

Wave Energy Converter Buoy with Variable Geometry that Improves Energy and Power Capture for Changing Sea States

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Abstract—This paper presents a nonlinear control design technique that capitalizes on an hour glass (HG) variable geometry wave energy converter (WEC). The HG buoy is assumed to operate in the heave motion of the wave. The unique interaction between the HG buoy and the wave creates a nonlinear cubic storage effect that produces actual energy storage or reactive power during operation. A multi-frequency Bretschneider spectrum wave excitation input is reviewed for the HG design both with constant and varying steepness angle profiles which demonstrates further increased power generation. Numerical simulations are performed to demonstrate the increase in power generation with changing sea states. The objective is to increase the power generation from multi-frequency nonlinear dynamic sources.

Keywords - wave energy converter; nonlinear control; energy storage

I. INTRODUCTION

There is a large untapped potential in ocean wave power which is estimated at 1-10 TW of future energy and power generation [1]. Various types of WEC devices are being reviewed by industry to harvest the ocean energies. For this investigation a WEC point absorber/buoy design which harvests energy omni-directional from a single offshore location in the ocean is considered. Typically, when waves interact with the WEC device at the resonance frequency, the device can absorb a significant amount of power/energy from the wave very efficiently. However, when the WEC is off-resonance with the impacting waves the WEC will operate less efficient. In addition, many of the current control designs and modeling efforts are based on linear techniques. In [2] a nonlinear hydrostatic model is developed. By exploiting the nonlinear static coupling between the buoy geometry and the potentially wideband frequency spectrum of incoming waves, the buoy design increases power/energy captured.

The future electric power grid will require new methods and tools to support and capitalize on high penetration of

renewable energy sources (RES). Wave energy is still in the early parts of research and development [4]. In a recent literature review by [4] the latest WEC technologies are discussed for; WEC types, generator types, implementation methods, validation approaches, and controller types.

WECs are devices that extract energy from waves in a body of water such as the ocean [5], [6]. The wave energy source is spatially, temporally, and energetically variable which translates to a predominant frequency of waves, wave heights, and widths of the wave frequency spectrum. Typically, when a wave impacts the WEC device at the resonance frequency, the device can absorb a significant amount of energy from the wave very efficiently. However, when the WEC is off resonance with the impacting waves the WEC operates much less efficient. Many control methods have been studied and investigated [4] on WEC systems. These control methods include; phase control, latching control, proportional plus integral (PI) control, optimal and predictive control (see [7]). Achieving increased power capture over a large range of sea states for stochastic wave profiles while minimizing additional power electronics and energy storage are some of the challenges needed to be addressed by current research. The main research effort in this paper is to increase the power/energy capture of the HG WEC beyond the results presented in [2], for a fixed steepness angle, or a single large wave angle adaptation in [8], by utilizing a *variable* steepness angle.

This paper is divided into five sections. In Section II a WEC with variable geometry is described. In Section III, the control system is discussed with the specific controller details given in [2], [3]. In Section IV multi-frequency numerical simulations are performed that demonstrates a proof-of-concept validation for the proposed HG WEC variable geometry design over a given sea state. Finally in Section V the results are summarized.

II. WEC GEOMETRY DEFINITION

Initially WEC point absorbers are popularly described with an approximate hydrodynamic model known as the Cummins' equation of motion [9]. For a heaving buoy this is given as

$$(\tilde{m} + \tilde{a}(\infty))\ddot{z} + \int_0^\infty h_r(\tau)\dot{z}(t-\tau)d\tau + kz = F_e + F_u. \quad (1)$$

Where \tilde{m} is the buoy mass, $\tilde{a}(\infty)$ is the added mass at infinite frequency, h_r is the radiation force term, k is the hydrostatic stiffness due to buoyancy, F_e is the excitation force, F_u is the control force, and z is the buoy's center of mass with respect to the mean water level position. In their simplest form linear WEC point absorbers can be defined for a regular wave, where the excitation force has only one frequency, ω , and it can be shown that the radiation term can be quantified using an added mass and a radiation damping term, each considered at a constant frequency only [5], [10]. The equation of motion for this simple case is expressed as

$$m\ddot{z} + c\dot{z} + kz = F_e + F_u \quad (2)$$

where m and c are constant mass and damping terms for a given excitation frequency, and k is the linear stiffness term. F_e is the input excitation force and F_u is the control force. This can be captured with a right-circular cylinder (RCC) geometry as shown in Fig. 1 (a). Specific to the RCC

