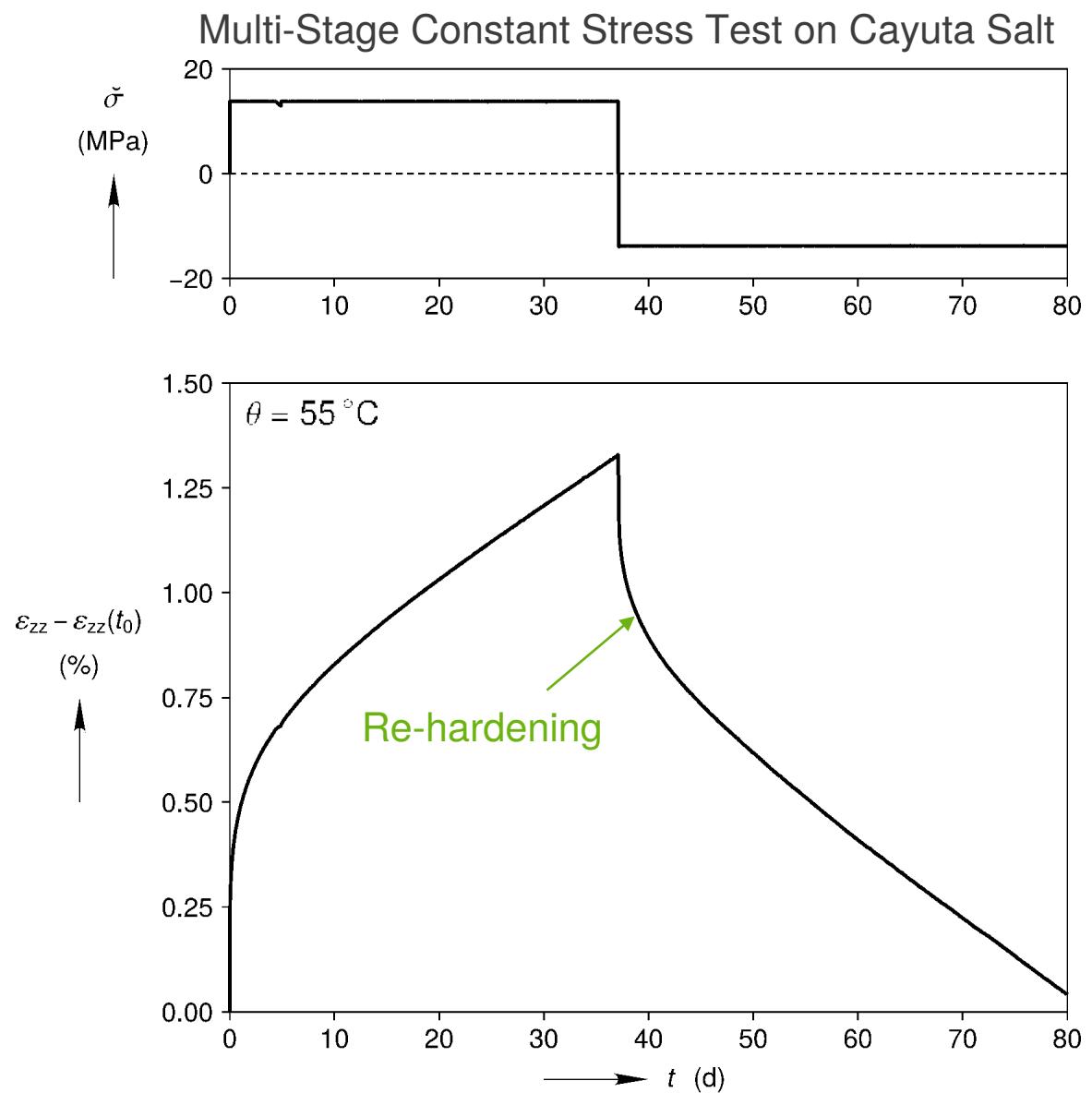
Re-hardening During Non-Monotonic Loading

Constant Strain Rate Test on Artificial Salt

$$\sigma_{rr} = 53 \text{ MPa}, \theta = 293 \text{ K}, \dot{\varepsilon}_{zz} = \pm 3.5 \times 10^{-5} \text{ 1/s}$$



Experimental measurements from Aubertin et al. (1999)



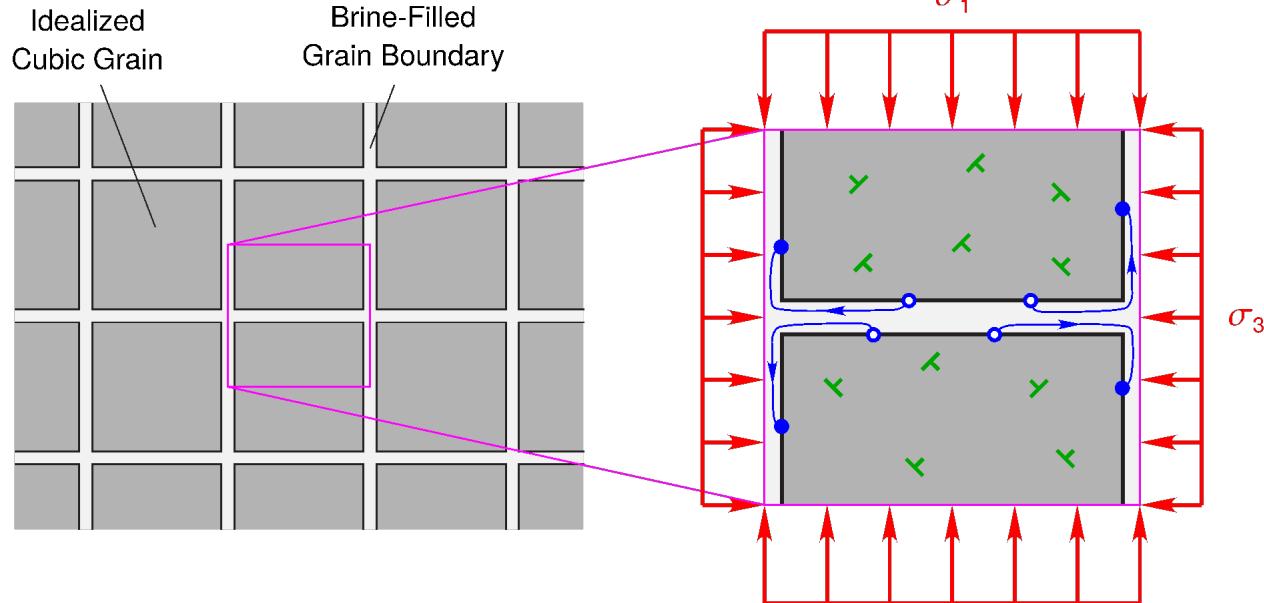
Experimental measurements from: Mellegard et al. (2007)

Model Overview

Viscoplastic Branches



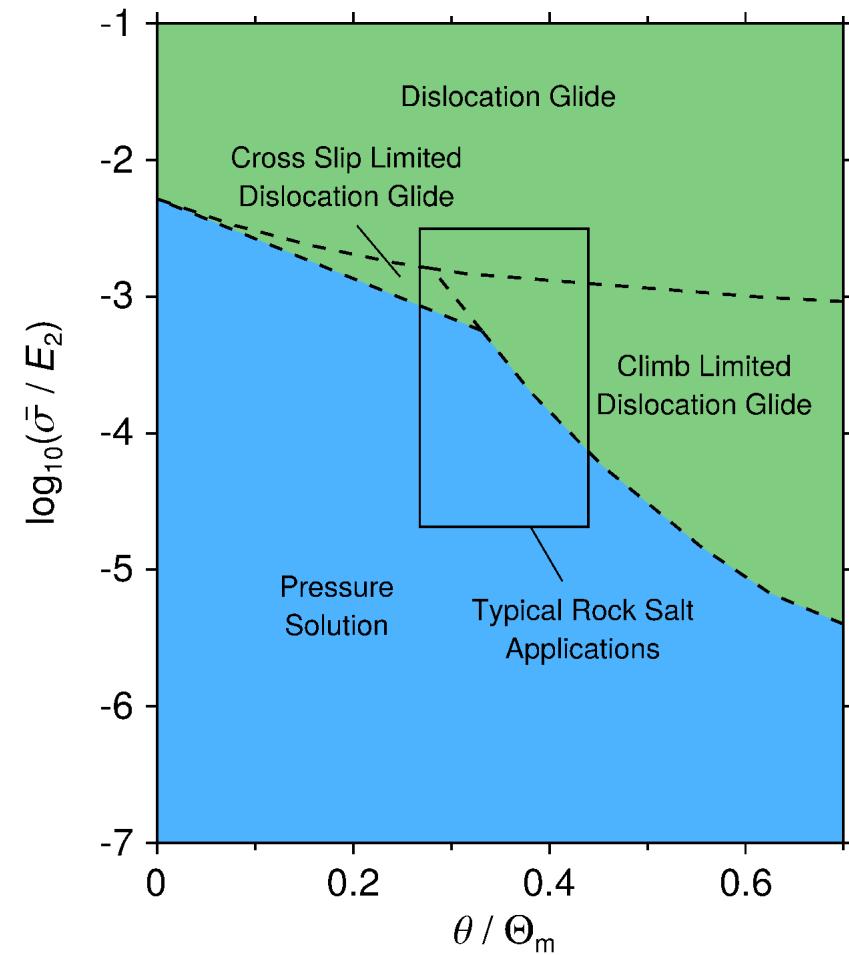
Pressure Solution and Dislocation Glide



Strain Rate Decomposition
(proportional, monotonic, loading)

$$\dot{\varepsilon}^{\text{vp}} = \dot{\varepsilon}^{\text{ps}} + \dot{\varepsilon}^{\text{dg}}$$

A (Rough) Steady-State Deformation Mechanism Map



$$\Theta_m = 1077 \text{ K}$$

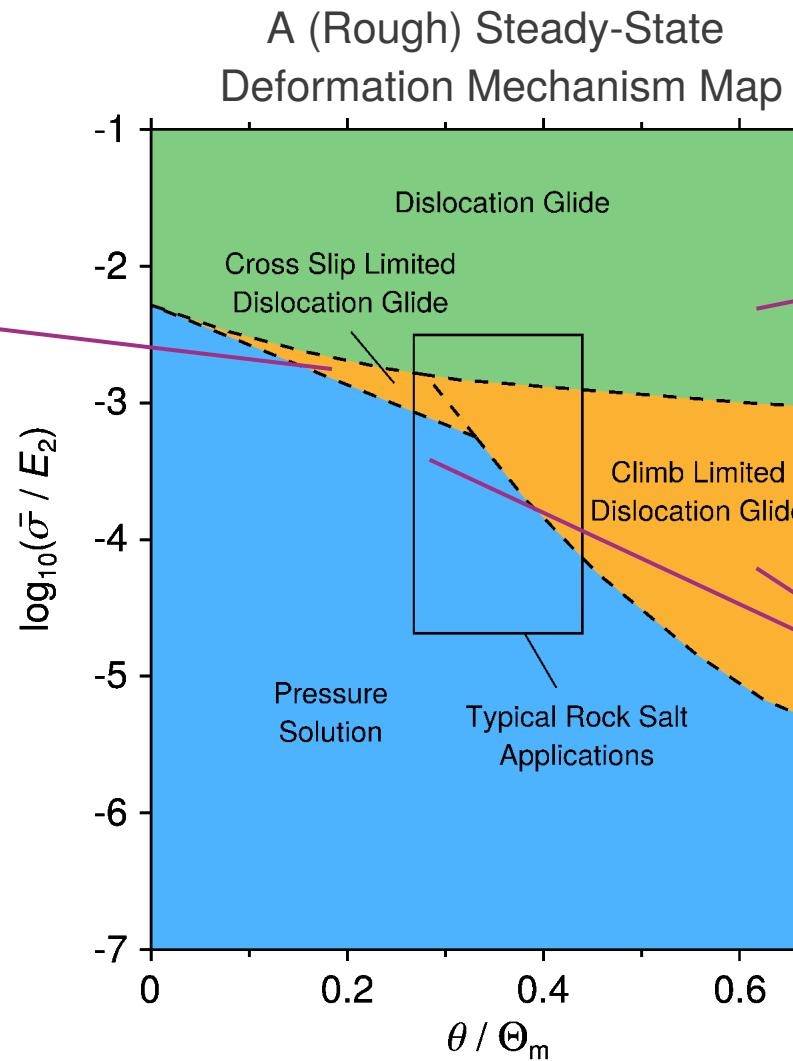
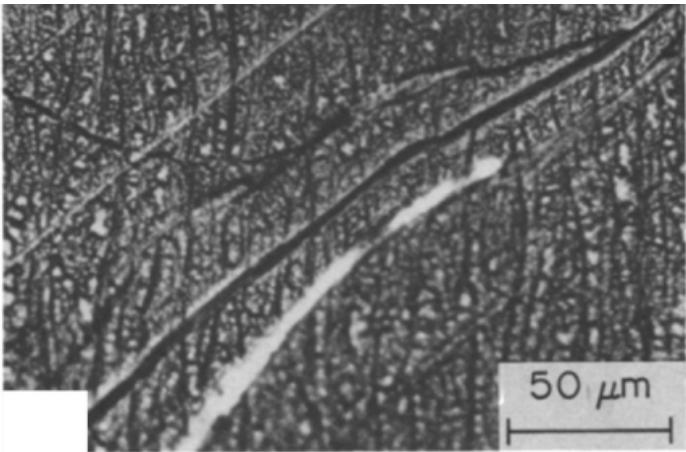
$$E_2 = 10 \text{ GPa}$$

Average grain size = 10 mm

Microstructural Observations

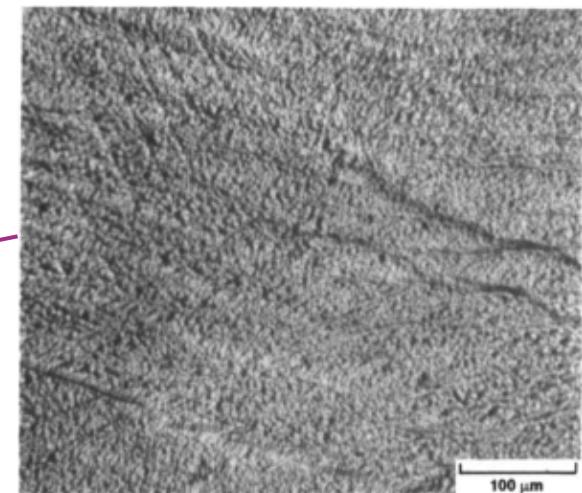


Wavy Slip Bands

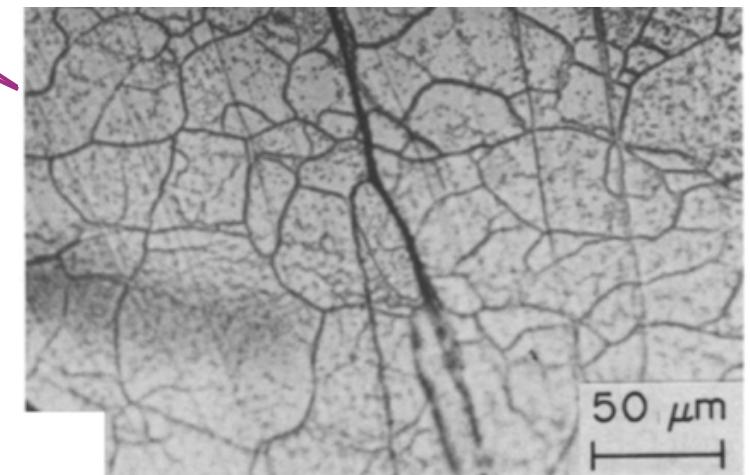


$\Theta_m = 1077$ K
 $E_2 = 10$ GPa
Average grain size = 10 mm

Uniform Dislocation Density



Subgrains



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233–242.

Carter, NL, Horsemann, ST, Russell, JE, and Handin, J (1993). Rheology of rocksalt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

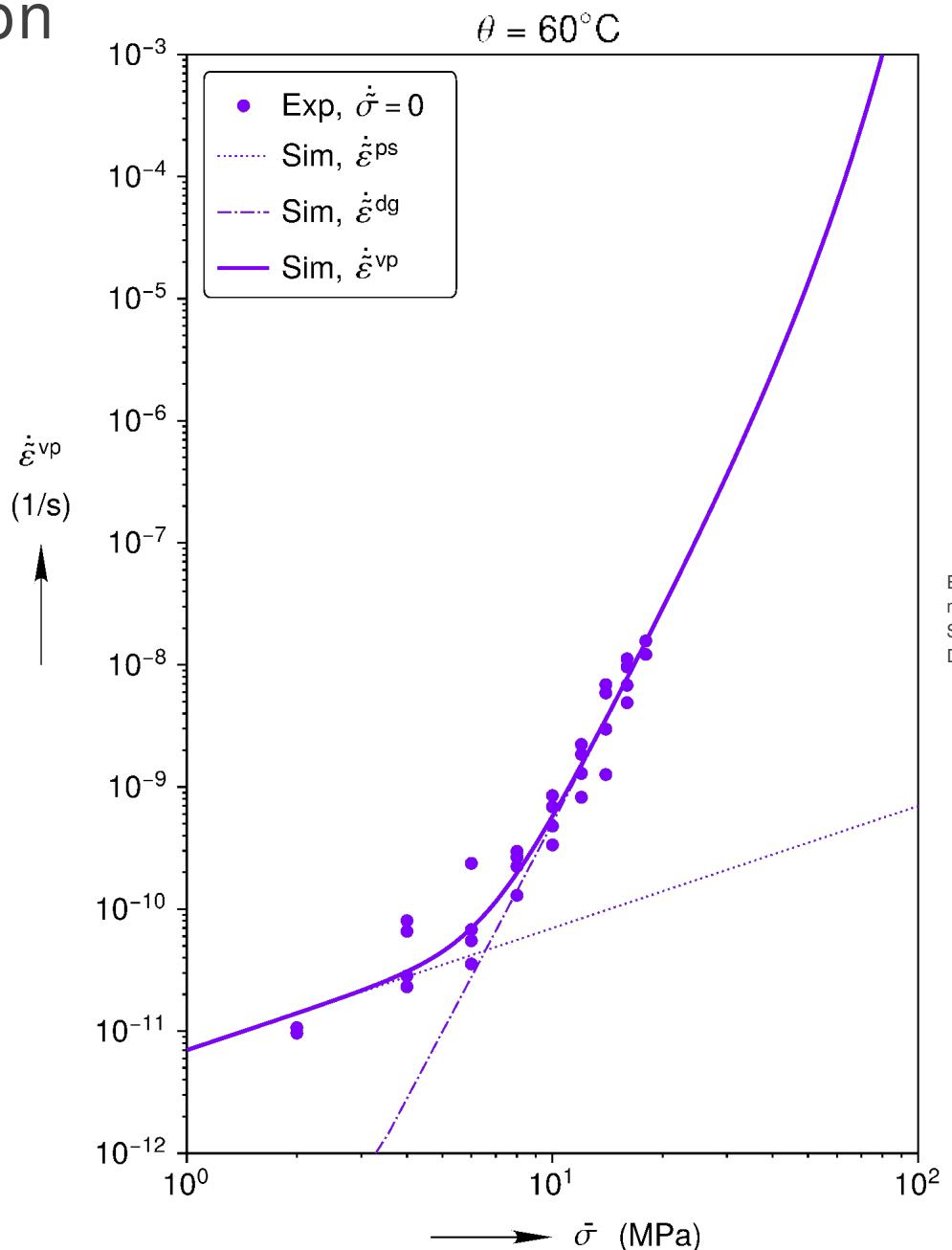
Steady-State Strain Rates

$$\dot{\varepsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\mathcal{E}}}^{\text{dg}} = Y_3 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}}{Y_4}\right) \right]^{Y_5}$$

Spies, C., Schutjens, P., Brzesowsky, R., Peach, C., Liezenberg, J., and Zwart, H. (1990). Experimental determination of constitutive parameters governing creep of rocksalt by pressure solution. In: Geological Society, London, Special Publications 54.1, pp. 215–227.

Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.



Experimental measurements from: Salzer et al. (2015) and Düsterloh et al. (2015).