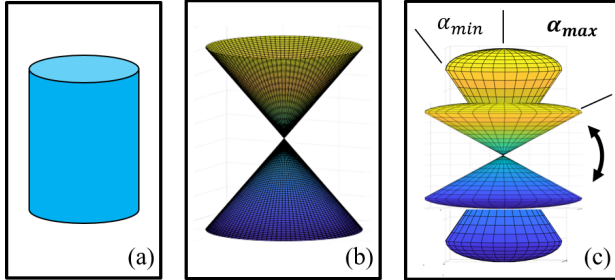


Fig. 1. WEC buoy design evolution: (a) RCC, (b) Fixed HG, and (c) variable HG with steepness angle, α .

buoy design is a single dominant resonating frequency as indicated by the linear nature of (2). To generate real power the reactive power is developed by incorporating power electronics (PE) and energy storage systems (ESS) which require additional complexity into the overall design. To address multiple frequency content in a typical incoming sea state wave profile the resonating controls can be made robust around the dominant frequency (see [11], [12] for more details). Alternatively, the wave spectrum can be decomposed into several dominant frequencies for which a proportional-derivative complex conjugate control (PDC3) system [12], [13], [14] can be designed to boost additional power capture. In an effort to simplify the design and still maintain improved power capture, in [2], a fixed hour glass (HG) nonlinear geometry buoy was investigated and for multiple frequency content the buoy interacts with the incoming waves as a nonlinear resonator where the HG geometry (see Fig. 1 (b))

serves as the reactive power. This simplifies the complexity of the PE and ESS elements in the design. However, for increasing and decreasing sea state wave profiles, it was discovered that if the HG geometry is varied with a changing steepness angle, α that another increase in power capture can be realized (see Fig. 1 (c)). In this paper option (c) is explored in more detail for a specific sea state.

The heave oscillations for a one degree-of-freedom (DOF) buoy relative to a reaction mass can be modeled simply with a power take-off (PTO) system consisting of an ideal linear actuator as part of the power conversion [15] from mechanical to electrical power. The reactive mass is submerged deep enough for its oscillations to be negligible in wave conditions of interest for power conversion [15]. The HG buoy nonlinear variable geometry is defined in Fig. 2 along with the corresponding range of parameters in Table I. The variable geometry can be adjusted according to incoming wave heights and durations with respect to sea states. With varying steepness angles both the radius and draft are either increased or decreased as needed. Figure 3 shows the cross-sectional diagram for both minimum and maximum extreme steepness angles. Both the wave elevation, η , and water line (equilibrium position), ζ , locations are indicated for reference. The maximum steepness angle (blue cross-section) accommodates smaller waves and draft displacements while the minimum steepness angle (gold cross-section) accommodates larger waves and draft displacements, respectively.

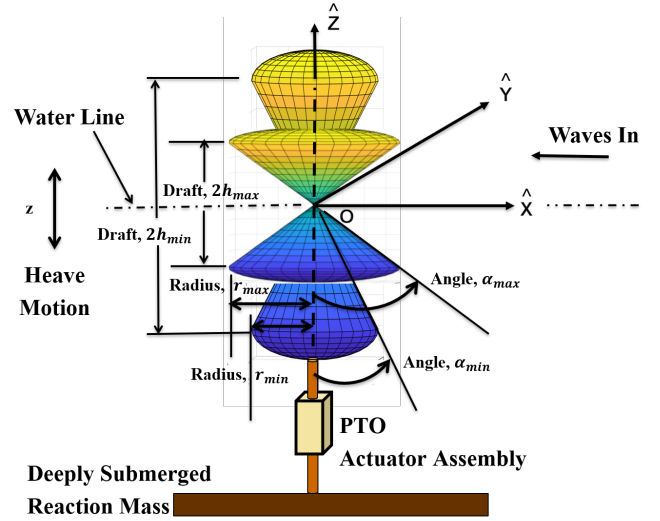


Fig. 2. Hourglass variable geometric buoy design with two configurations shown for maximum and minimum steepness angles which are superimposed on each other.