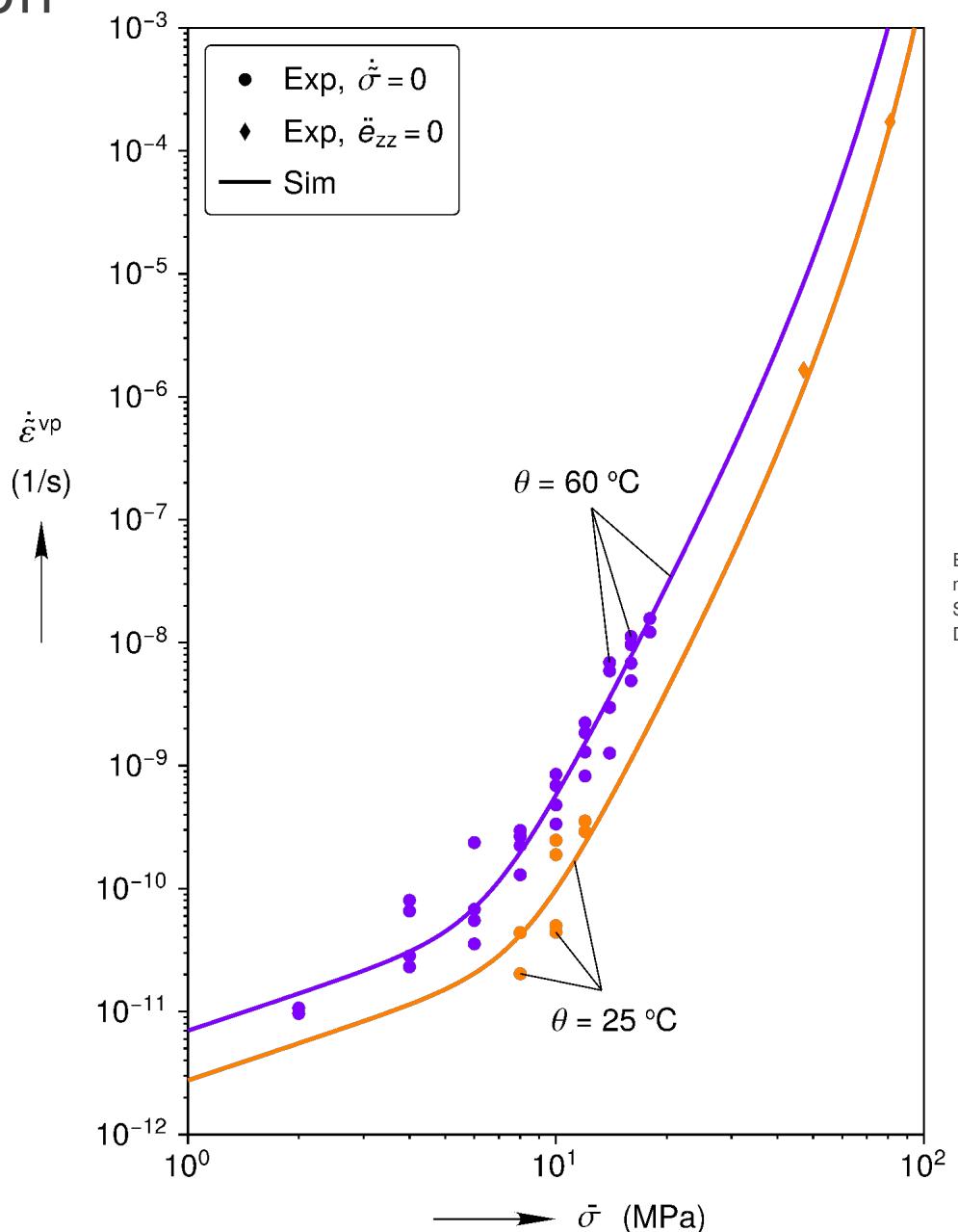
Steady-State Strain Rates

$$\dot{\varepsilon}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

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Spies, C., Schutjens, P., Brzesowsky, R., Peach, C., Liezenberg, J., and Zwart, H. (1990). Experimental determination of constitutive parameters governing creep of rocksalt by pressure solution. In: Geological Society, London, Special Publications 54.1, pp. 215–227.

Garofalo, F. (1963). An empirical relation defining the stress dependence of minimum creep rate in metals. In: Trans. AIME 227, pp. 351–356.





Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

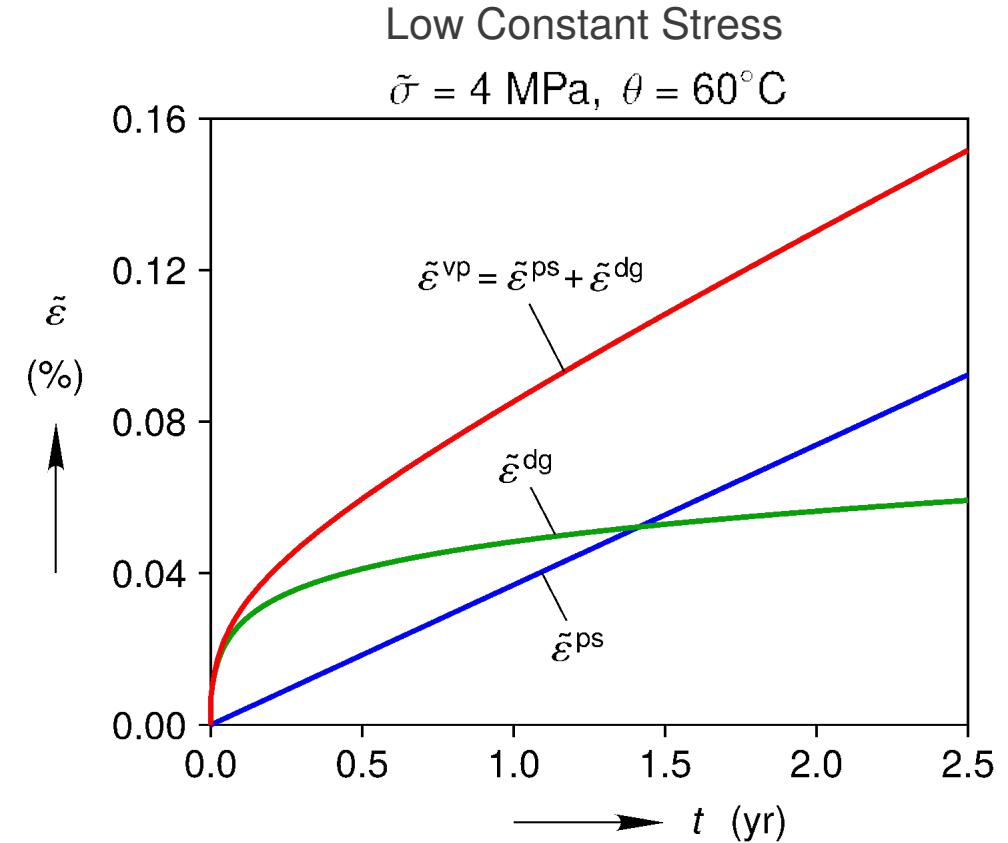
Strain Rates While Hardening



Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$





Strain Rates
(proportional, monotonic, loading)

$$\dot{\tilde{\varepsilon}}^{\text{ps}} = P_1 \exp\left(-\frac{P_2}{\theta}\right) \frac{\tilde{\sigma}}{\theta}$$

$$\dot{\tilde{\varepsilon}}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma} - \tilde{b}}{y}\right) \right]^{G_3}$$

Dislocation Glide Hardening



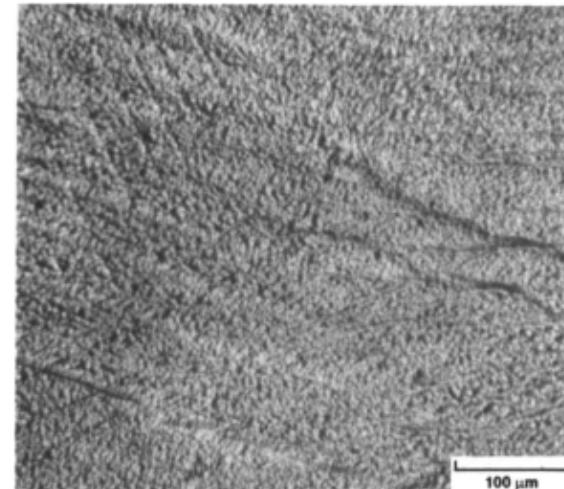
Equivalent Stress Decomposition
(proportional, monotonic, loading)

$$\tilde{\sigma} = \tilde{b} + \tilde{\sigma}^{dg}$$

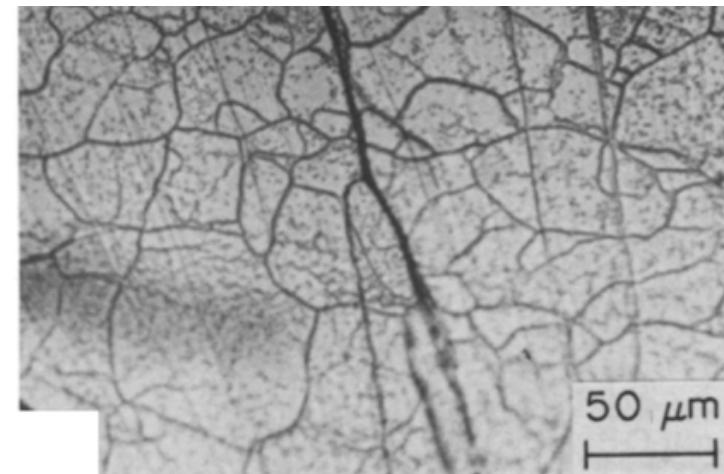
Drag Stress Contribution

$$\tilde{\sigma}^{dg} = y \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{dg}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$

Uniform Dislocation Density



Non-Uniform Dislocation Density



Raj, S. V. and Pharr, G. (1989). "Creep substructure formation in sodium chloride single crystals in the power law and exponential creep regimes". In: Materials Science and Engineering: A 122.2, pp. 233–242.
Carter, NL, Horsemann, ST, Russell, JE, and Handin, J (1993). Rheology of rocksalt. Journal of Structural Geology. Vol 15. No 9-10. pp 1257-1271.

Dislocation Glide Hardening Saturation (Steady-State)

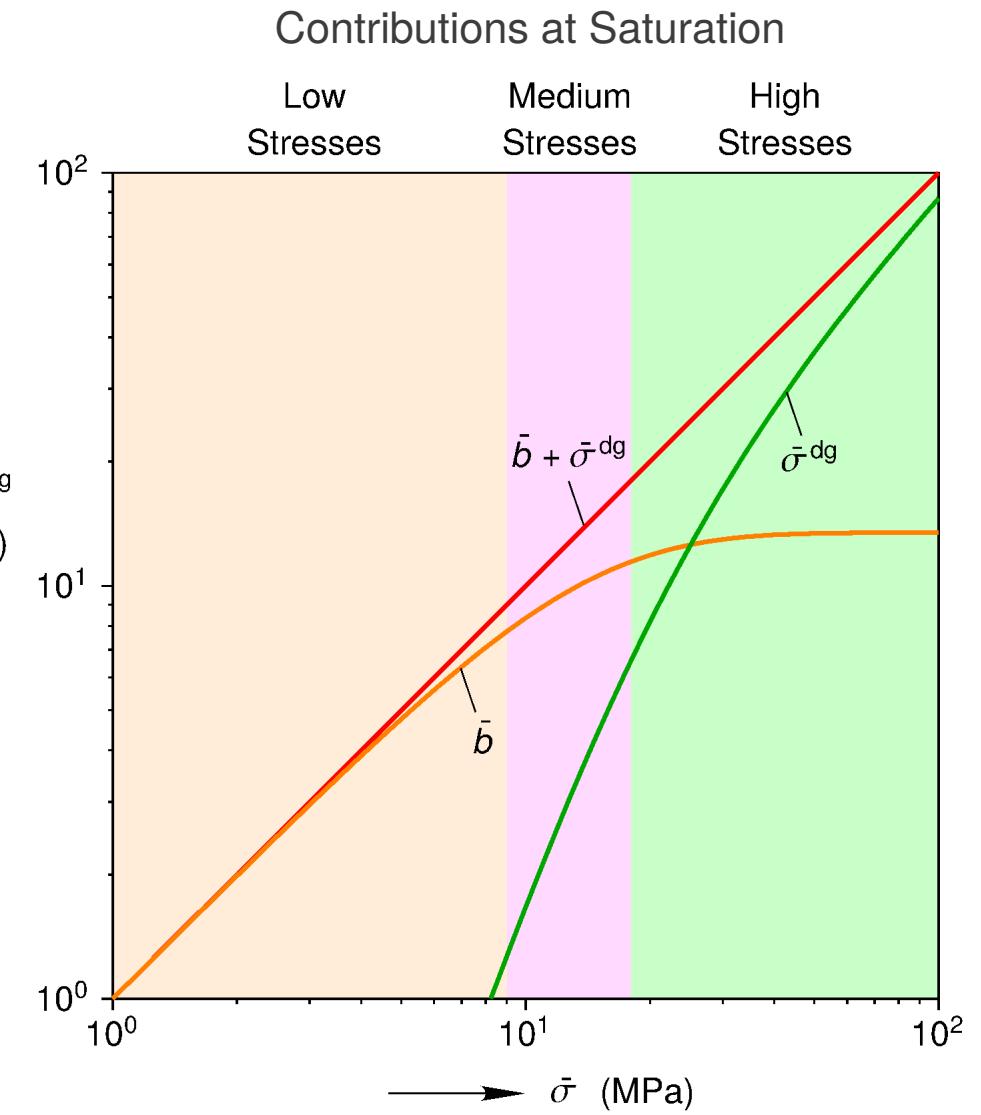


Equivalent Stress Decomposition
(proportional, monotonic, loading)

$$\bar{\sigma} = \bar{b} + \bar{\sigma}^{\text{dg}}$$

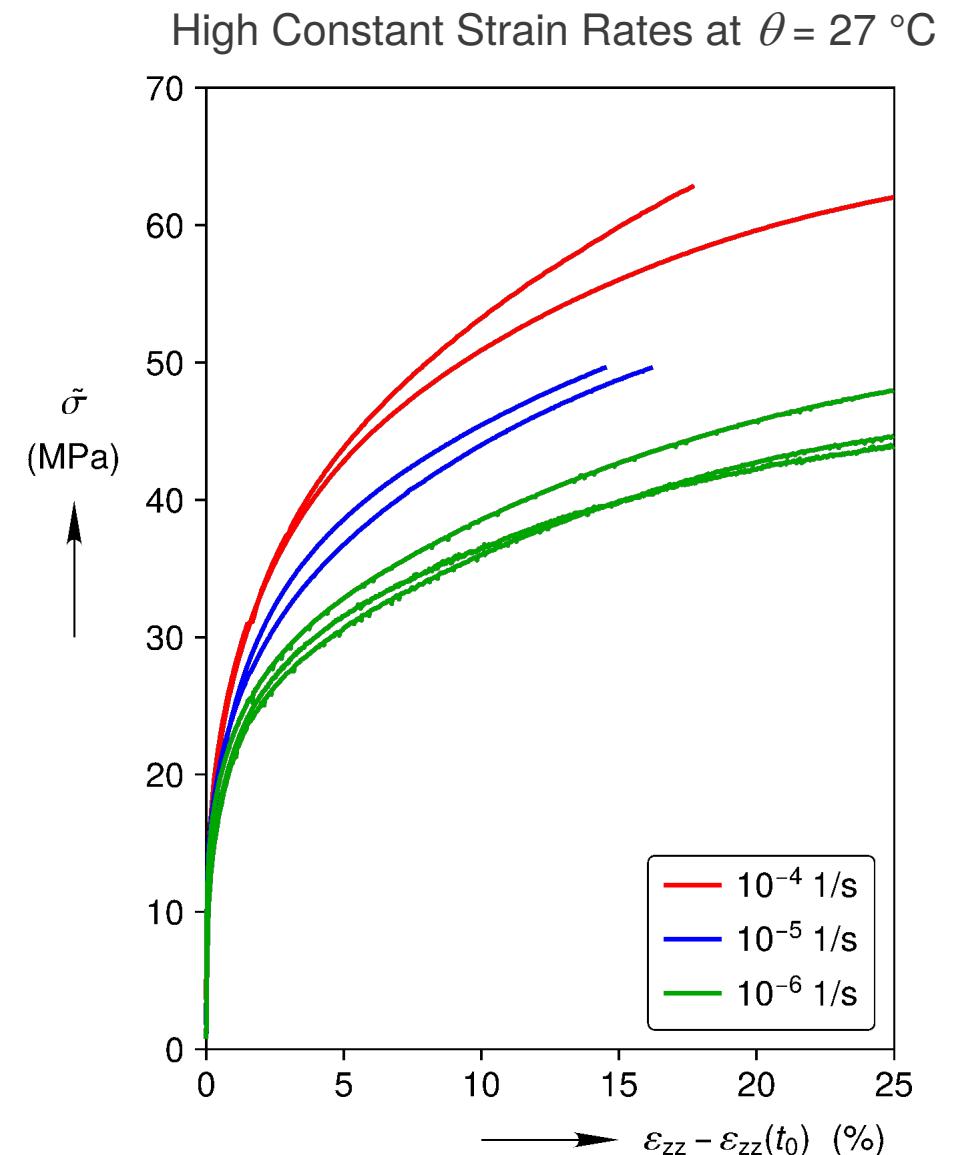
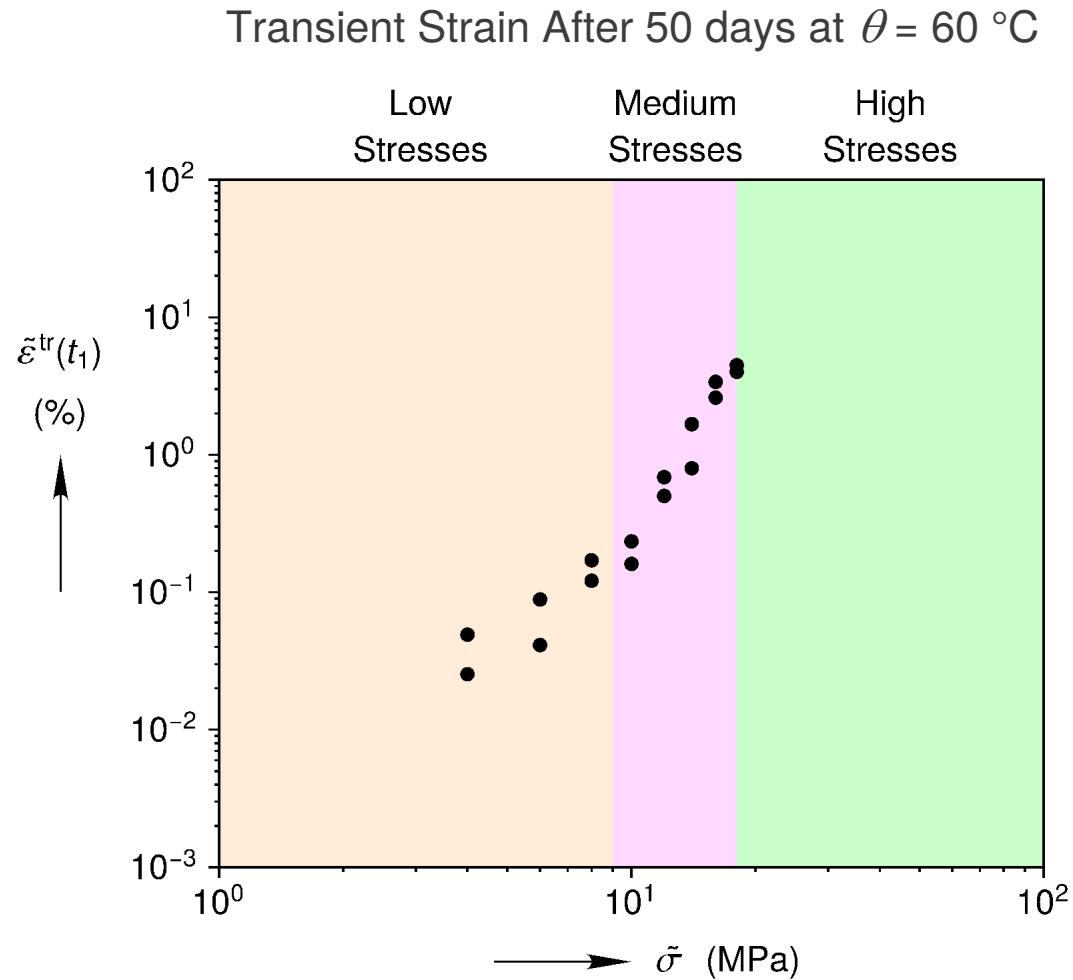
Drag Stress Contribution

$$\bar{\sigma}^{\text{dg}} = \bar{y} \sinh^{-1} \left\{ \left[\frac{\dot{\varepsilon}^{\text{dg}}}{G_1 \exp(-G_2/\theta)} \right]^{1/G_3} \right\}$$