TABLE I
WEC HOURGLASS VARIABLE GEOMETRY PARAMETERS

Buoy	r [m]	h [m]	α °
HG	5.72-10.0	8.18-2.68	35-75

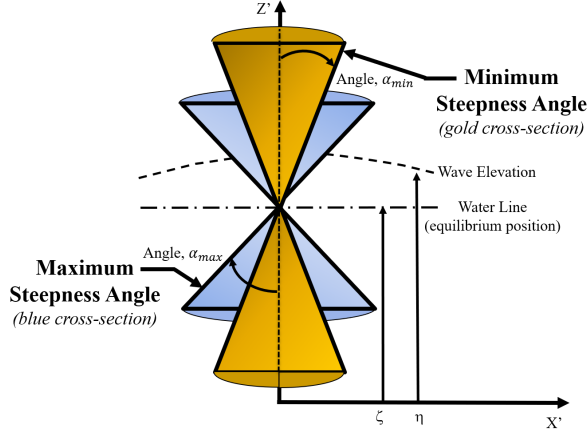


Fig. 3. Variable geometry HG buoy cross-sectional diagram.

III. NONLINEAR CONTROL DRIVEN BUOY DESIGN

At resonance, a WEC device operates at maximum energy absorption [2], [3]. In off-resonance the WEC absorbs less real power and will require reactive power to increase energy capture by enabling resonance. Practically, this can be achieved with model predictive control (MPC) [6], [16], [17], robust resonating control [11], or PDC3 [12], [13], [14]. These techniques require energy storage and power electronic elements. MPC will also need additional wave prediction as *a priori* input.

This paper utilizes a nonlinear control design [18], [2] to realize a nonlinear buoy with variable geometry [2] to produce the energy storage and reactive power through the nonlinear coupling between the buoy and wave interaction, thus reducing the need for energy storage and power electronic elements at the individual buoy level. Initial efforts [19], [20] are ongoing to determine the next steps for optimal use of power packet network technology [20], [21], [22]. For the overall collective or WEC arrays the goal is to maximize power while minimizing PE and ESS from the WEC arrays to onshore grid utility power delivery that are subject to varying sea states and/or wave inputs.

A cubic hardening spring as discussed in [2], [23], [24], can be created by defining the buoy shape as an HG geometry as shown in Fig. 2. The model uses a small body approximation [2], [8]. The summarized equation of motion is given as (see [2] for full details)

$$m\ddot{z} + c\dot{z} + K_{HG}(\alpha)\left[\frac{1}{3}z^3 - \eta z^2 + \eta^2 z\right] = \frac{1}{3}K_{HG}(\alpha)\eta^3 + F_u \quad (3)$$

which contains the cubic spring term given by $\frac{1}{3}K_{HG}(\alpha)z^3$. The wave elevation term is given as η . The parameter $K_{HG}(\alpha)$ is a function of the steepness angle α , buoy mass and geometric properties (all properties are given in Table I and [2]). The controller is defined simply as rate feedback or

$$F_u = -K_D \dot{z} \quad (4)$$

where K_D is the derivative gain. Note that the rate feedback control force will maximize the *real* power/energy capture

while the *reactive* power is realized with the HG buoy variable geometry modifications defined by the $K_{HG}(\alpha)\frac{1}{3}z^3$ term in (3).

The WEC variable geometry HG buoy control block diagram is illustrated in Fig. 4. A wave prediction buoy measures the wave height and direction. This is then transmitted into a wave prediction algorithm to determine the statistical estimate of wave height and duration as controller inputs. These are *a priori* inputs to the variable geometry WEC control system (also noted as dashed boxed lines in the diagram). The controllers provide two functions: i) using rate or velocity feedback control to the PTO device which generates real power at multi-resonances and ii) the servo controller uses the wave prediction estimator reference inputs along with the steepness angle feedback to adjust the variable geometry to increase additional power capture. Sensor modules that support the feedback controllers are also shown in Fig. 4. This additional power capture is demonstrated next which for this study assumes ideal servo tracking.