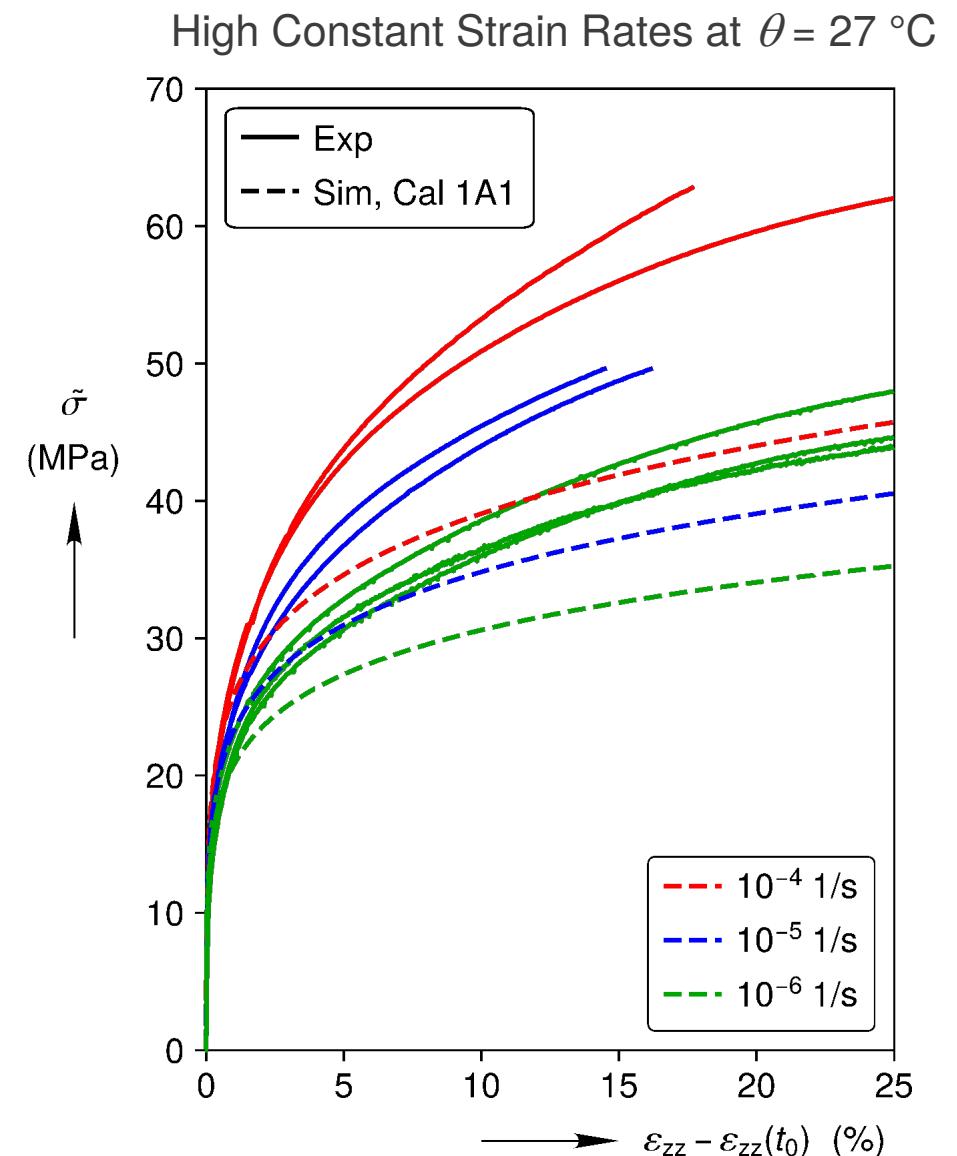
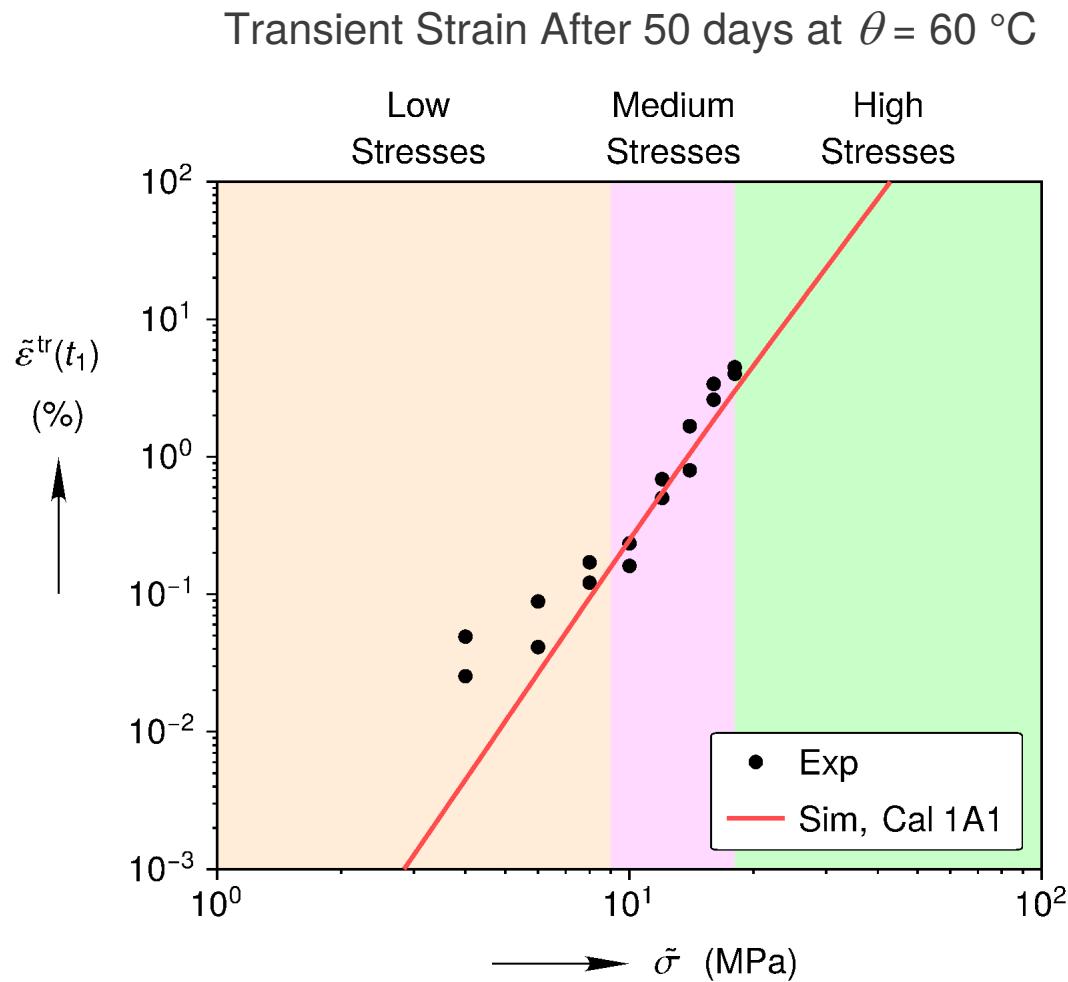


Hardening Calibrations

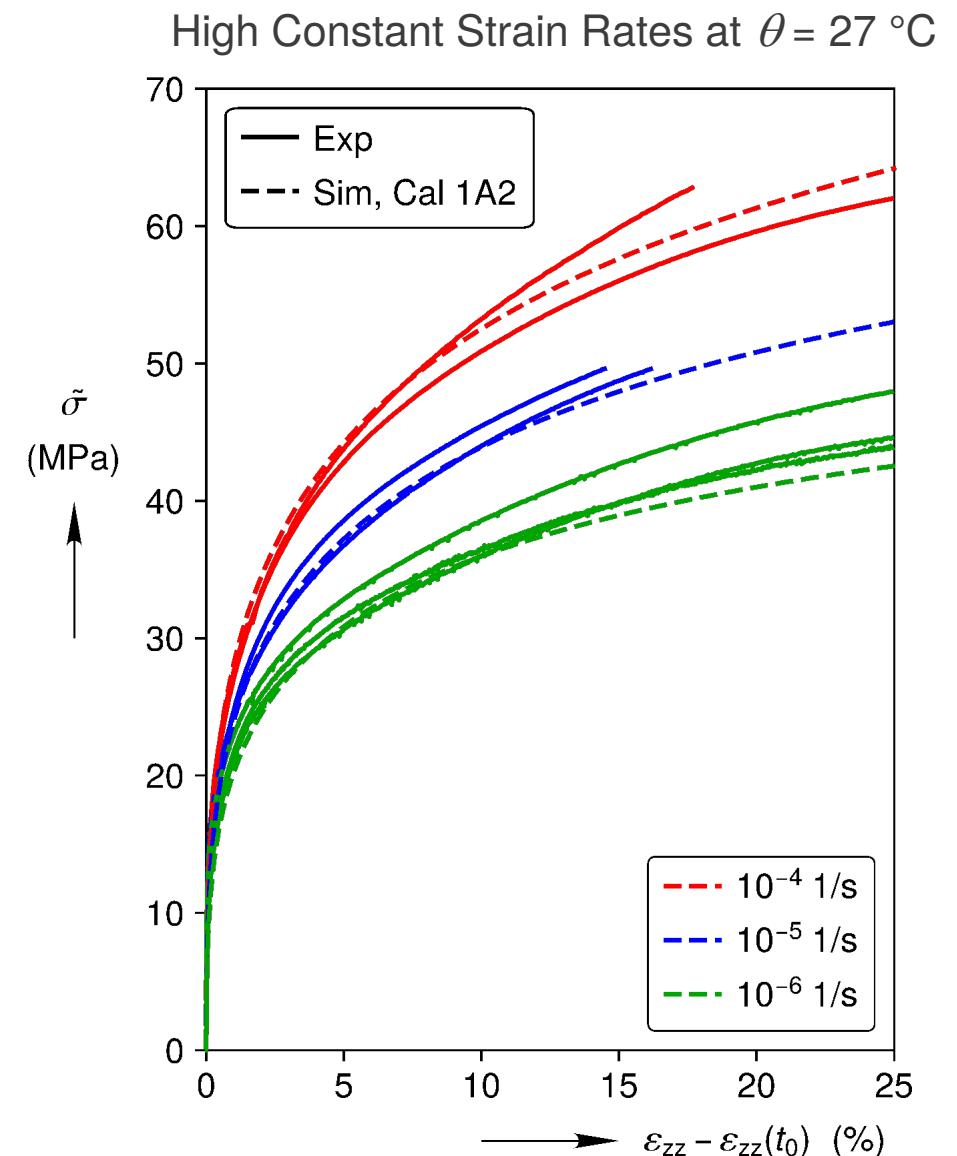
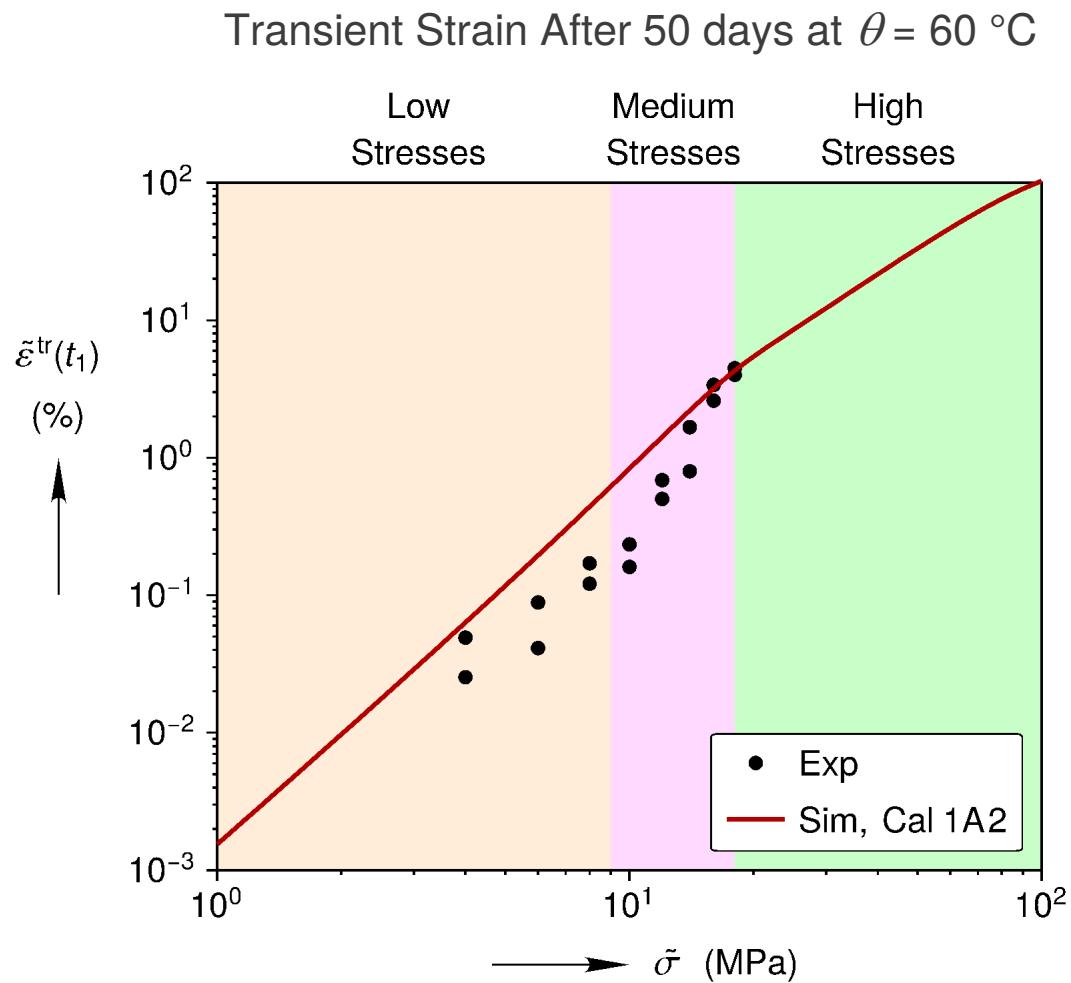
Selected Hardening Measurements on WIPP Salt



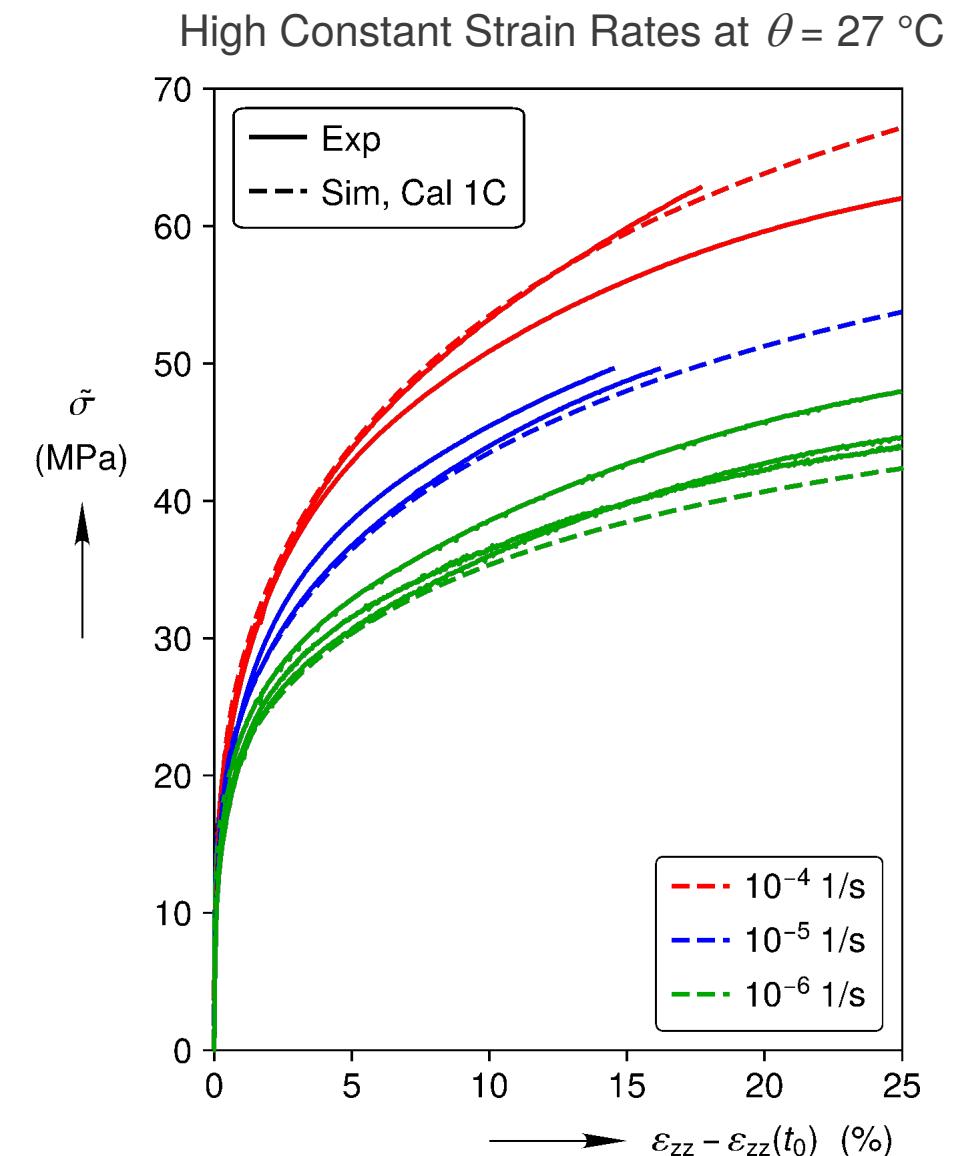
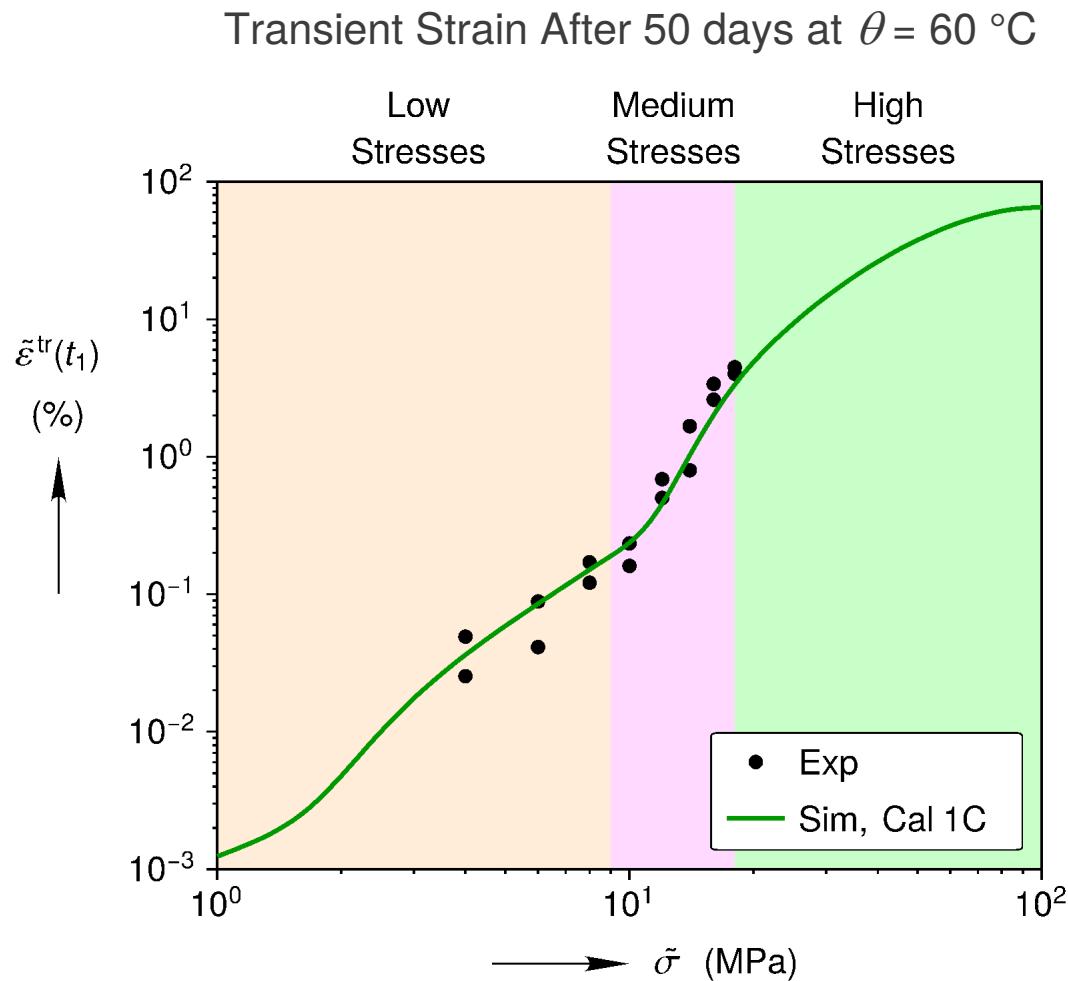
Calibration 1A1: Drag Stress Only



Calibration 1A2: Drag Stress Only



Calibration 1C: Drag Stress and Back Stress

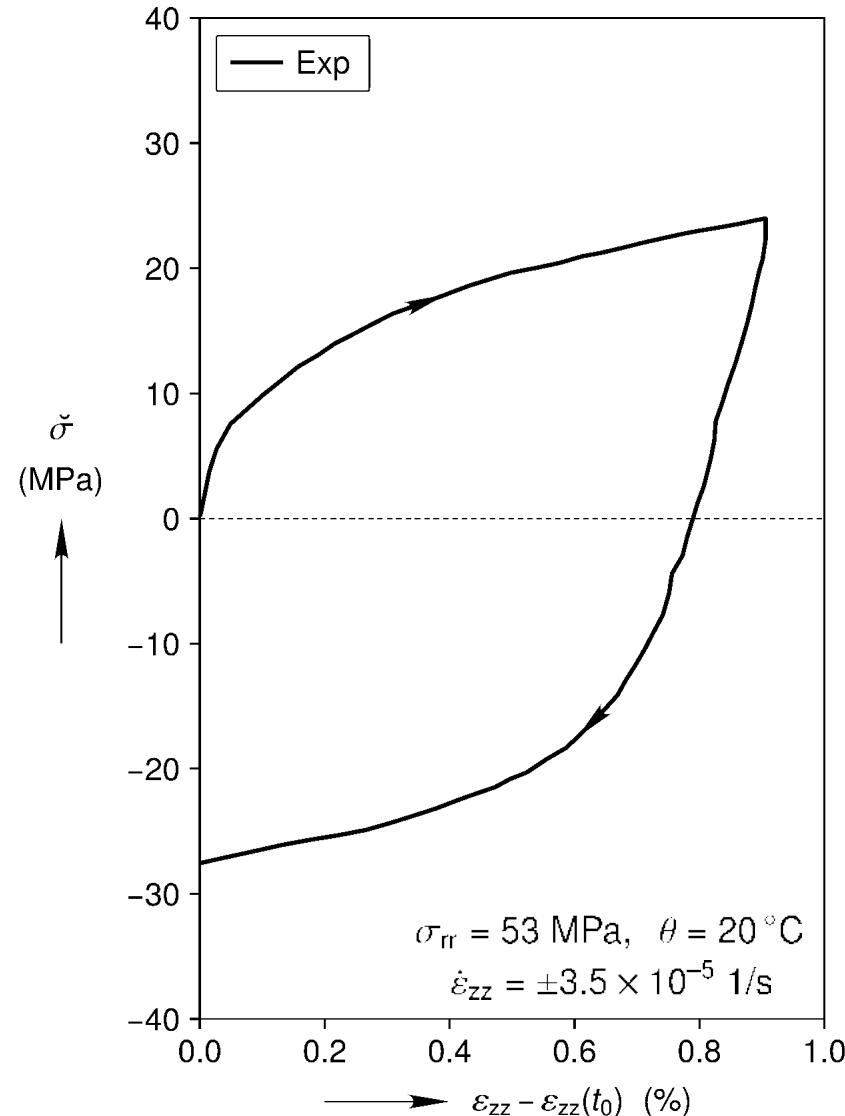


Partial Validations

Re-hardening during Non-Monotonic Loading

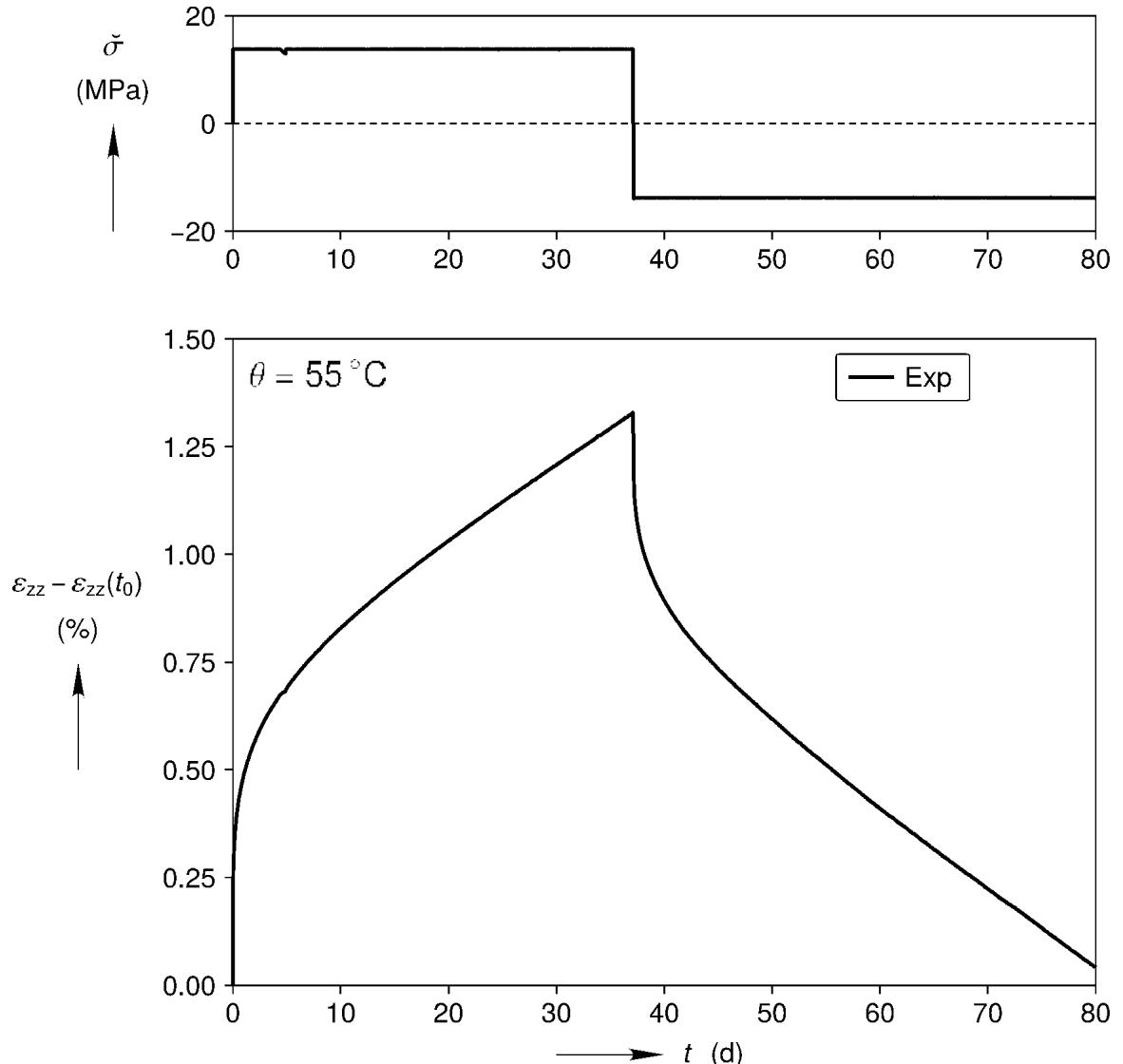


Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

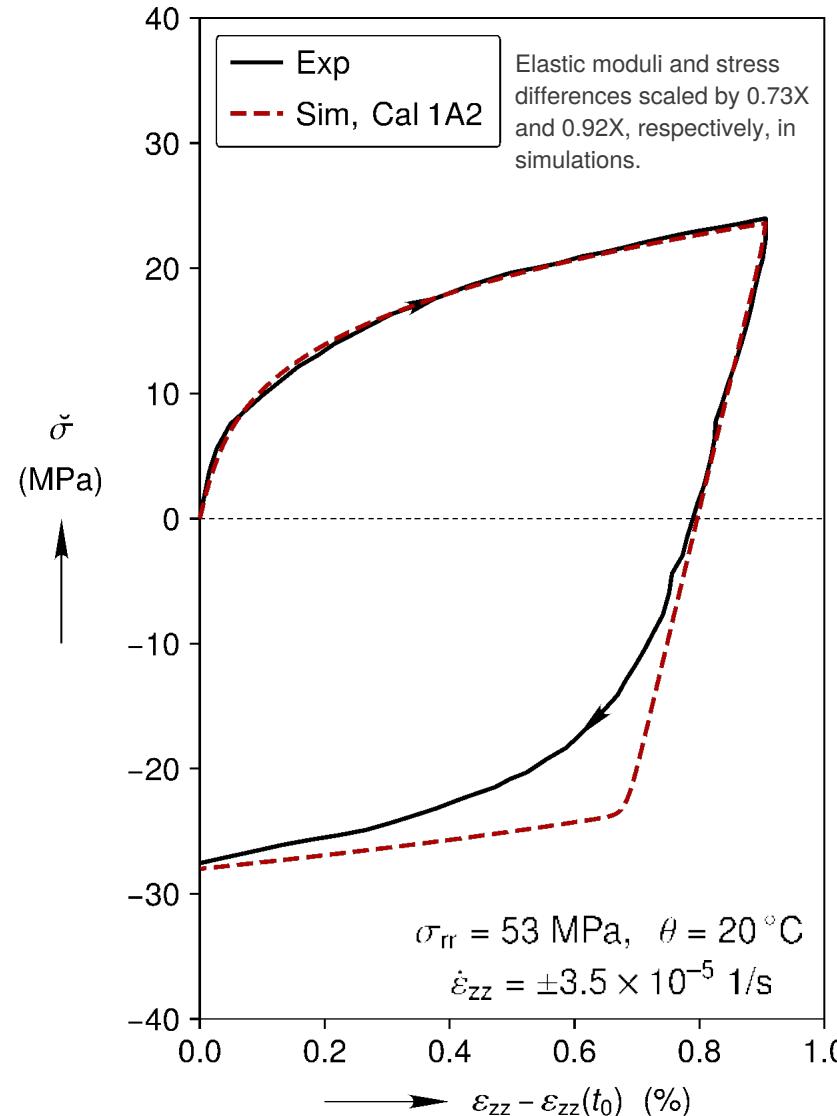
Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)

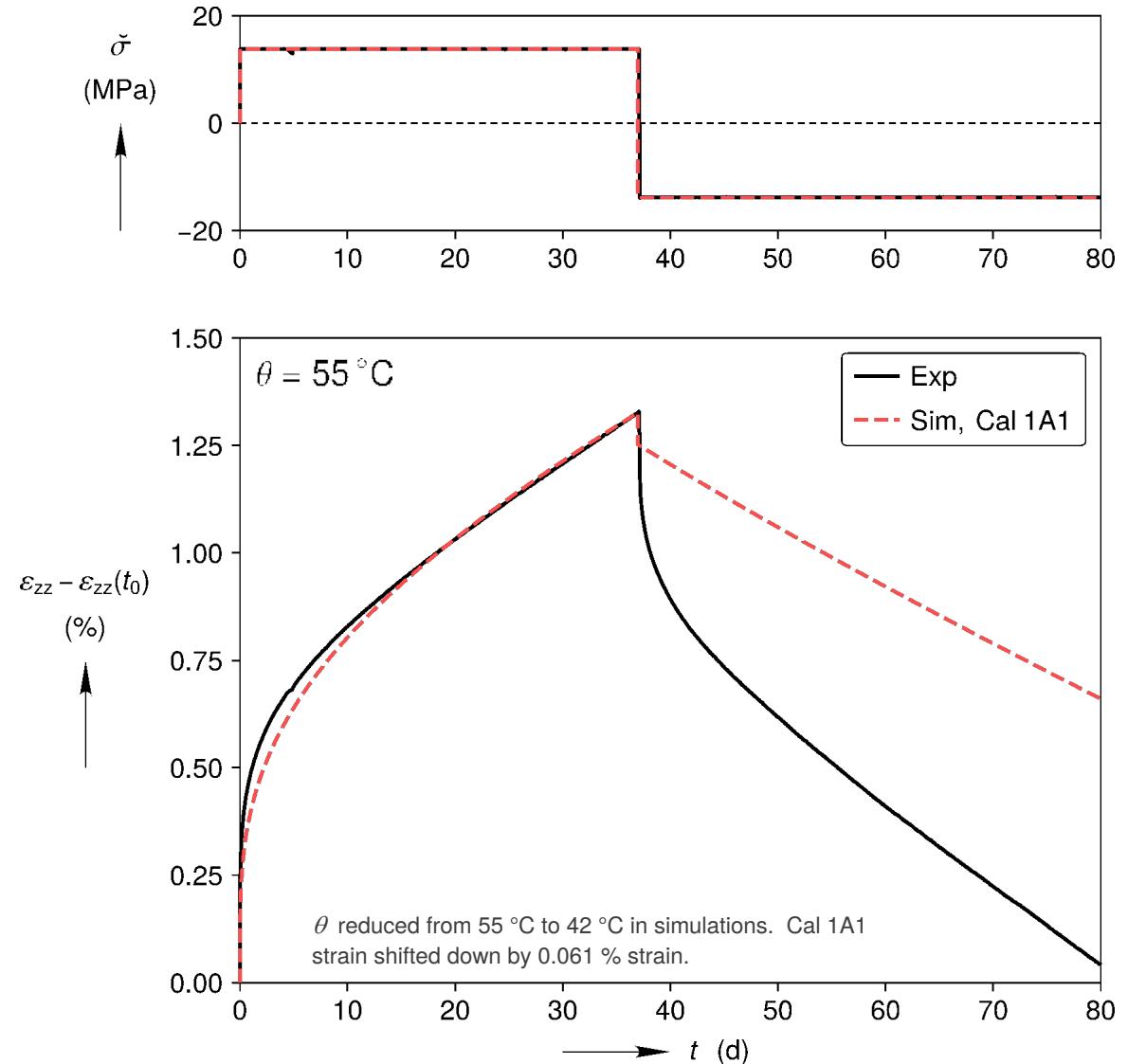
Re-hardening during Non-Monotonic Loading

Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

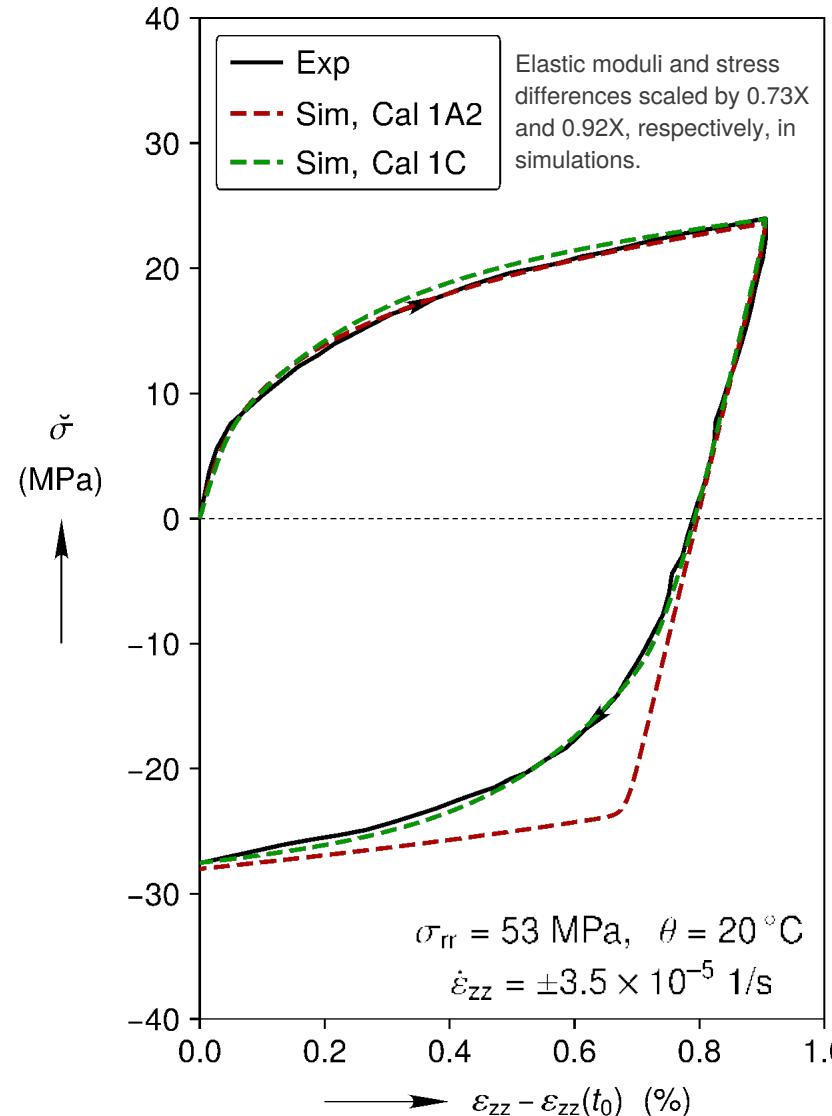
Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)

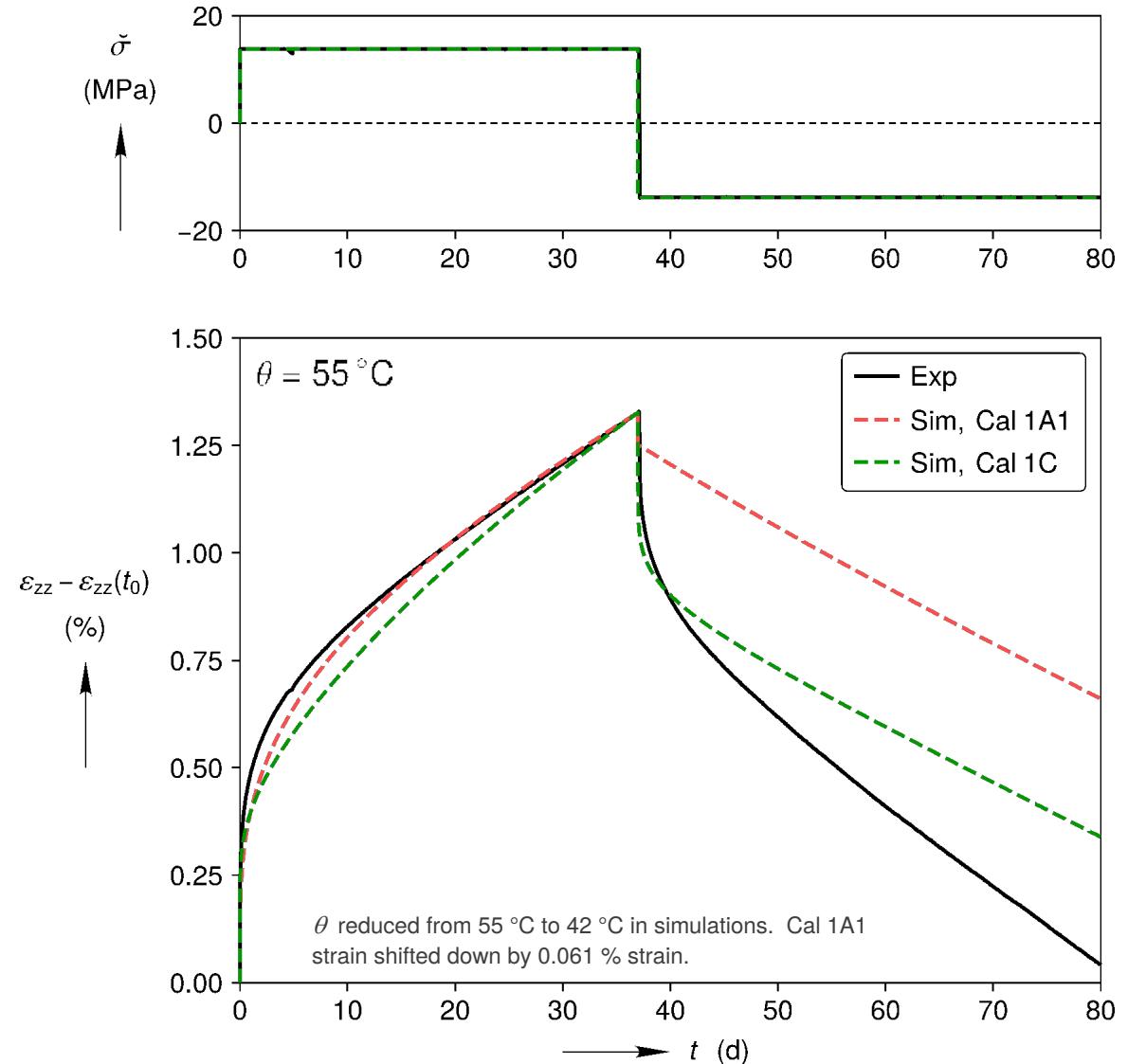
Re-hardening during Non-Monotonic Loading