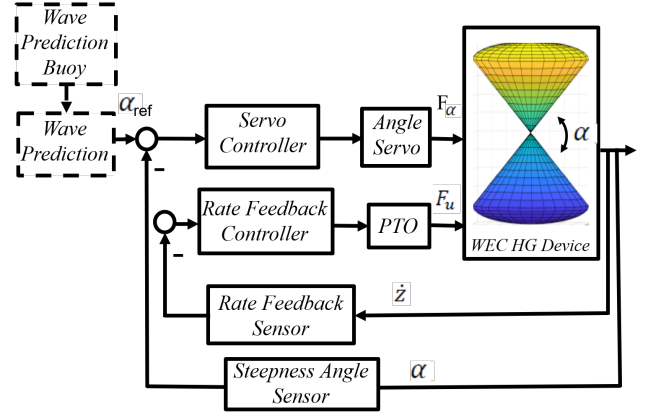


Fig. 4. Variable geometry HG buoy control block diagram.

IV. MULTI-FREQUENCY NUMERICAL SIMULATIONS

In [2] four varying Bretschneider sea states (SS) were investigated with a *constant* steepness angle. Five minute Bretschneider profiles were generated from the MATLAB toolbox [25]. In these cases, the steepness angle was increased statically in five degree increments until the HG buoy draft constraint was violated. The maximum safe angle was set to the previous value such that the HG buoy would not overtop or exit the water. The results are summarized in Table II for a fixed or *constant* steepness angle. In [8], the steepness angle was relaxed for a single larger wave in sea state 4 and increased power and energy capture was observed.

For this study, sea state 4 was further reviewed and a new scenario was defined that investigates the benefit of utilizing wave estimations with a slower update on α for the variable geometry HG buoy given in Figs. 1 (c), 2, and 3 where α was allowed to vary. An empirical optimization was performed over sea state 4 with the given wave input profile shown

TABLE II
BRETSCHNEIDER SPECTRUM SEA STATE RESULTS [2]

Sea State	Steepness Angle [$^{\circ}$]	Energy [MJ]
1	65	67.170
2	70	92.752
3	55	174.63
4	65	69.790

in Fig. 5. The steepness angle was adjusted in draft and radius to allow for increased power capture while staying within the constraints shown in Fig. 6. Referring to the variable geometry HG buoy control block diagram Fig. 4, the empirical optimization performed here would serve as a primitive wave prediction function to generate reference steepness angle inputs α_{ref} into the servo controller to produce the new variable geometry configuration subject to the incoming wave inputs.

An analysis was performed for a Bretschneider spectrum evaluation for both a *constant* and variable steepness angle HG WEC variable geometry design in Fig. 2. The following numerical simulation results were produced. These simulations demonstrate the reactive power requirements exploited as part of the HG buoy variable geometry. The simplicity in the design eliminates the need for PDC3 like algorithm implementation [14] complexity or traditional power electronics hardware and explicit energy storage devices.

A Bretschneider spectrum with $T_p = 11$ seconds and $H_s = 6.9$ meters was employed (SS4 in [2]) with the corresponding wave input shown in Fig. 5. The external forces and control forces are given in Figs. 7 and 8, respectively. The corresponding real and reactive powers are given in Figs. 9 and 10. Note that the HG buoy design inherently *produces* the required reactive power MVARs shown in Fig. 10 which again does not require addition PE and/or ESS devices. The energy captured is given in Table III. The buoy positions and velocities are given in Figs. 11 and 12, respectively.

The steepness angle results for the HG design are given in Fig. 6 versus a *constant* steepness angle, set at $\alpha = 65^{\circ}$, along with the constrained draft heights for SS4. As can be observed, for varying steepness angles, further increase in power and energy capture is achieved. These results would also be a viable option for other sea states (SS1-3 in [2]) and in other general sea state conditions and locations.