Constant Strain Rate Test on Artificial Salt



Experimental measurements from: Aubertin et al. (1999)

Multi-Stage Constant Stress Test on Cayuta Salt



Experimental measurements from: Mellegard et al. (2007)

Summary & Future Work

Summary & Future Work



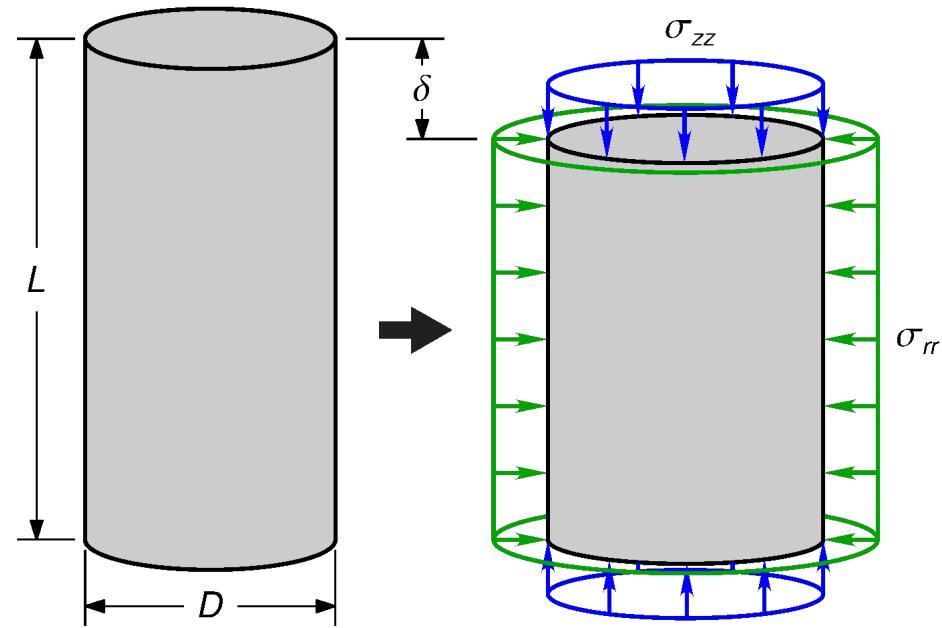
1. The new model is largely phenomenological, but key decisions were motived by micro-physical observations.
 1. Pressure solution and dislocation glide branches
 2. Drag stress hardening and back stress hardening
 3. Captures rock salt's viscoplastic behavior over a wide range of strain rates (10^{-12} to 10^{-4} 1/s)
 4. Predicts re-hardening behavior after non-monotonic loading
2. Future work
 1. Polish numerical implementation
 2. Simulate underground structures
 3. Add damage and healing

Extra Slides



1. Background
2. Model Overview
3. Hardening Calibrations
4. Partial Validation
5. Summary

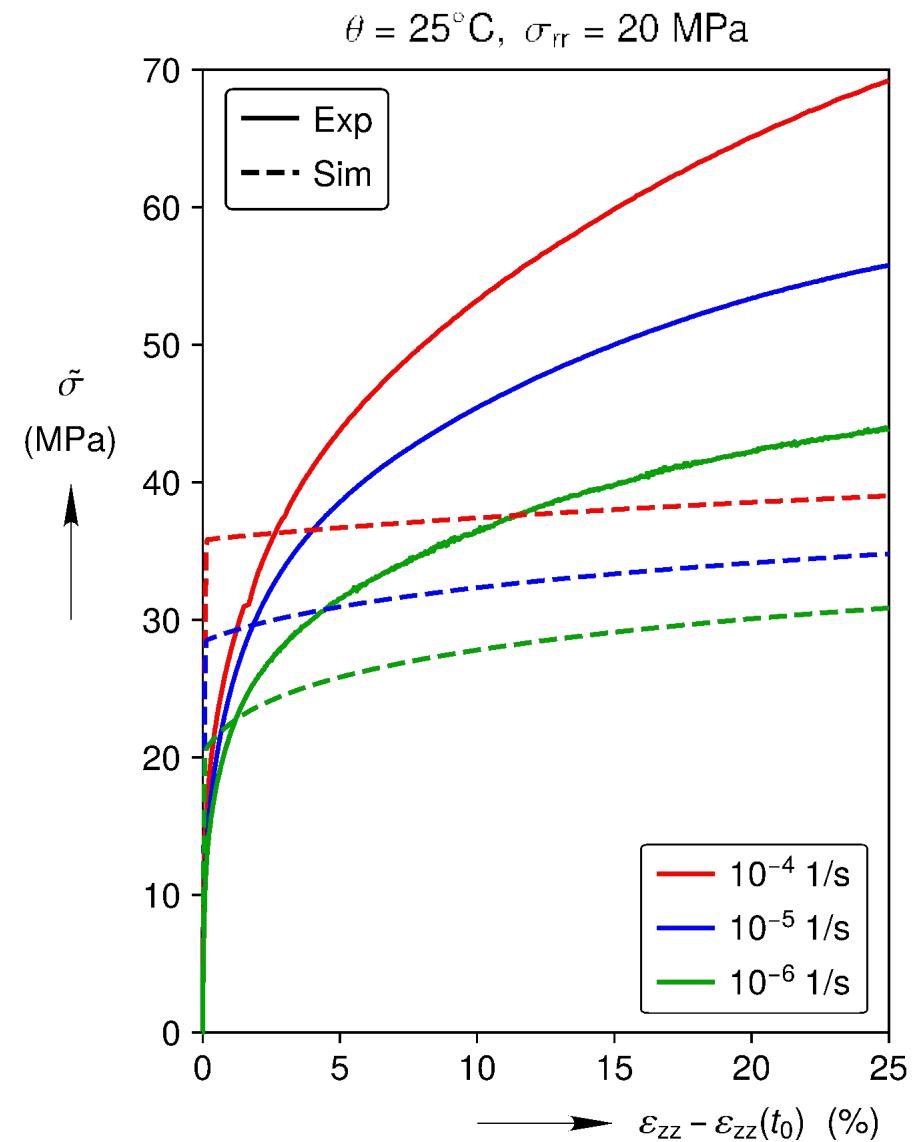
Damage-Free, High Strain Rate, Munson-Dawson Predictions



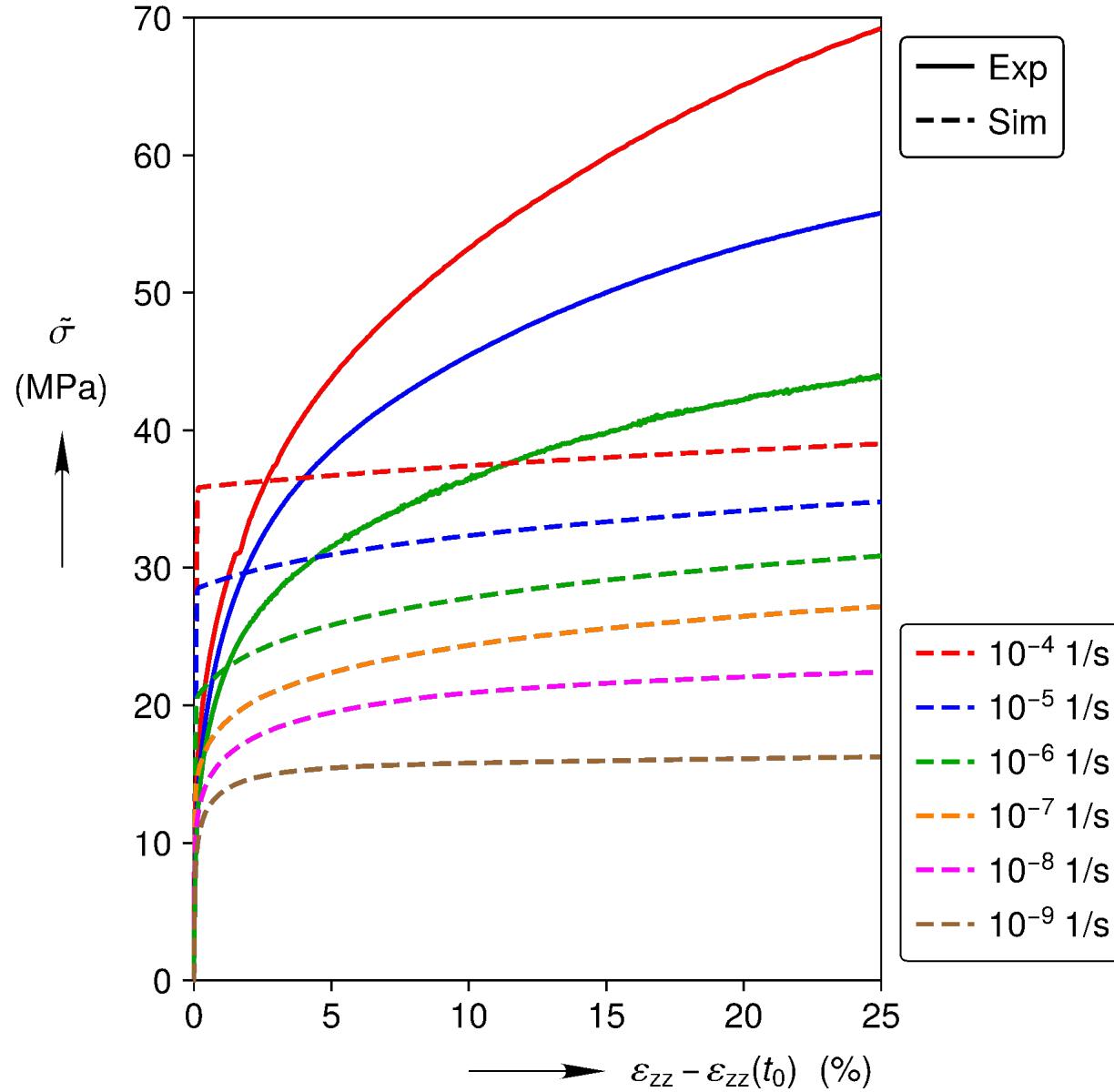
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

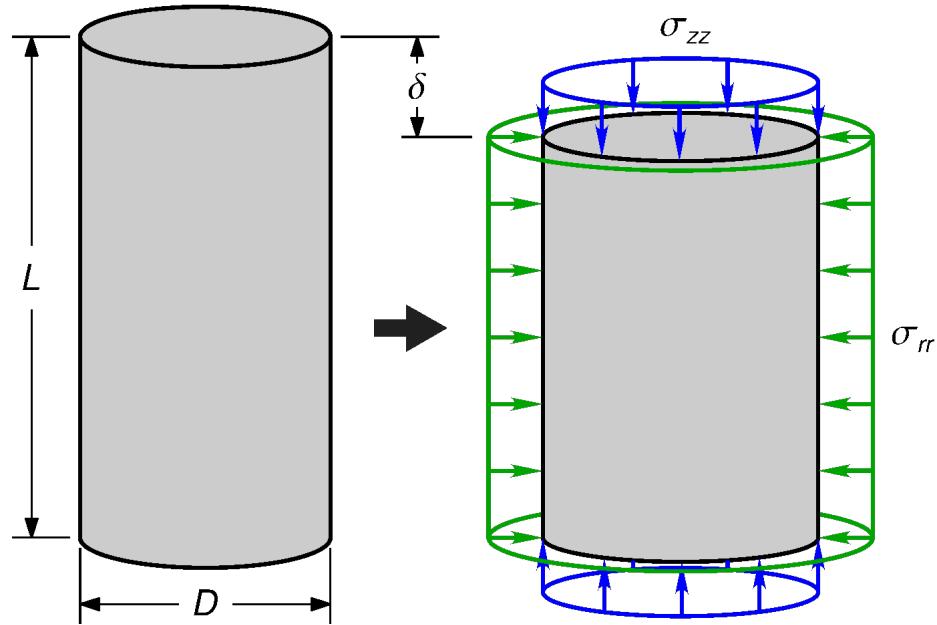
$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



Munson-Dawson Model Constant Strain Rate Predictions



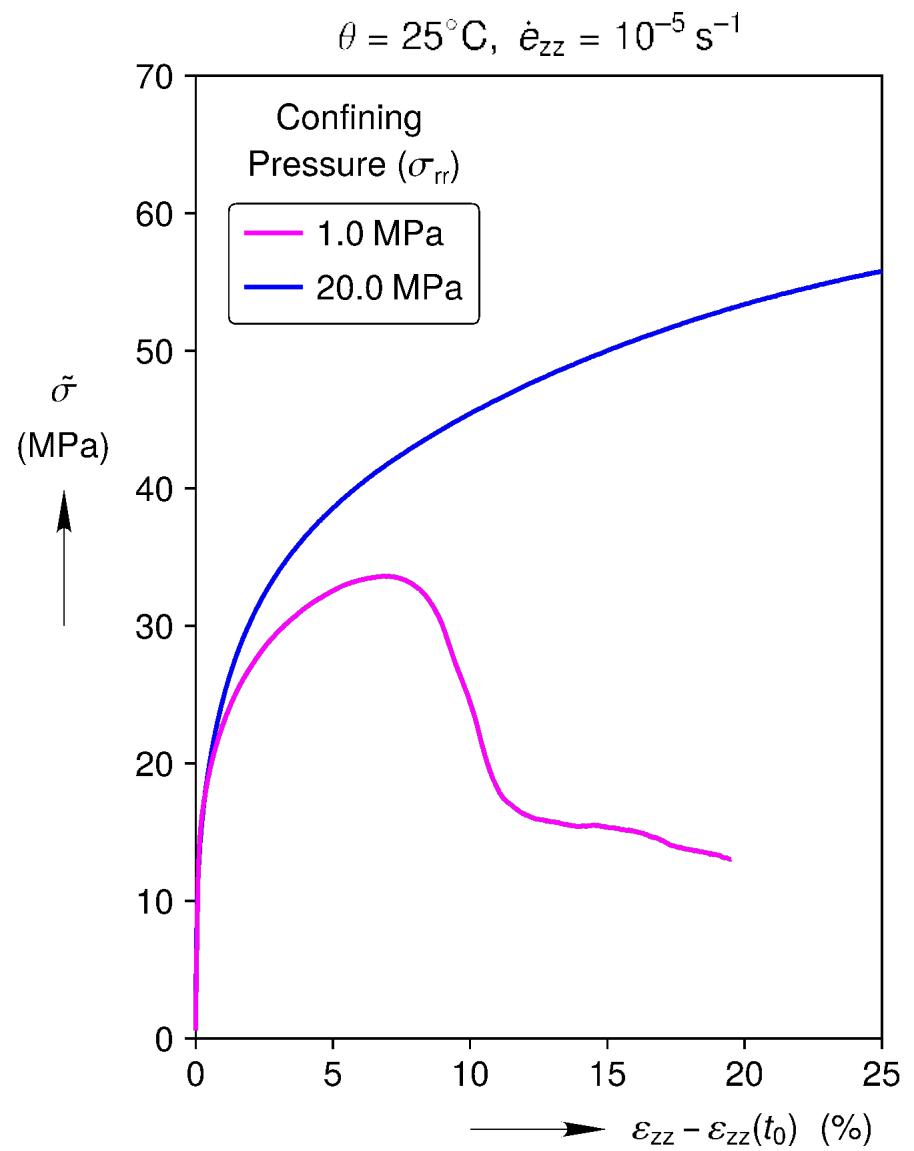
Damaged, High Strain Rate, Behavior



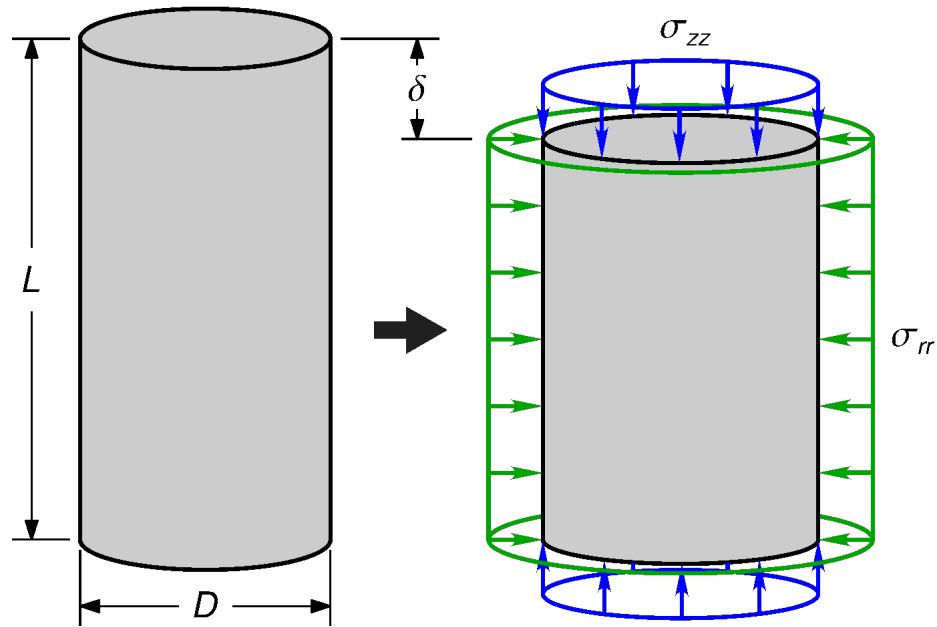
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