TABLE III
ENERGY CAPTURED BRETSCHNEIDER COMPARISON

Steepness Angle	Energy [MJ]
Constant (65°)	69.80
Variable	104.6

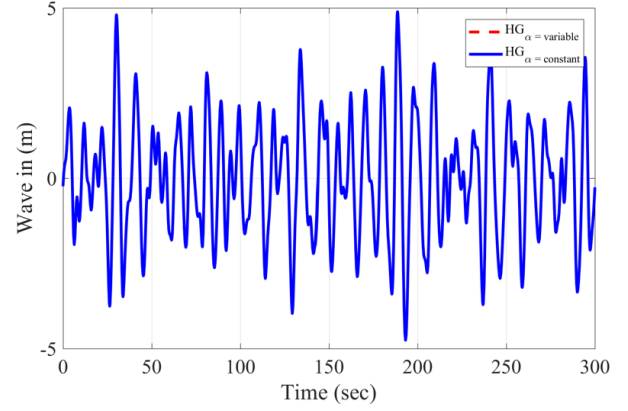


Fig. 5. Wave input profile.

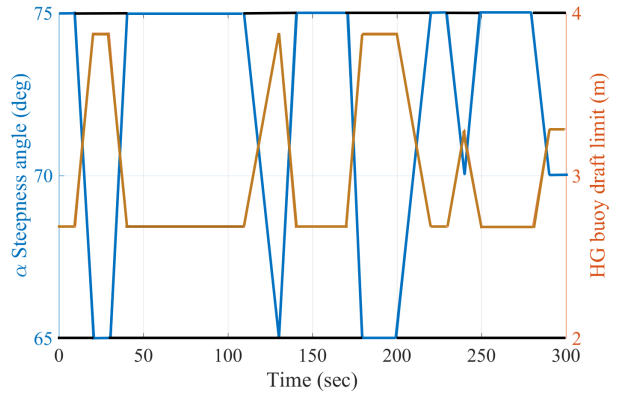


Fig. 6. Buoy steepness and draft limit specifications.

V. SUMMARY AND CONCLUSIONS

This paper presented a nonlinear control design technique that capitalizes on a WEC HG buoy with variable geometry. The unique interaction between the HG buoy and the wave creates a nonlinear cubic storage effect that produced actual energy storage or reactive power during operation. This design realizes a practical complex conjugate control (C3) strategy. A multi-frequency Bretschneider spectrum wave excitation input is reviewed for the HG design both with a constant and varying steepness angle profiles which demonstrated further increased power generation given the potential benefit of intermittent wave measurement previews. Numerical simulations were performed to demonstrate the increase in power and energy capture, respectively. By exploiting the nonlinear geometric/wave interactions in the HG WEC design resulted in implicitly including geometric energy storage/reactive power requirements with increased power generation.

Future work at the individual WEC designs will include detailed mechanisms and servo designs to realize the steepness angle HG buoy adjustments and corresponding parasitic power deductions from the overall increase from these promising power capture results. The current research investigations for the overall collective or WEC arrays are

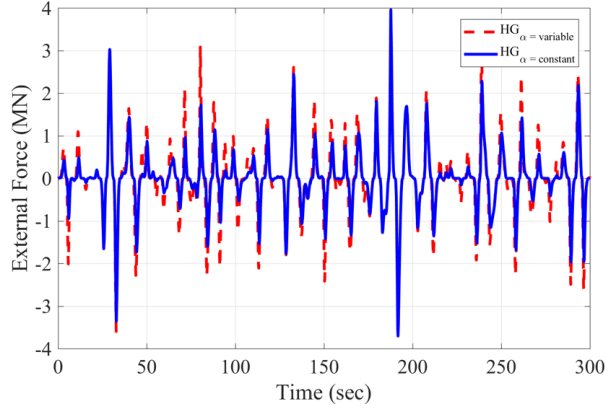


Fig. 7. External force responses.

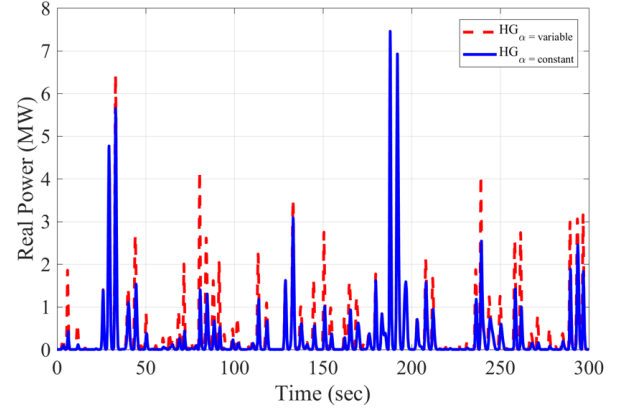


Fig. 9. Real power responses.