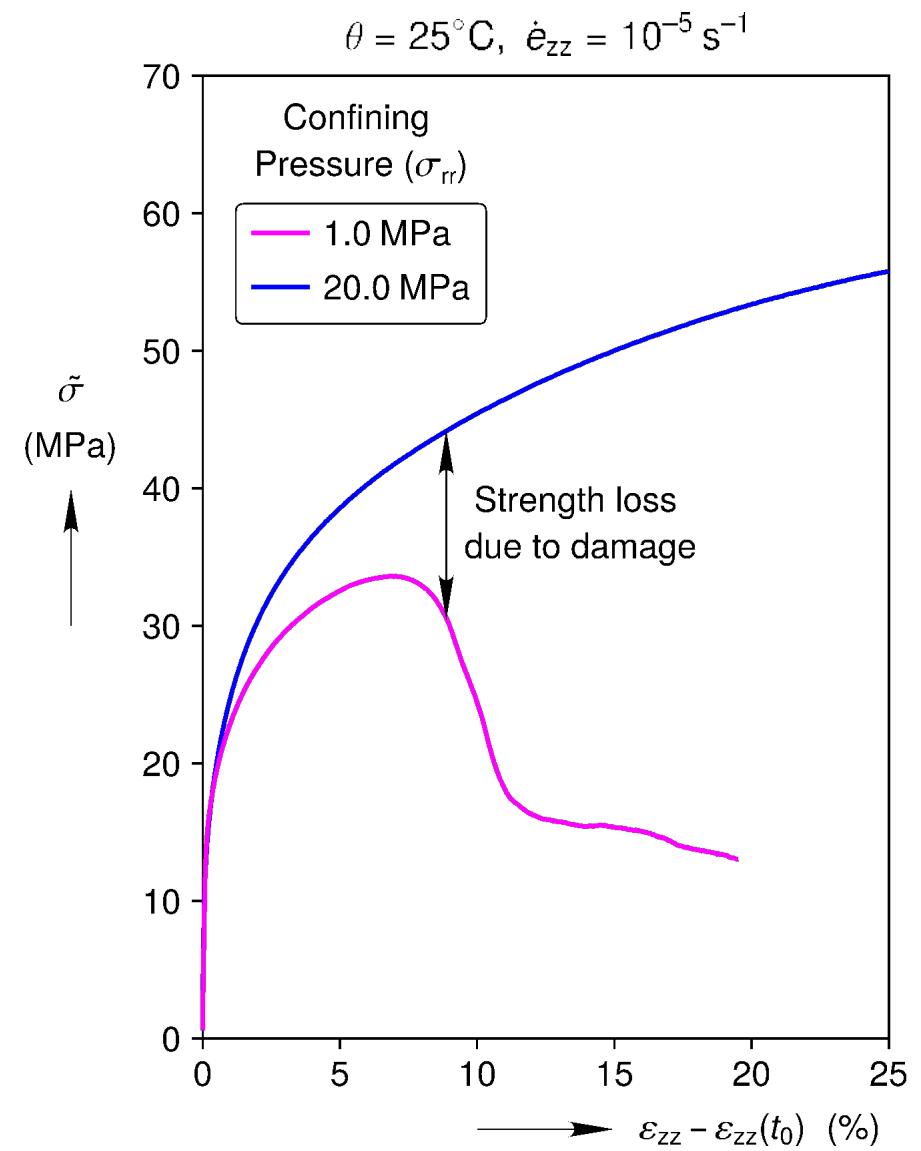
Damaged, High Strain Rate, Behavior



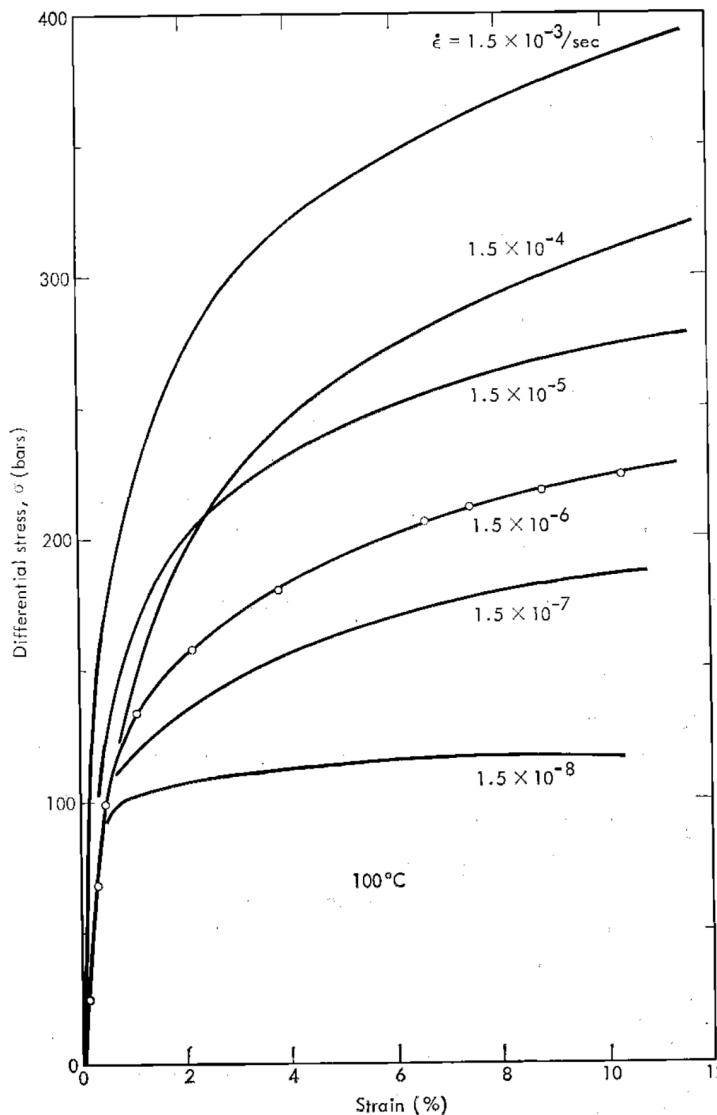
$$\frac{L}{D} = 2$$

$$\varepsilon_{zz} = \ln \left(1 + \frac{\delta}{L} \right)$$

$$\tilde{\sigma} = |\sigma_{zz} - \sigma_{rr}|$$



Constant Strain Rate Tests on Artificial Salt

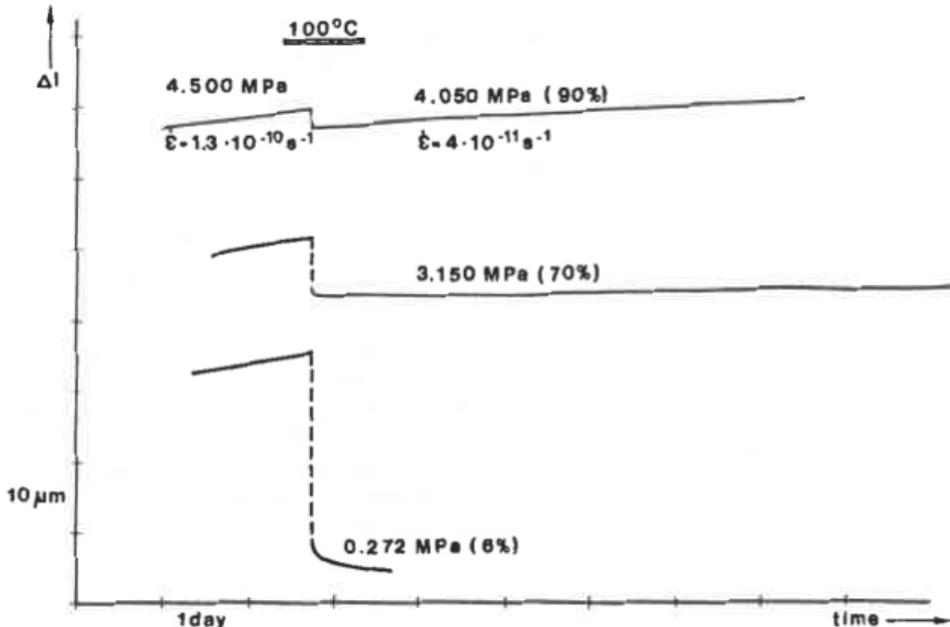


Heard, H. (1972): Steady-state flow in polycrystalline halite at pressure of 2 kilobars. *Flow and fracture of rocks*. Vol 16. pp 191-209.

Fig. 1. Differential stress-strain curves for polycrystalline halite extended at 2 kb, $\dot{\epsilon} = 1.5 \times 10^{-3}$ to 1.5×10^{-8} sec $^{-1}$, and 100°C.

Back Stress Measurements

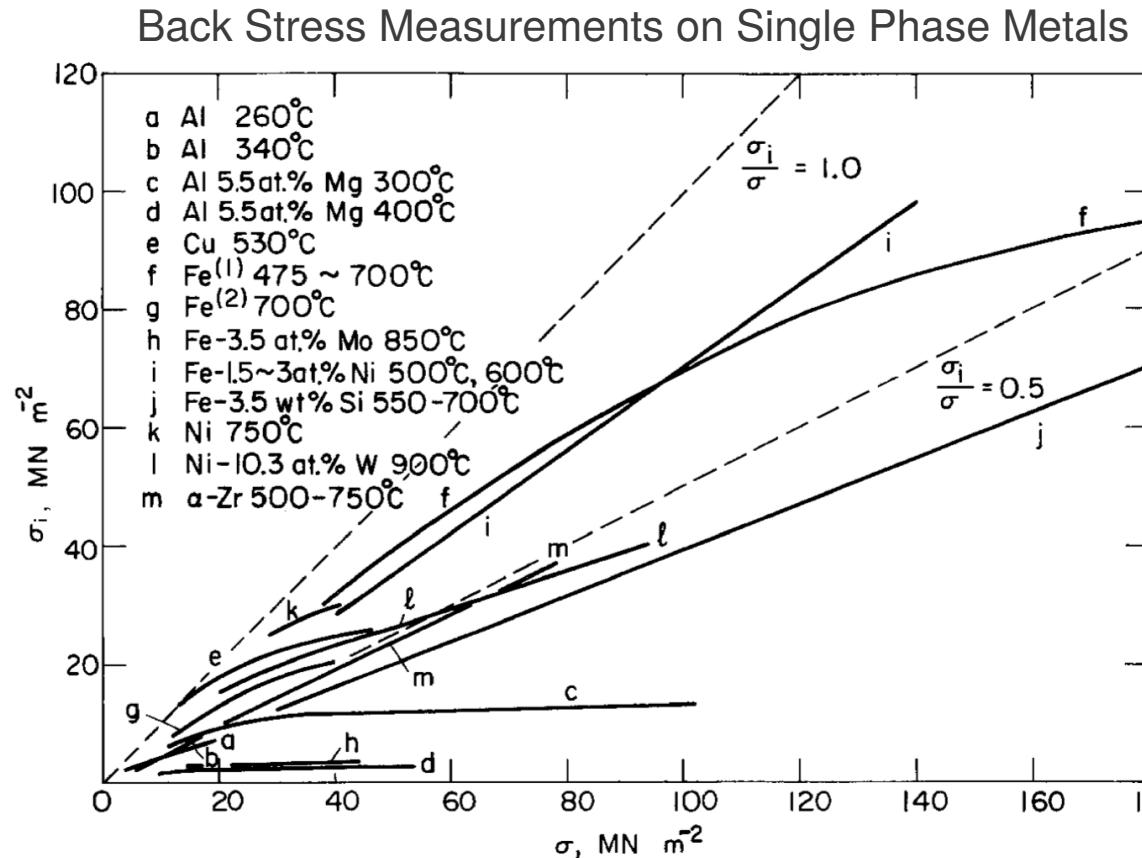
Creep Responses due to Small Stress Changes Asse Rock Salt



Hunsche, U. 1988. Measurement of creep in rock salt at small strain rates. Proceedings of the 2nd Conference on the Mechanical Behavior of Salt. Pg. 187-196

Dislocation Glide Strain Rate (proportional loading)

$$\dot{\varepsilon}^{\text{dg}} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{|\check{\sigma} - \check{b}|}{y}\right) \right]^{G_3} \text{sign}(\check{\sigma} - \check{b})$$



Takeuchi, S. and Argon, A. (1976). "Steady-state creep of single-phase crystalline matter at high temperature". In: Journal of Materials Science 11.8, pp. 1542-1566.

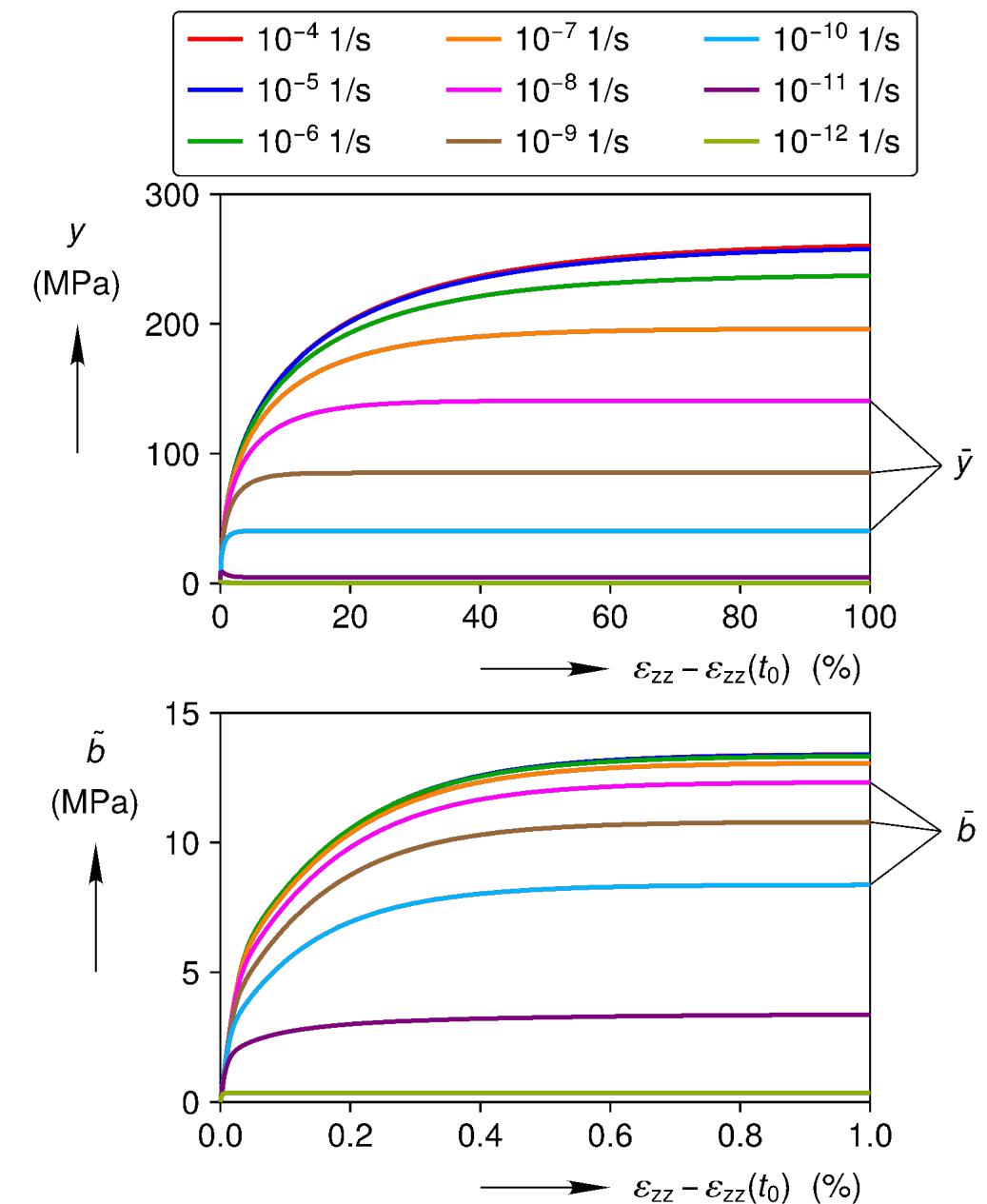
Dislocation Glide Hardening

Drag Stress Evolution Equation

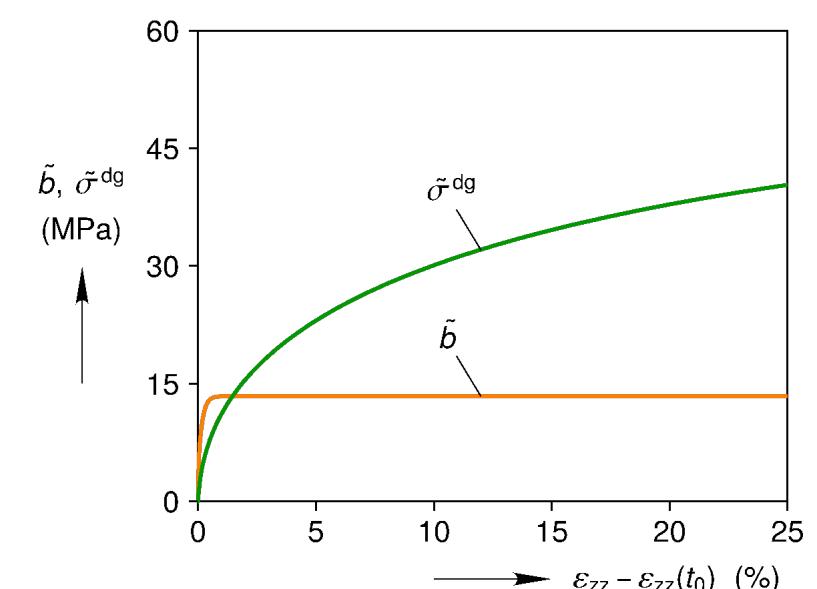
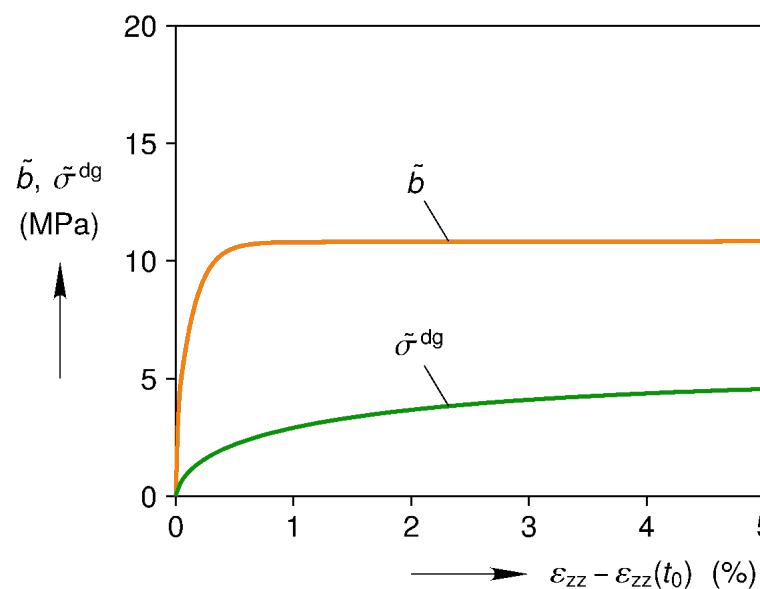
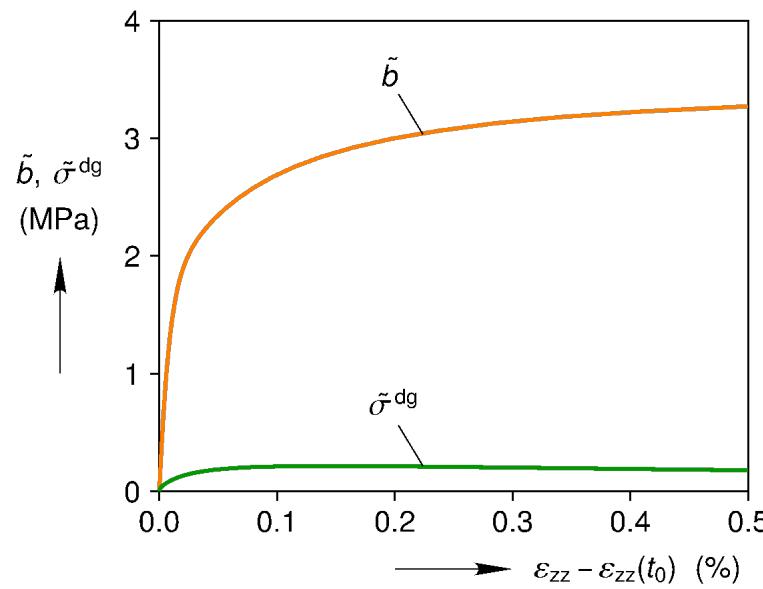
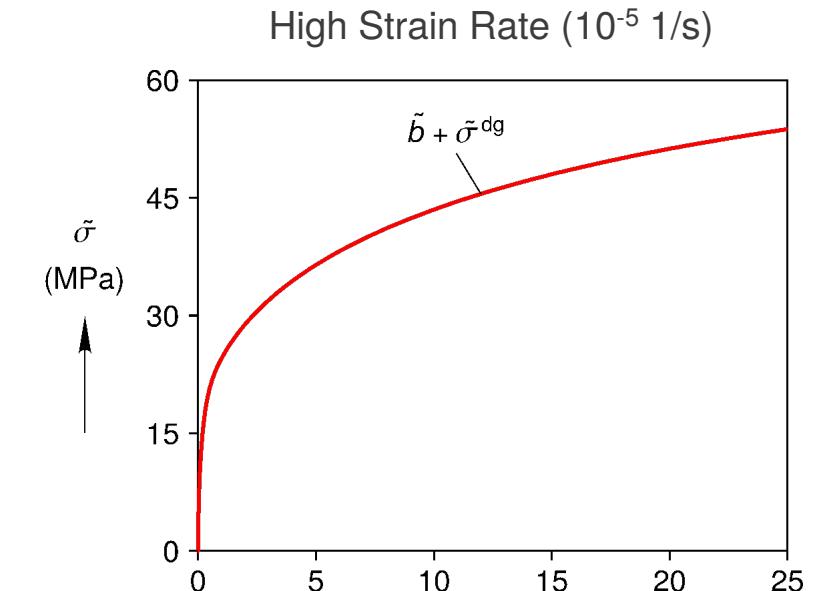
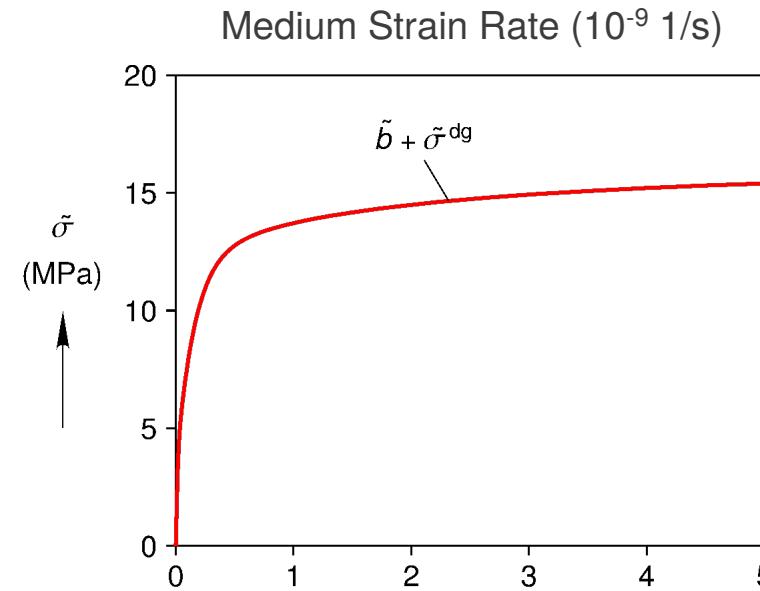
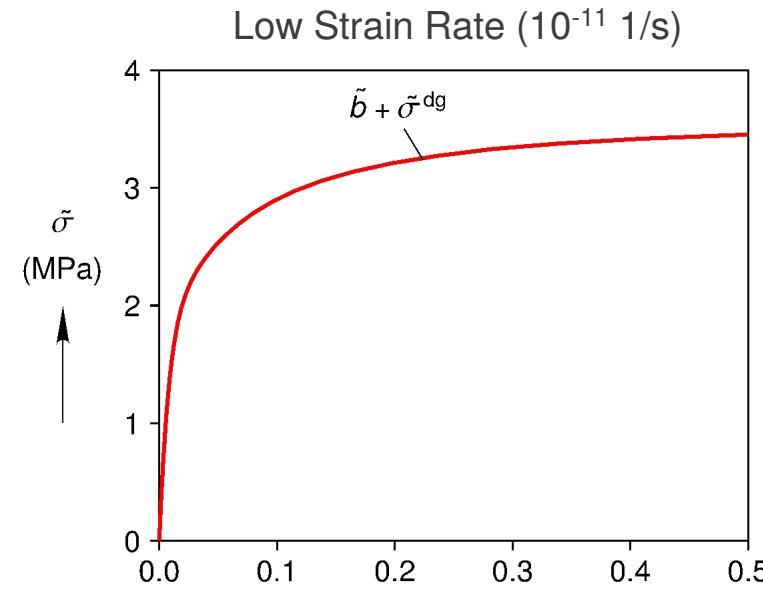
$$\dot{y} = Y_1 \left(\frac{Y_1}{y} \right)^{Y_2} \left(1 - \frac{y}{\bar{y}} \right) \dot{\varepsilon}^{\text{dg}}$$

Back Stress Evolution Equation
(proportional, monotonic, loading)

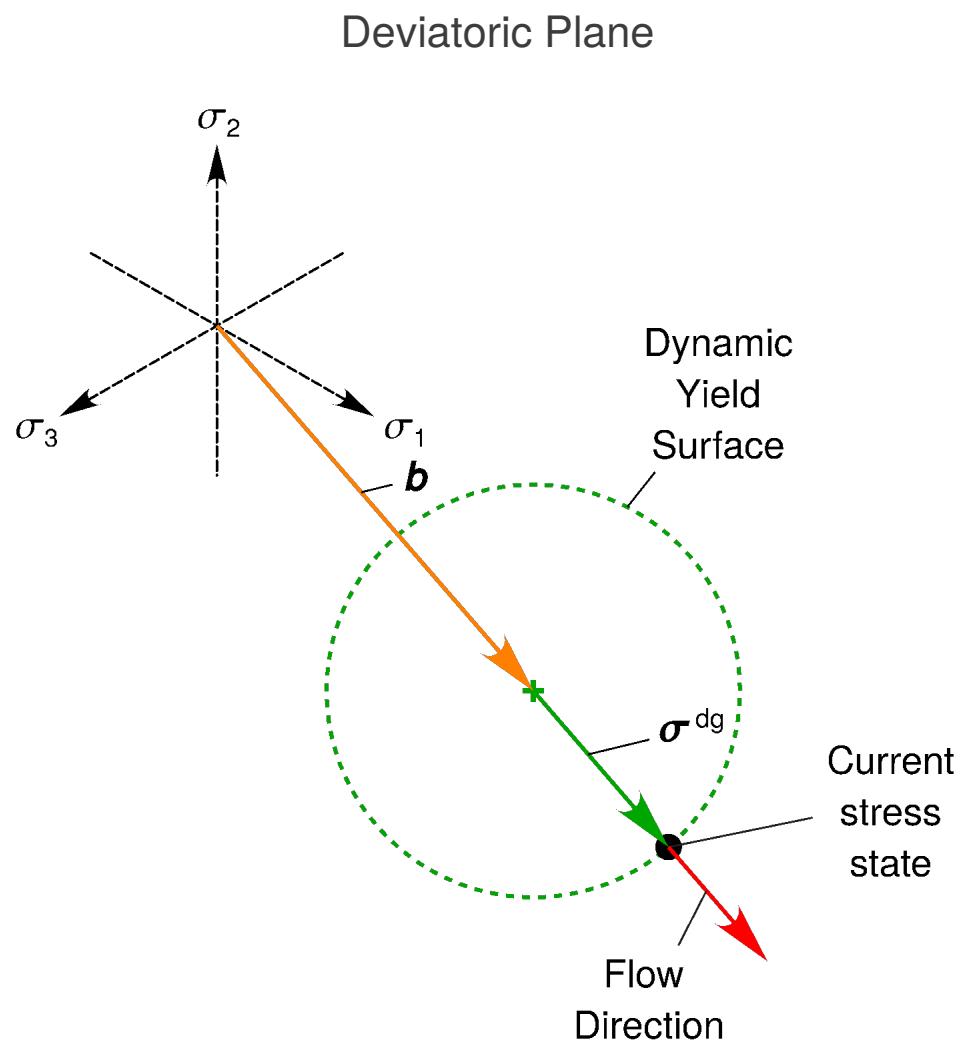
$$\dot{\tilde{b}} = B_1 \left(1 - \frac{\tilde{b}}{\bar{b}} \right) \dot{\varepsilon}^{\text{dg}}$$



Constant Strain Rate Behavior



Dislocation Glide Hardening



Dislocation Glide Stress

$$\sigma^{dg} = \sigma^{dev} - \mathbf{b}$$

$$\tilde{\sigma}^{dg} = \sqrt{\frac{3}{2} \sigma^{dg} : \sigma^{dg}}$$

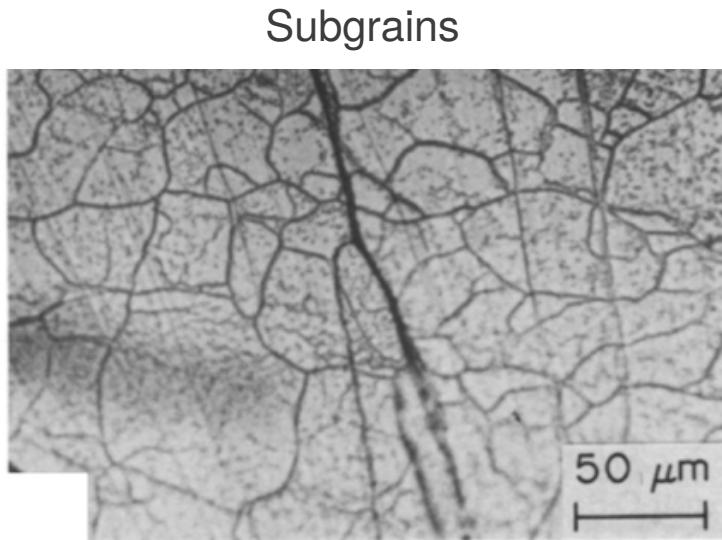
Flow Rule

$$\dot{\epsilon}^{dg} = \dot{\tilde{\epsilon}}^{dg} \frac{\partial \tilde{\sigma}^{dg}}{\partial \sigma}$$

Dislocation Glide Strain Rate

$$\dot{\tilde{\epsilon}}^{dg} = G_1 \exp\left(-\frac{G_2}{\theta}\right) \left[\sinh\left(\frac{\tilde{\sigma}^{dg}}{y}\right) \right]^{G_3}$$

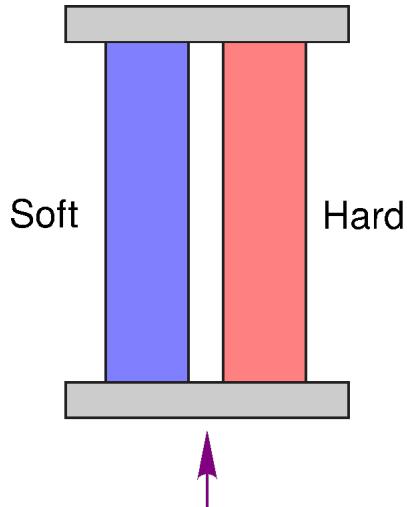
A “Gedankenexperiment”



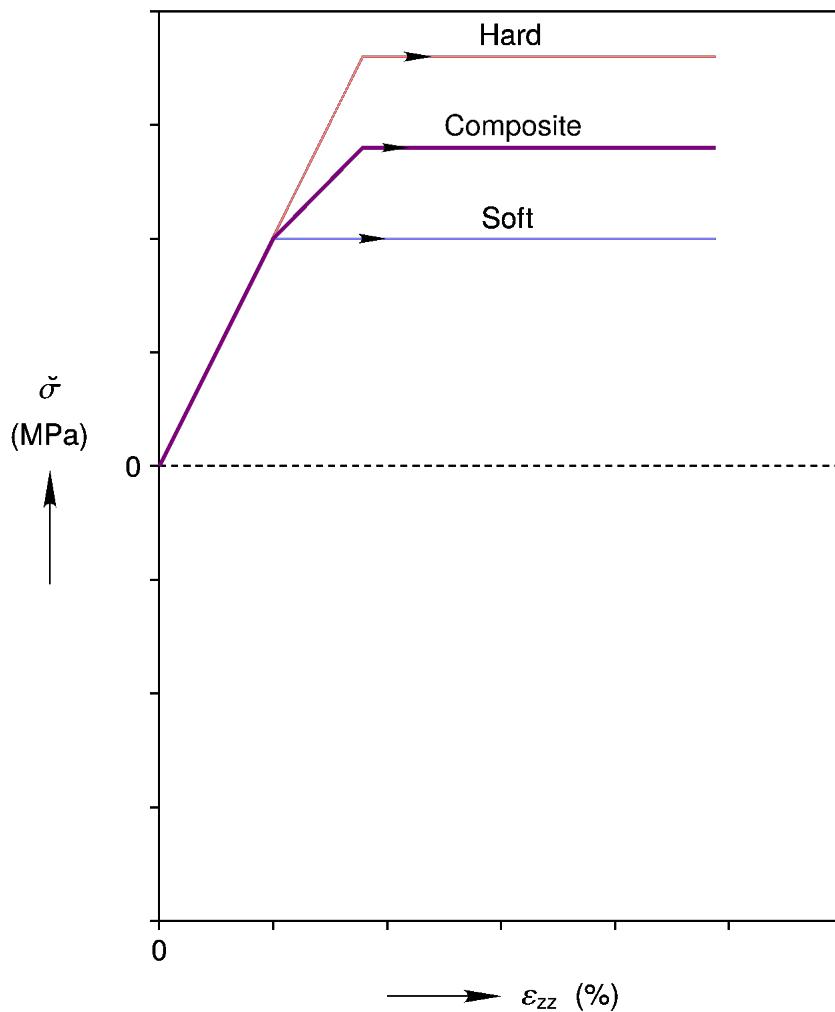
Subgrains

Schematic

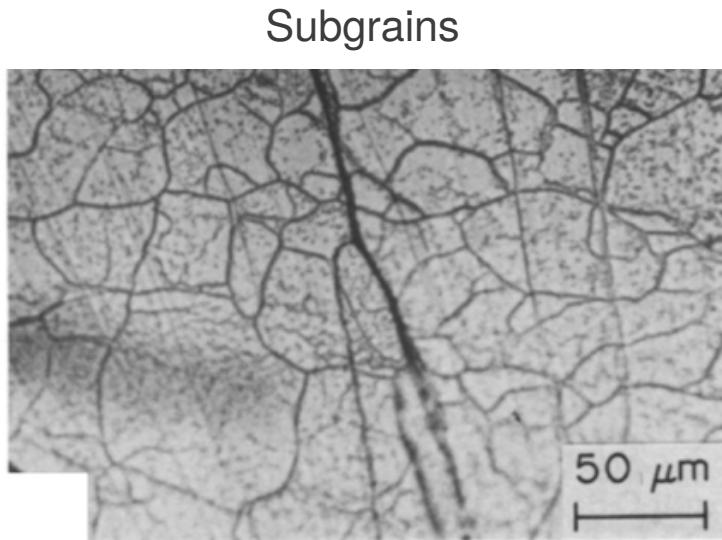
$$f = \check{\sigma} A$$



Mechanical Responses



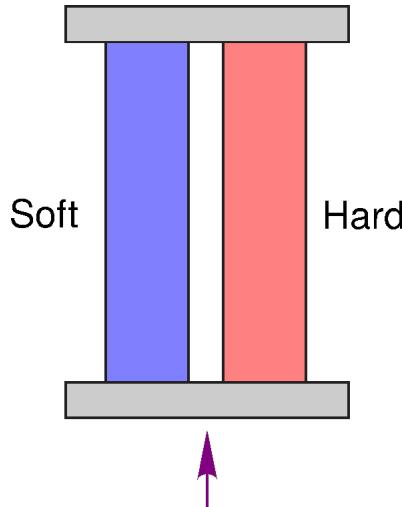
A “Gedankenexperiment”



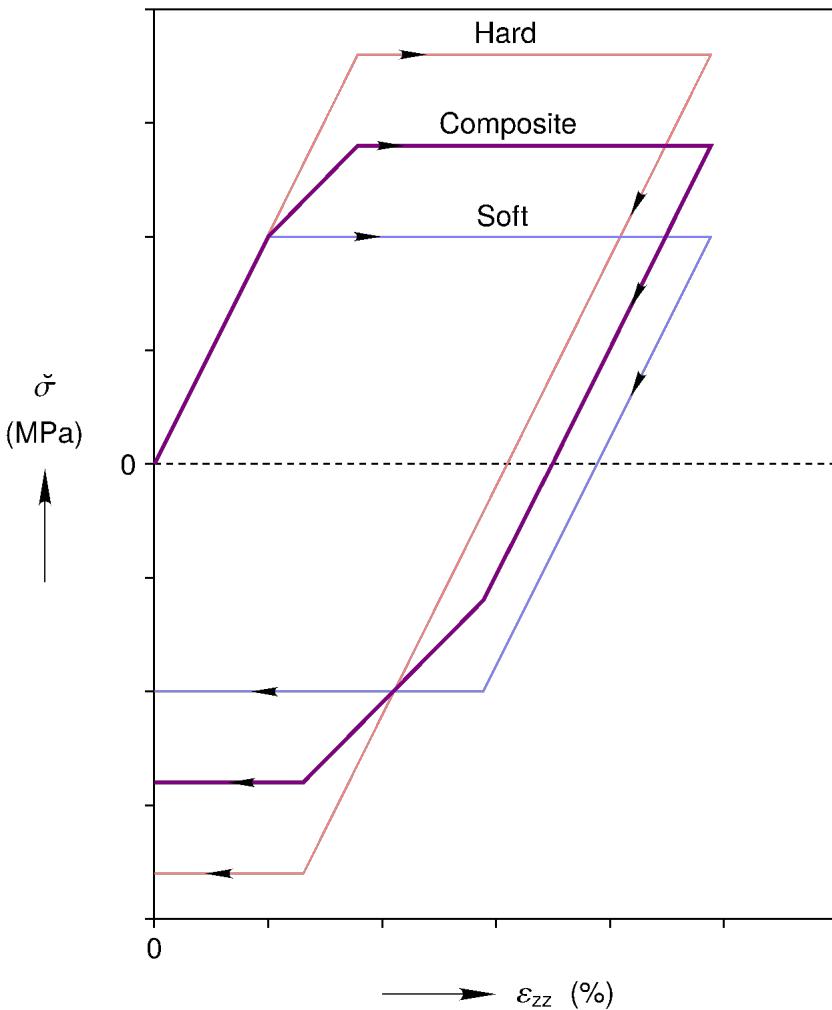
Subgrains

Schematic

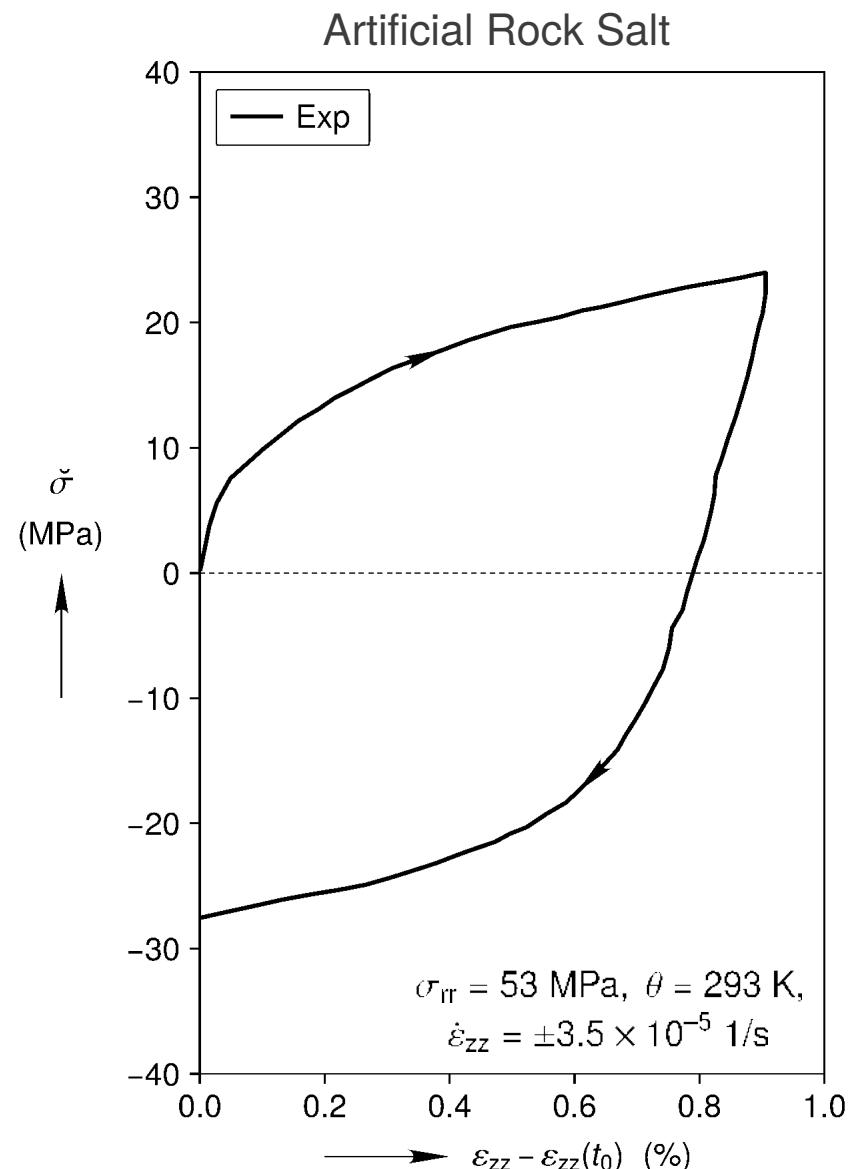
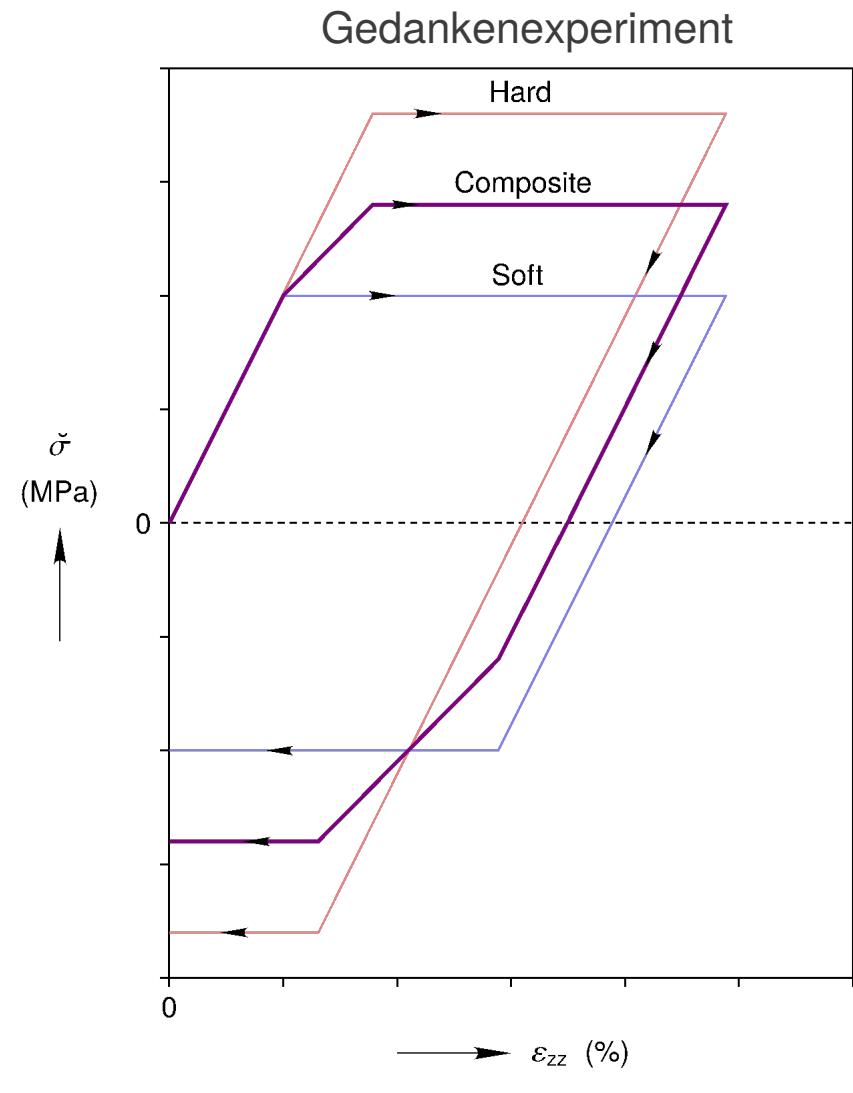
$$f = \check{\sigma}^* A$$