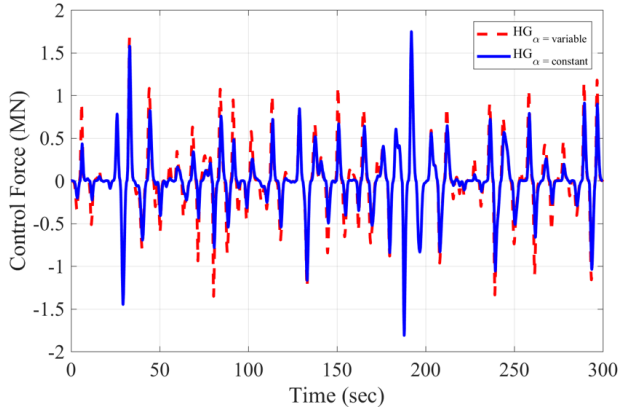


Fig. 8. Control force responses.

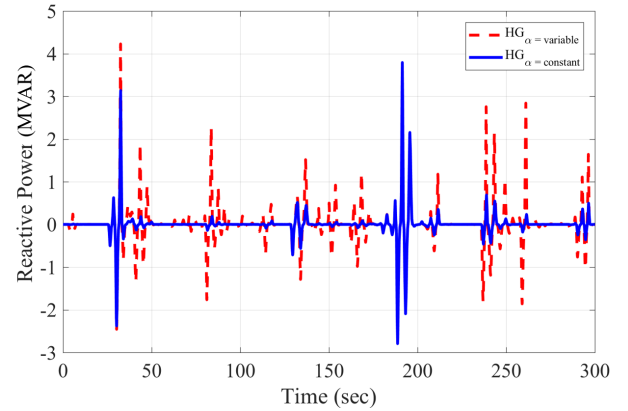


Fig. 10. Reactive power responses.

exploring optimal use of power packet network technology [20], [21], [22] which focuses on maximizing power while minimizing PE and ESS from the WEC arrays to onshore grid utility power interactions. Ultimately one would like to see renewable energy sources perform with high reliability, less complexity and in the case of water power production, higher power quality while being subject to changing wave sea states. This would help support universal deployment to many different regions and seasonal conditions throughout the world.

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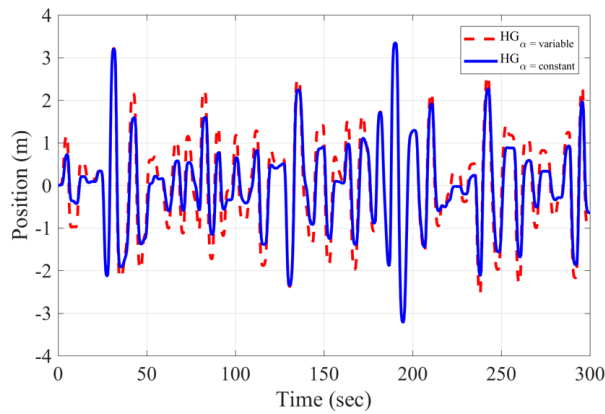


Fig. 11. Buoy position responses.

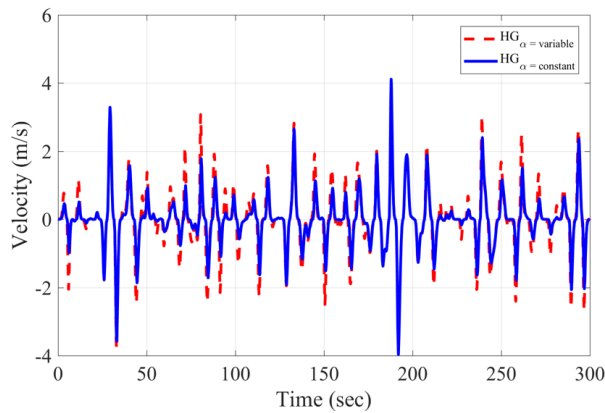


Fig. 12. Buoy velocity responses.

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