Mechanical Responses



Bauschinger Effect

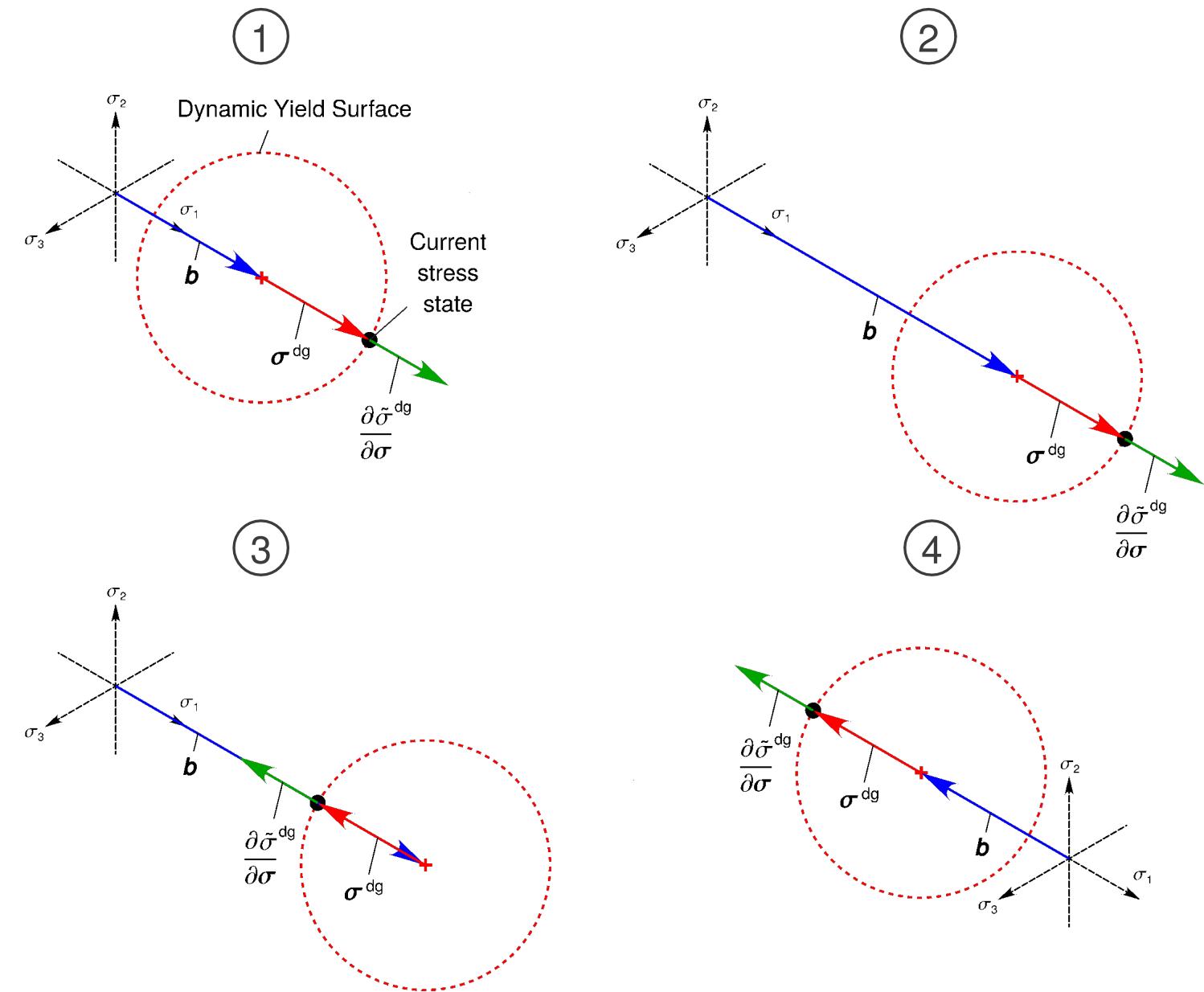
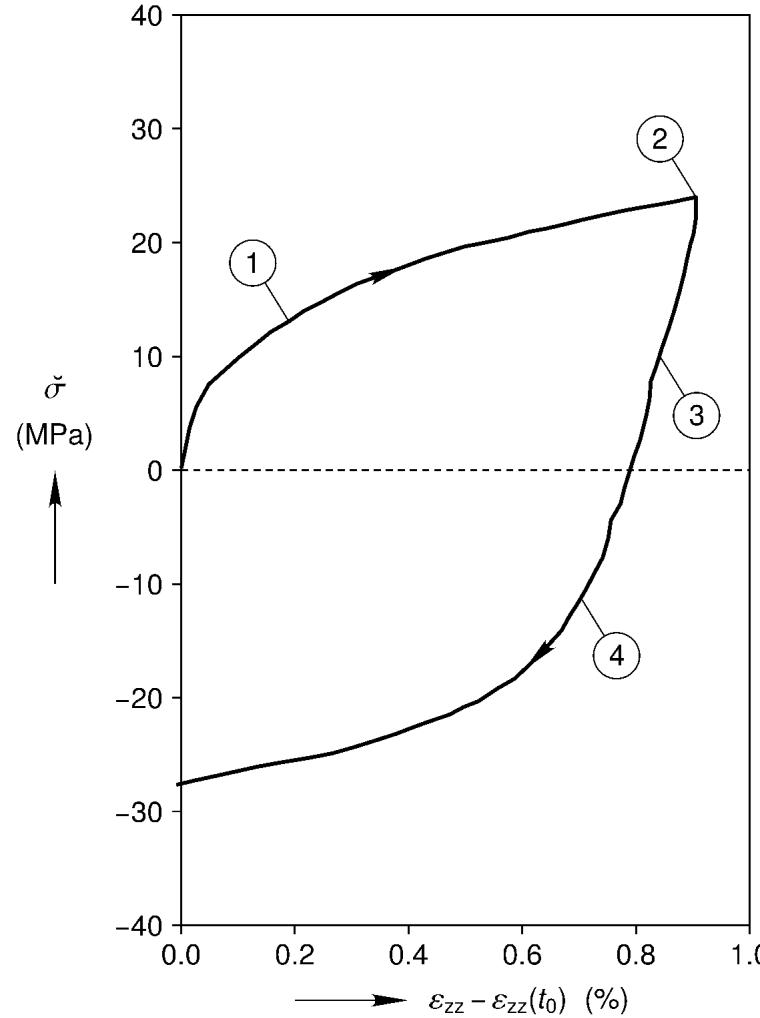


Experimental measurements from: Aubertin, M., Julien, M. R., Servant, S., and Gill, D.E. (1999) A rate-dependent model for the ductile behavior of salt rocks. Canadian Geotechnical Journal. Vol. 36. No. 4. pg. 660-674

Back Stress Hardening



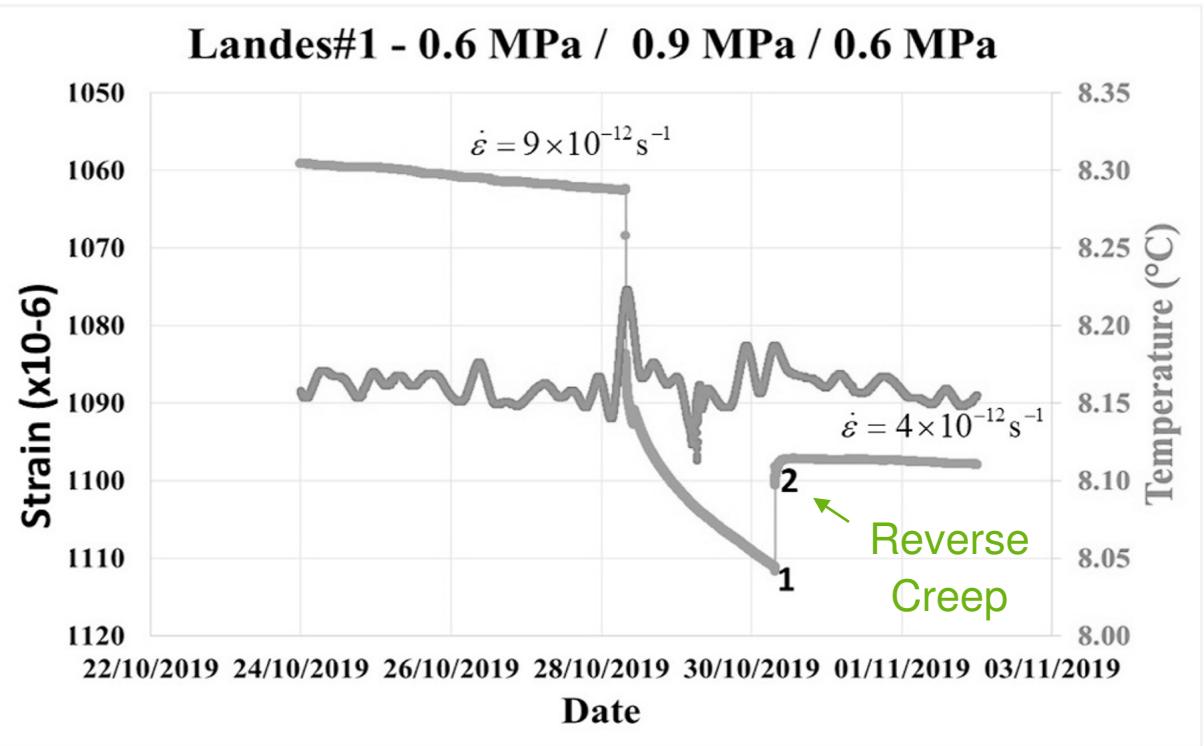
Mechanical Response
Artificial Rock Salt



Reverse Creep at Low Stress and Room Temperature

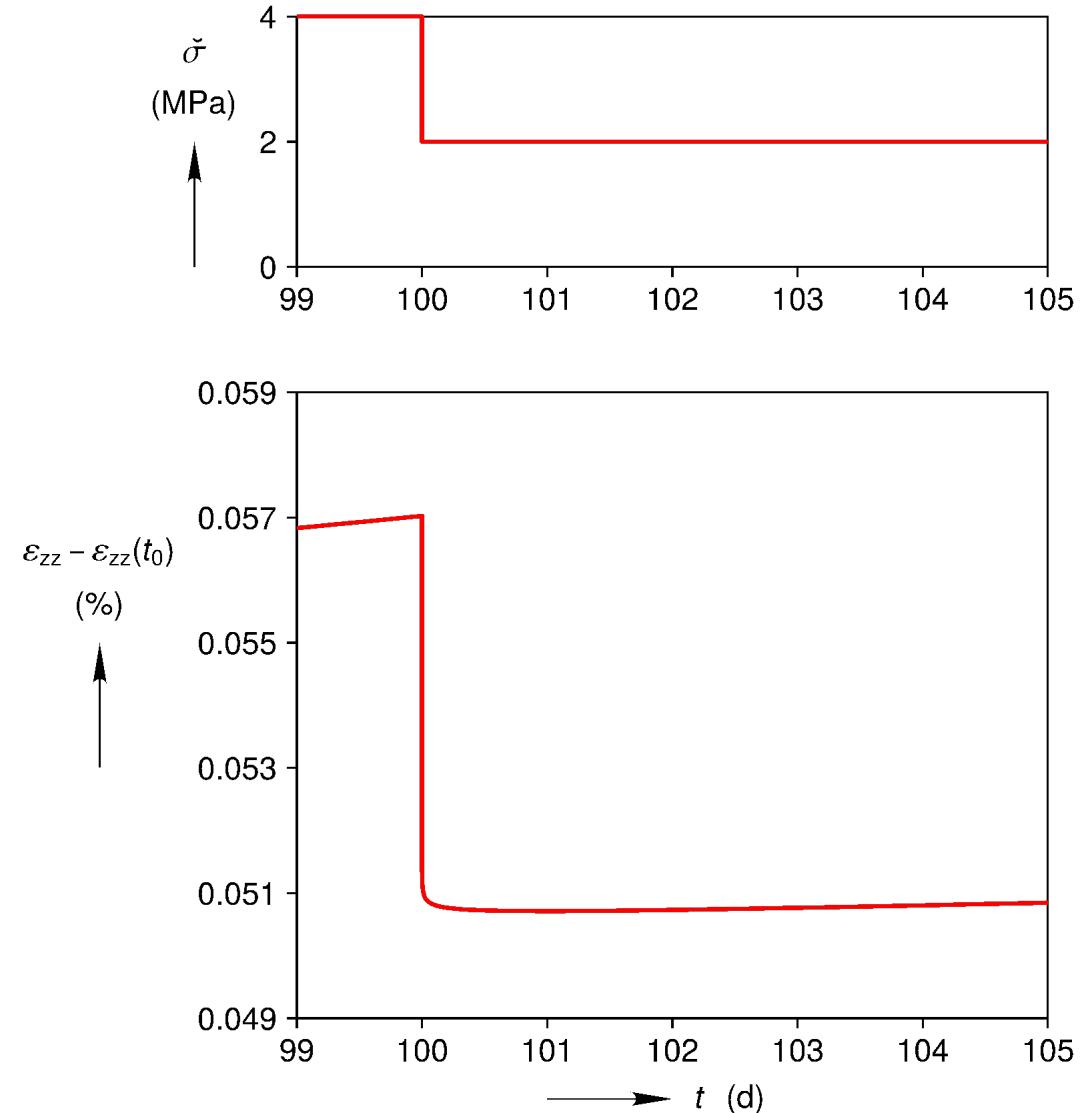


Multi-stage Constant Stress Test on Landes Salt

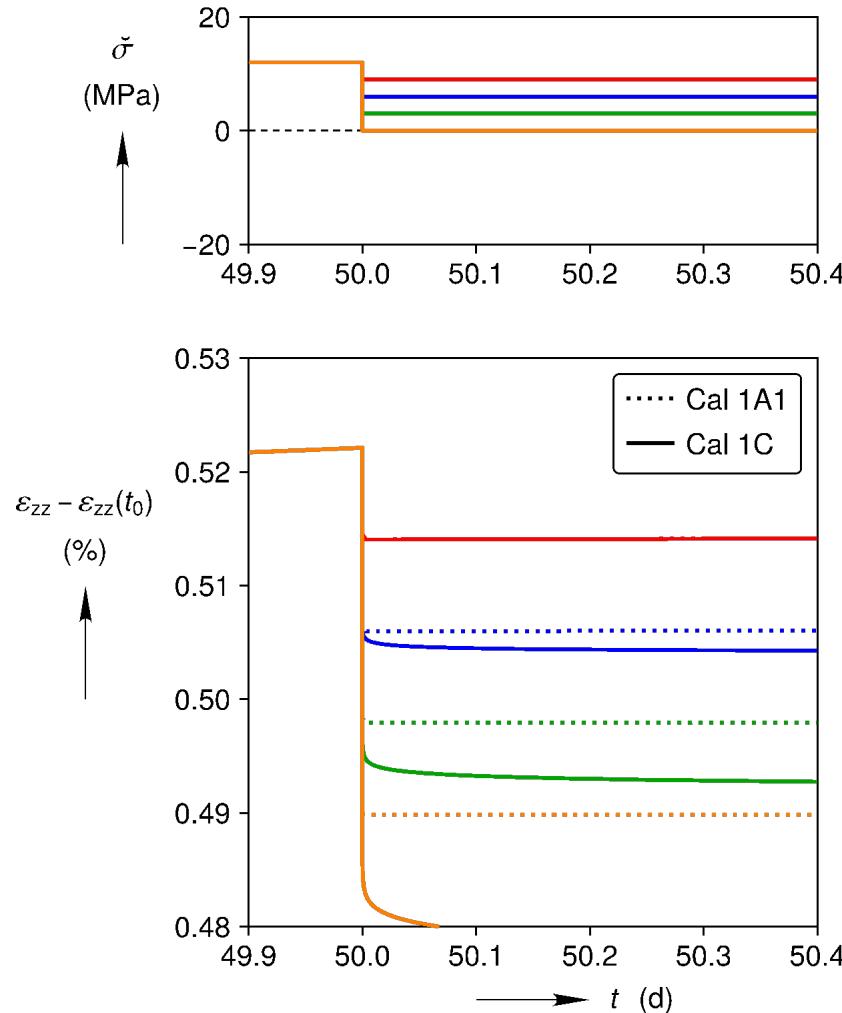
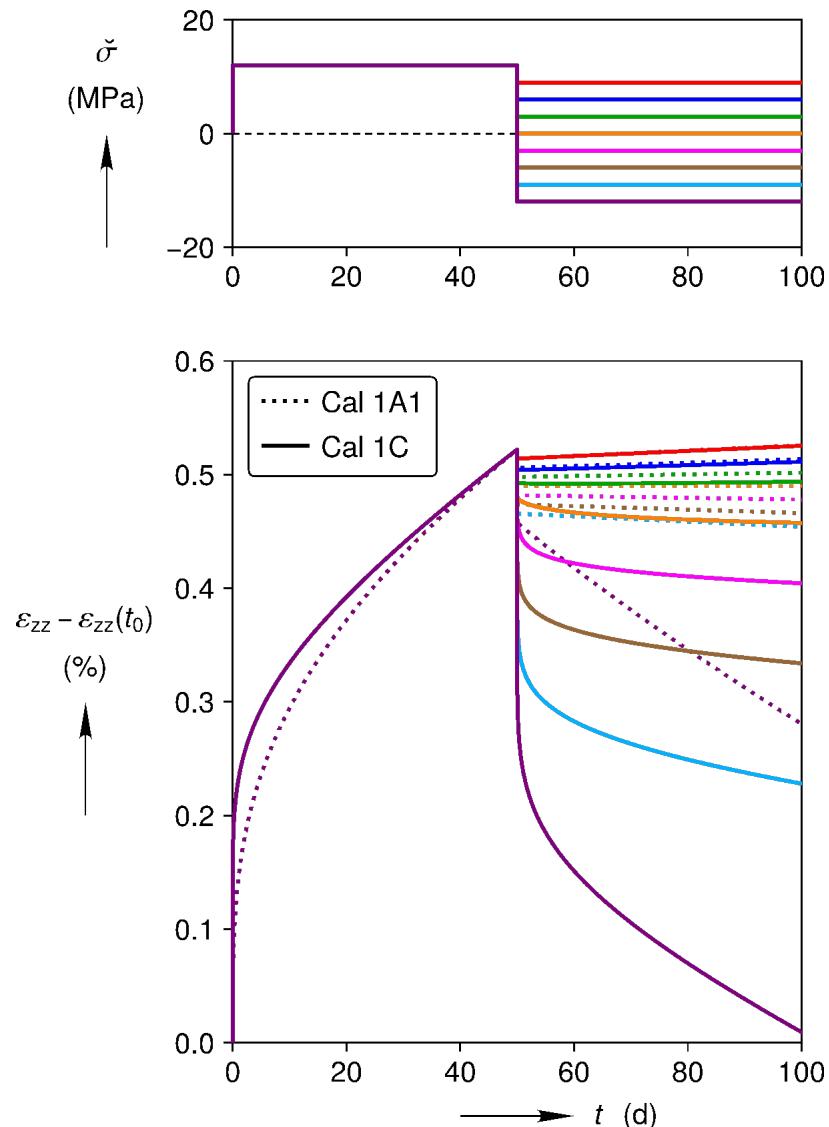


Gharbi, H., Bérest, P., Blanco-Martín, L., and Brouard, B. (Oct. 2020). "Determining upper and lower bounds for steady state strain rate during a creep test on a salt sample". In: International Journal of Rock Mechanics and Mining Sciences 134, p. 104452

Multi-stage Constant Stress Predictions (Cal 1C)
(different stresses than on Landes salt)



Room Temperature Stress Drop Behavior



Cal 1A1 strain shifted down by 0.067 % strain.

Gas Storage Cavern, Volume Loss with Wellhead Pressure Cycling

